

# Probing CP violation in neutrino oscillations with neutrino telescopes

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## Abstract

Measurements of flavor ratios of astrophysical neutrino fluxes are sensitive to the two yet unknown mixing parameters  $\theta_{13}$  and  $\delta$  through the combination  $\sin\theta_{13}\cos\delta$ . We extend previous studies by considering the possibility that neutrino fluxes from more than a single type of sources will be measured. We point out that, if reactor experiments establish a lower bound on  $\theta_{13}$ , then neutrino telescopes might establish an upper bound on  $|\cos\delta|$  that is smaller than one, and by that prove that CP is violated in neutrino oscillations. Such a measurement requires several favorable ingredients to occur: (i)  $\theta_{13}$  is not far below the present upper bound; (ii) The uncertainties in  $\theta_{12}$  and  $\theta_{23}$  are reduced by a factor of about two; (iii) Neutrino fluxes from muon-damped sources are identified, and their flavor ratios measured with accuracy of order 10% or better. For the last condition to be achieved with the planned  $\text{km}^3$  detectors, the neutrino flux should be close to the Waxman-Bahcall bound. It motivates neutrino telescopes that are effectively about 10 times larger than IceCube for energies of  $\mathcal{O}(100 \text{ TeV})$ , even at the expense of a higher energy threshold.

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## I. INTRODUCTION

One of the main goals of future neutrino experiments [1] is to observe CP violation in neutrino oscillations. The significance of such a measurement goes beyond the determination of a fundamental parameter of Nature: it can give further qualitative support to leptogenesis, the idea that the observed baryon asymmetry of the Universe has its source in a lepton asymmetry generated in neutrino interactions. In some scenarios, it is even quantitatively related to leptogenesis.

Neutrino telescopes [2], such as the IceCube experiment, aim to observe neutrinos coming from astrophysical sources. The experiments will provide information on the direction, energy, and flavor of the incoming neutrinos. In particular, ratios between fluxes of different flavors arriving to the detector can be measured. Ratios between these fluxes at the source are predicted by rather robust theoretical considerations.

The modifications of the flavor ratios between source and detector originate from neutrino oscillations. This means that the relations between the fluxes at the source and the fluxes at the detector depend on the neutrino parameters in a calculable way. Flavor measurements in neutrino telescopes can thus provide information on the neutrino mixing parameters [3, 4, 5, 6, 7, 8, 9]. In particular, there is sensitivity to two yet unknown parameters: the mixing angle  $\theta_{13}$  and the CP violating phase  $\delta$ .

CP violation in neutrino oscillations can, in principle, be observed via interference terms. For neutrinos coming from astrophysical sources, such interference terms are washed out, and the measured fluxes are therefore sensitive only to CP conserving parameters. Specifically, the measured flavor ratios are sensitive to the combination

$$\Delta_{13} \equiv \sin \theta_{13} \cos \delta. \tag{1}$$

Since  $\theta_{13}$  is experimentally bounded from above and known to be small, it is convenient to write the flavor ratios in the general form  $a + b\Delta_{13}$ , where  $a$  and  $b$  are known functions of the two measured parameters,  $\theta_{12}$  and  $\theta_{23}$ , but independent of  $\theta_{13}$  and  $\delta$ . The  $b\Delta_{13}$  term provides a small correction to the zeroth order prediction  $a$ . If  $\sin \theta_{13} = 0$ , or if CP violation is maximal, *i.e.*  $\delta = \pi/2$  or  $3\pi/2$ , the correction term is absent.

If  $\sin \theta_{13}$  is close to the present experimental upper bound, it is likely to be measured in near future reactor experiments [10]. In that case, if neutrino telescopes are able to exclude

a correction term as large as  $\pm b \sin \theta_{13}$ , they will establish that  $\cos \delta \neq \pm 1$  and, by that, will discover that CP is violated in neutrino interactions.

Our goal in this paper is to analyze whether such a discovery of CP violation by neutrino telescopes is at all possible. More concretely, we do the following. On the qualitative level, we find what types of sources and what types of flavor ratios provide the strongest sensitivity to the parameters of interest. On the quantitative level, we estimate the accuracy that is required in these measurements and in independent measurements of the mixing angles in order to establish that the CP violating phase is different from 0 and from  $\pi$ . Our final conclusion is that, with large  $\theta_{13}$  and near-maximal CP violation, and under some favorable circumstances, it may be possible for IceCube (or, more easily, for future, larger detectors) to establish CP violation in neutrino interactions.

## II. FLAVOR RATIOS AND MIXING PARAMETERS

Our goal in this section is to derive analytical expressions for neutrino flavor fluxes that can be measured in neutrino telescopes and, in particular, in IceCube.

Neutrino telescopes can identify the neutrino flavor ( $\alpha = e, \mu, \tau$ ) via its characteristic interaction topology [11, 12]. IceCube has an energy threshold  $\sim 100 \text{ GeV}$  for detecting muon tracks, and  $\sim 1 \text{ TeV}$  for detecting electron- and tau-related showers. Above an energy threshold  $\sim 1 \text{ PeV}$ , it is possible to distinguish between the electron-related electromagnetic showers and the tau-related hadronic showers. Finally, around  $E \sim 6.3 \text{ PeV}$ , the Glashow resonance may allow the identification of  $\bar{\nu}_e$  events [7, 13].

We denote the flux of  $\nu_\alpha + \bar{\nu}_\alpha$  measured at the detector by  $\phi_\alpha^d$ ; the flux of antineutrinos  $\bar{\nu}_\alpha$  is denoted by  $\bar{\phi}_\alpha^d$ . We consider the following flavor ratios:

$$R \equiv \frac{\phi_\mu^d}{\phi_e^d + \phi_\tau^d}, \quad (2)$$

$$S \equiv \frac{\phi_e^d}{\phi_\tau^d}, \quad (3)$$

$$T \equiv \frac{\bar{\phi}_e^d}{\phi_\mu^d}. \quad (4)$$

Below  $E \sim \text{PeV}$ , only  $R$  can be measured. At higher energies,  $S$  and perhaps  $T$  may become available.

We denote the flux of  $\nu_\alpha + \bar{\nu}_\alpha$  emitted from the source by  $\phi_\alpha^s$ . The relation between  $\phi_\alpha^s$  and  $\phi_\beta^d$  is given by

$$\phi_\beta^d = P_{\beta\alpha}\phi_\alpha^s, \quad (5)$$

where  $P_{\beta\alpha} \equiv P(\nu_\alpha \rightarrow \nu_\beta)$  is the transition probability from a flavor  $\nu_\alpha$  at the source to a flavor  $\nu_\beta$  at the detector.

For propagation over astronomical distance scales, the distance-dependent oscillatory terms average out, and  $P_{\beta\alpha}$  depends on mixing parameters only:

$$P_{\beta\alpha} = \sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2. \quad (6)$$

Here  $U$  is the unitary transformation that relates the neutrino interaction eigenstates  $\nu_\alpha$  ( $\alpha = e, \mu, \tau$ ) and mass eigenstates  $\nu_i$  ( $i = 1, 2, 3$ ):

$$|\nu_\alpha\rangle = U_{\alpha i}^* |\nu_i\rangle. \quad (7)$$

We parametrize the matrix  $U$  by three mixing angles,  $\theta_{12}$ ,  $\theta_{23}$  and  $\theta_{13}$ , and three CP violating phases,  $\delta$ ,  $\alpha_1$  and  $\alpha_2$ :

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\alpha_1/2} & & \\ & e^{i\alpha_2/2} & \\ & & 1 \end{pmatrix}, \quad (8)$$

where  $c_{ij} \equiv \cos \theta_{ij}$ ,  $s_{ij} \equiv \sin \theta_{ij}$ . It is clear from Eq. (6) that  $P_{\beta\alpha}$  is independent of the phases  $\alpha_{1,2}$ . It depends on the three mixing angles  $\theta_{ij}$  and on  $\delta$ .

Since it is experimentally known that  $\theta_{13}$  is small (see Table I), it is convenient to write down the flavor transition probabilities to first order in  $\Delta_{13}$  (see Eq. (1)) [14, 15, 16]:

$$\begin{aligned} P_{ee} &\simeq 1 - \frac{1}{2} \sin^2 2\theta_{12}, \\ P_{e\mu} &\simeq \frac{1}{2} \sin^2 2\theta_{12} \cos^2 \theta_{23} + \frac{1}{4} \sin 2\theta_{23} \sin 4\theta_{12} \Delta_{13}, \\ P_{\mu\mu} &\simeq 1 - \frac{1}{2} (\cos^4 \theta_{23} \sin^2 2\theta_{12} + \sin^2 2\theta_{23}) - \frac{1}{2} \sin 2\theta_{23} \cos^2 \theta_{23} \sin 4\theta_{12} \Delta_{13}, \\ P_{e\tau} &\simeq \frac{1}{2} \sin^2 2\theta_{12} \sin^2 \theta_{23} - \frac{1}{4} \sin 2\theta_{23} \sin 4\theta_{12} \Delta_{13}, \\ P_{\mu\tau} &\simeq \frac{1}{8} \sin^2 2\theta_{23} (4 - \sin^2 2\theta_{12}) + \frac{1}{8} \sin 4\theta_{23} \sin 4\theta_{12} \Delta_{13}. \end{aligned} \quad (9)$$

The remaining probabilities can be derived from  $P_{\alpha\beta} = P_{\beta\alpha}$  and  $\sum_\alpha P_{\alpha\beta} = 1$ .

### III. ASTROPHYSICAL NEUTRINO SOURCES AND FLAVOR RATIOS

We consider two types of sources:

- “Pion sources” (denoted by sub-index  $\pi$ ) provide the following flavor ratios:

$$\phi_e^s : \phi_\mu^s : \phi_\tau^s = 1 : 2 : 0. \quad (10)$$

As concerns the  $\nu_e - \bar{\nu}_e$  decomposition of  $\phi_e$ , the situation depends on whether the pions are produced mainly by  $pp$  or  $p\gamma$  interactions:

$$\frac{\bar{\phi}_\mu^s}{\phi_\mu^s} = \frac{1}{2}, \quad \frac{\bar{\phi}_e^s}{\phi_e^s} = \begin{cases} 1/2 & pp, \\ 0 & p\gamma. \end{cases} \quad (11)$$

- “Muon-damped sources” (denoted by sub-index  $\mu$ ) provide the following flavor ratios:

$$\phi_e^s : \phi_\mu^s : \phi_\tau^s = 0 : 1 : 0. \quad (12)$$

As concerns the  $\nu_\mu - \bar{\nu}_\mu$  decomposition of  $\phi_\mu$ , the situation depends on whether the pions are produced mainly by  $pp$  or  $p\gamma$  interactions:

$$\frac{\bar{\phi}_\mu^s}{\phi_\mu^s} = \begin{cases} 1/2 & pp, \\ 0 & p\gamma. \end{cases} \quad (13)$$

The expectation is that all sources where the initial stage of neutrino production is charged pion decays will undergo a transition from a “pion” to “muon-damped” flavor decomposition at high enough neutrino energies [17]. If energy losses are mainly due to synchrotron radiation and inverse compton emission, the transition region is expected to span about one decade in energy. The actual threshold energy cannot be determined model independently and, furthermore, is likely to differ from source to source. We assume here that, nevertheless, the transition is such that it will be possible to separate the neutrino events to lower-energy events from pion sources and higher-energy events from muon-damped sources.

The dependence of the flavor ratios at the detector on the mixing parameters can be obtained as follows. One starts from the fluxes at the source (in arbitrary units), Eqs. (10), (11), (12) and (13). Then, the fluxes at the detector can be found by using Eq. (5) and the expressions for the transition probabilities (6). Finally, the expressions are put in Eqs. (2), (3) and (4).

TABLE I: Experimental ranges of mixing angles [1]

Parameter	Best fit	$1\sigma$ range	‘Future’
$\sin^2 \theta_{12}$	0.31	0.29 – 0.33	$0.31 \pm 0.01$
$\sin^2 \theta_{23}$	0.47	0.40 – 0.55	$0.47 \pm 0.04$
$\sin^2 \theta_{13}$	0.00	$\leq 0.008$	$0.022 \pm 0.003$

## IV. DESCRIPTION OF ANALYSIS

### A. Numerical input

The current best fit values and  $1\sigma$  ranges of the mixing angles are given in Table I [1]. By the time that IceCube can carry out the measurements that we discuss in this work, it is likely that the knowledge – from other experiments – of the mixing angles will improve. Such progress is very significant for our purposes, as we see below. In particular, in order that the IceCube measurements will be able, even in principle, to show that  $\delta \neq 0$ , it is crucial that experiments establish that  $\sin \theta_{13} \neq 0$ . For the sake of our analysis, we assume that reactor experiments will measure  $\sin^2 2\theta_{13} = 0.090 \pm 0.013$  [10]. (This value for  $\theta_{13}$  corresponds to the current  $2\sigma$  allowed range [1].) For  $\theta_{12}$  and  $\theta_{23}$  we assume a factor of two improvement in the accuracy. The resulting ranges which we use to examine the question of whether IceCube can discover CP violation are given in the column labelled ‘Future’ in Table I. As concerns the phase, we assume that it will remain unconstrained.

To obtain an understanding of the dependence of the flavor ratios on the mixing parameter  $\Delta_{13}$ , we use the central values for the two measured angles,  $\theta_{12}$  and  $\theta_{23}$ , and apply the approximate relations (9). We obtain for the pion source

$$\begin{aligned}
 R_\pi &= 0.49 - 0.15\Delta_{13}, \\
 S_\pi &= 1.04 + 0.52\Delta_{13}, \\
 T_\pi &= \begin{cases} 0.52 + 0.28\Delta_{13} & pp, \\ 0.23 + 0.22\Delta_{13} & p\gamma, \end{cases} \tag{14}
 \end{aligned}$$

and for the muon-damped source

$$\begin{aligned}
R_\mu &= 0.62 - 0.49\Delta_{13}, \\
S_\mu &= 0.58 + 0.44\Delta_{13}, \\
T_\mu &= \begin{cases} 0.30 + 0.38\Delta_{13} & pp, \\ 0 & p\gamma, \end{cases} \tag{15}
\end{aligned}$$

We emphasize, however, that in our calculations we use the full dependence on the mixing angles [see Eq. (6)], and not just the leading order (in  $\Delta_{13}$ ) expressions, Eqs. (9), (14) and (15).

## B. Experimental errors

It is not yet clear whether all of the flavor ratios defined in Section II will indeed be available at IceCube (or any future neutrino telescope). We assume that  $R_\pi$ ,  $R_\mu$  and  $S_\mu$  will be measured, and consider cases where  $S_\pi$  and  $T_\mu$  are available or not.

The goal of this work is not to obtain a detailed realistic estimate of the accuracies that are *expected* in the relevant measurements. Such an estimate depends on both features of the astrophysical neutrinos that are not yet known (*e.g.* the actual total flux), and features of the detectors that will only become clear when these neutrinos are observed. The main goal here is to find the accuracies that are *required* in order to establish that CP is violated.

We thus consider the following experimental accuracies in the measurements of the various flavor ratios:

1.  $R_\pi$ : we consider hypothetical accuracies of 5%, 10% or 20%. If the flux is close to the Waxman-Bahcall bound, then we expect  $\mathcal{O}(100)$  events, and an error of order 10% seems realistic;
2.  $S_\pi$ : In the cases that it is available, we relate the accuracy to that of  $R_\pi$ , by assuming a Poisson distribution of the number of events for each neutrino flavor. We neglect issues of efficiency in detecting tracks versus showers. This leads to  $\Delta S_\pi/S_\pi = \sqrt{S_\pi(1 + S_\pi^{-1})^2/(1 + R_\pi^{-1})}(\Delta R_\pi/R_\pi)$ . Using central values from Eq. (14), we obtain  $\Delta S_\pi/S_\pi = 1.2(\Delta R_\pi/R_\pi)$ ;

TABLE II: Scenarios for experimental accuracies

Scenario	$\Delta R_\pi/R_\pi$	$\Delta R_\mu/R_\mu$	$\Delta S_\mu/S_\mu$	$(\Delta S_\pi/S_\pi)$	$(\Delta T_\mu/T_\mu)$
(5, 5)	5	5	7	6	5
(5, 10)	5	10	13	6	10
(5, 20)	5	20	27	6	20
(10, 10)	10	10	13	12	10
(10, 20)	10	20	27	12	20
(20, 20)	20	20	27	24	20

3.  $R_\mu$ : we consider hypothetical accuracies which are at best the same as the error on  $R_\pi$  and at worst 20%;

4.  $S_\mu$ : Following the same line of thought as for  $S_\pi$ , we use  $\Delta S_\mu/S_\mu = \sqrt{S_\mu(1 + S_\mu^{-1})^2/(1 + R_\mu^{-1})(\Delta R_\mu/R_\mu)}$ . Using central values from Eq. (15), we obtain  $\Delta S_\mu/S_\mu = 1.3(\Delta R_\mu/R_\mu)$ ;

5.  $T_\mu$ : In the cases that it is available, we assume  $\Delta T_\mu/T_\mu = \Delta R_\mu/R_\mu$ .

The various scenarios can be defined by the assumed accuracies in  $R_\pi$  and  $R_\mu$ : We denote by  $(a, b)$  a scenario where the errors are  $\Delta R_\pi/R_\pi = a\%$  and  $\Delta R_\mu/R_\mu = b\%$ . The six scenarios that we consider are presented in Table II.

We thus consider a hypothetical set of measurements –  $R$ ,  $S$ ,  $T$  and  $\sin^2 \theta_{ij}$  – which provide information on  $\theta_{ij}$  and  $\delta$ . The statistical procedure by which this information is extracted is described in the following section.

### C. Statistical procedure

Given a measurement of an observable  $Y^{\text{meas}} = \langle Y \rangle \pm \sigma_Y$ , we construct  $\chi^2(\theta_{ij}, \delta) = \sum_Y \left[ \frac{\langle Y \rangle - Y(\theta_{ij}, \delta)}{\sigma_Y} \right]^2$ , where  $Y(\theta_{ij}, \delta)$  represents the theoretical description of the  $Y$  observable. The uncertainty  $\sigma_Y$  is given in Table I for  $\sin^2 \theta_{ij}$  and in Table II for  $R$ ,  $S$  and  $T$ . A



statistical handling of the parameters is performed by analyzing the quantity  $\Delta\chi^2(\theta_{ij}, \delta) = \chi^2 - \min_{\theta_{ij}, \delta} \{\chi^2\}$ .

We define the  $N$ -dimensional “ $\alpha\%$  CL acceptance region”, for a subset of  $N$  out of the four mixing parameters  $(\theta_{ij}, \delta)$ , by the region in the  $N$  parameter space for which  $\Delta\chi_{\text{marg}}^2 < C^{-1}(\alpha, N)$ . Here  $\Delta\chi_{\text{marg}}^2$  is obtained by marginalizing  $\Delta\chi^2$  with respect to the  $4 - N$  redundant parameters and  $C^{-1}(\alpha, N)$  is the inverse chi-square CDF with  $N$  degrees of freedom, evaluated at the point  $\alpha$ . We have compared this procedure to the more computationally demanding FC construction, (as described in [18] and demonstrated, for example, in [19]) under the assumption of gaussian measurement errors, for several sample configurations. We have found a reasonable agreement between our simplified method and the full FC routine, with the former tending in general to supply slightly more conservative acceptance regions.

We define the “ $\alpha\%$  CL acceptance interval”, for a specific parameter, by the set of parameter values for which the condition  $\Delta\chi_{\text{marg}}^2 < C^{-1}(\alpha, 1)$  is satisfied, with  $\Delta\chi_{\text{marg}}^2$  given by marginalizing  $\Delta\chi^2$  with respect to all of the other parameters.

An “ $\alpha\%$  CL fraction of coverage” is further defined for a specific parameter as the percentage of the parameter range that is included in the  $\alpha\%$  CL acceptance interval. The lower is this fraction, the stronger is the exclusion power of the experiment with respect to the relevant parameter.

We say that a specific value of a parameter is excluded with  $\alpha\%$  confidence, if this value is not contained in the corresponding  $\alpha\%$  acceptance interval. This notion will be used below, when we discuss the prospects of various measurement scenarios do exclude CP conservation in neutrino oscillations.

## V. RESULTS

### A. Neglecting uncertainties in $\theta_{12}$ and $\theta_{23}$

To understand the abilities and difficulties that are intrinsic to the measurements by neutrino telescopes, we first carry out an analysis where  $\theta_{12}$  and  $\theta_{23}$  are held fixed at their current best fit values. In the next section, we will study the implications of the uncertainties in these angles.

We begin by choosing specific values for the parameters  $\theta_{13}$  and  $\delta$ , which we call “true parameters”. Concretely, we assume a true value  $\theta_{13} = 0.15$ , and consider mainly three possibilities for the true value of  $\delta$ : the two CP conserving ones ( $\delta = 0, \pi$ ) and the maximally CP violating one ( $\delta = \pi/2$ ). We evaluate the flux ratios that theoretically correspond to these mixing parameters. For the sake of illustration, we assume that the experimental measurements will obtain these flux ratios as their central values, with errors as specified for each of our six scenarios. We then perform a fit to  $\theta_{13}$  and  $\delta$  (obtaining, of course, the “true values” as the best-fit parameters, but with acceptance regions that are different between the various scenarios).

The resulting 90% CL acceptance regions in the  $\theta_{13} - \delta$  plane are presented, for the six scenarios, in Figs. 1, 2 and 3. As can be seen in the figures, for some cases, the neutrino telescope measurements can mildly improve our knowledge of  $\theta_{13}$  compared to the reactor constraint.

As concerns  $\delta$ , the 90% CL fraction of coverage in case that *all* the relevant observables will be measured is shown in Fig. 4, for true  $\theta_{13} = 0.15$  and scanning values of true  $\delta$  between 0 and  $\pi$ . Since only CP-even quantities are considered, the results for  $\delta = \pi + \theta$  are equal to those for  $\delta = \pi - \theta$ . We can make the following statements:

1. If the neutrino telescope measurements reach the accuracy assumed in this work, they are likely to exclude a certain range of  $\delta$ .
2. If the Dirac phase is small (that is close to 0 or  $\pi$ ), the excluded range will be quite significant.
3. The combination of all available observables is usually significantly more efficient than partial combinations.
4. The power of combining measurements is particularly significant as resolutions get worse and in the large phase ( $\delta \sim \pi/2$ ) case.
5. If only  $R_\pi$  is measured, no range of  $\delta$  will be excluded.

The main question that we are asking is the following: Given a hypothetical situation where  $\delta \sim \pi/2$ , will IceCube be able to establish CP violation, that is, exclude 0 and  $\pi$  from the acceptance interval in  $\delta$ ? The answer depends of course on which of the various

scenarios described in Table II, if any, will indeed be achieved in the experiment. The main lessons that we draw from our calculations are the following:

**(5,5):** Measuring  $R_\mu$  and  $S_\mu$  with an accuracy that is significantly better than 10 percent will enable a discovery of CP violation in neutrino oscillations.

**(5,10):** With this scenario, the sensitivity to CP violation is only marginally affected if either  $T_\mu$  or  $S_\pi$  are removed from the analysis. Studying the acceptance interval for  $\delta$ , one finds that CP violation may be established even without either  $T_\mu$  or  $S_\pi$ . This result will be further qualified when we elaborate on the scenario, below.

**(10,10):** If both  $T_\mu$  and  $S_\pi$  are measured, with an accuracy  $\sim 10\%$ , than the required accuracy on  $R_\pi$  can be somewhat relaxed.

**((5,10,20),20):** If the flavor ratios from muon-damped sources cannot be measured with an accuracy significantly better than 20%, then even an excellent measurement of flavor ratios from pion sources will not exclude CP conservation.

We learn that the (5, 10) scenario gives a reasonable sense of the minimal required set of measurements and accuracies in order that a discovery that CP is violated in neutrino oscillations will become possible. Further insight into the role of each of the five observables in achieving this goal is given in Fig. 5, depicting the flavor ratios as a function of  $\delta$  and the  $\chi^2$  composition for true  $\delta = \pi/2$ . While measurements of  $R_\pi$  and  $R_\mu$  at the assumed accuracies suffice to exclude  $\delta = \pi$ , at least one of  $S_\pi$  or  $T_\mu$  needs to be added in order to exclude  $\delta = 0$ .

The probability that CP conserving values of  $\delta$  will be excluded as a function of the true  $\delta$ , within the four scenarios (5,5), (5,10), (5,20) and (10,10), is shown in Fig. 6. To produce this plot, we generated a large sample (1000) of random sets of observables with the prescribed statistics, then checked for each realization whether  $\delta = 0$  or  $\pi$  is contained in the resulting acceptance interval. For example, with zero uncertainties in  $\theta_{12}$  and  $\theta_{23}$ , the conditional probability to exclude CP conservation in the (10,10) scenario given maximal phase is about 50%. Note that statistical fluctuations may lead to erroneous exclusion of CP conservation even with  $\sin \delta = 0$ . The fact that the (10,10) scenario is more likely than (5,20) to establish CP violation is suggestive for future detector optimizations: If the errors on  $\theta_{12}$  and  $\theta_{23}$  at the time of analysis are significantly reduced, then it may be preferable to improve the detection efficiency at the higher range of the spectrum,  $E > 100 \text{ TeV}$ , even at the cost of somewhat weaker efficiency at lower energies.

## B. Taking into account uncertainties in $\theta_{12}$ and $\theta_{23}$

As a first step in this analysis, we considered the present ranges for  $\theta_{12}$  and  $\theta_{23}$  (see Table I). The potential of neutrino telescopes to exclude a range of  $\delta$  can be seen from Fig. 4 (upper right panel). The impact of the uncertainties in  $\theta_{12}$  and  $\theta_{23}$  can be understood by comparing it to the upper left panel. We learn that, with present accuracies, the excluded ranges are weaker by 30-50% compared to the idealized case of zero uncertainties. (The importance of this ingredient in the analysis was noted in [20].)

As a second step, we assumed experimental errors on  $\sin^2 \theta_{12}$  and  $\sin^2 \theta_{23}$  that are reduced by a factor of two compared to the present (see Table I). The results are shown in Fig. 4 (lower panel). By comparing to the upper right panel, we learn that such an improvement will entail an exclusion power stronger by about 20% compared to the situation that present uncertainties remain.

Concerning the probability that CP violation will be established, we repeat the analysis with the present and with the assumed future uncertainties for the four leading scenarios. The results are shown in Fig. 6. Without an improvement in the determination of  $\theta_{12}$  and  $\theta_{23}$ , only the very optimistic scenario (5,5) allows a discovery. With the assumed improvements, the more realistic (5,10) scenario also has over 30% probability to make such a discovery. The (5,20) and (10,10) scenarios are not powerful enough to do so.

## C. Discussion

A related analysis has been performed previously in Refs. [6, 8], which highlighted the synergy between neutrino telescopes and terrestrial experiments. The conclusion in Refs. [6, 8] regarding the impact of neutrino telescopes on the issue of CP violation is more pessimistic than ours. The main difference lies in the fact that Refs. [6, 8] consider the information of one type of sources at a time, and indeed we agree with the pessimistic conclusion in this case. What we show, however, is that by combining the two types of sources that we considered, the ability to exclude CP conservation improves considerably. Actually, if this combination of sources is indeed available (and the experimental accuracy is similar to or better than our (10,10) scenario), the exclusion power that neutrino telescopes have on  $\delta$  will be comparable to the proposed superbeams [21]. (This situation actually

reinforces the point made in [6]: since the  $\delta$ -dependencies of the IceCube and the superbeam measurements are different, the information from the two will be complimentary.)

Ref. [22] points out that variations in the flavor ratios between sources can reach the ten percent level and consequently play an important role in the investigation of the mixing parameters from astrophysical neutrinos. In particular, the resulting uncertainties may wash-out the effects of the  $\Delta_{13}$  terms, especially in the case of low  $\theta_{13}$ . We agree that flavor composition uncertainties at the source would tighten greatly the requirements on the experimental precision. There are two reasons, however, why we think that this issue may have only limited consequences for our purposes. First, by the time that this analysis can be carried out in IceCube, the theoretical analysis of neutrino spectra, which is only at its beginning [17, 22], is likely to improve considerably. In particular, higher quality electromagnetic data, from radio to TeV photon energies, will become available. Second, our study is relevant only for the case of large  $\theta_{13}$  where, as we have argued, 10% accuracy might be just enough for our purposes if a global analysis of flavor-dependent spectrum will be possible.

The general trends reflected in our results can be simply understood, based on Eqs. (14) and (15). We rewrite them as follows:

$$\begin{aligned}
 R_\pi &= 0.49 [1 - 0.05(s_{13}/0.15) \cos \delta], \\
 S_\pi &= 1.04 [1 + 0.08(s_{13}/0.15) \cos \delta], \\
 R_\mu &= 0.62 [1 - 0.12(s_{13}/0.15) \cos \delta], \\
 S_\mu &= 0.58 [1 + 0.11(s_{13}/0.15) \cos \delta], \\
 T_\mu &= 0.30 [1 + 0.19(s_{13}/0.15) \cos \delta].
 \end{aligned}
 \tag{16}$$

We learn the following:

- The ratios related to muon-damped sources are more sensitive to the  $\cos \delta$ -dependent terms than those related to pion sources;
- To be sensitive to the  $\cos \delta$ -dependent terms, the accuracy should be of order 10% or better;
- The required accuracy scales with  $s_{13}$ . If, for example,  $s_{13} \sim 0.05$ , sensitivity to  $\cos \delta$  will be achieved only with accuracy better than 5%, which seems out of reach for IceCube.

## VI. CONCLUSIONS

We have studied the potential of combining measurements of flavor ratios in neutrino telescopes with observation of  $\theta_{13} \neq 0$  by reactor experiments in constraining  $\delta$ , the CP violating phase in the lepton mixing matrix. We reached the following conclusions:

- Since the neutrino telescopes are sensitive only to the combination  $\Delta_{13} \equiv \sin \theta_{13} \cos \delta$ , they can constrain  $\delta$  only if  $\sin \theta_{13}$  is not too small [6].
- Neutrino telescope may exclude at 90% CL up to 30% of the a-priori allowed range for  $\delta$ , even with present accuracies in  $\theta_{12}$  and  $\theta_{23}$ .
- Since the  $\Delta_{13}$ -term is maximized in size for  $\cos \delta = \pm 1$ , the exclusion region is largest if CP is nearly conserved [6].
- Reduced uncertainties in  $\theta_{12}$  and  $\theta_{23}$  can enlarge the excluded region to about 50% of the a-priori allowed range, and give sensitivity even for  $\cos \delta \sim 0$ .
- Measuring flavor ratios of fluxes from muon-damped sources will further strengthen the exclusion power (compared to measurements based on solely pion sources). Their significance is particularly important for  $\cos \delta \sim 0$ .

A more specific question that we posed is whether, in case that the CP violating phase  $\delta$  is large ( $\sim \pi/2$ ), the measurements of flavor ratios among neutrino fluxes from astrophysical sources can establish that the phase is indeed different from 0 or  $\pi$ , and by that prove that CP is violated in neutrino interactions. Our conclusions regarding this question are the following:

- $\sin \theta_{13}$  must be large, between current  $1 - 2\sigma$  upper bounds.
- The neutrino flux must not be lower than the Waxman-Bahcall bound. If the flux is smaller, a larger neutrino telescope may still achieve this goal, within a reasonable time scale ( $\lesssim 10$  years).
- Neutrino flux from muon-damped sources must be identified, and the related flavor ratios measured with accuracy better than 10%.

- The uncertainties on  $\theta_{12}$  and  $\theta_{23}$  must be reduced by other experiments by a factor of about two.

Even if all these conditions are met, the probability of excluding CP conservation in neutrino oscillations is at best 60%.

The strongest sensitivity to  $\cos \delta$  arises in flavor ratios related to muon-damped sources. On the theoretical side, a more careful study of the transition at high energy from pion-source to muon-damped source is important for better understanding of this crucial ingredient in our analysis [22]. On Nature's side, the lower the transition energy, and the sharper the transition, the higher statistics of events from muon-damped source that will become available and, consequently, the better chances are that a neutrino telescope will contribute significantly to understanding CP violation in neutrino oscillations. Finally, on the experimental side, a neutrino telescope that is effectively ten times bigger than IceCube, for neutrino energy  $\sim 100$  *TeV* (see Section V A), is well motivated by our arguments.

The fact that establishing CP violation in IceCube, an experiment under construction, is not manifestly impossible is exciting. While a combination of several favorable circumstances is required to achieve such a goal, it is worth to refine this analysis, to prepare for a fortunate case that these circumstances are fulfilled by the parameters of Nature and by the capabilities of neutrino telescopes.

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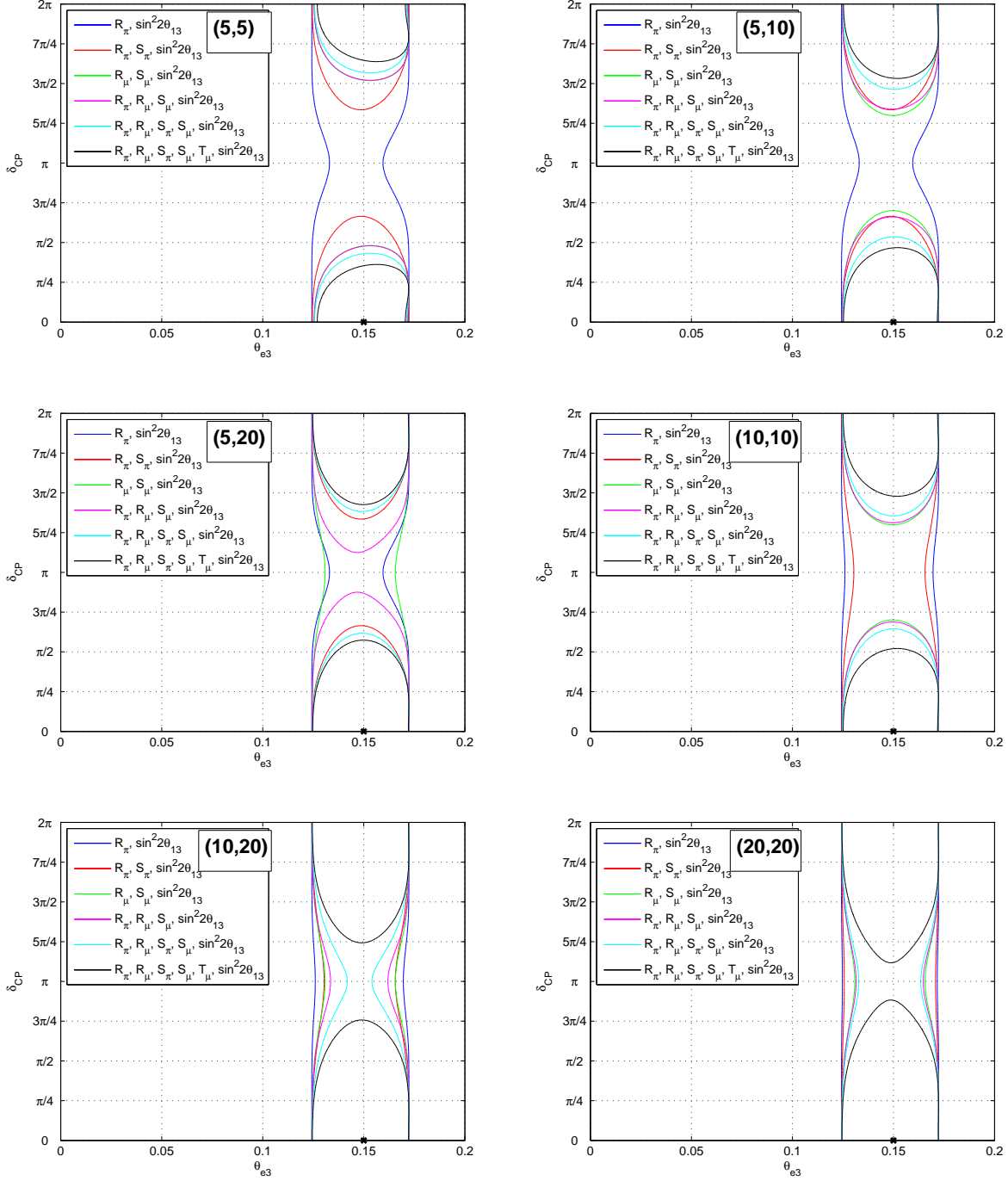


FIG. 1: 90%CL (2 d.o.f.) allowed regions for true  $\delta_{CP} = 0$ .

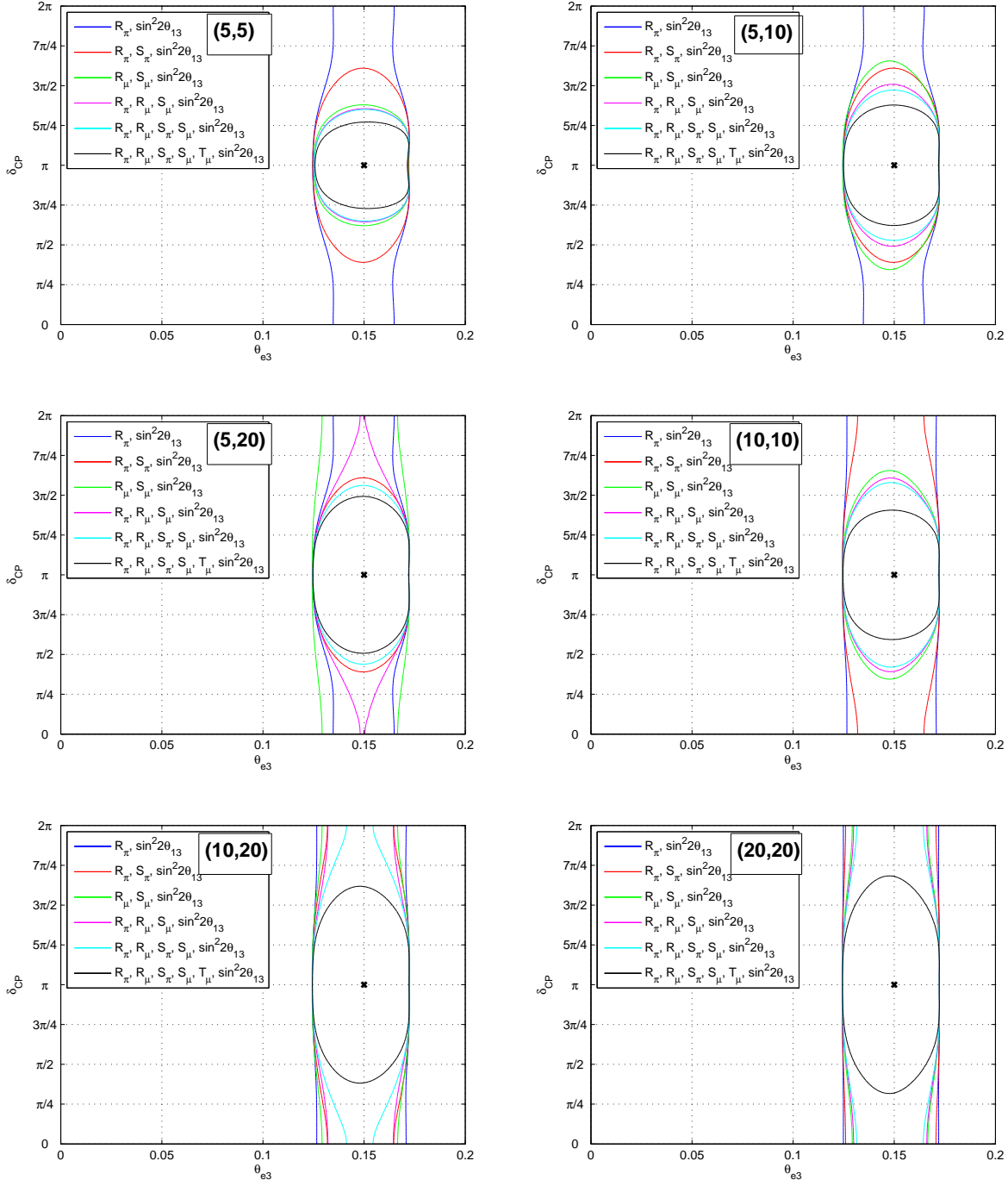


FIG. 2: 90%CL (2 d.o.f.) allowed regions for true  $\delta_{CP} = \pi$ .

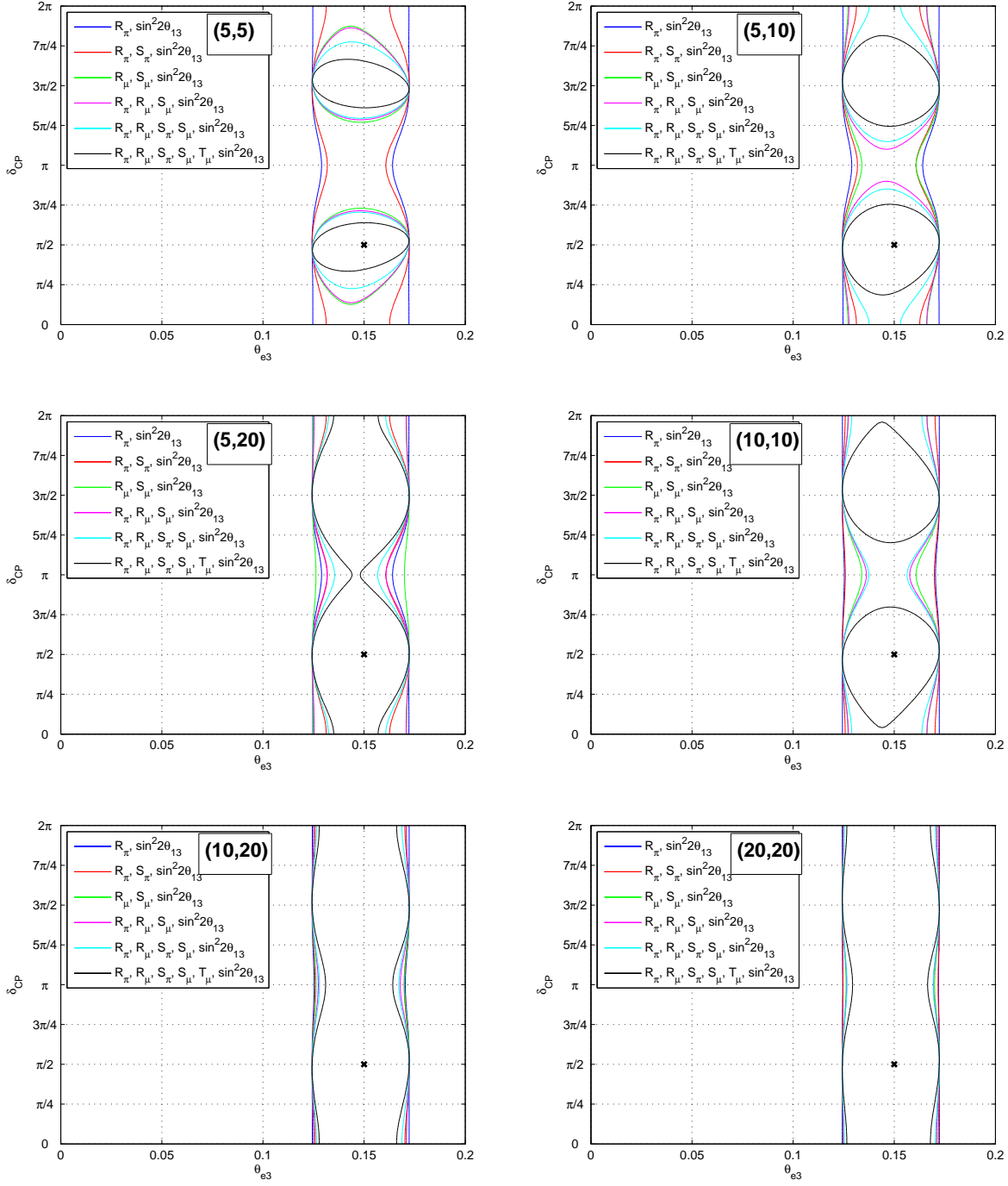


FIG. 3: 90%CL (2 d.o.f.) allowed regions for true  $\delta_{CP} = \pi/2$ .

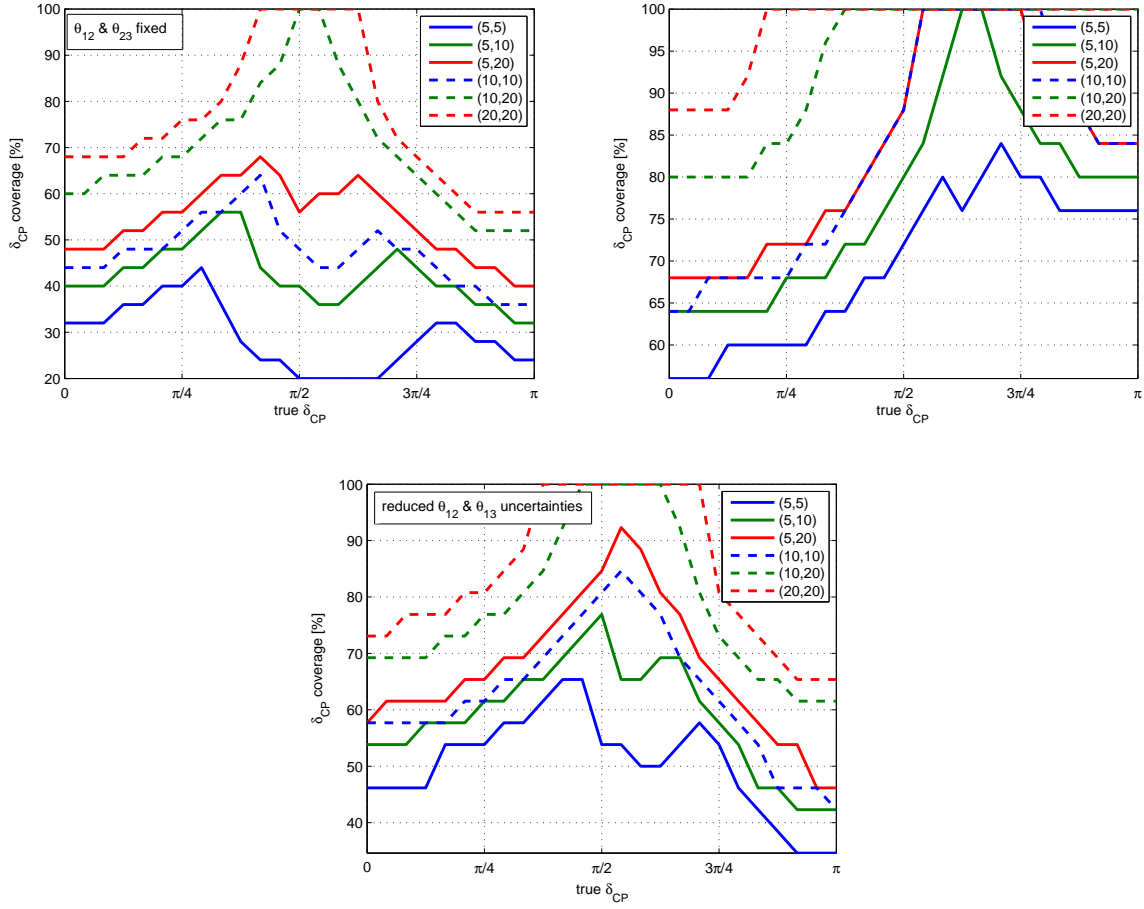


FIG. 4: 90% CL fraction of coverage for  $\delta$  in the six scenarios defined in Table II. The three panels differ in the uncertainties attributed to  $\theta_{12}$  and  $\theta_{23}$  (see Table I): (upper left) Zero uncertainties; (upper right) Present uncertainties; (bottom) ‘Future’ uncertainties.

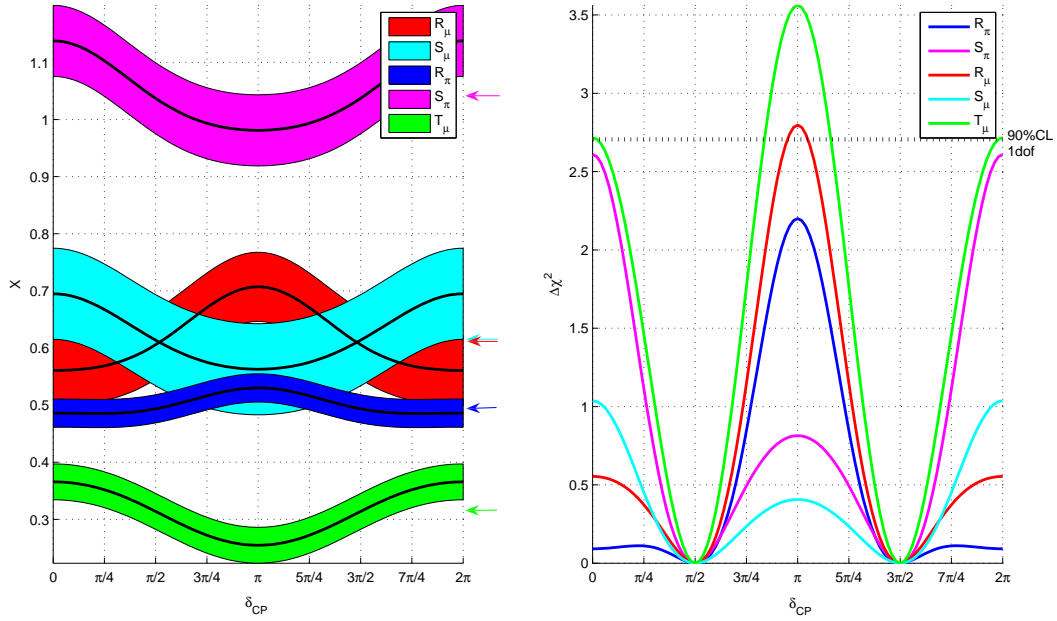


FIG. 5: The (5,10) scenario: (left) The flavor ratios as a function of  $\delta$  and their one-sigma range (arrows mark the central values corresponding to  $\delta = \pi/2$ ); (right) The  $\chi^2$  composition for true  $\delta = \pi/2$ . Both panels correspond to  $\theta_{12}$  and  $\theta_{23}$  fixed at their best-fit values,  $\theta_{13} = 0.15$ .

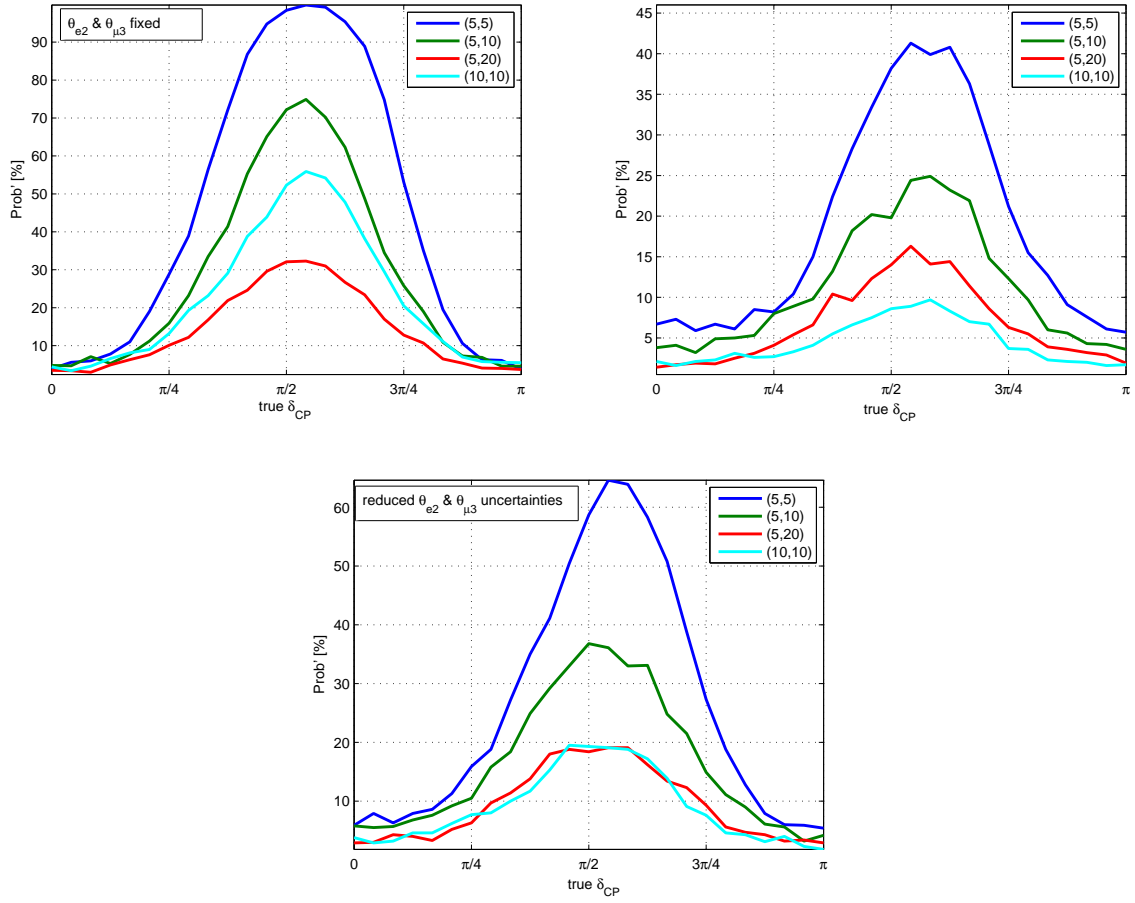


FIG. 6: Probability to exclude CP conservation with 90%CL. The three panels differ in the uncertainties attributed to  $\theta_{12}$  and  $\theta_{23}$  (see Table I): (upper left) Zero uncertainties; (upper right) Present uncertainties; (bottom) ‘Future’ uncertainties.