Newton law corrections and instabilities in $f(R)$ gravity with the effective cosmological constant epoch

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We consider class of modified $f(R)$ gravities with the effective cosmological constant epoch at the early and late universe. Such models pass most of solar system tests as well they satisfy to cosmological bounds. Despite their very attractive properties, it is shown that one realistic class of such models may lead to significant Newton law corrections at large cosmological scales. Nevertheless, these corrections are small at solar system as well as at the future universe. Another realistic model with acceptable Newton law regime shows the matter instability.

I. INTRODUCTION

Much attention has been paid recently to the study of modified $f(R)$ gravity (for review, see [1]). Such theory which may be related with string effective action [2]) may successfully describe the dark energy epoch [3, 4, 5, 6, 7, 8]. It is remarkable that even the form of $f(R)$ gravity may be reconstructed from the known universe expansion history[9]. Hence, this approach suggests the gravitational alternative for dark energy. It may be considered as proposal for new gravity theory which could be more exact than usual General Relativity at current/future universe. If it is so, such a theory should pass the known solar system tests [4, 5, 6, 10] as well as cosmological bounds. Unfortunately, despite the significant progress in the construction of more-less acceptable models, the totally satisfactory theory has not been yet proposed.

Recently, $f(R)$ gravity with the early/late-time effective cosmological constant epoch was proposed [7, 9, 11, 12]. The very attractive, simple versions of such theory [11, 12] seem to show quite satisfactory behaviour from the cosmological point of view (the models of ref.[7, 9] are quite complicated). As well they seem to satisfy (most) of solar system tests. Nevertheless, some deviations from General Relativity may be expected. Specifically, the model [11] may show large Newton law corrections at cosmological scales. Nevertheless, for limited range of parameters these corrections are small in Solar System. As well they become small at the future universe. On the same time, the model of ref.[12] may lead to matter instability in the proposed range of parameters. This indicates that such theories which show remarkably beatiful behaviour as ΛCDM cosmologies should be extended, perhaps, introducing more parameters.

II. THE NEWTON LAW CORRECTIONS IN F(R) GRAVITY WITH AN EFFECTIVE COSMOLOGICAL CONSTANT EPOCH

The action of general $f(R)$ gravity (for a review, see [1]) is given by

$$S = \frac{1}{\kappa^2} \int d^4x \sqrt{-g} \left(R + f(R) \right) . \tag{1}$$

Here $f(R)$ is an arbitrary function. The general equation of motion in $f(R)$-gravity with matter is given by

$$\frac{1}{2} g_{\mu\nu} F(R) - R_{\mu\nu} F'(R) - g_{\mu\nu} \Box F'(R) + \nabla_\mu \nabla_\nu F'(R) = -\frac{\kappa^2}{2} T_{(m)\mu\nu} . \tag{2}$$

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Here $F(R) = R + f(R)$ and $T_{(m)\mu\nu}$ is the matter energy-momentum tensor.

By introducing the auxilliary field A one may rewrite the action (1) in the following form:

$$S = \frac{1}{\kappa^2} \int d^4x \sqrt{-g} \left\{ (1 + f'(A))(R - A) + A + f(A) \right\} . \tag{3}$$

From the equation of motion with respect to A, it follows $A = R$. By using the scale transformation $g_{\mu\nu} \to e^\sigma g_{\mu\nu}$ with $\sigma = -\ln(1 + f'(A))$, we obtain the Einstein frame action [4]:

$$\begin{aligned} S_E &= \frac{1}{\kappa^2} \int d^4x \sqrt{-g} \left\{ R - \frac{3}{2} \left(\frac{F''(A)}{F'(A)} \right)^2 g^{\rho\sigma} \partial_\rho A \partial_\sigma A - \frac{A}{F'(A)} + \frac{F(A)}{F'(A)^2} \right\} \\ &= \frac{1}{\kappa^2} \int d^4x \sqrt{-g} \left(R - \frac{3}{2} g^{\rho\sigma} \partial_\rho \sigma \partial_\sigma \sigma - V(\sigma) \right) , \end{aligned} \tag{4}$$

$$V(\sigma) = e^\sigma g\left(e^{-\sigma}\right) - e^{2\sigma} f\left(g\left(e^{-\sigma}\right)\right) = \frac{A}{F'(A)} - \frac{F(A)}{F'(A)^2} . \tag{5}$$

Here $g\left(e^{-\sigma}\right)$ is given by solving $\sigma = -\ln(1 + f'(A)) = \ln F'(A)$ as $A = g\left(e^{-\sigma}\right)$. After the scale transformation $g_{\mu\nu} \to e^\sigma g_{\mu\nu}$, there appears a coupling of the scalar field σ with the matter. For example, if the matter is the scalar field Φ with mass M, whose action is given by

$$S_\phi = \frac{1}{2} \int d^4x \sqrt{-g} \left(-g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - M^2 \Phi^2 \right) , \tag{6}$$

there appears a coupling with σ in the Einstein frame:

$$S_{\phi E} = \frac{1}{2} \int d^4x \sqrt{-g} \left(-e^\sigma g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - M^2 e^{2\sigma} \Phi^2 \right) . \tag{7}$$

The strength of the coupling is of the gravitational coupling κ order. Unless the mass of σ, which is defined by

$$m_\sigma^2 \equiv \frac{1}{2} \frac{d^2 V(\sigma)}{d\sigma^2} \tag{8}$$

is large, there appears the large correction to the Newton law.

More exactly, in the Einstein frame, matter fields give a source term for the scalar field σ like

$$J_\sigma = e^{a\sigma} \rho . \tag{9}$$

Here ρ is the energy density (in the Jordan frame). Now we consider the fluctuations from the background of $\sigma = \sigma_0$ (σ_0 is not always a constant):

$$\sigma = \sigma_0 + \delta\sigma . \tag{10}$$

For simplicity, we consider the limit where the spacetime is almost flat and consider the point like souces

$$J_\sigma = \rho_0^{(1)} e^{a^{(1)} \sigma(x)} \delta\left(x - x^{(1)}\right) + \rho_0^{(2)} e^{a^{(2)} \sigma(x)} \delta\left(x - x^{(2)}\right) . \tag{11}$$

Then by the propagation of $\delta\sigma$, we find the following correlation function

$$\left\langle e^{a^{(1)} \sigma(x^{(1)})} e^{a^{(1)} \sigma(x^{(2)})} \right\rangle \sim e^{\left(a^{(1)} + a^{(2)}\right) \sigma_0 + a^{(1)} a^{(2)} G_\sigma(x_1, x_2)} . \tag{12}$$

Here $G_\sigma(x_1, x_2)$ is the correlation function of σ. When the mass of σ is small, we have

$$G_\sigma(x_1, x_2) = \frac{\kappa^2}{12\pi(x^{(1)} - x^{(2)})^2} , \tag{13}$$

and

$$\left\langle e^{a^{(1)} \sigma(x^{(1)})} e^{a^{(1)} \sigma(x^{(2)})} \right\rangle \sim e^{\left(a^{(1)} + a^{(2)}\right) \sigma_0 + \frac{a^{(1)} a^{(2)} \kappa^2}{12\pi(x^{(1)} - x^{(2)})^2}} . \tag{14}$$

At the long range where $(x^{(1)} - x^{(2)})^2$ is large enough compared with κ^2, we find

$$\left\langle e^{a^{(1)}\sigma(x^{(1)})} e^{a^{(1)}\sigma(x^{(2)})} \right\rangle \sim e^{\left(a^{(1)}+a^{(2)}\right)\sigma_0} \left(1 + \frac{a^{(1)}a^{(2)}\kappa^2}{12\pi(x^{(1)}-x^{(2)})^2} + \cdots \right) \ . \tag{15}$$

Then there appears the long range force and the strength of the coupling is given by $e^{\left(a^{(1)}+a^{(2)}\right)\sigma_0} a^{(1)} a^{(2)} \kappa^2$. If the coupling is very small, the correction to the Newton law might be not so small.

Recently very interesting $f(R)$ model has been proposed by Hu and Sawicki [11]. In the model $f(R)$ is given by

$$f_{HS}(R) = -\frac{m^2 c_1 \left(R/m^2\right)^n}{c_2 \left(R/m^2\right)^n + 1} \ , \tag{16}$$

which satisfies the condition

$$\lim_{R \to \infty} f_{HS}(R) = \text{const} \ ,$$
$$\lim_{R \to 0} f_{HS}(R) = 0 \ , \tag{17}$$

The estimation of ref.[11] suggests that R/m^2 is not so small but rather large even in the present universe and $R/m^2 \sim 41$. Then we have

$$f_{HS}(R) \sim -\frac{m^2 c_1}{c_2} + \frac{m^2 c_1}{c_2^2} \left(\frac{R}{m^2}\right)^{-n} \ , \tag{18}$$

which gives an "effective" cosmological constant $-m^2 c_1/c_2$. The effective cosmological constant generates the accelerating expansion in the present universe. Then

$$H^2 \sim \frac{m^2 c_1 \kappa^2}{c_2} \sim (70\text{km/s}\cdot\text{pc})^2 \sim \left(10^{-33}\text{eV}\right)^2 \ . \tag{19}$$

In the intermediate epoch, where the matter density ρ is larger than the effective cosmological constant,

$$\rho > \frac{m^2 c_1}{c_2} \ , \tag{20}$$

there appears the matter dominated phase (such phase may occur for other modified $f(R)$ gravity as well [7, 8]) and the universe could expand with deceleration. Hence, above model leads to the effective ΛCDM cosmology like models [7, 9].

Some remark is in order. The approximate expression for the Hu-Sawicky model should be taken with great care. The reason is that at very small curvatures where the (non-perturbative) function $f(R)$ goes to zero, the approximation breaks down (the corresponding function f may become singular).

Due to the scalar field in (4), an extra (fifth) force could manifest itself. It could violate the Newton law. The Newton law is well understood and its correction should be very small at least in the present universe. If the mass of σ is large enough in the present universe, the problem could be avoided. We now investigate the model by assuming $A/m^2 = R/m^2 \gg 1$ since $R/m^2 \sim 41$ even in the present universe. Then one gets

$$\sigma \sim -\frac{nc_1}{c_2}\left(\frac{A}{m^2}\right)^{-n-1} \ , \quad V(\sigma) \sim \frac{m^2 c_1}{c_2} - \frac{(n+1)m^2 c_1}{c_2^2}\left(\frac{A}{m^2}\right)^{-n} \ , \tag{21}$$

and

$$m_\sigma^2 \equiv \frac{1}{2}\frac{d^2 V(\sigma)}{d\sigma^2} = -\frac{1}{2}\left(\frac{d\sigma}{dA}\right)^{-3}\frac{d^2\sigma}{dA}\frac{dV}{dA} + \frac{1}{2}\left(\frac{d\sigma}{dA}\right)^{-2}\frac{d^2 V}{dA^2} = \frac{1}{2}\left\{\frac{A}{F'(A)} - \frac{4F(A)}{(F'(A))^2} + \frac{1}{F''(A)}\right\} \sim \frac{m^2}{2nc_1}\left(\frac{A}{m^2}\right)^{n+2} \tag{22}$$

First we consider the universe at very large scales, where $R \sim \left(10^{-33}\,\text{eV}\right)^{-2}$ and therefore $R/m^2 \sim 41$. If c_1 is not so small and/or n is not so large, $R/m^2 \sim 41$, we find m_σ should be very small $m_\sigma \sim 10^{-33}$ eV. Therefore, the correction to the Newton law is large. Note that $\sigma_0 \sim 0$ in (15) for the model [11]. Since $a_{1,2} \sim 1$, the correction to the Newton law could be not so small.

Although m_σ could be very small at large scales since R_0 is very small, R_0 can be larger near or in the star. Since $1\,g \sim 6 \times 10^{32}\,\text{eV}$ and $1\,\text{cm} \sim \left(2 \times 10^{-5}\,\text{eV}\right)^{-1}$, the density is about $\rho \sim 1\text{g}/\text{cm}^3 \sim 5 \times 10^{18}\,\text{eV}^4$ inside the earth. This shows that the magnitude of the curvature could be $R_0 \sim \kappa^2 \rho \sim \left(10^{-19}\,\text{eV}\right)^2$ and therefore $R_0/m^2 \sim 10^{28}$. Hence, in case $n = 2$, we find $m_\sigma \sim 10^{19}\,\text{GeV}$, which is very large and the correction to the Newton law is very small.

Even in air, one finds $\rho \sim 10^{-6}\text{g}/\text{cm}^3 \sim 10^{12}\,\text{eV}^4$, which gives $R_0 \sim \kappa^2 \rho \sim \left(10^{-25}\,\text{eV}\right)^2$ and $R_0/m^2 \sim 10^{16}$. In case $n = 2$, $m_\sigma \sim 10^{-1}\,\text{eV}$, which gives a correlation length (Compton wave length) about $1\,\mu\text{m}$. Thus, the correction to the Newton law could not be observed on the earth for such a model.

What happens in the solar system? In the solar system, there could be interstellar gas. Typically, in the interstellar gas, there is one proton (or hydrogen atom) per $1\,\text{cm}^3$, which shows $\rho \sim 10^{-5}\,\text{eV}^4$, $R_0 \sim 10^{-61}\,\text{eV}^2$, and therefore $R_0/m^2 \sim 10^4$. Then for $n = 2$, we find $m_\sigma \sim 10^{-25}\,\text{eV}$, which corresponds to the correlation length of $10^{18}\,\text{m} \sim 100\,\text{pc}$. Then the correction to the Newton law could be observed. In case $n = 8$, however, we find $m_\sigma \sim 10^{-13}\,\text{eV}$, which corresponds to the correlation length of $10^6\,\text{m}$, which is less than the radius of earth ($\sim 10^7\,\text{m}$). Then the correction to the Newton law could not be observed. Hence, some sub-class of above theory may pass known solar sytem tests at the scales of the solar system order.

In (22), the Einstein frame was considered (4). However, similar conclusions may be made also in Jordan frame. By multiplying (2) with $g^{\mu\nu}$, one obtains

$$-3\Box F'(R) - RF'(R) + 2F(R) = -\frac{\kappa^2}{2}T \ . \tag{23}$$

Here $T \equiv T_{(m)\rho}^{\ \ \rho}$. The equation (23) corresponds to Eq.(39) in [11]. Now we consider the background where R is a constant $R = R_0$, that is, (anti-)de Sitter space which can be obtained by solving the algebraic equation

$$-R_0 F'(R_0) + 2F(R_0) = 0 \ . \tag{24}$$

Since $\mathrm{e}^{-\sigma} = F'(R)$, one gets

$$\delta R = -\frac{F'(R)}{F''(R)}\delta\sigma \ . \tag{25}$$

Consider the fluctuation

$$R = R_0 + \delta R \ , \tag{26}$$

which leads to

$$\Box\delta\sigma - \frac{1}{3}\left(\frac{F'(R_0)}{F''(R_0)} - R_0\right)\delta\sigma = -\frac{\kappa^2}{6F'(R_0)}T \ . \tag{27}$$

One may consider the point source

$$T = T_0 \delta(x) \ . \tag{28}$$

Then the solution of (27) is given by

$$\delta\sigma = -\frac{\kappa^2 T_0}{6F'(R_0)}G(m^2, |x|) \ . \tag{29}$$

Here

$$m^2 \equiv \frac{1}{3}\left(\frac{F'(R_0)}{F''(R_0)} - R_0\right) \ , \quad \left(\Box - m^2\right)G(m^2, |x|) = \delta(x) \ . \tag{30}$$

If $m^2 < 0$, there appears tachyon and there could be some instability. Even if $m^2 > 0$, when m^2 is small, $\delta R \neq 0$ at long ranges, which generates the large correction to the Newton law. In case of [11], we find, when $R/m^2 \gg 1$ as in the present universe,

$$m^2 \sim \frac{m^2 c_2^2}{3n(n+1)c_1}\left(\frac{R_0}{m^2}\right)^{n+2} \ . \tag{31}$$

Compared this expression (31) with (22) by putting $A = R_0$, we find $m^2 \sim m_\sigma^2$. Then the correction to the Newton law is the same.

In [11], it is assumed $R/m^2 \gg 1$ but it might be interesting to study the model assuming $R/m^2 \ll 1$, which may correspond to the future universe. When $A/m^2 = R/m^2 \ll 1$, the potential $V(\sigma)$ (4) is given by

$$V(\sigma) \sim (1 - n) c_1 m^2 \left(A/m^2\right)^n \ , \tag{32}$$

and we find

$$\sigma \sim -\ln\left(1 - nc_1 \left(A/m^2\right)^{n-1}\right) \ . \tag{33}$$

Let us consider the case $n > 1$ and $0 < n < 1$ separately.

In case $n > 1$, when A is small, (33) can be written as

$$\sigma \sim nc_1 \left(A/m^2\right)^{n-1} \ , \tag{34}$$

and therefore

$$V(\sigma) \sim (1 - n) c_1 m^2 \left(\frac{\sigma}{nc_1}\right)^{n/(n-1)} \ . \tag{35}$$

Then

$$m_\sigma^2 \sim \frac{n-2}{2n(n-1)c_1} \left(\frac{\sigma}{nc_1}\right)^{-1+1/(n-1)} \ . \tag{36}$$

Note $m_\sigma > 0$ if $c_1 > 0$. Eq.(34) shows that σ is small when A/m^2 is small. Then the mass m_σ becomes large when the curvature $R \sim A$. Therefore the scalar field does not propagate at large ranges and the Newton law could not be violated.

On the other hand, in case $n < 1$, for small A, we find

$$\sigma \sim -(n-1) \ln \frac{A}{m^2} + \ln(-nc_1) \ . \tag{37}$$

When A is small, σ is negative and large. Eq.(37) shows

$$V(\sigma) \sim (1 - n) c_1 m^2 \left(-nc_1\right)^{n/(n-1)} \mathrm{e}^{-n\sigma/(n-1)} \ . \tag{38}$$

In order that the potential being real, c_1 should be negative. Since

$$m_\sigma^2 \sim \frac{n^2 c_1 m^2}{1 - n} \left(-nc_1\right)^{n/(n-1)} \mathrm{e}^{-n\sigma/(n-1)} \ , \tag{39}$$

the squared mass m_σ^2 is negative since $c_1 < 0$, which shows that σ is tachyon and unstable. Tachyon is inconsistent with quantum theory. Classically if we consider the perturbation with respect to σ, the perturbation becomes large. Since σ is related with the curvature by $\sigma = -\ln F'(A) = -\ln F'(R)$, the instability may indicate the solution where by the perturbation, the curvature of the universe could become large.

Hence, it seems there may be significant correction to Newton law in the $f(R)$ gravity model under consideration at cosmological scales. It is remarkable that such correction becomes negligible in the future, at least, for some range of parameters.

III. THE ABSENCE OF MATTER INSTABILITY

There may exist another type of instability (so-called matter instability) in $f(R)$ gravity [10]. The example of the model without such instability is given in [4] (for related discussions of matter instability, see, [13]). Let us show that current and related models are free from such instability. The instability might occur when the curvature is rather large, as on the planet, compared with the average curvature in the universe $R \sim \left(10^{-33}\,\mathrm{eV}\right)^2$. By multipling Eq.(2) with $g^{\mu\nu}$, one obtains

$$\Box R + \frac{F^{(3)}(R)}{F^{(2)}(R)} \nabla_\rho R \nabla^\rho R + \frac{F'(R)R}{3F^{(2)}(R)} - \frac{2F(R)}{3F^{(2)}(R)} = \frac{\kappa^2}{6F^{(2)}(R)} T \ . \tag{40}$$

Here $T \equiv T_{(m)\rho}^{\ \rho}$. We consider a perturbation from the solution of the Einstein gravity:

$$R = R_0 \equiv -\frac{\kappa^2}{2}T > 0 \ . \tag{41}$$

Note that T is negative since $|p| \ll \rho$ on the earth and $T = -\rho + 3p \sim -\rho$. Then we assume

$$R = R_0 + R_1 \ , \quad (|R_1| \ll |R_0|) \ . \tag{42}$$

Now one can get

$$0 = \Box R_0 + \frac{F^{(3)}(R_0)}{F^{(2)}(R_0)}\nabla_\rho R_0 \nabla^\rho R_0 + \frac{F'(R_0)R_0}{3F^{(2)}(R_0)} - \frac{2F(R_0)}{3F^{(2)}(R_0)} - \frac{R_0}{3F^{(2)}(R_0)}$$

$$+ \Box R_1 + 2\frac{F^{(3)}(R_0)}{F^{(2)}(R_0)}\nabla_\rho R_0 \nabla^\rho R_1 + U(R_0)R_1 \ ,$$

$$U(R_0) \equiv \left(\frac{F^{(4)}(R_0)}{F^{(2)}(R_0)} - \frac{F^{(3)}(R_0)^2}{F^{(2)}(R_0)^2}\right)\nabla_\rho R_0 \nabla^\rho R_0 + \frac{R_0}{3}$$

$$- \frac{F^{(1)}(R_0)F^{(3)}(R_0)R_0}{3F^{(2)}(R_0)^2} - \frac{F^{(1)}(R_0)}{3F^{(2)}(R_0)} + \frac{2F(R_0)F^{(3)}(R_0)}{3F^{(2)}(R_0)^2} - \frac{F^{(3)}(R_0)R_0}{3F^{(2)}(R_0)^2} \ . \tag{43}$$

If $U(R_0)$ is positive, since $\Box R_1 \sim -\partial_t^2 R_1$, the perturbation R_1 is exponentially large and the system becomes unstable. One may regard $\nabla_\rho R_0 \sim 0$ if it is assumed the matter is almost uniform as inside the earth.

For the model (16), by assuming $R_0/m^2 \gg 1$, it follows

$$U(R_0) \sim -\frac{m^2 c_2^2}{3c_1 n(n+1)}\left(R_0/m^2\right)^{n+2} \ , \tag{44}$$

which is large and negative if $c_1 > 0$. Hence, there is no instability in the sense of ref.[10]. When $c_1 < 0$, however, there could be an instability. In first ref. of [13], a simple condition for the stability in a sense of [10] was given, that is, theory is stable if $F''(R_0) = f''(R_0) > 0$ but unstable if $F''(R_0) = f''(R_0) < 0$. Now we have

$$F''(R_0) \sim \frac{n(n+1)m^2 c_1}{c_2^2}\left(\frac{R_0}{m^2}\right)^{-n-2} \ . \tag{45}$$

Then $F''(R_0) \sim -1/U(R_0) > 0$ if c_1 is positive and theory is not stable.

As one more example satisfing the conditions (17), we now consider

$$f_A(R) = -\frac{m^2 c_1}{c_2}\left(1 - \mathrm{e}^{-\frac{c_2\left(R/m^2\right)^n}{c_2\left(R/m^2\right)^n+1}}\right) \ , \tag{46}$$

The asymptotic behaviors of (46) are identical with the model (16) when R is large,

$$f_A(R) \sim f_{HS}(R) \to -\frac{m^2 c_1}{c_2} \ , \tag{47}$$

and when R is small

$$f_A(R) \sim f_{HS}(R) \to -m^2 c_1\left(R/m^2\right)^n \ . \tag{48}$$

Then asymptotic behaviors of the universe does not change and the correction to the Newton law could be large when R is large and small when R is small. The instability is also absent, as one can reobtain the results identical with (32-44).

Another example is

$$f_B(R) = -f_0 \mathrm{e}^{-\frac{\tilde{m}^4}{R^2}} \ , \tag{49}$$

with a positive constants f_0 and m^4. As in the model (16) in [11], we may assume $R/m^2 \gg 1$ from the early universe to the present universe. Even in the model (47), we assume $R^2 \gg \tilde{m}^4$. Then by expanding $f_B(R)$ with respect to m^4/R^2, we find

$$f_B(R) \sim -f_0 + f_0\frac{\tilde{m}^4}{R^2} \ . \tag{50}$$

By comparing (50) with (18), we may identify

$$n \leftrightarrow 2 , \quad f_0 \leftrightarrow \frac{m^2 c_1}{c_2} , \quad f_0 \tilde{m}^4 \leftrightarrow \frac{m^6 c_1}{c_2^2} . \tag{51}$$

Hence, f_0 plays the role of the cosmological constant if $f_0 > 0$

$$H^2 \sim f_0 \sim (70 \text{km/s} \cdot \text{pc})^2 \sim \left(10^{-33} \text{eV}\right)^2 . \tag{52}$$

Thus, the accelerated expansion of the present universe could be generated by the effective cosmological constant f_0. As in (20), in the earlier but not primordial universe, the matter density ρ is larger than the effective cosmological constant f_0. Hence, there occurs the matter dominated phase and the universe could have expanded with deceleration. The aymptotic behavior when the curvature is large is identical with the model (16), the correction to the Newton law could be not so small.

We now investigate also the case that the curvature is small. Then for the model (47), we obtain

$$V(\sigma) = \left(-\frac{A^4}{2 f_0 \tilde{m}^4} + \frac{A^6}{4 f_0 \tilde{m}^8}\right) e^{-\frac{\tilde{m}^4}{A^2}} , \quad \sigma \sim \frac{2 f_0 \tilde{m}^4}{A^3} e^{-\frac{\tilde{m}^4}{A^2}} . \tag{53}$$

and

$$\frac{d^2 V(\sigma)}{d\sigma^2} = -\left(\frac{d\sigma}{dA}\right)^{-3} \frac{d^2\sigma}{dA^2} \frac{dV}{dA} + \left(\frac{d\sigma}{dA}\right)^{-2} \frac{d^2 V}{dA^2} \sim \frac{A^{10}}{416 \tilde{m}^{12} f_0^3} e^{m^4/A^2} . \tag{54}$$

If f_0 is positive, $m_\sigma \equiv (1/2)(d^2 V/d\sigma^2)$ is positive and large when the curvature $R = A$ is small and therefore there is no large corrrection to the Newton law. We should note, however, if f_0 is negative, which corresponds to the model in (16) m_σ^2 becomes negative and there could occur an instability. On the other hand, when the curvature is large, $U(R_0)$ in (43) has the following form:

$$U(R_0) \sim -\frac{R_0^4}{18 f_0 \tilde{m}^4} R_0 , \tag{55}$$

which is negative and large and therefore there is no instability. In fact, since

$$f_B''(R_0) \sim \frac{6 f_0 \tilde{m}^4}{R_0^4} > 0 , \tag{56}$$

the condition from first ref. of [13] is satisfied.

Recently another interesting $f(R)$ model was proposed in [12], where

$$F_{AB}(R) = R + f_{AB}(R) = \frac{R}{2} + \frac{1}{2a} \ln \left[\cosh(aR) - \tanh(b) \sinh(aR)\right] , \tag{57}$$

with positive constants a and b (for first $f(R)$ models with log-terms, see first ref. in [6]).

Since the correction to the Newton law has been studied in [12], we now investigate the possible instability for the model (57). Since

$$F''(R) = 2a \frac{(1 - \tanh(b))}{(1 + \tanh(b))} e^{-2aR} , \tag{58}$$

it is positive. Then the condition [13] seems to be satisfied and therefore theory seems to be consistent.

When the curvature R is large, one finds

$$F_{AB}(R) \sim R + \frac{1}{2a} \ln \frac{1 - \tanh(b)}{2} + \frac{(1 + \tanh(b)) e^{-2aR}}{2a (1 - \tanh(b))} + \mathcal{O}\left(e^{-4aR}\right) . \tag{59}$$

Then $U(R_0)$ (43) has the following form:

$$U(R_0) \sim -\frac{e^{2aR_0} (1 - \tanh(b))}{6a (1 + \tanh(b))} \left(1 + 2 \ln \frac{1 - \tanh(b)}{2}\right) . \tag{60}$$

If $1 + 2 \ln \left((1 - \tanh(b))/2\right) > 0$, $U(R_0)$ is very large and negative and therefore there is no instability. In [12], b is choosen to be $b \gtrsim 1.2$, so

$$1 + 2 \ln \frac{1 - \tanh(1.2)}{2} = -3.97 < 0 , \tag{61}$$

and therefore the matter instability seems to occur. This indicates that such model should be considered in the other range of parameters.

IV. DISCUSSION

In the present letter we considered some solar system tests for several modified gravities which satisfy to conditions (16). These theories show very realistic cosmological behaviour and may easily lead to ΛCDM cosmology. It is shown that the theory (15) passes known solar system tests as well as cosmological bounds. Signficant Newton law corrections appear only beyond the solar system scales as well as for specific values of curvature power which puts some bound for such theory. Theory (59) which has an acceptable Newton law regime shows the matter instability in the proposed range of the parameters. Thus, the suggested class of models seems to be very realistic and looks like the alternative for ΛCDM. More accurate and detailed check of cosmological bounds for such theories should be done but in any case it is expected that some (combination/extension) of such theories may fit with observable cosmological data.

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