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Theory of nonlinear particle acceleration at shocks and self-generation of the magnetic field

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Abstract: We present some recent developments in the theory of particle acceleration at shock fronts in the presence of dynamical reaction of the accelerated particles and self-generation of magnetic field due to streaming instability. The spectra of accelerated particles, the velocity, magnetic field and temperature profiles can be calculated in this approach anywhere in the precursor and in the downstream region. The implications for the origin of cosmic rays and for the phenomenology of supernova remnants will be discussed.

Introduction

The paradigm of supernova remnants as sources of the galactic component of cosmic rays heavily relies on the fact that the mechanism of particle acceleration at collisionless shocks should take place in the nonlinear regime. The nonlinearity manifests itself in at least two respects: 1) the efficiency of acceleration necessary to explain the origin of cosmic rays needs to be large enough that the dynamical reaction of the accelerated particles is not negligible; 2) the accelerated particles are responsible for the self-generation of the magnetic disturbances which in turn scatter the particles thereby allowing their acceleration to the maximum energies observed in cosmic ray data, most notably the KASCADE data in the knee region [1].

A special emphasis should be given to the fact that protons may be energized to energies of $10^5 - 10^6$ GeV only if the magnetic field in the shock vicinity is amplified by a factor few hundreds and reorganized in the form of a flat power spectrum which may lead to Bohm diffusion. Such amplification may take place through streaming instability induced by the super-Alfvenic drift of accelerated particles in the frame of the plasma upstream of the shock [2, 3, 4]. Magnetic field amplification may also take place due to firehose instability [5].

A comprehensive theory of particle acceleration in the nonlinear regime, with both dynamical reaction of the accelerated particles and magnetic amplification taken into account has been recently formulated in [6, 7]. In [8] the calculation of the maximum momentum for the nonlinear regime was presented.

Here we illustrate the basic formalism and the phenomenological implications of the theoretical approach of [7].

The formalism

The two basic equations needed in this section are the equation of conservation of momentum and the transport equation for the accelerated particles. In the upstream plasma, conservation of momentum reads:

$$\xi_c(x) = 1+\frac{1}{\gamma_g M_0^2}-U(x)-\frac{1}{\gamma_g M_0^2}U(x)^{-\gamma_g}, \quad (1)$$

where $\xi_c(x) = P_{CR}(x)/\rho_0 u_0^2$ and $U(x) = u(x)/u_0$ and we used conservation of mass $\rho_0 u_0 = \rho(x)u(x)$ (here ρ_0 and u_0 refer to the density and plasma velocity at upstream infinity, while $\rho(x)$ and $u(x)$ are the density and velocity at the location x upstream. M_0 is the sonic Mach number at upstream infinity). The pressure in the form of accelerated particles is defined as

$$P_{CR}(x) = \frac{1}{3}\int_{p_{inj}}^{p_{max}} dp\, 4\pi p^3 v(p) f(x,p), \quad (2)$$

and $f(x,p)$ is the distribution function of accelerated particles. Here p_{inj} and p_{max} are the injection and maximum momentum. The function f vanishes at upstream infinity. The distribution function satisfies the following transport equation in the reference frame of the shock:

$$\frac{\partial}{\partial x}\left[D(x,p)\frac{\partial}{\partial x}f(x,p)\right] - u\frac{\partial f(x,p)}{\partial x} + \frac{1}{3}\left(\frac{du}{dx}\right) p \frac{\partial f(x,p)}{\partial p} + Q(x,p) = 0.$$

[6] and [9] showed that an excellent approximation to the solution $f(x,p)$ has the form

$$f(x,p) = f_0(p)\exp\left[-\frac{q(p)}{3}\int_x^0 dx' \frac{u(x')}{D(x',p)}\right], \quad (3)$$

where $f_0(p) = f(x=0,p)$ is the cosmic rays' distribution function at the shock and $q(p) = -\frac{d\ln f_0(p)}{d\ln p}$ is its local slope in momentum space.

The function $f_0(p)$ can be written in a very general way as found by [10]:

$$f_0(p) = \left(\frac{3R_{tot}}{R_{tot}U_p(p)-1}\right)\frac{\eta n_0}{4\pi p_{inj}^3} \times \exp\left\{-\int_{p_{inj}}^{p}\frac{dp'}{p'}\frac{3R_{tot}U_p(p')}{R_{tot}U_p(p')-1}\right\}.$$

Here we introduced the function $U_p(p) = u_p/u_0$, with

$$u_p = u_1 - \frac{1}{f_0(p)}\int_{-\infty}^{0} dx(du/dx)f(x,p)\ , \quad (4)$$

where u_1 is the fluid velocity immediately upstream (at $x = 0^-$). We used $Q(x,p) = \frac{\eta n_{gas,1}u_1}{4\pi p_{inj}^2}\delta(p-p_{inj})\delta(x)$, with $n_{gas,1} = n_0 R_{tot}/R_{sub}$ the gas density immediately upstream ($x = 0^-$) and η the fraction of the particles crossing the shock which are going to take part in the acceleration process. In the expressions above we also introduced the compression factor at the subshock $R_{sub} = u_1/u_2$ and the total compression factor $R_{tot} = u_0/u_2$. If the upstream plasma only evolves adiabatically, the two compression factors are related through the following expression ([10]):

$$R_{tot} = M_0^{\frac{2}{\gamma_g+1}}\left[\frac{(\gamma_g+1)R_{sub}^{\gamma_g} - (\gamma_g-1)R_{sub}^{\gamma_g+1}}{2}\right]^{\frac{1}{\gamma_g+1}}, \quad (5)$$

where M_0 is the Mach number of the fluid at upstream infinity and γ_g is the ratio of specific heats for the fluid. The parameter η in Eq. 4 contains the very important information about the injection of particles from the thermal pool. The injection is modelled as proposed in [11]:

$$\eta = \frac{4}{3\pi^{1/2}}(R_{sub}-1)\xi^3 e^{-\xi^2}. \quad (6)$$

Here ξ is a parameter that identifies the injection momentum as a multiple of the momentum of the thermal particles in the downstream section ($p_{inj} = \xi p_{th,2}$). The latter is calculated self-consistently from the Rankine-Hugoniot relations at the subshock. For the numerical calculations that follow we always use $\xi = 3.5$, that corresponds to a fraction of order 10^{-4} of the particles crossing the shock to be injected in the accelerator.

The scattering properties of the background plasma are described by the scalar function $D(p)$, the diffusion coefficient. Once $\mathcal{F}(x,k)$ is known, the diffusion coefficient is known in turn ([2]):

$$D(x,p) = \frac{4}{\pi}\ \frac{r_L\ v}{3\ \mathcal{F}}\ . \quad (7)$$

From the latter equation, where r_L stands for the Larmor radius of particles of momentum p, it is clear that the diffusion coefficient tends to Bohm's expression for $\mathcal{F} \to 1$. The expected saturation level for the overall energy density of the perturbed magnetic field can be easily evaluated from the fact that

$$\frac{B_0^2}{8\pi}\int\frac{dk}{k}\sigma\ \mathcal{F}(k,x) = v_A\frac{dP_{CR}}{dx}. \quad (8)$$

Integration of this equation is straightforward when non-linear effects on the fluid are neglected so that u and v_A are both spatially constant. One obtains $\delta B^2/8\pi = (v_A/u)P_{CR}$, or, in terms of amplification of the ambient magnetic field:

$$\left(\frac{\delta B}{B_0}\right)^2 = 2\ M_A\ \frac{P_{CR}}{\rho_0 u_0^2}\ , \quad (9)$$

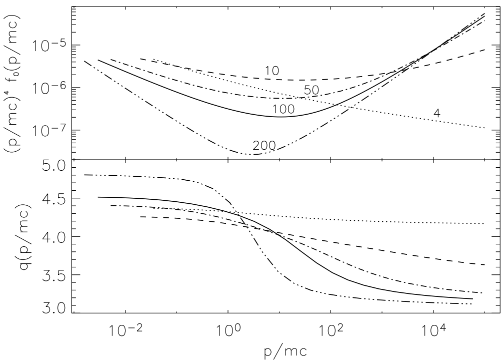

Figure 1: Spectrum and slope at the shock location as functions of energy for $p_{max} = 10^5 \mathrm{mc}$ and magnetic field at upstream infinity $B_0 = 1\mu G$. The curves refer to Mach numbers at upstream infinity ranging from $M_0 = 4$ to $M_0 = 200$: dotted for $M_0 = 4$, dashed for $M_0 = 10$, dot-dashed for $M_0 = 50$, solid for $M_0 = 100$ and dot-dot-dashed for $M_0 = 200$.

with $M_A = u_0/v_A$ the Alfvénic Mach number.

It is worth stressing that for $P_{CR}/\rho_0 u_0^2 \sim 1$ and $M_A \gg 1$, the predicted amplification of the magnetic field exceeds unity. CLearly this implies that the quasi-linear theory used here loses validity and that a more accurate description, though very difficult to achieve should be sought.

The set of equations of conservation of momentum and transport equation with a diffusion coefficient determined as described above can be solved by using the iterative procedure described in detail in [6, 7].

Results

The spectra of the accelerated particles for Mach numbers at upstream infinity ranging from $M_0 = 4$ to $M_0 = 200$ are shown in Fig. 1 for a background magnetic field at upstream infinity $B_0 = 1\mu G$ (the result is however independent of the strength of the background magnetic field). In the bottom part of the same figure we plot the slope of the spectrum as a function of momentum.

For low Mach numbers and at given p_{max} the modification of the shock due to the reaction of the accelerated particles is small (see for instance the case $M_0 = 4$). For the strongly modified case (e.g. $M_0 = 200$) the asymptotic spectrum of the accelerated particles is very flat, tending to $p^{-\alpha}$ with $\alpha = 3.1 - 3.2$ for $p \to p_{max}$.

We need to comment on the issue of the shape of the spectrum of cosmic rays accelerated in supernova remnants: the spectra illustrated in Fig. 1 are the spectra in proximity of the shock surface and therefore the ones which are relevant for the calculation of the spectra of secondary radiation produced by the accelerated particles. In general the spectra that are observed by an observer far upstream are more complex to determine: at each time during the supernova evolution particles at the maximum momentum can escape to upstream infinity (this is a peculiar aspect of non-linear theory) carrying away a sizeable fraction of the total energy (due to the flat spectra). At each time the instantaneous spectrum that escapes to upstream infinity is a narrow function centered around $p_{max}(t)$, where t is the age of the remnant. If the maximum momentum decreases with time (in the Sedov phase this is the case) then a spectrum is built due to the overlap of many delta-function-like spectra leaving the acceleration region from upstream. In the simple model considered in [12] this overlap leads to a power law spectrum p^{-4}, despite the fact that the spectrum at the shock may be concave as illustrated in Fig. 1. In addition to these particles that leave the system from upstream, the distant observer will also measure the cosmic ray spectrum which is kept in the supernova shell and is eventually liberated at later times, possibly suffering adiabatic energy losses. The spectrum observed at the Earth is likely to be a complex superposition of the particles escaping from upstream, those leaving the system at the end of the supernova evolution, and summed over all supernova events occurred during a confinement time of cosmic rays in the Galaxy.

The diffusion coefficient associated with the self-generated waves is given by Eq. 7. We plot this diffusion coefficient at the shock location in Fig. 2 for Mach numbers $M_0 = 10$ (dashed lines) and $M_0 = 100$ (solid lines). For comparison, we also plot the corresponding Bohm diffusion coefficient $D_B(p) \propto v(p)p$ in the unperturbed magnetic field B_0, for $B_0 = 1\mu G$ and $B_0 = 10\mu G$.

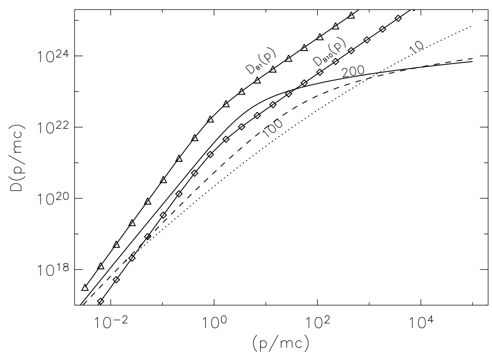

Figure 2: The self-generated diffusion coefficient at the shock location $x = 0^-$ as a function of the particle momentum for Mach numbers $M_0 = 10$ (dotted line), $M_0 = 100$ (dashed line) and $M_0 = 200$ (solid line). Also plotted is the Bohm diffusion coefficient corresponding to $B_0 = 1\mu G$ (solid line with triangles) and $B_0 = 10\mu G$ (solid line with diamonds). The y-axis is in units of $\mathrm{cm}^2\mathrm{s}^{-1}$.

As stressed above, in the regime we considered, the fluctuations in the magnetic field become strongly non linear. The dynamical role of the amplified field remains however negligible as the highest values of $\delta B^2/8\pi\rho_0 u_0^2$, reached close to the shock front, are of the order of $10^{-2} - 10^{-3}$.

Conclusions

We developed a mathematical formalism that allows us to calculate the spectrum of particles accelerated at a collisionless non-relativistic shock taking into acount both the nonlinear dynamical reaction of the accelerated particles and the magnetic field amplification by streaming instability. The approach has also recently been generalized to allow us to determine self-consistently the maximum momentum reached by the particles in this fully nonlinear acceleration regime [8].

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