Scale–dependent Galaxy Bias

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Abstract. We present a simple heuristic model to demonstrate how feedback related to the galaxy formation process can result in a scale-dependent bias of mass versus light, even on very large scales. The model invokes the idea that galaxies form initially in locations determined by the local density field, but the subsequent formation of galaxies is also influenced by the presence of nearby galaxies that have already formed. The form of bias that results possesses some features that are usually described in terms of stochastic effects, but our model is entirely deterministic once the density field is specified. Features in the large-scale galaxy power spectrum (such as wiggles that might in an extreme case mimic the effect of baryons on the primordial transfer function) could, at least in principle, arise from spatial modulations of the galaxy formation process that arise naturally in our model. We also show how this fully deterministic model gives rise to apparently stochasticity in the galaxy distribution.

1. Introduction

Thanks to large-scale spectroscopic surveys such as the Anglo-Australian 2dF Galaxy Redshift Survey (2dFGRS: Norberg et al. 2001; Wild et al. 2004; Conway et al. 2005) and the Sloan Digital Sky Survey (Zehavi et al. 2002; Tegmark et al. 2004; Swanson et al. 2007) it is now well established that the clustering of galaxies depends subtly on their internal properties. Since galaxies of different types display different spatial distributions it follows that not all galaxies can trace the distribution of underlying dark matter. In other words galaxies are biased tracers of the cosmological mass distribution. Theories of cosmological structure formation must explain the relationship between galaxies and the distribution of gravitating matter which probably yields important clues to the process by which they were assembled.

Galaxy formation involves complex hydrodynamical and radiative processes alongside the merging and disruption of dark matter haloes. This entails a huge range of physical scales that poses extreme challenges even for the largest supercomputers. The usual approach is therefore to encode the non-gravitational physics into a series of simplified rules to be incorporated in a code which evolves the dark matter distribution according to Newtonian gravity (e.g. Benson et al. 2000). This "semi-analytic" approach has many strengths, including the ability to make detailed models for direct testing against observations, but it difficult to use it to make models with which one can make inferences from data. For this reason, simplified analytical models of bias are still extremely useful if one hopes to proceed from observations to theory rather than vice-versa.

In the new era of "precision cosmology" the presence of bias is more an obstacle than a key to understanding (Zheng & Weinberg 2007). Attempts to infer parameter values from cosmological observations are hampered by the unknown relationship between visible objects and the underlying mass fluctuations they trace. For example, the relatively weak residual baryon acoustic oscillations (BAO) one expects to be present in the matter power spectrum (Pen 1998; Meiksin, White & Peacock 1999; Blake & Glazebrook 2003; Eisenstein et al. 2005; Seo & Eisenstein 2005; Wang 2006) are potentially extremely important diagnostics of the presence of dark energy if they can be observed at high redshift. However, when matter fluctuations are inferred from galaxy statistics, the form and evolution of bias must be understood and controlled if the required level of accuracy is to be reached. Here again simplified anaytical models have an important role to play.

In this paper we introduce a simple yet general theoretical model which can describe various aspects of galaxy bias is a unified way. We describe biasing models in general in the next section. In Section 3 we present our model and in Sections 4 and 5 we describe a couple of applications. We discuss the results in Section 6.

2. From Local Bias to the Halo Model

The idea that galaxy formation might be biased goes back to the realization by Kaiser (1984) that the reason Abell clusters display stronger correlations than galaxies at a given separation is that these objects are selected to be particularly dense concentrations of matter. As such, they are very rare events, occurring in the tail of the distribution function of density fluctuations. Under such conditions a "high-peak" bias prevails: rare high peaks are much more strongly clustered than more typical fluctuations (Bardeen et al. 1986). More generally, in *local bias* models, the propensity of a galaxy to form at a point where the total (local) density of matter is ρ is taken to be some function $f(\rho)$ (Coles 1993; Fry & Gaztanaga 1993).

It is possible to place stringent constraints on the effect this kind of bias can have on galaxy clustering statistics without making any particular assumption about the form of f. In particular, it can be shown that the large-scale two–point correlation function of galaxies typically tends to a constant multiple of the mass autocorrelation function in these models. Coles (1993) proved that under weak conditions on the form of $f(\rho)$ as discussed in the introduction, the large-scale biased correlation function of galaxies would generally have a leading-order term proportional to $\xi_{\rm m}(r)$. In other words, one cannot change the large-scale slope of the correlation function of locally-biased galaxies with respect to that of the mass. This was a serious problem for the standard cold dark matter model of times past (which had $\Omega_0 = 1$ and $\Lambda = 0$) because there is insufficient power in the matter spectrum in this model to match observations unless one incorporates a strongly scale dependent bias (Bower et al. 1993).

The local bias "theorem" was initially proved for biasing applied to Gaussian fluctuations only and did not necessary apply to galaxy clustering where, even on large scales, deviations from Gaussian behaviour are significant. Steps towards the plugging of this gap began with Fry & Gaztanaga (1993) who used an expansion of f in powers of the dimensionless density contract δ and weakly non-linear (perturbative) calculations of $\xi_{\rm m}(r)$ to explore the statistical consequences of biasing in more realistic (i.e. non-Gaussian) fields. Based largely on these arguments, Scherrer & Weinberg (1998) showed explicitly that non-linear evolution always guarantees the existence of a linear leading-order term regardless of the form of f, thus strengthening the original argument of Coles (1993) at the same time as confirming the validity of the theorem in the non-linear regime. A similar result holds under the hierarchical ansatz, as discussed by Coles et al. (1999).

It is worth noting that the original form of the local bias theorem has a minor loophole: for certain peculiar forms of f the leading order term is proportional to $[\xi_{\rm m}(r)]^2$ (Coles 1993). However, $\xi_{\rm m}(r)$ must be a convex function of r because its Fourier transform, the power spectrum, is non-negative definite (i.e. it can be positive or exactly zero). Higher order terms in $\xi_{\rm m}^n$ therefore fall off more sharply than $\xi(r)$ on large scales so this loophole does not have any serious practical consequences for large-scale structure.

Such results greatly simplify attempts to determine cosmological parameters using galaxy clustering surveys, as well as facilitating the interpretation of any specific features in large-scale clustering statistics because they require the galaxy spectrum to have the same shape as the underlying mass spectrum. This reduces the possible effect of bias to a single parameter which can be estimated and removed by marginalisation. On the other hand, it results in a drastic truncation of the level of complexity in the assumed relationship between galaxies and dark matter.

In hierarchical models, galaxy formation involves the formation of a dark matter halo, the settling of gas into the halo potential, and the cooling and fragmentation of this gas into stars. This all happens within a population of haloes which is undergoing continuous merging and disruption. Rather than attempting to model these stages in one go by a simple function f of the underlying density field it is better to study the dependence of the resulting statistical properties on the various ingredients of this process. Bardeen et al. (1986), following Kaiser (1984), pioneered this approach by calculating detailed statistical properties of high-density regions in Gaussian fluctuations fields. Mo & White (1996) and Mo et al. (1997) went further along this road by using an extension of the Press-Shechter (1974) theory to calculate the correlation bias of halos, this making an attempt to correct for the dynamical evolution absent in the Bardeen et al. approach. The extended Press-Schechter theory forms the basis of many models for halo bias in the subsequent literature (e.g. Matarrese et al. 1997; Moscardini et al. 1998; Tegmark & Peebles 1998).

It is worth stressing that by "local bias" we mean some form of coarse–graining to select objects on a galaxy scale. In the earlier models described above, galaxy correlations arise because the underlying matter field is correlated but the process of galaxy formation does not itself influence the formation of structure on scales larger than this resolution scale. More recent developments involve the Halo Model (Seljak 2000; Peacock & Smith 2000; Cooray & Sheth 2002; Neyrinck & Hamilton 2005; Blanton et al. 2006; Schulz & White 2006; Smith, Scoccimarro & Sheth 2006, 2007). This model generally assumes that galaxy properties are derived from the underlying mass or halo field. Some degree of scale–dependence then arises because galaxies interact on the scale of an individual halo to provide some degree of self-organisation within the resolution scale. This model has scored some notable successes at explaining features in observed galaxy correlations.

It has also been suggested that bias might not be a deterministic function of ρ, and that consequently there is a stochastic element in the relationship between mass and light (Dekel & Lahav 1999).

In the following sections we present a model that extends a number of these different lines of thought. In particular we consider the possibility that large-scale interactions between galaxies or proto-galaxies might induce a significant scale dependent bias that is qualitatively different from that which arises even in the halo model.

3. Self-interacting Galaxy Formation

As described in the previous section, the idea of local bias models is that the density of matter at a given spatial position $\mathbf{x}$ is responsible for generating the propensity that a galaxy will form there (after suitable coarse-graining of the density field). In its simplest terms we can represent this idea in terms of a galaxy fluctuation field

$$\delta_{\rm g}(\mathbf{x}) \equiv \frac{n(\mathbf{x})}{\bar{n}} - 1, \tag{1}$$

where $n(\mathbf{x})$ is the number density of galaxies at $\mathbf{x}$ and $\bar{n}$ is the mean number density of galaxies. The simplest way to account for discreteness is to use the Poisson cluster model of Layzer (1956) in which galaxies form with a probability proportional to $\delta_{\rm g}$. If there are interactions within the resolution scale then the Poisson model does not necessarily hold (Coles 1993). In order to keep the presentation of our model as simple as possible we ignore discreteness effects and restrict ourselves to large scale clustering properties. In local bias theories the galaxy field is a deterministic function of the local matter density field at the same point $\mathbf{x}$.

Our model for scale–dependent bias has the form:

$$\delta_{\rm g}(\mathbf{x}) = \delta_{\rm s}(\mathbf{x}) + \alpha \int h(\mathbf{x}-\mathbf{x}')\delta_{\rm g}(\mathbf{x}')d\mathbf{x}' \tag{2}$$

In this equation the field $\delta_{\rm s}(\mathbf{x})$ represents a "seed" field and the second term models the interactions. In a realistic situation the parameter α might well be stochastic, varying in a complicated way from galaxy to galaxy, but for simplicity we will assume it to be a constant in this paper. In principle a galaxy may either enhance or suppress the formation of others around it so α may be either positive or negative. In the absence of interactions (i.e. taking $\alpha = 0$), the model reduces to a standard biasing picture where the clustering of galaxies is, at some level, reducible directly to the clustering ot the mass. In the "no-bias" case the seed field will simply be the underlying density fluctuation field, i.e. $\delta_{\rm s} = \delta_{\rm m}$. Galaxies could then form as a Poisson sampling of the mass field as suggested by Layzer (1956). For linear bias models, we would take $\delta_{\rm s} = b\delta_{\rm m}$. In such cases the resulting galaxy spectrum $P_{\rm g}(k) = b^2 P_{\rm m}(k)$ for all k. In general local bias models we might take the seed field to be some local function $f(\delta_{\rm m})$, as described in the previous section. In these cases $P_{\rm g}(k) \simeq b^2 P_{\rm m}(k)$ for small k via the local bias theorems. More realistically perhaps, $\delta_{\rm s}$ could be the "halo field". Explicitly in this case, and indeed implicitly in the other cases discussed above, $\delta_{\rm s}$ does possess a filtering scale of its own, with the width of the smoothing kernel representing the characteristic size of a galaxy halo.

If the seed field is simply the halo field, the galaxies do not form a Poisson sample; the distribution of galaxies within a given halo is a degree of freedom within the halo model which must be fixed by reference to observations (Seljak 2000; Peacock & Smith 2000; Cooray & Sheth 2002). The seed field might also include stochastic terms (Dekel & Lahav 1999; Blanton et al. 1999; Matsubara 1999), i.e. terms which can not be expressed as any function of $\rho_{\rm m}$ but which might instead be modelled as random variables. The first term on the right hand side of equation (2) therefore includes the traditional bias models discussed in the previous section. If $\alpha = 0$ we recover models in which the clustering of galaxies is, at some level, reducible directly to the clustering of the mass. In such cases if the seed field were uncorrelated then all these models would produce uncorrelated galaxies.

If $\alpha = 0$ and the seed field is uncorrelated then all these models would produce uncorrelated galaxies. If $\alpha \neq 0$, however, then we have a qualitatively different form of bias. The galaxy field then not only depends on the seed field, but also on the galaxy field itself. This "bootstrap" effect allows a greater degree of flexibility in modelling galaxy correlations. In particular, even if the seed field were completely uncorrelated, interactions could produce a non-zero galaxy-galaxy correlation function in the bootstrap model. This can not happen in local bias models. In this respect our model is similar to the autoregressive (AR) models used to simulate time series: these are correlated processes that are seeded by random (uncorrelated) noise. More relevantly for cosmology, as we shall see shortly, the bootstrap model allows us to generate scale-dependent bias that violates the theorems referred to in Section 2. The initial seed field $\delta_s(\mathbf{x})$ plays the same role as the "innovation" in autoregressive time series models.

The presence of the kernel in equation (2) gives the model the ability to generate non-local interactions if it extends over a relatively large scale. The kernel $h(\mathbf{y})$ determines the size of the zone of influence of one galaxy on the formation of others in its neighbourhood; we denote this scale by R_h. Just as with the parameter α, we take this scale to be constant for simplicity. Note, however, that since both the scale and level of feedback may be difficult to predict given only the ambient density field, it may be more realistic to model the kernel scale as stochastic variable.

The filter should be defined in such a way that it preserves the statistical homogeneity of the density field and does not lead to diverging moments. For sensible filters h will have the following properties: $h =$ constant $\simeq R_h^{-3}$ if $|\mathbf{x}-\mathbf{x}'| \ll R_h$, $h \simeq 0$ if $|\mathbf{x}-\mathbf{x}'| \gg R_h$, $\int h(\mathbf{y};R_h)d\mathbf{y} = 1$. We discuss a couple of specific examples in the subsequent sections of this paper.

The integral on the right hand side of equation (2) represents the galaxy fluctuation field convolved with a low pass filter. One can write (2) in the form

$$\delta_{\rm g}(\mathbf{x}) = \delta_{\rm s}(\mathbf{x}) + \alpha\delta_{\rm g}(\mathbf{x}; R_{\rm h}). \tag{3}$$

The filtered field, $\delta_g(\mathbf{x}; R_h)$, may be obtained by convolution of the "raw" galaxy density field with some function h having a characteristic scale R_h:

$$\delta_{\rm g}(\mathbf{x}; R_h) = \int \delta_{\rm g}(\mathbf{x}')h(|\mathbf{x}-\mathbf{x}'|; R_h)d\mathbf{x}'. \tag{4}$$

To recover the local bias model with $\alpha \neq 0$ we simply take $h(\mathbf{x}-\mathbf{x}') = \delta_D(\mathbf{x}-\mathbf{x}')$ in which case $\delta_{\rm g} = \delta_{\rm s}/(1-\alpha) = b\delta_{\rm s}$. Scale independence and linearity of the bias are therefore both recovered in this limit.

Equation (2) is a Fredholm integral equation of the second type. Assuming that the interaction kernel h is well-behaved we can solve it quite straightforwardly. Defining the Fourier transform of $\delta_{\rm s}(\mathbf{x})$ to be $\tilde{\delta}_{\rm m}(\mathbf{k})$ etc and using the convolution theorem, the k-space version of the equation (2) is seen to be

$$\tilde{\delta}_{\rm g}(\mathbf{k}) = \tilde{\delta}_{\rm s}(\mathbf{k}) + \alpha\tilde{h}(\mathbf{k})\tilde{\delta}_{\rm g}(\mathbf{k}), \tag{5}$$

which gives a solution for $\tilde{\delta}_{\rm g}(\mathbf{k})$:

$$\tilde{\delta}_{\rm g}(\mathbf{k}) = \frac{\tilde{\delta}_{\rm s}(\mathbf{k})}{1-\alpha\tilde{h}(\mathbf{k})}. \tag{6}$$

The power spectrum of the filtered field is given by

$$P(k; R_h) = \tilde{h}^2(k; R_h)P_{\rm g}(k), \tag{7}$$

where $P_{\rm g}(k)$ is the power spectrum of the galaxy field. Assuming that $h(\mathbf{y})$ is isotropic, the galaxy-galaxy power spectrum can be expressed as

$$P_{\rm g}(k) = \frac{P_{\rm s}(k)}{|1-\alpha\tilde{h}(k)|^2}, \tag{8}$$

where $k = |\mathbf{k}|$. It is clear that the kernel can imprint features into the power spectrum through the dependence on $\tilde{h}(k)$, even in the case where $P_{\rm s}(k)$ is completely flat. This means it is considerably more general than the simpler models discussed above. It possesses some features that resemble the cooperative galaxy formation model of Bower et al. (1993) but with significantly more generality. We shall illustrate some of its properties in the following sections.

4. Bogus Baryon Wiggles?

In this section we present an extreme example of scale–dependent bias which is based on the idea that some violent astrophysical process connected with galaxy formation (such as the ionizing radiation produced by quasar activity) could seriously influence the propensiy of galaxies to form in the neighbourhood of a given object. This concept is not new (Rees 1988; Babul & White 1991), and has been recently revived in a milder form (Pritchard, Furlanetto & Kamionkowski 2006).

To give an illustration of the extreme effects that could arise in the galaxy power spectrum, consider the extreme example where the zone of influence of a galaxy (or quasar) has a sharp edge similar to an HII region. We can use our model to describe this situation if we adopt a kernel which has the form of a *"top hat'* filter, with a sharp cut off, defined by the relation

$$h_{\rm T}(|\mathbf{x}-\mathbf{x}'|;R_{\rm h}) = \frac{3}{4\pi R_h^3}\Theta\Big(1-\frac{|\mathbf{x}-\mathbf{x}'|}{R_h}\Big), \tag{9}$$

where Θ is the Heaviside step function: $\Theta(y)=0$ for $y\le 0$ and $\Theta(y)=1$ for $y>0$. The form of the kernel in Fourier space is then

$$\tilde{h}_{\rm T}(k;R_h) = \frac{3(\sin kR_h - kR_h\cos kR_h)}{(kR_h)^3}\ . \tag{10}$$

Oscillatory features can be generated in the galaxy power spectrum by this form of interaction and with a suitable choice of scale R_h they could even mimic the BAOs mentioned in the Introduction.

To establish the required parameters we refer to the 2dFGRS redshift-space power spectrum data given in Table 2 of Cole et al. (2005) for the 2dFGRS. We do not attempt to fit the small-scale clustering in this data set. This could be done by fiddling with the form of $\delta_{\rm s}$, but our interest lies here in illustrating the large–scale behaviour only. We also ignore redshift–space distortions. In Cole et al. (2005), the error bars on the spectrum are derived from the diagonal elements of the covariance matrix calculated from model lognormal density fields. The model power spectrum for these lognormal fields has $\Omega_{\rm m}h=0.168$, $\Omega_{\rm b}/\Omega_{\rm m}=0.17$ and $\sigma_8^{\rm g}=0.89$ and agrees very well with the best fit model for the overall 2dFGRS power spectrum. This model, convolved with the 2dFGRS survey window function, is also given in Table 2 of Cole et al. (2005) and plotted in Figure 1 (solid line) & Figure 2. Using the full covariance matrix, Cole et al (2005) find $\chi^2/\text{d.o.f}=37/33$ for $k<0.2\ h\text{Mpc}^{-1}$. As this analysis is for illustrative purposes only , we do not perform a full likelihood analysis, rather we calculate the χ^2 for the same model using only the error bars. In this case the fit is characterized by $\chi^2/\text{d.o.f}=12/33$. As discussed in Section 5 of Cole et al (2005), since the convolution with the survey window function causes the errors to be correlated, resulting in a very low value of χ^2. The goodness of fit does however provide a useful benchmark for our alternative explanation of the wiggles seen in $P(k)$.

In order to explain the shape of the galaxy spectrum using only galaxy interactions and without the baryon oscillations, we use a top-hat kernel for our biasing model and fit it to the same data using the Eisenstein & Hu (1998) transfer functions and assuming $n_s = 1$, $h = 0.72$ and $\Omega_b = 0$. In other words we use an underlying cosmology without baryon oscillations and seek to explain the shape of the galaxy spectrum using only galaxy interactions. Our best fit cosmological parameters are $\Omega_m = 0.23$, $\sigma_8^g = 0.85$ and for the bias model we get $\alpha = 0.25$ and $R_h = 114$Mpc. This model has $\chi^2/\text{d.o.f} = 9/33$. The value of χ^2 is again very low due to correlations between the data points, but a comparison with the result of the previous paragraph for which the same problem also holds, demonstrates that the fit is if anything marginally better for our model than for the reference model used by Cole et al. (2005).

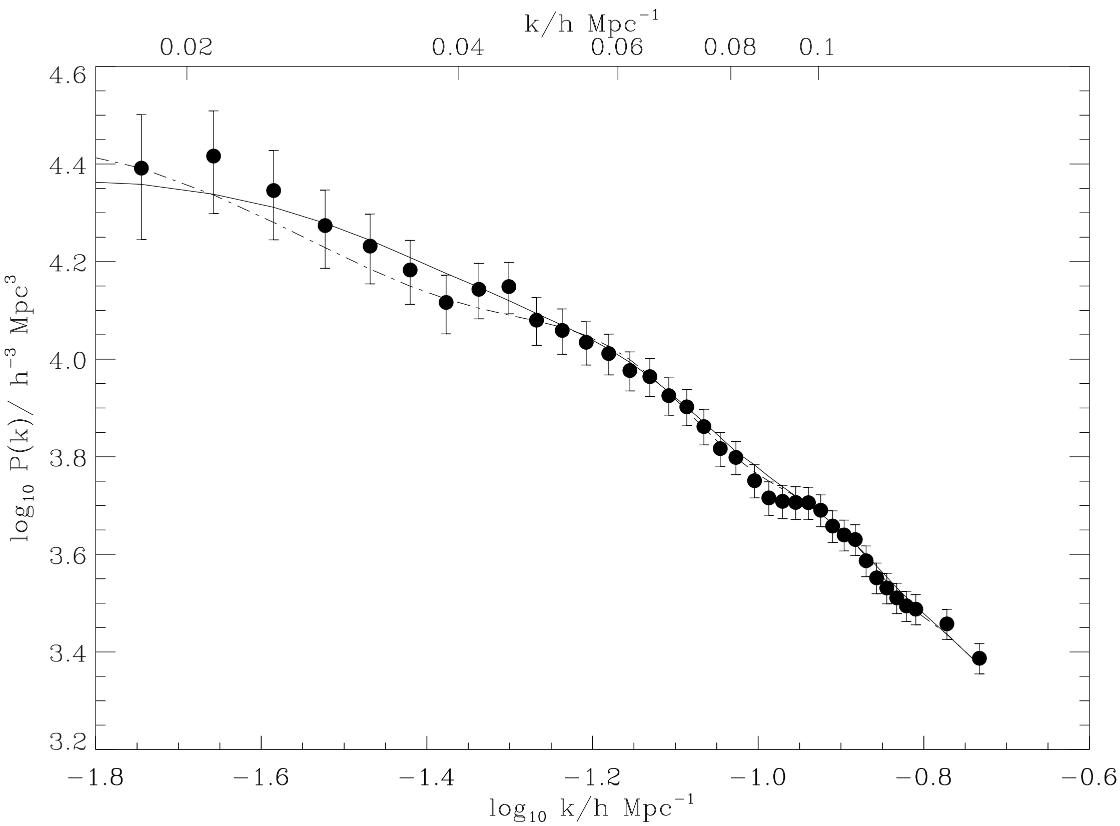

Figure 1. The black filled circles and the associated error bars are the 2dFGRS power-spectrum data given in Table 2 of Cole et al. (2005). The black solid lines in both plots denote the reference power spectrum convolved with the window function also given in Table 2, with $\Omega_m h = 0.168$, $\Omega_b/\Omega_m = 0.17$ and $\sigma_8^{gal} = 0.89$. The dashed lines are for the best fit model with biasing but no baryons.

Of course one does not know for sure whether and how ionization influences galaxy formation, but this example illustrates that in principle the observed wiggles in the galaxy power spectrum could have an astrophysical rather than cosmological origin. This would pose problems for their use as cosmological probes. On the other hand, the scale required is very large. Rees (1988) pointed out that a quasar of luminosity Luv lasting for a time t_Q produces sufficient energetic photons to ionize all the baryons

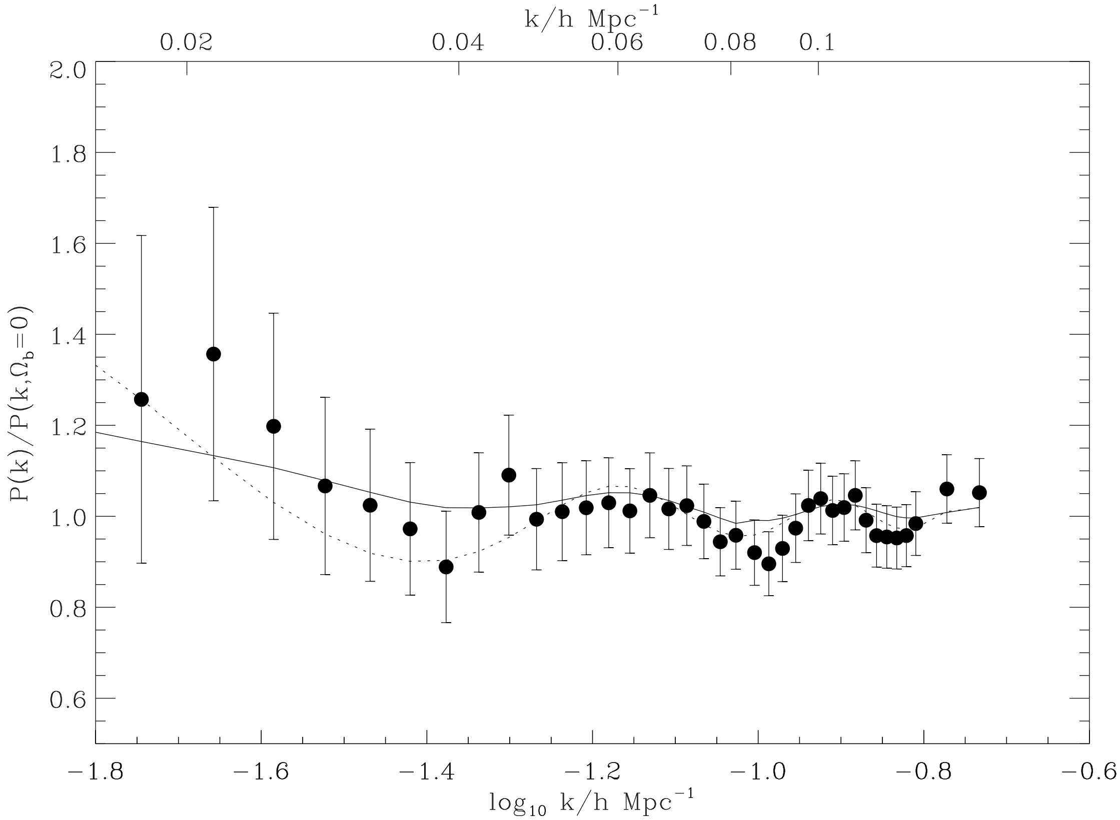

Figure 2. As Figure 1, but with the data and the curves divided by a model with $b = 1$, $\Omega_{\rm m} = 0.23$ and $\Omega_{\rm b} = 0$.

within a radius

$$R_h \simeq 67 \left(\frac{L_{\rm uv}}{10^{46}\,{\rm erg\,s^{-1}}}\right)^{1/3} \left(\frac{t_{\rm Q}}{2\times 10^9\,{\rm yrs}}\right)^{1/3} {\rm Mpc}. \tag{11}$$

In order to be able to contribute at a redshift z, the ionizing photons must have been emitted in less than the lifetime of the Universe at that redshift, $t(z)$. This places a minimal requirement that $t_Q < t(z)$. In the concordance cosmology, $t(z = 3) \simeq 2.2$ Gyrs, $t(z = 6) \simeq 0.95$ Gyr and $t(z = 10) = 0.48$ Gyr. The actual lifetime of quasars may well depend on their mass, but recent estimates suggest $t_Q \simeq 10^8$ yrs is more likely than 10^9 yrs (Mclure & Dunlop 2004). If this is the case then equation (11) implies that the corresponding value is more like $R_h \simeq 25$ Mpc by equation (11); for this value of t_Q the required ionization could easily have been achieved early, but the scale of resulting wiggles would be relatively small. For $R \simeq 100$ Mpc one needs to push the parameters excessively hard: a high value of $t_Q > 2 \times 10^9$ and a redshift of reionization $z < 3$ would be necessary. This seems to be at odds with the general consensus that reionization of the Universe happened relatively early (Becker et al. 2001; Fan et al. 2002).

There are other problems with this model. Quasars have a range of lifetimes and luminosities. Their radiation may also be beamed rather than isotropic. And in any case it is not known to what extent the galaxy formation process is sensitive to this form of feedback anyway. Moreover, the baryon acoustic oscillations inferred from galaxy clustering have the same characteristic scale as that derived from cosmic microwave background observations. This would be a sheer coincidence in our model.

This model is therefore unlikely to be the correct interpretation of observed wiggles, but it does at least demonstrate that large-scale interactions can have a significant impact on the shape of the clustering power spectrum. Notice also that even if the scale R_h is not sufficiently large to match the observed oscillations, any non-zero astrophysical effect could seriously degrade the ability to recover cosmological information from galaxy surveys. Mass tracers selected in some way other than counting galaxies may well display clustering that is less susceptible to this type of feedback bias. Galaxy clusters may be detected not only detected through X-ray emission or Sunyaev-Zel'dovich measurements, both of which are sensitive to the properties of the extremely hot gas the clusters contain. If these properties vary systematically on large scales then scale-dependent bias may also apply to such objects. However, the strong non-linear merging and heating processes that create this intracluster gas are likely to swamp any primordial effects generated on smaller scales. One would therefore expect cluster correlations to be less vulnerable to astrophysical modulation than galaxy correlations; complementary observations on the same length scales could be be used to identify and eliminate this source of uncertainty.

5. Scale-dependence versus Stochasticity

Even if the scale and form of the interaction kernel do not produce very large scale features in the galaxy correlation function or power spectrum, it is still possible for scale–dependence to manifest itself in more subtle ways. In particular, it is possible for scale-dependence to appear as a form of stochastic bias (Dekel & Lahav 1999) even though the relationship (2) is entirely deterministic once the density field is specified.

To see how this happens consider a simplified version of our general model in which the seed field δ_s is simply the matter density field δ_m. Let us assume explicitly that the fields were are considering are filtered on a scale R_0 to represent the selection of galaxy sized objects. Let the scale of feedback–induced interactions be $R_{\rm F}$, so that

$$\delta_{\rm m}(R_0) = \delta_{\rm g}(R_0) - \alpha\delta_{\rm g}(R_{\rm F}). \tag{12}$$

It is straightforward to see that

$$\begin{aligned}\langle\delta_{\rm m}\delta_{\rm g}\rangle &= \langle\delta_{\rm g}(R_0)^2\rangle - \alpha\langle\delta_{\rm g}(R_{\rm F})\delta_{\rm g}(R_0)\rangle \\ &= \langle\delta_{\rm g}^2\rangle\left(1 - \alpha\frac{\langle\delta_{\rm g}(R_{\rm F})\delta_{\rm g}(R_0)\rangle}{\langle\delta_{\rm g}(R_0)^2\rangle}\right)\end{aligned} \tag{13}$$

and

$$\begin{aligned}\langle\delta_{\rm m}^2\rangle &= \langle\delta_{\rm g}(R_0)^2\rangle + \alpha^2\langle\delta_{\rm g}(R_{\rm F})^2\rangle - 2\alpha\langle\delta_{\rm g}(R_0)\delta_{\rm g}(R_{\rm F})\rangle \\ &= \langle\delta_{\rm g}^2\rangle\left\{1 + \alpha^2\frac{\langle\delta_{\rm g}(R_{\rm F})^2\rangle}{\langle\delta_{\rm g}(R_0)^2\rangle} - 2\alpha\frac{\langle\delta_{\rm g}(R_0)\delta_{\rm g}(R_{\rm F})\rangle}{\langle\delta_{\rm g}(R_0)^2\rangle}\right\},\end{aligned} \tag{14}$$

where we have dropped the dependence on R_0 in the terms outside the curly brackets. It is useful to define the quantities

$$\gamma \equiv \frac{\langle \delta_{\rm g}(R_{\rm F})\delta_{\rm g}(R_0)\rangle}{\langle \delta_{\rm g}(R_0)^2\rangle} \qquad (15)$$

and

$$\omega^2 \equiv \frac{\langle \delta_{\rm g}(R_{\rm F})^2\rangle}{\langle \delta_{\rm g}^2(R_0)\rangle}, \qquad (16)$$

so that the cross-correlation coefficient between the mass and galaxy fluctuation fields is

$$r \equiv \frac{\langle \delta_{\rm m}\delta_{\rm g}\rangle}{\langle \delta_{\rm g}^2\rangle^{1/2}\langle \delta_{\rm m}^2\rangle^{1/2}} = \frac{1-\alpha\gamma}{(1+\alpha^2\omega^2-2\alpha\gamma)^{1/2}}. \qquad (17)$$

To provide a simple illustrative model we assume a *Gaussian filter*:

$$h_{\rm G}(|\mathbf{x}-\mathbf{x}'|;R_{\rm F}) = \frac{1}{(2\pi R_{\rm F}^2)^{3/2}}\exp\Big(-\frac{|\mathbf{x}-\mathbf{x}'|^2}{2R_{\rm F}^2}\Big), \qquad (18)$$

for which the appropriate window function is

$$\tilde{h}_{\rm G}(kR_{\rm F}) = \exp\Big[-\frac{(kR_{\rm F})^2}{2}\Big]. \qquad (19)$$

We then need to tackle quantities of the form

$$\langle \delta_{\rm g}(R_1)\delta_{\rm g}(R_2)\rangle = \frac{1}{2\pi^2}\int dk k^2 P_{\rm g}(k)\exp[-k^2(R_1^2+R_2^2)], \qquad (20)$$

which can be evaluated straightforwardly if we assume, for simplicity, that the (unsmoothed) galaxy power spectrum is a power-law: $P_{\rm g}(k)\propto k^n$. In this case we find that

$$\langle \delta_{\rm g}(R_0)\delta_{\rm g}(R_{\rm F})\rangle = \sigma^2\left(\frac{2R_0^2}{R_0^2+R_{\rm F}^2}\right)^{n+3/2}, \qquad (21)$$

where σ^2 is the variance of the unsmoothed density field. This gives

$$\gamma = \left(\frac{2R_0^2}{R_0^2+R_{\rm F}^2}\right)^{n+3/2} \qquad (22)$$

and

$$\omega^2 = \left(\frac{R_0}{R_{\rm F}}\right)^{(n+3)}. \qquad (23)$$

Note that if $R_0 = R_{\rm F}$ so that the feedback scale is no larger than a galaxy scale then $\omega = 1$, $\gamma = 1$ and consequently $r = 1$. If, however, $R_{\rm F} > R_0$ then $\gamma < 1$. However, it is always true that $\omega^2 > \gamma$ so that $(1-\alpha\gamma)^2 < 1+\alpha^2\omega^2-2\alpha\gamma$ and consequently that $r < 1$. The larger the value of $R_{\rm F}$ compared to R_0 the smaller the resulting value of r.

Assuming the fields $\delta_{\rm m}$ and $\delta_{\rm g}$ are jointly Gaussian one can express the conditional distribution of one given a specific value of the other. Suppose the (unconditional) variance of $\delta_{\rm g}$ is σ^2 then the variance after conditioning on $\delta_{\rm m} = a$, say, reduces to $\sigma^2(1-r^2)$. Only if $|r| = 1$ is there no scatter in the relationship. For this reason a value of $r < 1$ is usually taken to indicate the presence of stochastic bias (e.g. Tegmark

& Bromley 1999), but in this case the scatter in the relationship between $\delta_{\rm m}$ and $\delta_{\rm g}$ arises from non-locality in a fully deterministic way. This suggests that considerable care needs to be exercised in the interpretation of measured values of r : they may be indicative of scale–dependence rather than stochastic effects.

If we instead look at the galaxy and matter fields (assuming $\delta_s = \delta_m$) in Fourier space the situation is quite different. In this case, by equation (8) we get

$$P_{\rm g}(k) = b^2(k) P_{\rm m}(k) \tag{24}$$

with $b(k) = 1 - \alpha \tilde{h}(k)$. The cross-spectrum in Fourier space is usually defined to be $P_{\rm mg} = r(k)b(k)P_{\rm m}$ (Tegmark & Bromley 1998) for stochastic bias, with $r(k)$ playing a role analogous to the correlation coefficient discussed above. In this case, however, it reduces to $P_{\rm mg} = b(k)P_{\rm m}$ indicating a complete absence of stochasticity. The apparent stochasticity in real space is actually due to non-locality, but the model is local (and linear) in Fourier space so no stochasticity appears in this representation. This is an example of a phenomenon noted by Matsubara (1999).

6. Discussion and Conclusions

In this paper we have presented a new model for scale-dependent astrophysical bias. Although it is inspired to some extent by Bower et al. (1993), this model is considerably more general and easier to use. In the absence of any more complete theory of galaxy formation we hope it will provide a useful way to parametrise the possible level and scale of interactions so that they can be determined from observations and eliminated from cosmological considerations.

We illustrated the generality of this model by pushing it to an extreme and showing that it can produce features that mimic baryon oscillations. Although the required effect is quite small in amplitude it does require astrophysical processes to be coordinated over very large scales. This, together with the concordance between clustering observations and the cosmic microwave background, suggests that the observed wiggles have a primordial origin. Nevertheless, in the precision era, any scale dependence in clustering bias could seriously degrade the business of cosmological parameter estimation. However, as we have argued in Section 4, different forms of mass tracer are unlikely to suffer from this bias to the same extent as galaxies. Using complementary observations should provide sufficient data to estimate the parameters in our bias model. This will not only allow us to learn whether there is significant evidence for scale-dependent bias at all but also, by marginalization, provide a way to remove this uncertainty from cosmological studies. Some of the observations will go towards estimating and eliminating a nuisance parameter rather than reducing the statistical uncertainty in interesting ones so the existence of scale-dependent bias will degrade the cosmological value of surveys to some extent even if it can be modelled satisfactorily.

As a second, less extreme example of our approach we showed how non–locality in the feedback relationship described by equation (2) bears many of the hallmarks of stochastic bias. In particular, although our model is deterministic once the density field is specified, it is characterized by an imperfect correlation between galaxy and mass fluctuations. The difference between our model and a truly stochastic one is that in our case the residuals are not random but correlated through the interaction terms. One might learn more from observations by looking for correlated scatter than by giving up and treating them as completely stochastic. In any case the model we have presented shows up a terminological deficiency: stochasticity and non-locality can be easily confused.

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