

Constraining white-dwarf kicks in globular clusters

Jeremy Heyl¹

¹*Department of Physics and Astronomy, University of British Columbia, Vancouver, British Columbia, Canada, V6T 1Z1*
Email: heyjl@phas.ubc.ca; Canada Research Chair

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ABSTRACT

The wind of an asymptotic-giant-branch stars is sufficiently strong that if it is slightly asymmetric, it can propel the star outside of the open cluster of its birth or significantly alter its trajectory through a globular cluster; therefore, if these stellar winds are asymmetric, one would expect a deficit of white dwarfs of all ages in open clusters and for young white dwarfs to be less radially concentrated than either their progenitors or older white dwarfs in globular clusters. This latter effect has recently been observed. Hence, detailed studies of the radial distribution of young white dwarfs in globular clusters could provide a unique probe of mass loss on the asymptotic giant branch and during the formation of planetary nebulae both as a function of metallicity and a limited range of stellar mass.

Key words: white dwarfs — stars : AGB and post-AGB — globular clusters : general — stars: mass loss — stars: winds, outflows

1 INTRODUCTION

Spruit (1998) proposed that white dwarfs can acquire their observed rotation rates from mild kicks generated by asymmetric winds toward the end of their time on the asymptotic giant branch (AGB) (Vassiliadis & Wood 1993). Fellhauer et al. (2003) invoked these mild kicks to explain a putative dearth of white dwarfs in open clusters (e.g. Weidemann 1977; Kalirai et al. 2001). Unfortunately from the dynamics of open clusters it is difficult to probe the kicks because the white dwarfs simply leave the cluster. Although one can probe the dynamics of AGB winds directly through observations of masers, attempts to look for asymmetries are dogged by the variability of the star itself which makes it difficult to constrain any relative motion between the centre of mass of the wind and that of the star. The dynamics of globular clusters provide a unique environment to probe white dwarf kicks because both the magnitude of the expected kick and the velocity dispersion of the giants within the cluster are on the order of kilometers per second, so one would expect a significant signal. On the other hand the escape velocity of the cluster may be several times larger so most of the white dwarfs remain in the cluster to measure after the kick. The expected signature of white dwarf kicks has been observed in M4 and NGC 6397 (Davis et al. 2006).

This letter will examine how white dwarf kicks affect the radial distribution of young white dwarfs in globular clusters through analytic and Monte Carlo calculations of the phase-space distribution function of stars in the cluster.

2 CALCULATIONS

Clusters of stars can typically be modelled with a lowered isothermal profile (or King model) (Michie 1963; King 1966)

$$f = \frac{dN}{d^3x d^3v} = \begin{cases} \rho_1 (2\pi\sigma^2)^{-3/2} \left(e^{\epsilon/\sigma^2} - 1 \right) & \text{if } \epsilon > 0 \\ 0 & \text{if } \epsilon \leq 0 \end{cases} \quad (1)$$

where $\epsilon = \Psi - \frac{1}{2}v^2$, Ψ is the gravitational potential, σ is a characteristic velocity dispersion and ρ_1 is a characteristic density. Integrating the distribution function over velocity yields the density distribution

$$\rho = \rho_1 \left[e^{\Psi/\sigma^2} \operatorname{erf} \left(\frac{\sqrt{\Psi}}{\sigma} \right) + \sqrt{\frac{4\Psi}{\pi\sigma^2}} \left(1 + \frac{2\Psi}{3\sigma^2} \right) \right]. \quad (2)$$

The density is typically constant within the core radius, $r_0 = \sqrt{9\sigma^2/(4\pi\rho_0)}$ where ρ_0 is the central density of the cluster. The gravitational potential can be solved self-consistently with Eq. (2) (Binney & Tremaine 1987) using

$$\frac{d}{dr} \left(r^2 \frac{d\Psi}{dr} \right) - 4\pi G \rho r^2 \quad (3)$$

to give a model for the cluster. Because the distribution function depends only on constants of the motion (the energy), it is constant in time as well.

With time the kinetic energy within the cluster approaches equipartition between the various stars such that $m_i \sigma_i^2 = m_j \sigma_j^2$ (Spitzer 1987). The progenitors of young white dwarfs will be the most massive main-sequence stars in a cluster at the time, so they will typically have $\sigma_{\text{TO}} < \sigma_c$, where σ_c is the mean velocity dispersion of the cluster; furthermore, because these progenitors will only have a small

fraction of the mass of the cluster, they can be considered as massless tracers; their phase space density will be given by Eq. (1) with $\sigma = \sigma_{\text{T0}}$. The gravitational potential and core radius will be determined by Eq. (1)–Eq. (3) with $\sigma = \sigma_c$.

During their time on the AGB, the stars may lose a large fraction of their mass suddenly and asymmetrically (Vassiliadis & Wood 1993; Spruit 1998; Fellhauer et al. 2003) and receive an impulsive kick. To model this kick, the original phase-space density can be convolved with a kick distributed as a Gaussian:

$$f_{\text{final}} = \frac{1}{(\pi\sigma_k^2)^{3/2}} \int d^3v' \exp\left[-\frac{(\mathbf{v} - \mathbf{v}')^2}{2\sigma_k^2}\right] f(x, v'). \quad (4)$$

Although this convolved distribution function can be expressed in closed form (see the appendix), two key results are important. First, the final distribution function depends on the value of the gravitational potential and the velocity separately and not on the conserved energy alone. Second, in the limit of large Ψ or small σ_k the distribution approaches a lowered isothermal profile with $\sigma_f^2 \approx \sigma_{\text{T0}}^2 + \sigma_k^2$.

Because the distribution no longer depends on constants of the motion alone, the distribution function itself will depend on time, so a Monte Carlo realization of the kicked distribution function is helpful to make further progress. Specifically, for a variety of values of $\Psi(0)$, σ_{T0} and σ_k , ten thousand stars are drawn from Eq. (1) and given a three-dimensional velocity kick drawn from a Gaussian of width of σ_k . Finally, each star is evolved forward along its orbit to a random phase; this yields an estimate of the radial distribution of the young white dwarfs in the cluster after the kick as well as the fraction of young white dwarfs that might escape the cluster. The potential of the cluster is fixed to be the solution to Eq. (3) for $\sigma = 1$; this assumes that the evolving AGB stars and young white dwarfs make a negligible contribution to the mass of the cluster and that the observations of the young white dwarfs occur within a relaxation time of their formation.

The result of the Monte Carlo realization is a list of radial positions at the moment of the kick and at a random time later for those stars that cannot escape the cluster. The density distribution given by Eq. (2) provides a natural characterization of the positions of the particles that do not manage to escape the cluster. The best distribution is obtained by maximizing the Kolmogorov-Smirnov probability (e.g. Press et al. 1992) that the list of radial positions is drawn from Eq. (2) with a particular value of σ_f ; in this way the escape fraction and the best-fit σ_f provide an approximation of the distribution of the young white dwarfs (see Fig. 1).

3 RESULTS

The numerical results depicted in Fig. 2 confirm the analytic expectations. For small kicks the final distribution is well characterized by

$$\sigma_f^2 \approx \sigma_{\text{T0}}^2 + \sigma_k^2 \quad (5)$$

and in general $\sigma_f^2 < \sigma_{\text{T0}}^2 + \sigma_k^2$; therefore, $\sigma_f^2 - \sigma_{\text{T0}}^2$, the estimate used by Davis et al. (2006), provides a firm lower limit on the kick velocity required to puff up a stellar distribution within a cluster. A kick of a given size is less effective

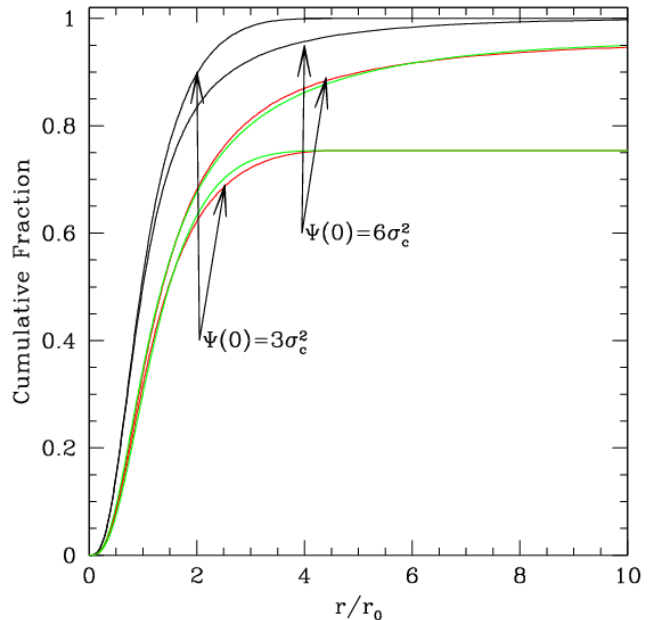


Figure 1. The cumulative radial distribution for $\sigma_{\text{T0}} = \sigma_k = 0.7\sigma_c$ with $\Psi(0) = 3\sigma_c^2$ and $6\sigma_c^2$. The black curve follows the initial distribution, the red curve follows the phase-mixed distribution after the kick. The green curves give the best fitting functions of the form Eq. (2): a model with $\Psi(0) = 3\sigma_c^2$ yields a escape fraction of 25% and $\sigma_f = 0.86\sigma_c$, and the model with $\Psi(0) = 6\sigma_c^2$ gives an escape fraction of 4.4% and $\sigma_f = 0.76\sigma_c$.

at changing the radial distribution of stars within clusters with deeper potential wells than in shallow wells.

The second diagnostic is the fraction of the stars that escape the cluster after receiving a kick. The escape fraction is approximately proportional to $\sigma_k^2/\sigma_{\text{T0}}^2$, the relative change in the kinetic energy of the stars due to the kick. Furthermore, the fraction decreases exponentially with increasing values of $\Psi(0)$ and increases exponentially with increasing values of the initial velocity dispersion. Lighter stars are much more likely to escape from clusters with more shallow potential wells than heavier stars that are initially more centrally concentrated. More importantly and in contrast with the results for open clusters (Fellhauer et al. 2003), the escape fraction is typically quite small; it did not exceed 25% for any of the models considered (not surprisingly the largest number of escapees came from a model with $\Phi(0) = 4\sigma_c^2$ and $\sigma_k = \sigma_{\text{T0}} = 0.8\sigma_c$).

Depending on the distribution of stellar masses in the cluster the ratio of σ_{T0} to σ_c may vary. For simplicity, the mass function can be assumed to be a power law,

$$\frac{dN}{dM} = AM^{-\alpha} \text{ for } M_{\text{min}} \leq M \leq M_{\text{T0}} \quad (6)$$

and zero elsewhere. This yields a mean mass of

$$\bar{M} = \frac{1-\alpha}{2-\alpha} M_{\text{T0}} \frac{\left(\frac{M_{\text{min}}}{M_{\text{T0}}}\right)^{2-\alpha} - 1}{\left(\frac{M_{\text{min}}}{M_{\text{T0}}}\right)^{1-\alpha} - 1}. \quad (7)$$

If $M_{\text{min}} = 0.1M_{\text{T0}}$ and $\alpha = -2.25$ (a Salpeter initial mass function), $\bar{M} \approx 0.23M_{\text{T0}}$ and $\sigma_{\text{T0}} = 0.5\sigma_c$.

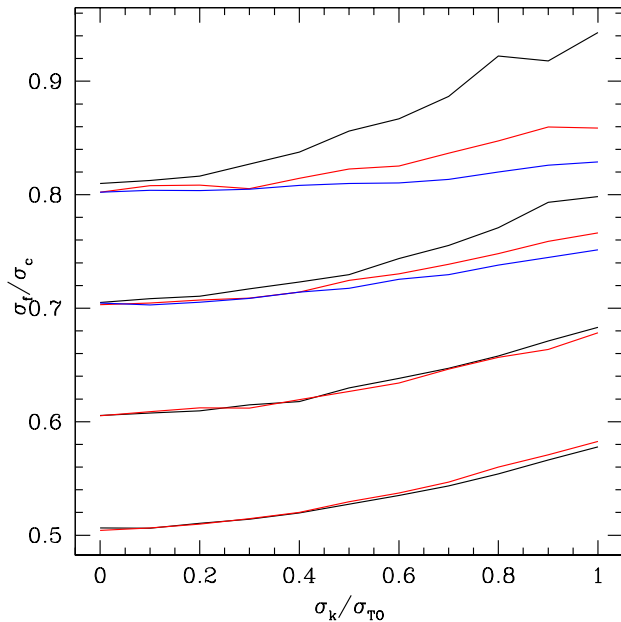


Figure 2. The final values of the best fitting σ for the phase-mixed stellar distributions as a function of the initial value of $\sigma_{\text{TO}}/\sigma_c$, $\Psi(0)/\sigma_c^2$ and $\sigma_k/\sigma_{\text{TO}}$. From top to bottom the sets of curves are for $\sigma_{\text{TO}}/\sigma_c = 0.8, 0.7, 0.6$ and 0.5 . Within each set the curves give $\Psi(0)/\sigma_c^2 = 4, 6$ and 8 in black, red and blue respectively.

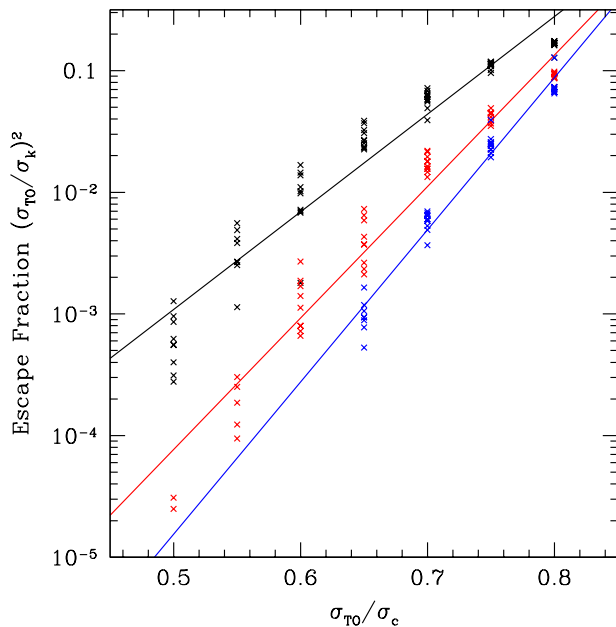


Figure 3. The fraction of stars that escape the cluster after receiving a kick. The escape fraction is typically proportional to the square of the relative size of the kick, so this dependence has been removed. The black, red and blue points give the results for $\Psi(0)/\sigma_c^2 = 4, 6$ and 8 , respectively, for various values of σ_k ranging from $0.1\sigma_{\text{TO}}$ to σ_{TO} , and values of σ_{TO} from 0.5 to $0.8\sigma_c$. Where the points are absent the escape fraction was less than one part in 10^4 .

Typically when the line-of-sight velocity dispersion of a cluster is determined, the result is dominated by the brightest stars, that is the giants and the stars near the turnoff; therefore, the line-of-sight velocity dispersion measured near the centre of the cluster is a direct determination of the value of σ_{TO} . Typically, $\sigma_{\text{los}} \rightarrow \sigma_{\text{TO}}$ as $\Psi(0) \rightarrow \infty$. Specifically,

$$\sigma_{\text{los}} \approx \sigma_{\text{TO}} \left[1 - \exp \left(- \frac{(\Psi(0)/\sigma_{\text{TO}}^2)^{0.85}}{1.45} \right) \right] \quad (8)$$

to within two percent for $\Psi(0) > 2\sigma_{\text{TO}}^2$. For $\Psi(0) = 4\sigma_{\text{TO}}^2$, $\sigma_{\text{los}} = 0.89\sigma$. By looking at the distribution of stellar masses in the cluster, the value of σ_c can be estimated from σ_{TO} as in Eq. (6) and (7). As Tab. 1 shows, the line-of-sight velocity dispersion of a globular cluster is typically a few kilometers per second, similar to the proposed kick velocities; the two clusters observed by Davis et al. (2006) have velocity dispersions of about 4 kms^{-1} , possibly smaller than the expected kicks. Furthermore, the effect could be observable in all of the clusters listed with the possible exceptions of 47 Tuc, Omega Cen and M22.

4 CONCLUSIONS

Studies of the radial distribution of stars near the turnoff and young white dwarfs within globular clusters can provide a unique handle on the asymmetry of the rapid mass loss while stars are on the asymptotic giant branch. In the case of an open cluster as studied by Fellhauer et al. (2003), the assumed dearth of young white dwarfs could only provide a lower limit on the kick; to get a better result requires searching for the white dwarfs that have just left the cluster. In a globular cluster one can get a quantitative estimate of the typical kick velocity from the observed distributions because very few stars typically escape the cluster; furthermore, by comparing the distributions in globular clusters of different ages and metallicities one can probe the mass loss for a variety of stellar masses and chemical compositions. By comparing the distribution of white dwarfs of different ages up to and greater than the relaxation time for the cluster, the process of dynamical relaxation of the white dwarf population can be constrained observationally.

Such a study requires observations of many stars throughout a globular cluster, specifically the accurate radial positions of the brightest main-sequence stars (near the turnoff) and the brightest white dwarfs. Because information from the central regions of the cluster is most helpful, understanding or at least minimizing the effects of confusion is crucial; the space-based observations presented in (Davis et al. 2006) indeed found evidence of white dwarf kicks in NGC 6397 and M4. Further studies of resolved stellar populations in both open clusters and globular clusters could provide further probes of this violent stage in the evolution of stars like our Sun.

APPENDIX : KICKED PHASE-SPACE DISTRIBUTION FUNCTION

The convolution of the lowered isothermal profile distribution function with a Gaussian kick in velocity is given by

Table 1. Kinetic parameters for a few nearby globular clusters whose radial stellar distribution could be probed with JWST (Richer *priv. comm.*). The concentration or the ratio of the core radius to the tidal radius ($c = \log_{10} r_t/r_c$) are from Harris (1996), a dash denotes that the core has collapsed. The values of $\Psi(0)/\sigma_c^2$ are from McLaughlin & van der Marel (2005) with the exception of NGC 6837 where it was inferred from the concentration. The value of σ_c was obtained by assuming that $\bar{M}/M_{\text{TO}} = 0.3/0.8 = 0.375$. Refs: (a) McLaughlin et al. (2006); (b) Reijns et al. (2006); (c) Peterson & Latham (1986); (d) Pryor & Meylan (1993).

Cluster	σ_{los} [km/s]	σ_c [km/s]	c	$\Psi(0)/\sigma_c^2$
NGC 104 (47 Tuc)	11.6±0.8 ^(a)	19	2.03	8.60±0.10
NGC 5139 (Omega Cen)	15 ^(b)	25	1.61	6.20±0.20
NGC 6121 (M4)	3.9±0.7 ^(c)	6.4	1.59	7.40±0.10
NGC 6397	3.5±0.2 ^(d)	5.7	—	—
NGC 6656 (M22)	8.5±1.9 ^(c)	14	1.31	6.50±0.20
NGC 6752	4.5±0.5 ^(d)	7.3	—	—
NGC 6809 (M55)	4.2±0.5 ^(d)	6.8	0.76	4.50±0.10
NGC 6838 (M71)	2.8±0.6 ^(c)	4.6	1.15	5.4

$$f_{\text{final}} = \frac{1}{(\pi\sigma_k^2)^{3/2}} \int d^3v' \exp\left[-\frac{(\mathbf{v}-\mathbf{v}')^2}{2\sigma_k^2}\right] f_{\text{initial}}(x, v'). \quad (9)$$

Because both the kick and basic distribution function are Gaussian it is not surprising that the final distribution function can be expressed in closed form. If there was no restriction on the velocities in the distribution function in Eq. (1), the convolution would also be a Gaussian with $\sigma_f^2 = \sigma^2 + \sigma_k^2$; however, the restriction on the velocities complicates the result:

$$f_{\text{final}} = N \left\{ \frac{2\sigma_k\sigma_f}{v} e^{-(\Psi+v^2/2)/\sigma_k^2} \left(1 - e^{-2\sqrt{2\Psi}v/\sigma_k^2}\right) + \sqrt{2\pi}\sigma \exp\left[\frac{\epsilon}{\sigma_f^2} + \frac{\sigma_k\Psi}{\sigma^2\sigma_f^2} + \frac{v\sqrt{2\Psi}}{\sigma_k^2}\right] [g(v) + g(-v)] \right\} \quad (10)$$

where

$$N = \frac{\rho_1}{4\sqrt{2\pi^2}\sigma\sigma_f^3} \exp\left[-\frac{v\sqrt{2\Psi}}{\sigma_k^2} - 1\right] \quad (11)$$

$$g(\Psi, v) = \text{erf}\left(\frac{\sigma^2v + \sigma_f^2\sqrt{2\Psi}}{\sqrt{2}\sigma\sigma_k\sigma_f}\right) \quad (12)$$

In the limit of large Ψ or small σ_k the distribution approaches a lowered isothermal profile with $\sigma \rightarrow \sigma_f$.

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