

Disappearing cosmological constant in $f(R)$ gravity

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For higher-derivative $f(R)$ gravity where R is the Ricci scalar, a class of models is proposed which produce viable cosmology different from the LambdaCDM one at recent times and satisfy cosmological, Solar system and laboratory tests. These models have both flat and de Sitter space-times as particular solutions in the absence of matter. Thus, a cosmological constant is zero in flat space-time, but appears effectively in a curved one for sufficiently large R . A 'smoking gun' for these models would be small discrepancy in values of the slope of the primordial perturbation power spectrum determined from galaxy surveys and CMB fluctuations. On the other hand, a new problem for dark energy models based on $f(R)$ gravity is pointed which is connected with possible overproduction of new massive scalar particles (scalarons) arising in this theory in the very early Universe.

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I. INTRODUCTION

Continuing investigation of dark energy (DE) properties in the Universe (see the recent review [1] for the definitions of what is usually called the effective DE energy density ρ_{DE} and pressure p_{DE} from the observational point of view) has shown that its properties are very close to those of an exact cosmological constant Λ that has $\rho_\Lambda = -p_\Lambda = \Lambda/8\pi G = \text{const} > 0$.¹ In particular, if $w_{DE} \equiv p_{DE}/\rho_{DE}$ is assumed to be constant, then $|w_{DE} + 1| < 0.1$ (1σ error bars) or even smaller, see [3] for the analysis of the most recent observational data using different techniques.² However, for more generic DE models with $w_{DE} \neq \text{const}$, the same analysis does not exclude varying w_{DE} including even temporal phantom behaviour of DE ($w_{DE} < -1$) at recent redshifts $z < 0.3$. The latter behaviour (in other words, breaking of the weak energy condition for DE), if confirmed by future, more exact data, may not be explained in the scope of physical DE models according to the terminology of [1] (e.g., the quintessence ones) and requires some kind of geometrical DE (otherwise dubbed modified gravity), see also recent reviews [4].

Among geometrical DE models one of the most simplest ones is the $f(R)$ class of gravity models with the Lagrangian density

$$L = \frac{f(R)}{16\pi G} + L_m, \quad (1)$$

where L_m describes all non-gravitational kinds of matter including non-relativistic (cold) dark matter and the metric variation is assumed.³ $f(R)$ may be an arbitrary function subjected, however, to some stability conditions discussed below. For $f(R) = R - 2\Lambda$, it reduces to the Einstein gravity with a cosmological constant. Thus, it contains the standard cosmological Λ CDM model as a particular case. However for $f''(R) \neq 0$, in addition to the massless spin-2 graviton, this class of models contains a scalar particle, dubbed scalaron in [5], which rest-mass is $M^2(R) = (3f''(R))^{-1}$ in the WKB-regime $|M^2| \gg R^2$, $R_{\mu\nu}R^{\mu\nu}$ (here and below prime means differentiation with respect to an argument). It is neither a tachyon, not a ghost for $f''(R) > 0$ in this regime. Finally, graviton is not a ghost if $f'(R) > 0$. All these properties can be easily obtained either directly, or (in the absence of L_m) using conformal equivalence of equations of motions for this class of models to those of the Einstein gravity interacting with a minimally coupled scalar field ϕ

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¹ The sign conventions are: the metric signature $(+ - - -)$, the curvature tensor $R^\sigma_{\mu\rho\nu} = \partial_\nu \Gamma^\sigma_{\mu\rho} - \dots$, $R_{\mu\nu} = R^\sigma_{\mu\sigma\nu}$, so that the Ricci scalar $R \equiv R^\mu_{\mu} > 0$ for the de Sitter space-time and the matter-dominated cosmological epoch; $c = \hbar = 1$ is assumed throughout the paper.

² Note that the assumption $w_{DE} = \text{const} < 0$ and not equal to $-1, -2/3, -1/3$ is not very natural in the presence of non-relativistic matter since it requires DE models with rather specific potentials, see e.g. [2].

³ The Palatini variation of (1) leads to completely different equations of motion, even the number of degrees of freedom (i.e. particle content) is not the same. Thus, two models with the same Lagrangian density (1) but different ways of variation should be studied and compared with data as two separate models; their similarity is illusory.

with some potential $V(\phi)$ which form is uniquely determined by $f(R)$ in all points where $f'(R) \neq 0$ [6]. However, I shall not use this conformal equivalence below, in particular, because it may be misleading in the presence of other kinds of matter.

In this paper Eq. (1) is considered as a purely phenomenological effective Lagrangian density describing geometrical DE. However, in principle, it may arise either due to quantum fluctuations of all fields including gravity (as was supposed in early papers like [5]), or as a result of reduction from higher dimensions to 4D in some variant of modern string/M-theory (see e.g. [7] in this respect).

First papers on cosmological models in $f(R)$ gravity appeared already in 1969-1970 [8]. Then, among other results, this class of models with $f(R) = R + R^2/6M^2$ plus some small non-local terms (which are crucial for reheating after inflation) was used to construct the first internally self-consistent cosmological model possessing a (quasi-)de Sitter (latter dubbed inflationary) stage in the early Universe with slow-roll decay, a graceful exit to the subsequent radiation-dominated Friedmann-Robertson-Walker (FRW) stage (through an intermediate matter-dominated one) and sufficiently effective reheating in the regime of narrow parametric resonance [5] (see [9] for more details). It is for this model that calculations of scalar (adiabatic) perturbations generated during inflation were first done [10]. Moreover, with all recent observational data taken into account, it still remains among viable cosmological models: as follows from the final results for both scalar and tensor perturbations [11] (see also [12, 13, 14]), the model predictions for the slope of the primordial spectrum of scalar perturbations n_s and the tensor/scalar ratio r are $n_s - 1 = -2N^{-1} = -0.04 (N/50)^{-1}$, $r = 12N^{-2} = 0.0048 (N/50)^{-2}$ where N is the number of e-folds between the first Hubble radius crossing of the present inverse comoving scale 0.05 Mpc^{-1} and the end of inflation – well in agreement with the present data (of course, in the case of r we have an upper limit only). The only free parameter of this model – the scalaron rest-mass M – is determined from the normalization of the primordial scalar spectrum. If we take it from the best fit to the combined WMAP3-SDSS measurements [15], then $M = 2.8 \times 10^{-6} (N/50)^{-1} M_{Pl}$ where $M_{Pl} = 1/\sqrt{G}$ (by the way, this lies inside the range conjectured in [11]).

Due to remarkable *qualitative* similarity between the present DE and primordial DE that supported inflation in the early Universe, all inflationary models may be applied to the description of the present DE, too, after changing numerical values of their microscopical parameters only.⁴ Actually, it had been done already for practically all models (sometimes in the inverse historical order). The same occurred to models based on $f(R)$ gravity beginning from [16], and then it was proposed in [17] to use $f(R)$ models with $f(R)$ diverging (or finite but non-analytic) at $R \rightarrow 0$ for description of the present DE. However, after much agitation on this particular class of models, it was proven that they are either non-viable, or practically indistinguishable from the standard Λ CDM model, see [14, 18] for such rather pessimistic conclusions as well as for extensive lists of publications on this topic.

This does not completely close the way to construct a viable DE model in $f(R)$ gravity observationally distinguishable from the Λ CDM model, but suggests to abandon the hypothesis of divergence of $f(R)$ at $R = 0$, as well at any other value of R , and to return to the natural assumption that $f(R)$ is regular in this point. Moreover, an interesting and intriguing possibility is $f(0) = 0$ while $f \rightarrow R - 2\Lambda$ for $R \gg \Lambda$. This behaviour corresponds to an effective cosmological constant existing in a sufficiently curved space-time but 'disappearing' in the flat one – that explains the title of the paper. In other words, in such a model the observed DE (close to the cosmological constant for sufficiently large R) is a purely curvature induced effect. It is totally unrelated to quantum vacuum energy in flat space-time that should be zero due to some other symmetry.⁵ However, the price to pay is that flat space-time becomes unstable with a characteristic time of the order of the present Universe age.

Of course, it is much more difficult to construct a viable DE model in $f(R)$ gravity as compared to general scalar-tensor gravity since the former contains only one arbitrary function while the latter has two functions and provides much place for viable DE models, see e.g. [20] for reconstruction of such models from different kinds of observational data and [21] for models which admit recent phantom behaviour of DE. Another source of problems is that it is rather non-trivial to satisfy laboratory and Solar system tests in $f(R)$ gravity since it formally represents the limiting case $\omega = 0$ of scalar-tensor gravity where ω is the Brans-Dicke parameter [22], though sometimes this limit should be taken carefully (see [23] for recent reconsideration). However, this may be considered even as an advantage of DE $f(R)$ models since it makes easier to falsify them. So, in the next section a trial 3-parametric form of $f(R)$ is introduced which realizes the 'disappearing cosmological constant' possibility and behaviour of its FRW solutions is investigated. In Sec. 3 laboratory and Solar system tests, as well as dynamics of small perturbations are considered. Sec. 4 contains conclusions and discussion of problems and further tests of this model.

⁴ This does not mean that the present and primordial DE should necessarily be the same kind of matter, like in the so called quintessential inflationary models. I am only speaking about possibility to use the same kind of theoretical models in both cases.

⁵ So, this model easily realizes a possibility of 'degravitation of the cosmological constant' in the scalar sector which was recently proposed and tried to be achieved in the tensor sector in [19] but faced with very serious technical problems due to non-locality of equations.

II. GEOMETRICAL DARK ENERGY MODEL AND BEHAVIOUR OF ITS FRW SOLUTIONS

Field equations following from (1) can be written in the following Einsteinian form (though gravity itself is not the Einstein one):

$$R_{\mu}^{\nu} - \frac{1}{2}\delta_{\mu}^{\nu}R = -8\pi G \left(T_{\mu(m)}^{\nu} + T_{\mu(DE)}^{\nu} \right) \quad (2)$$

where

$$8\pi GT_{\mu(DE)}^{\nu} \equiv F'(R)R_{\mu}^{\nu} - \frac{1}{2}F(R)\delta_{\mu}^{\nu} + (\nabla_{\mu}\nabla^{\nu} - \delta_{\mu}^{\nu}\nabla_{\rho}\nabla^{\rho})F'(R), \quad F(R) \equiv f(R) - R \quad (3)$$

and $T_{\mu(m)}^{\nu}$ follows from variation of L_m and satisfies the generalized conservation law $T_{\mu;\nu(m)}^{\nu} = 0$ separately (since the left-hand side of Eq. (2) and the right-hand side of Eq. (3) satisfy this condition, too). There exists a subtlety in this representation that is discussed below. The trace of Eq. (2) reads

$$3\nabla_{\mu}\nabla^{\mu}f' - Rf' + 2f = 8\pi GT_m. \quad (4)$$

Constant curvature solutions (de Sitter ones for $R > 0$) are roots of the algebraic equation $Rf' = 2f$.

Let us take $f(R)$ in the following 3-parametric form:

$$f(R) = R + \lambda R_0 \left(\left(1 + \frac{R^2}{R_0^2} \right)^{-n} - 1 \right) \quad (5)$$

with $n, \lambda > 0$ and R_0 of the order of the presently observed effective cosmological constant. Then $f(0) = 0$ (the cosmological constant 'disappears' in flat space-time) and $R_{\mu}^{\nu} = 0$ is always a solution of Eq. (2) in the absence of matter, but $f''(0)$ is negative – flat space-time is unstable. For $|R| \gg R_0$, $f(R) = R - 2\Lambda(\infty)$ where the high-curvature value of the effective cosmological constant is $\Lambda(\infty) = \lambda R_0/2$. The equation for de Sitter solutions having $R = const = R_1 = x_1 R_0$, $x_1 > 0$ can be written in the form

$$\lambda = \frac{x_1(1+x_1^2)^{n+1}}{2((1+x_1^2)^{n+1} - 1 - (n+1)x_1^2)}. \quad (6)$$

Below, by x_1 I will mean the *maximal* root of Eq. (6). So, instead of specifying λ , one may take any value of x_1 and then determine the corresponding value of λ . It follows from the structure of Eq. (6) that $x_1 < 2\lambda$. Thus, the effective cosmological constant at the de Sitter solution $\Lambda(R_1) = R_1/4 < \Lambda(\infty)$. On the other hand, $x_1 \rightarrow 2\lambda$ in both limiting cases x_1 fixed, $n \gg 1$ and $x_1 \gg 1$, n fixed. In these cases the Universe evolution becomes indistinguishable from that in the Λ CDM model.

Let us now consider the stability conditions

$$f'(R) > 0, \quad f''(R) > 0, \quad R \geq R_1. \quad (7)$$

Note that they are imposed not in the whole space of solutions but only on a trajectory of the evolution of our Universe from very large and positive R in the past to $R = R_1$ in the infinite future. The quantum meaning of these conditions (graviton is not a ghost, scalaron is not a tachyon) has already been mentioned in Sec. 1. However, violation of these conditions in the course of purely classical evolution is undesirable, too. If $f'(R) = 0$ for some finite $R > R_1$, a universe generically becomes strongly anisotropic and inhomogeneous at some finite moment of time [24, 25] (the same happens to the Einstein gravity + a non-minimally coupled scalar field, preventing G_{eff} from changing sign [26]). In the point where $f''(R) = 0$, some weak singularity occurs which will be considered elsewhere. In terms of the conformal equivalence mentioned above, $dR/d\phi$ diverges at this point.

It can be shown that it is sufficient to satisfy the conditions (7) for $R = R_1$ and then they will be valid over the whole interval $[R_1, \infty)$. Correspondingly, this gives two necessary conditions for parameters of the form (5):

$$(1+x_1^2)^{n+1} > 1 + (2n+1)x_1^2, \quad x_1^2 > 1/(2n+1). \quad (8)$$

To these inequalities, the condition of the stability of the future de Sitter stage has to be added. It follows from variation of Eq. (4) and reads (since the condition $f''(R_1) > 0$ is already assumed to be satisfied) [27]:

$$f'(R_1) > R_1 f''(R_1). \quad (9)$$

This condition is stronger than the first of inequalities (7) at $R = R_1$, so it substitutes it. For our model it produces the requirement:

$$(1 + x_1^2)^{n+2} > 1 + (n + 2)x_1^2 + (n + 1)(2n + 1)x_1^4 . \quad (10)$$

It can be proven that it implies the second of inequalities (8), too. So, it is sufficient to check this inequality only. In particular, for $n = 1$ it reads $x_1 > \sqrt{3}$ that leads to $\lambda > 8/3\sqrt{3}$. In addition, the value of x_1 saturating (10) is also the point where $\lambda(x_1)$ in Eq. (6) reaches minimum.

Now turn to the model evolution at the matter dominated stage in the regime $R \gg R_0$. By construction, the model (5) satisfies the conditions

$$|F| \ll R, \quad |F'(R)| \ll 1, \quad R|F''(R)| \ll 1 \quad (11)$$

for $R \gg R_0$. Due to this, it is possible to solve Eq. (4) iteratively in this regime and corrections to the standard Einsteinian (actually, even Newtonian) behaviour $R = R^{(0)} = 8\pi GT_m \propto a^{-3}$ appear to be small ($a(t)$ is a FRW scale factor). Thus, the model does not possess the Dolgov-Kawasaki instability [28]. These corrections are of two types – matter induced ones and free scalaron oscillations. The former ones follow from direct iteration of Eq. (4):

$$R = R^{(0)} + \delta R_{ind} + \delta R_{osc}, \quad \delta R_{ind} = (RF'(R) - 2F(R) - 3\nabla_\mu \nabla^\mu F'(R))_{R=R^{(0)}} \quad (12)$$

where ∇_μ is taken with respect to the unperturbed metric, too. For $R \gg R_0$, $\delta R_{ind} \approx const = -2F(\infty) = 2\lambda R_0 = 4\Lambda(\infty)$ and $8\pi GT_{\mu(DE)}^\nu \approx \Lambda(\infty)\delta_\mu^\nu$. Thus, DE behaves as a positive cosmological constant at redshifts $z \gg 1$.

Let us now return to the subtle point in the definition of the DE energy-momentum tensor (3) pointed above. It consists in the following: what is the constant G in it? For the model involved, G coincides with $G_{eff}(\infty)$ – the value of the Newtonian gravitational constant measured in a Cavendish-type experiment made in an environment with space-time curvature $R \gg R_0$. In the next section, we will discuss under what restriction on n this is achieved in laboratory experiments already made. Moreover, it coincides with G_{eff} at the radiation-dominated stage in the Universe during the period of the Big Bang nucleosynthesis (BBN) $t \sim (1 - 100)$ s.⁶ So, BBN predictions remain unchanged in this DE model, too. Thus, such a choice of G in the left-hand side of (3) is very natural for this model. Taking another constant, say $G(df/dR)_{now}^{-1}$ where the subscript 'now' means the present moment, results in adding a 'tracking' component to DE which is proportional to the Einstein tensor and may have any sign. That is why with our definition (3), there is no effect of ρ_{DE} becoming negative at sufficiently large redshifts (with w_{DE} diverging at the moment when $\rho_{DE} = 0$) like that proposed in the recent paper [29].

More unexpected is the behaviour of the second small correction δR_{osc} . It satisfies the equation

$$\frac{3}{a^3} \frac{d}{dt} \left(a^3 \frac{d}{dt} \left(F''(R^{(0)}) \delta R_{osc} \right) \right) + \delta R_{osc} = 0 . \quad (13)$$

For $R \gg R_0$, the WKB-approximation may be used. Then the solution is

$$\delta R_{osc} = C a^{-3/2} (F''(R^{(0)}))^{-3/4} \sin \left(\int \frac{dt}{\sqrt{3F''(R^{(0)})}} \right) \quad (14)$$

with $C = const$. The integral here is just $\int M(R^{(0)}) dt$ where M is the scalaron mass introduced in Sec. 1. At the matter-dominated regime:

$$a \propto t^{2/3}, \quad R^{(0)} = \frac{4}{3t^2}, \quad F'' \propto t^{4n+4}, \quad M(R) \propto t^{-2n-2}, \quad (15)$$

$$\delta R_{osc} \propto t^{-3n-4} \sin(const \cdot t^{-2n-1}), \quad \frac{\delta a}{a} \propto t^n \sin(const \cdot t^{-2n-1}). \quad (16)$$

Similar behaviour continues during the radiation-dominated stage:

$$R^{(0)} \propto t^{-3/2}, \quad M(R) \propto t^{-3(n+1)/2}, \quad \delta R_{osc} \propto t^{-\frac{9n}{4}-3} \sin(const \cdot t^{-(3n+1)/2}). \quad (17)$$

⁶ Note that at $t \sim 100$ s, after antimatter – positrons – annihilation, T_m is $\sim 10^{-3}$ g cm⁻³ only, similar to conditions on the Earth.

Thus, though oscillations of the scale factor remain small as $t \rightarrow 0$, oscillations of R grow to the past and finally violate the assumption $|\delta R_{osc}| \ll R^{(0)}$. Note that the energy of scalaron oscillations at that moment $\rho_{osc} \sim (\delta R_{osc})^2/GM^2(R)$ is still much less than T_m since $R \ll M^2$ for $R \gg R_0$. Investigation of further FRW evolution of the model to the past is blocked by the stability problem: R can become less than R_1 and even negative during oscillations, so the conditions (7) are violated. For $n > 1$, it is possible to choose x_1 or λ in such a way that $f'(R) > 0$ for all real R . But $f''(R)$ always becomes zero at $R = \pm R_2$ where $R_2 = R_0/\sqrt{2n+1} < R_1$ and negative for $|R| < R_2$, and it is not possible to avoid this property without abandoning the assumption of the cosmological constant disappearance in flat space-time $f(0) = 0$.

Therefore, first, to avoid $M(R)$ becoming too large, say larger than M_{Pl} , the function $f(R)$ in (5) has to be modified at $R \rightarrow \infty$. The simplest way is to add the term $R^2/6M^2$ to (5) where the value of M (which will be the limiting value of $M(R)$ for $R \rightarrow \infty$) may be taken just that which is needed for the $R + R^2$ inflationary model mentioned in Sec. 1. This term will be negligible for the present DE.

Second, a new serious problem for DE models in $f(R)$ gravity appears which has not been considered before: to avoid destroying radiation- and matter-dominated FRW stages, some mechanism in the early Universe should work to prohibit overproduction of scalarons. In the $R + R^2$ inflationary model such mechanism does exist – gravitational creation of non-conformally-invariant particles and antiparticles (though not gravitons!) by oscillations of R . However, in the model (5) it is not possible to use it in full until the question how to evolve through the point where $f''(R) = 0$ is solved. The only way to avoid this problem at all is to assume that $|\delta R_{osc}| < 8\pi GT_m$ just from the very beginning of early evolution of the Universe (or at least from the moment when the effective Lagrangian density (1) becomes valid). In any case this makes the constant C in Eq. (14) very small, practically zero at present and greatly reduces possible phase space for this DE model.

Still, if this problem is solved and C may be put zero somehow, then a viable FRW background is obtained once the condition (10) is satisfied. Moreover, since it follows from the present observational data that $\Lambda(R_1)$ is not too much different from $\Lambda(\infty)$, one can choose x_1 close to 2λ . Then the whole FRW evolution is analytically described by the first iteration formula (12) where the background FRW metric is that of the Λ CDM model:

$$a^{(0)} \propto \sinh^{2/3} \left(\frac{3}{2} H_0 t \right), \quad H^{(0)} = H_0 \coth \left(\frac{3}{2} H_0 t \right), \quad R^{(0)} = 3H_0^2 \left(4 + \frac{1}{\sinh^2(\frac{3}{2} H_0 t)} \right) \quad (18)$$

where $H \equiv \dot{a}/a$, $H_0^2 = \Lambda(\infty)/3 = \lambda R_0/6$. Knowing δR_{ind} , it is straightforward to obtain δH and $\delta a/a$.

III. LABORATORY AND SOLAR SYSTEM TESTS AND DYNAMICS OF INHOMOGENEITIES

The main problem with laboratory and Solar system tests of gravity for all $f(R)$ DE models irrespective of the form of $f(R)$ is that this class of models represents the limiting case $\omega \rightarrow 0$ of scalar-tensor gravity as was mentioned above. As a result, would scalaron be massless, an additional ‘fifth force’ would show itself in laboratory experiments and the first post-Newtonian parameters would have the values $\beta = 1$, $\gamma = 1/2$ that is not admissible. However, this problem is qualitatively the same as it occurs for the string theory dilaton which in the massless low-energy limit corresponds to scalar-tensor gravity with $\omega = -1$ that is excluded, too. As is well known, that problem is solved by assuming that the dilaton is sufficiently massive. The same one has to assume about the scalaron in $f(R)$ DE models. Namely, let us choose model parameters in such a way that for any laboratory or Solar system test of gravity having a characteristic scale L and made in an environment with some non-relativistic matter density ρ_m , the scalaron mass satisfies the strong inequality $M(R(\rho_m))L \gg 1$. Under this condition deviations from the equation $R = 8\pi GT_m$ (and the Poisson equation, too) are small both inside and outside the Sun and other compact bodies. There have been hopes that this requirement may be circumvented using the so called ‘‘Chameleon’’ effect [30] (see also [31]). However, until recently it has not been shown to what extent can it be important in construction of viable $f(R)$ gravity models, see e.g. [14]. For the model (5), $M(\rho_m) \propto \rho_m^{n+1}$ for $R \gg R_0$ with $M(R_0) \sim R_0^{1/2} \sim 10^{-28} \text{ cm}^{-1}$.

For the most recent and best Cavendish-type experiment [32], taking $L \approx 50 \mu\text{m}$ and $\rho_m \approx 10^{-12} \text{ g cm}^{-3}$ (corresponding to a vacuum of $\approx 10^{-6}$ torr achieved there), we obtain the sufficient condition $n \geq 1$ (rounded to a larger integer). In the case of light deflection or the Shapiro time delay by the Sun, the main contribution to $\gamma - 1$ is from distances $r \sim R_\odot \approx 7 \times 10^{10} \text{ cm}$. If we take the Solar corona density at this distance ($\sim 10^{-15} \text{ g cm}^{-3}$) as ρ_m , then already $n \geq 0.5$ would be sufficient. In the case $0.5 < n < 2$ the condition $M(R)r \gg 1$ can be violated farther from the Sun. However, the scalar component of Solar gravitational field has been already screened by the Yukawa damping factor $\exp(-\int M(r) dr)$ by that distance. Clearly more careful calculation is needed here with the detailed account of interplanetary matter profile in the Solar system. The same refers to such tests like Lunar laser ranging. In any case, all known laboratory and Solar system tests of gravity are certainly satisfied for $n \geq 2$ and probably this

condition may be softened up to $n \geq 1$. However, it will be shown below that there is no necessity in such softening due to limits from the large-scale structure of the Universe.

Finally, let us turn to the evolution of weak inhomogeneities in the linear regime. It follows from equations for perturbations, either obtained for scalar-tensor gravity with the limit $\omega \rightarrow 0$ taken in them, see e.g. [20], or from those directly derived for $f(R)$ theory ([12, 13, 33] and other papers), that the equation for density perturbations $\delta_m \equiv \delta\rho_m/\rho_m$ in the non-relativistic matter component (cold dark matter + baryons) during the matter-dominated stage has the form

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G_{eff}\rho_m\delta_m = 0 \quad (19)$$

in the limit $k \gg aH$ where $k \equiv |\mathbf{k}|$ (spatial dependence $\exp(i\mathbf{k}\mathbf{r})$ is assumed) and $G_{eff} = G/f'(R)$ for $k \ll M(R)a$ and $G_{eff} = 4G/3f'(R)$ for $k \gg M(R)a$ ($f'(R) \approx 1$ for $R \gg R_0$). This increase of G_{eff} in 4/3 times is just how the 'fifth force' due to additional scalar gravity shows itself at large scales and low matter density.

Because of this, δ_m grows as usually, $\propto t^{2/3}$, before the time moment t_k when $k = M(R)a$, but after that this law changes to $\delta_m \propto t^{\frac{\sqrt{33}-1}{6}}$ that continues up to t_Λ – the end of the matter-dominated stage when $\ddot{a} = 0$. Using (15), we obtain $t_k \propto k^{-1/2(n+\frac{2}{3})}$. As a result, $\delta_m(k)$ acquires an additional growth factor during the matter-dominated stage (compared to the Λ CDM model) proportional to

$$\left(\frac{t_\Lambda}{t_k}\right)^{\frac{\sqrt{33}-1}{6}-\frac{2}{3}} \propto k^{\frac{\sqrt{33}-5}{4(3n+2)}}. \quad (20)$$

This additional increase occurs at small redshifts and is not seen in CMB fluctuations (apart from some features in a few low multipoles which are bounded by cosmic variance). E.g., for $n = 2$, $k/a_{now}H_{now} = 300$ and $z_\Lambda = 0.7$, the redshift $z_k(t_k) = 2.5$. As a result, there arises some discrepancy between values of the slope n_s of the primordial power spectrum determined from galaxy surveys on one side (assuming the standard evolution of perturbations) and CMB fluctuations on the other:

$$\Delta n_s = n_s^{(gal)} - n_s^{(CMB)} = \frac{\sqrt{33}-5}{2(3n+2)}. \quad (21)$$

For comparison, Δn_s is equal to 0.074 for $n = 1$ and 0.047 for $n = 2$. Note that the limit $n \rightarrow 0$ corresponds not to the Λ CDM model but to $F(R) \propto \ln R$ at large R .

At present no such discrepancy is seen, see e.g. [15], so we may conservatively bound $\Delta n_s < 0.05$ that leads to $n \geq 2$ in (5). Of course, a more exact numerical calculation is needed for larger values of n since then the values of z_k lie rather close to unity, so the formula (21) becomes too approximate.

IV. CONCLUSIONS AND DISCUSSION

Thus, in contrast to numerous unsuccessful previous attempts to construct a viable DE model in $f(R)$ gravity using a function f divergent or non-analytic at $R = 0$, it appears possible to achieve this goal with a regular $f(R)$ satisfying the condition $f(0) = 0$ which means the absence of a 'bare' cosmological constant in flat space-time. On the other hand, DE in this model behaves itself as an effective cosmological constant for large R if $F(R) = f - R \rightarrow const$ at $R \rightarrow \infty$. This model passes laboratory and Solar system tests of gravity if its parameter n is sufficiently large ($n \geq 2$ seems to be the sufficient, but probably not necessary condition), though analysis of gravitational radiation from double pulsars may add some new restriction. But it is clear already that limits from large-scale structure arising due to the anomalous growth of linear perturbations at recent redshifts are more critical and lead to a stronger limit on n . Just the opposite, any discrepancy Δn_s between values of the slope of the primordial perturbation spectrum obtained from galaxy and CMB data may serve as a strong argument for such model. Then Eq. (21) directly relates Δn_s to n .

However, more deep theoretical analysis of this class of models has uncovered its new serious problem not considered before: how to avoid an overabundance of new scalar particles arising in $f(R)$ gravity (dubbed scalarons) which can be generated in the very early Universe. Mathematically this means that the coefficient C in Eq. (14) should be practically zero at present, i.e. almost the whole degree of freedom has to be suppressed. Otherwise, a FRW solution in this model cannot have sufficiently long radiation- and matter-dominated stages because it hits the weak singularity where $f''(R) = 0$. Note that this difficulty is of a more subtle type than those of linear stability considered before. It is rather a problem of a measure of initial conditions in the early Universe leading to the standard cosmological evolution almost up to the present time. Anyway it remains an important topic for further study.

While this paper was being prepared for publication, two papers [34, 35] appeared where similar DE models in $f(R)$ gravity possessing the property $f(0) = 0$ were proposed. Our model is closer to that in [34].

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