On the origin of thermal string gas

Kari Enqvist, Niko Jokela, Esko Keski-Vakkuri and Lotta Mether

Department of Physical Sciences, University of Helsinki and Helsinki Institute of Physics,P. O. Box 64, FIN-00014 University of Helsinki, Finland.

Abstract. We investigate decaying D-branes as the origin of the thermal string gas of string gas cosmology. We consider initial configurations of low-dimensional branes and argue that they can time evolve to thermal string gas. We find that there is a range in the weak string coupling and fast brane decay time regimes, where the initial configuration could drive the evolution of the dilaton to values, where exactly three spacelike directions grow large.

E-mail: kari.enqvist@helsinki.fi, niko.jokela@helsinki.fi, esko.keski-vakkuri@helsinki.fi,lotta.mether@helsinki.fi

1. Introduction

The origin of the primordial perturbations is one of the key problems in cosmology. A scale invariant spectrum could indicate an early de Sitter universe with a large cosmological constant, but the latest WMAP data [1] with a spectral index of $n_s =$ 0.958 ± 0.016 very much favors a dynamical origin. There should be an order parameter, a field, whose value changes during inflation and thus gives rise to a slight deviation from the exact scale invariance. This is of course what slow-roll inflation models [2] are famous for, but there could be alternatives to scalar field inflation. In particular, string theoretical effects could manifest themselves in the early universe, making the primordial perturbation a testing ground for fundamental physics. Attempts along these lines include pre-Big Bang cosmology [3] and various scenarios that involve dynamics of branes 4, as well as string gas cosmology 5, which assumes that the early universe consisted of a collection of strings in thermal equilibrium. String gas cosmology originates in an early proposal of Brandenberger and Vafa [6] for a natural explanation of the dimensionality of spacetime. Since then, the scenario has been developed in various directions and the framework has been applied to address other cosmological issues such as dark matter and, most recently, structure formation [7].

String gas cosmology focuses in particular on the effect of the string winding degrees of freedom and the implications of T-duality. The basic postulate of the model is that the universe starts out as a small compact space, usually taken to be a string size torus for simplicity, which is filled with a thermal gas of closed strings at a temperature near the Hagedorn temperature T_H , which is believed to be the maximal temperature for a perturbative string ensemble. Already a simple qualitative consideration of the string gas scenario has some very fundamental implications on the picture of the early universe. The degrees of freedom in the string gas consist of the three types of closed string modes: momentum modes (describing the center of mass motion of strings), winding modes (expressing the number of times a string is wound around a given torus one cycle), and oscillatory modes. The energy of momentum modes is quantized in units of the inverse torus radii, while that of winding modes is directly proportional to the radii and the oscillatory modes have radius independent energies. Thus, the string spectrum is invariant under T-duality, *i.e.*, the inversion of the torus radii, along with the interchange of the corresponding momentum and winding quantum numbers. This symmetry implies the existence of an effective minimal length scale, which in turn suggests a resolution of both the spatial and temperature singularities of the early universe.

But what could be the origin of the thermal string gas? Let us break the question of the initial conditions into two subquestions: (i) the origin of the spacetime itself, and (ii) the origin of the matter or energy in the spacetime. The first one is the question of how to resolve the initial spacelike Big Bang singularity. The most famous scenario is the no-boundary proposal [8], but there have been other more recent approaches. We only mention the idea of disappearance of space by closed string tachyon condensation, the time reversed process interpreted as emergence of space from nothing [9] (see also [10]). This scenario has recently been connected with string gas cosmology in [11]. In this paper we address the second question. How did the hot string gas come to exist – what creates the flux of energy from the initial singularity?‡

In a recently proposed scenario, energy in a spacetime can originate from a decaying brane, for which an initial condition can be prepared at the origin of time [14]. Loosely speaking, the spacetime comes to existence with a great amount of stored energy which gets released immediately. It is also conceptually interesting that brane decay is an example of a process which can be replaced by a microscopic statistical model, where time is absent and thus "time evolution" can be modeled as an emergent concept [15].

In the present paper we study specifically if a thermal gas of closed strings could arise as a consequence of the decay of unstable D-branes. This then provides a specific initial condition scenario for the thermal string gas in string gas cosmology at a *finite* origin of time (rather than introducing the thermal gas by *fiat*). One virtue of the scenario is that there is freedom in choosing the details of the initial condition, and this is reflected in the subsequent development. For example, a space filling brane appears to produce string-brane gas [16], while a configuration of lower dimensional branes can produce pure string gas. The details of the thermalization leave room for more detailed analysis and possible interesting effects. We focus on the time evolution of the dilaton and the string coupling. We find that the evolution is internally consistent and can lead to favorable values of the dilaton for three dimensions to grow large.

There then exist two complementary scenarios: emergence of spacetime by closed string tachyon condensation, and emergence of matter in the spacetime by open string tachyon condensation (brane decay). Future work will show if and how the two can be fused together for a complete scenario of initial conditions.

The plan of the paper is as follows. In Section 2 we briefly recapitulate the relevant features of string gas cosmology pertaining the dilaton. In Section 3 we consider unstable brane configurations as candidates of initial state. Section 4 investigates the time evolution of the dilaton – internal consistency conditions and the question of whether favorable values can be reached naturally from the brane initial state.

[‡] Note that the second paper in [11] also addresses this question and presents an argument for creation of a thermal distribution. However, the approach in [11] is very different from ours. For an oscillatory scenario, see [12]; for one involving branes but which differs from the setup in the present paper, see [13].

2. String gas cosmology and dilaton gravity

String gas cosmology is usually discussed within the framework of dilaton gravity, which is the simplest modification of Einstein gravity that respects the T-duality symmetry. The replacement of the full string theory by this low-energy effective theory is motivated by making the cosmologically not unreasonable assumption of slowly varying fields (*i.e.*, the adiabatic approximation), which allows one to stay at tree level in α' . Furthermore weak string coupling (*i.e.*, $g_s \ll 1$) is assumed. Let us recapitulate its main features.

Assuming no flux and critical string dimension, the dilaton gravity action in the string frame reads

$$S_{\text{string}} = \frac{1}{2\kappa_D^2} \int d^D x \sqrt{-G} e^{-2\phi} \left(R + 4(\nabla\phi)^2 \right) \,, \tag{1}$$

where κ_D is the *D*-dimensional reduced gravitational constant, *D* being the spacetime dimension, *G* is the determinant of the background spacetime metric, *R* the usual *D*dimensional Ricci scalar, and ϕ the dilaton. Assuming furthermore as in FRW cosmology a homogeneous spacetime, the background fields *G* and ϕ are at most functions of time, and the background metric can be written in the familiar form $ds^2 = -dt^2 + \sum_i a_i^2(t) dx_i^2$. Rewritten in terms of the number of e-folds $N_i = \log a_i(t)$ and the shifted dilaton $\varphi \equiv 2\phi - \sum_i N_i$, the action (1) takes a form that is manifestly invariant under the T-duality transformation $N_i \to -N_i$, $\varphi \to \varphi$, $(\phi \to \phi - N_i)$. When coupled to the matter action of a gas of free strings

$$S_m = \int dt \sqrt{-G_{00}} \ F(N_i, \beta \sqrt{-G_{00}}), \tag{2}$$

where F is the string gas free energy, and varied with respect to the fields, the dilaton gravity action yields the evolution equations of string gas cosmology [17], given by

$$\dot{\varphi}^2 - \sum_{i=1}^{D-1} \dot{N}_i^2 = e^{\varphi} E \tag{3}$$

$$\ddot{N}_i - \dot{\varphi}\dot{N}_i = \frac{1}{2}e^{\varphi}P_i \tag{4}$$

$$\ddot{\varphi} - \sum_{i=1}^{D-1} \dot{N}_i^2 = \frac{1}{2} e^{\varphi} E \quad .$$
(5)

Here E = F + TS is the total energy of the string gas and $P_i = -\partial F / \partial N_i$ the pressure in the *i*-th direction multiplied by the total volume.

The cosmology that emerges from these equations is determined by the behavior of the string gas energy and pressure as functions of the scale factor. Due to the different form of the scale factor dependence of the energy levels of the winding and momentum modes, the winding modes give a negative contribution to the total pressure, while the momentum modes give a positive one. Hence Eq. (4) implies that winding modes tend to prevent expansion, whereas the momentum modes induce it. In the assumed initial state, where the universe is string scale sized and filled with a dense string gas with a temperature close to the Hagedorn temperature, the energy is nearly constant, $E \sim T_H S$, and the numbers of winding and momentum modes are equal, so that the total pressure vanishes. As a consequence the scale factor remains constant on average, making this initial phase a semi-stable period. The dilaton, however, is slowly decreasing, as will be discussed in more detail in Section 4. In the absence of winding modes, space is free to expand, in which case the temperature drops, so that the massive string modes eventually go out of equilibrium and the universe enters a standard radiation dominated era. During this period, the radii evolve as $a_i \sim t^{2/D}$ as is usual during radiation domination, while the original dilaton $\phi = (\varphi + \sum_i N_i)/2$ approaches a constant.

3. Unstable branes as the origin of the string gas

3.1. Brane decay and closed string emission

The D-branes of bosonic string theory are unstable. As a sign of this, the spectrum of open strings on a brane contains a tachyonic mode. Supersymmetric Type II theories also contain unstable (non-BPS) D-branes, and pairs of stable (BPS) D-branes of opposite charge become unstable at subcritical separation. The branes decay to closed strings which then interact and thermalize under suitable circumstances. Thus it is natural to consider them as an initial state for hot string gas. This is of course not enough – the question is what can be gained by introducing unstable branes?

An immediate bonus is that they provide at least one possible answer to some conceptual shortcomings of string gas cosmology. Since we believe that the Universe originates from a Big Bang, it has a finite history. While string gas cosmology may provide a finite history for three large dimensions, the gas itself is implicitly assumed to have an infinite history. On the other hand, for unstable branes it is possible to introduce initial conditions at some finite point in time. These initial conditions fall into three categories: (i) a scenario, where the unstable brane is first created as a condensate of incoming closed strings, (ii) a scenario, where the unstable brane pops out from imaginary time, (iii) a scenario, where the brane initial state is prepared by a complex time contour at the initial spacelike singularity.

The consequence of having different possibilities for controllable initial conditions for hot string gas is that one can ask if they will leave an imprint – even better, a signature of observable interest. But before getting there, one must examine some intrinsic consistency conditions in order to classify the allowed possibilities. This is the goal of the present paper.

Let us briefly review some facts relating to the decay of D-branes in bosonic string theory§. Assuming weak coupling $g_s \ll 1$, an unstable D-brane acts as a classical timedependent source for closed string fields. The final state for a *p*-dimensional brane is then a coherent state of closed strings [19],

$$|\psi\rangle \sim :\exp\left\{-i\sum_{s}\int d^{p+1}x \ J_s(x)\cdot\phi_s(x)\right\} : |0\rangle , \qquad (6)$$

where $J_s(x)$ are source terms for closed string fields $\phi_s(x)$, and the expression contains a sum over all possible fields. For the so called full brane decay with a finite characteristic time scale

$$\tau \sim -\ln[\sin(\pi\lambda)] \tag{7}$$

with a tunable parameter $0 \le \lambda \le 1/2$ controlling the lifetime, the source J_s in the exponent of eq. (6) becomes (after a Fourier transformation to energy coordinate)

$$\tilde{J}_s = \pi T_p \frac{\sin(E_s \ln(\lambda))}{\sinh(\pi E_s)} , \qquad (8)$$

where T_p is the tension of the *p*-dimensional brane. The tension is inversely proportional to the closed string coupling constant, so that at weak coupling the brane stores a large energy density.

Consider first non-compact space and branes, so that there are no winding modes and

$$E_s = E_s(N, k_\perp) = \sqrt{4l_s^{-2}(N-1) + \vec{k}_\perp^2} , \qquad (9)$$

where l_s is the string length. The total energy and number of emitted closed strings from the decay of a D*p*-brane are [19]

$$\frac{\bar{E}}{V_p} = \mathcal{N}_p^2 \sum_{N=0}^{\infty} d(N) \int \frac{d^{25-p} k_{\perp}}{(2\pi)^{25-p}} E_s(N, k_{\perp}) \bar{n}(N, k_{\perp})$$
(10)

$$\frac{\bar{N}}{V_p} = \mathcal{N}_p^2 \sum_{N=0}^{\infty} d(N) \int \frac{d^{25-p} k_\perp}{(2\pi)^{25-p}} \bar{n}(N, k_\perp) , \qquad (11)$$

where

$$\bar{n}(N,k_{\perp}) = \frac{|\tilde{J}_s|^2}{2E_s(N,k_{\perp})} , \qquad (12)$$

and the overall coefficient abbreviates $\mathcal{N}_p^2 = \pi^{11}(2\pi)^{2(6-p)}$. In (10) and (11) the sum is over all final closed string states of symmetric oscillator excitations between left- and right-moving sectors and d(N) is the density of states at level N.

 \S For an extensive review, we refer the reader to [18].

Note that the number distribution \bar{n} deviates from thermality. However, for large E_s ,

$$|\tilde{J}_s|^2 \sim e^{-2\pi E_s}$$
, (13)

so that in the decay the production of highest energy states is close to the thermal distribution at the Hagedorn temperature $T_H = 1/2\pi$. The total number and energy of strings produced in the decay depends on the contribution from the highest energies, due to the exponential growth of the density of left-right symmetric closed string states

$$d(N) \sim N^{-27/4} e^{4\pi\sqrt{N}}$$
 (14)

By using $E_s \sim 2\sqrt{N} + \frac{\vec{k}_{\perp}^2}{2\sqrt{N}}$ we can evaluate the large energy behavior of (10) and (11). After performing the momentum integrals one finds that the total amount of energy per unit *p*-volume carried by all the closed string modes emitted during the rolling of the tachyon [20] is infinite for $p \leq 2$. The reason for the divergence is the breakdown of perturbation theory. Since an unstable D*p*-brane has a finite energy, the total energy carried by the closed strings cannot really be infinite, so higher order corrections must cut it off so that the total emitted energy is finite.

On the other hand, finite result for $p \geq 3$ means that the single closed string channel does not carry away all of the initial energy of the brane. Since multi-string emission channels are suppressed by powers of the string coupling, the result means that higher dimensional branes do not decay completely – the final state contains a lower dimensional brane. Hence the decay must be inhomogeneous. The process is not fully understood at the moment [21].

If we have a D*p*-brane with all its tangential and perpendicular directions compactified on a torus, then it is related to the D0-brane via T-duality, and hence we expect that similar results will hold for this system as well. In particular since under a T-duality transformation momentum along a circle gets mapped to the winding charge along the dual circle, we expect that (for low-dimensional branes) all the energy of the D*p*-brane wrapped on a torus is converted into closed string radiation. In particular, most of the energy is stored in the highly wound closed string modes of mass $\sim g_s^{-1}$ [22].

3.2. A proposal for the initial state

For simplicity, we focus on bosonic string theory. Initially, all spacelike directions are compactified on the torus T^{25} with equal string scale radii in all directions. The most natural initial state would be a space filling unstable D25-brane (or a stack of them). However, as we have seen before, branes with $p \geq 3$ presumably decay into lower dimensional branes, and the process is poorly understood at the moment. When the decay process becomes better known, we expect that space filling branes could serve as an initial state for string-brane gas cosmology [16]. For a pure string gas without any branes, we are thus directed to consider lower dimensional branes which decay completely. A simple initial configuration consists of a D1-brane wrapped in direction X^1 , D1-brane wrapped in direction X^2 etc. up to D1brane wrapped in direction X^{25} . The energy gets released predominantly in the form of wound strings. The net winding number is zero if there is no background electric field on the branes. For homogeneity we assumed equal number of D1-branes at every direction, with all having the same λ (decay equally fast).

As mentioned in the introduction, there are different prescriptions for the brane decay (and the associated rolling of the tachyonic mode in its effective potential). One way is to first form the brane as a time reversed version of the decay. Another possibility is to adopt a complex time contour integration prescription for the computation of closed string production in the decay, essentially corresponding to nucleating the brane from imaginary time [19]. (However, at present time there exist no proposals for assigning a probability measure for the nucleation event.) In this case the spacetime is initially empty, until the brane suddenly appears and decays into closed strings. The third possibility is to use the time reflection symmetry of the tachyonic mode and identify the spacetime points under a (C)PT reflection into a Lorentzian orbifold. The resulting orbifold has an initial spacelike singularity, where the brane can be prepared to nucleate by a variant of the complex contour integration argument of [19] (see [14] for more discussion).

The decay can be adjusted by the parameter $0 \le \lambda \le 1/2$ which controls the lifetime (7). However, for a generic choice of λ , the lifetime is very short, not too different from the string scale. Therefore the brane decay is more like an explosion. Furthermore, the spacetime stress tensor for the outgoing closed strings (also called tachyon matter [23]) has been computed and is known to quickly settle to zero total pressure [18], just as in the initial phase of string gas cosmology. Hence it is also natural to assume that during the brane decay the volume of the spacelike torus is essentially unchanged.

In the analysis of the decay, one assumes zero coupling for the closed strings. This is of course an idealization, and the strings will interact and backreact to the decay. Furthermore, the interactions will quickly drive the end state of the decay towards thermality. The thermalization time scale $t_{\rm th}$ is estimated by

$$t_{\rm th} \simeq (\sigma \bar{n} v_\perp)^{-1} \simeq \frac{\sqrt{l_s E_s}}{g_s^4} l_s^{-1} \gg \tau \simeq l_s^{-1} , \qquad (15)$$

where \bar{n} is the string density (formally divergent by (11), but regulated to be $\sim l_s^{-3}$), σ is the cross section for string interactions, $\sim g_s^4 l_s^2$ for an interaction with a two-string final state [24, 25], and $v_{\perp} \sim 1/\sqrt{l_s E_s}$ is the velocity of the slowly moving heavy strings of mass E_s . (An inspection of the distribution of the produced closed strings indicates that it is not very far from thermal distribution at Hagedorn temperature, to start with.) This crude estimate of the string interaction rate suggests that in the weak coupling limit the thermalization timescale is much longer than the brane decay timescale. Hence brane decay and the subsequent thermalization of closed strings can be discussed separately. After the thermalization, the standard string cosmological evolution as described in (3)-(5) presumably takes over. Obviously we are assuming that the earlier stages, brane decay and the dynamics of thermalization, have a minimal effect on the initial thermal state of the string gas. This could be challenged in many ways. Even so, there is at least one interesting effect to address.

During the decay, the brane sources the low-energy effective fields, in particular the dilaton time evolves. One might worry that the dilaton ends up being too large contradicting the initial assumption of weak string coupling and insignificant backreaction. More importantly for the string gas, previous studies of the evolution equations (3)-(5) have found that there is a narrow window for preferred initial conditions of the dilaton in order to avoid too early freeze-out or too many large space directions in the end [26, 25]. An interesting potential application of the added brane decay stage would be to drive the dilaton into the preferred initial range from generic initial values.

4. Dilaton time evolution and the dimensionality of spacetime

4.1. Time evolution of the dilaton during brane decay

So far we have argued that our proposed setup of decaying branes produces a universe that looks qualitatively like the initial state of string gas cosmology. In addition, we need to make sure that the process is internally consistent with respect to the weak coupling assumption, and that it creates a final state that fulfills the adiabatic and weak coupling assumptions of dilaton gravity.

During the brane decay, the evolution of the dilaton is governed by an equation of motion, which is given by the details of the decay scenario. After the decay, there follows a period of thermalization, during which the exact evolution of the dilaton is unknown. However, one can argue that the explicit form of the distribution of the background strings has a only a small effect on the propagation of the dilaton. What counts are the ensemble averaged quantities such as the mean comoving energy of the gas, which does not change during thermalization. Hence, we are led to assume that the dilaton should reach values that are consistent with the dilaton gravity era already at the end of the brane decay process.

Assuming any backreaction that the produced strings might generate during the decay process is negligible (as argued in the preceding Section 3.2, this is the case at

least for thermal backreaction), the dilaton equation of motion during brane decay reads

$$-\partial_t^2 \phi = a \Big[\frac{1}{1 + \hat{\lambda} e^t} + \frac{1}{1 + \hat{\lambda} e^{-t}} - 1 \Big] , \qquad (16)$$

where $\hat{\lambda} = \sin \pi \lambda$ and *a* is a positive constant, which is related to the initial tension of the unstable brane. From now on we set a = 1. Solving Eq. (16) the time dependence of the dilaton is found to be

$$\phi(t) = Li_2(-\hat{\lambda}^{-1}e^t) - Li_2(-\hat{\lambda}e^t) + C_1t + C_2 , \qquad (17)$$

where $Li_2(z)$ is the dilogarithm, and C_1 and C_2 are constants of integration. In our notation, the branes begin to decay at t = 0 and have lifetimes of $\tau = -\log \hat{\lambda}$, which are input parameters. The constants C_1 and C_2 in Eq. (17) are determined by assigning to $\phi(t)$ and $\dot{\phi}(t)$ some initial or final values (corresponding to initial values for the string gas cosmology era).

The dilaton gravity approximation is valid for $\dot{\varphi}(t) \gtrsim -1$ [26] and, in addition, there are some constraints on the dilaton following from the equations of motion (3)-(5). Firstly, since the energy E is positive, Eq. (3) implies that the dilaton time derivative $\dot{\varphi}$ can never reach zero and must be strictly positive or strictly negative at all times. Usually $\dot{\varphi} < 0$ is assumed, since $\dot{\varphi} > 0$ could spoil the adiabatic and weak coupling assumptions. Secondly, it follows from Eq. (5) that $\ddot{\varphi} > 0$, and thus $\dot{\varphi}$ grows with time and approaches 0 as $t \to \infty$. Thus it is relevant to consider final values for the dilaton time derivative in the range $-1 \lesssim \dot{\varphi}(\tau) < 0$. Note that the dilaton solution above is for the string frame dilaton ϕ , whereas these values are for the shifted dilaton $\varphi = 2\phi - \sum_i N_i$. Given that the brane decay is very fast, $\tau \simeq l_s^{-1}$, it is nevertheless reasonable to assume that the radii stay fixed during the brane decay, in which case these are simply related as $\dot{\varphi}(\tau) = 2\dot{\phi}(\tau)$. As for the value of the dilaton itself, both the brane decay process and the string gas cosmology scenario require it to be negative, in order to preserve weak coupling.

In Figs. 1 and 2, we have plotted the dilaton evolution during brane decay for different initial values and for different final values of its derivative, respectively. The figures address the consistency of the weak string coupling. As shown in Fig. 1, starting from large negative values of the dilaton, the typical decay time (here $5t_s$) is too short for the dilaton to grow significantly. On the other hand, Fig. 2 shows that different choices for dilaton time derivatives at the end of the decay cause no significant change in the evolution either. Fig. 3 illustrates the dependence of the dilaton evolution on the lifetime of the decaying branes.

4.2. Can the dilaton evolution lead to three large dimensions?

String gas cosmology has been advocated as a mechanism that dynamically generates a universe with precisely three large spatial dimensions [6, 17]. This argument is based on



Figure 1. $\phi(t)$ plotted for initial values $e^{\phi(0)} \simeq 1$, 10^{-4} , 10^{-8} and 10^{-12} , with lifetime $\tau = 5t_s$ and $\dot{\phi}(\tau) = -1$.



Figure 2. $\phi(t)$ plotted for $\dot{\varphi}(\tau) = 0, -0.25, -0.5, -0.75$ and -1, where the uppermost curve corresponds to $\dot{\varphi}(\tau) = 0$. Here we have chosen $\phi(0) = -20$, corresponding to $e^{\phi(0)} = 10^{-8}$, and $\tau = 5t_s$.

the observation that the winding modes give a negative contribution to the string gas pressure, and thus oppose expansion. In order for a spatial dimension to be able to grow large, the winding modes wrapped around this dimension must therefore be annihilated by intersecting with winding modes of opposite orientation. Since string worldsheets are two-dimensional, the argument goes, a pair of strings have non-zero probability of intersecting only in four or less spacetime dimensions, so that at most three spatial dimensions can grow large. The conclusion of this qualitative consideration has been confirmed in various studies [27, 16], but when cosmological dynamics, in particular the



Figure 3. $\phi(t)$ plotted for $\tau = 1, 5, 7$ and 9, with initial value $\phi(0) = -20$ and $\dot{\varphi}(\tau) = -1$. Note that each curve ends at the value of t corresponding to the decay time τ .

effect of the dilaton, are taken into consideration, the simple argument no longer holds.

In a numerical study of the Boltzmann equations governing the string annihilation in a dilaton gravity background [26], it was found that the coupling to the dilaton, which is rolling towards weak coupling, in general causes the strings to freeze out too fast for the anisotropic annihilation to take place. In particular, for initial conditions that admit a large number of winding modes (*i.e.*, for small initial values of the dilaton), the strings tend to freeze out so that all dimensions remain small, whereas for a small initial number of winding modes (large dilaton) all strings typically annihilate and the whole compact space grows large. Only for a very narrow range of intermediate initial conditions is it likely that three dimensions grow large.

In the previous Section, we studied the evolution of the dilaton during brane decay, and found that it can indeed lead to values that are consistent with the dilaton gravity picture. Let us now take one step further and investigate under what circumstances it might lead to the values favored for three dimensions to grow large. As argued above, it is reasonable to assume that the dilaton gravity era and the study of the Boltzmann equations set in just after the branes have decayed. In the analysis of [26], the direction of time is chosen so that $\dot{\varphi} < 0$, as is done here. Furthermore, the initial value of the derivative is chosen to be $\varphi(\tau) = -1$, which is just the borderline value allowed by the dilaton gravity approximation. In this case, there is a gap of initial values $\Delta \varphi(\tau) \simeq 0.5$, around the value $\varphi(\tau) \sim -2.5$, for which three dimensions are likely to become unwrapped [26].

In Fig. 4, we have plotted a number of combinations of $\phi(t)$ and τ , all of which



Figure 4. $\phi(t)$ plotted for initial values $\phi(0) = -0.5, -6, -15, -27$, with $\tau = 1, 5, 7, 9$, correspondingly, and $\dot{\varphi}(\tau) = -1$.

lead the dilaton to values within the favored range. Thus we see that it is possible to reach the favored range of initial values for a variety of decay configurations. In particular, the figure shows that one can start from very weak coupling and still reach the preferred range for a decay time $\tau \simeq 10t_s$ that is short in comparison to the time of thermalization, which once again verifies that the decay may be discussed separately from the other dynamics. Thus we may conclude that in our scenario the somewhat arbitrary fine-tuning of the dilaton initial value that is required in the study of [26] in order to generate three large dimensions, is given a more physical interpretation in terms of the lifetimes of the unstable branes.

5. Discussion

In this paper we discussed the issue of the origin of thermal string gas, whose existence is usually assumed in string gas cosmology. We studied low-dimensional unstable Dbranes wrapped on a torus in bosonic string theory, and found that the string gas is naturally produced by the decay process of the branes. There are several appealing features in brane decay that will carry over to string gas cosmology. For this scenario to hold up, however, it was essential to establish that the model is internally consistent. We focused on the time evolution of the dilaton and the string coupling, and found that the evolution is internally consistent and can lead to favorable values of the dilaton for three dimensions to grow large. Brane decay thus provides a natural initial condition for the string gas, and in particular, the initial values of the dilaton can now be given a physical interpretation in terms of the lifetimes of the decaying branes. While such features certainly seem promising, it should be borne in mind that there are a number of other open issues that must be resolved in order to fully understand the remnant of brane decay. We initially assume a weak string coupling so that any backreaction can be neglected and focus only on low-dimensional branes that are wrapped on a torus with all its radii equal. For homogeneity, we made the assumption that there is an equal number of branes at every direction of the torus, each decaying equally fast. Lifetime being of order string time, we furthermore assumed that the radii stay fixed during the decay process. We then found that during the decay, the dilaton grows and the interactions with emitted closed strings should be taken into account. By estimating the order of magnitude of the string scattering rate we are led to argue that the thermalization timescale is much longer than the decay time, and hence treating the brane decay and string dynamics separately is justified. Obviously these assumptions can be challenged in many ways.

An interesting aspect of our proposal is the origin of the entropy of the string gas. The unstable brane is a coherent state (6), which is a pure state. (More precisely, for the collection of 25 D1-branes each wound around a different spacelike circle, the state is a superposition of the coherent states of each brane, but still a pure state.) On the other hand the state can be expanded in the basis of the closed string modes, $|\psi\rangle = \sum_s A_s |\psi\rangle_s$, where the coefficients A_s give probability amplitudes for decaying into a closed string mode ψ_s . The density matrix ρ_0 of the state thus has the expansion $\rho_0 = \sum_{rs} A_r A_s^* |\psi_r\rangle \langle \psi_s|$. On the other hand, the thermal gas of strings in the end is described by the usual canonical ensemble density matrix, a mixed state with a diagonal expansion in the above basis.

A natural suggestion for making contact with the mixed state of the thermal gas is to adopt a variant of the coarse-graining proposal of [28], which considered gravitons produced as a squeezed (pure) state out of initial vacuum. Applying a random phase approximation eliminates the off-diagonal elements of the density matrix, leaving a diagonal mixed ensemble with the entropy depending on the average occupation numbers per mode according to the familiar formula, and reducing to the thermal entropy as the occupation number spectrum thermalizes.

In our context, we correspondingly assume that the phases of the off-diagonal elements are randomly distributed and averaged out. This leaves only the diagonal elements, and the density matrix will reduce to that of a thermal canonical ensemble as the string gas thermalizes. We leave this and other straightforward extensions for future work.

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References

- [1] D. N. Spergel *et al.*, arXiv:astro-ph/0603449.
- [2] For a recent review, see A. Linde, arXiv:0705.0164 [hep-th].
- [3] For a recent review, see M. Gasperini and G. Veneziano, arXiv:hep-th/0703055.
- [4] For a review, see F. Quevedo, Class. Quant. Grav. 19 (2002) 5721 [arXiv:hep-th/0210292].
- [5] For a review, see R. H. Brandenberger, arXiv:hep-th/0509099.
- [6] R. H. Brandenberger and C. Vafa, Nucl. Phys. B 316 (1989) 391.
- [7] T. Battefeld and S. Watson, Rev. Mod. Phys. 78 (2006) 435 [arXiv:hep-th/0510022];
 R. H. Brandenberger, arXiv:hep-th/0702001.
- [8] J. B. Hartle and S. W. Hawking, Phys. Rev. D 28, 2960 (1983); See also A. Vilenkin, arXiv:gr-qc/0204061, and references therein.
- J. McGreevy and E. Silverstein, JHEP 0508, 090 (2005) [arXiv:hep-th/0506130]; Y. Nakayama,
 S. J. Rey and Y. Sugawara, arXiv:hep-th/0606127; S. Hellerman and I. Swanson,
 arXiv:hep-th/0611317.
- [10] S. Hellerman and X. Liu, arXiv:hep-th/0409071; S. Hellerman Swanand I. son, arXiv:hep-th/0612051; H. Yang and B. Zwiebach, 0508, JHEP 046 (2005)[arXiv:hep-th/0506076]; Η. Yang and B. Zwiebach, JHEP 0509, 054(2005)[arXiv:hep-th/0506077].
- [11] R. H. Brandenberger, A. R. Frey and S. Kanno, arXiv:0705.3265 [hep-th]; R. H. Brandenberger, A. R. Frey and S. Kanno, arXiv:0706.1104 [hep-th].
- [12] T. Biswas, R. Brandenberger, A. Mazumdar and W. Siegel, arXiv:hep-th/0610274.
- [13] B. D. Chowdhury and S. D. Mathur, Class. Quant. Grav. 24 (2007) 2689 [arXiv:hep-th/0611330].
- [14] S. Kawai, E. Keski-Vakkuri, R. G. Leigh and S. Nowling, Phys. Rev. Lett. 96 (2006) 031301 [arXiv:hep-th/0507163].
- [15] V. Balasubramanian, N. Jokela, E. Keski-Vakkuri and J. Majumder, Phys. Rev. D 75, 063515 (2007) [arXiv:hep-th/0612090].
- [16] S. Alexander, R. H. Brandenberger and D. Easson, Phys. Rev. D 62, 103509 (2000) [arXiv:hep-th/0005212].
- [17] A. A. Tseytlin and C. Vafa, Nucl. Phys. B **372** (1992) 443 [arXiv:hep-th/9109048].
- [18] A. Sen, Int. J. Mod. Phys. A 20 (2005) 5513 [arXiv:hep-th/0410103].
- [19] N. Lambert, H. Liu and J. M. Maldacena, arXiv:hep-th/0303139.
- [20] A. Sen, JHEP **0204** (2002) 048 [arXiv:hep-th/0203211].
- [21] A. Sen, Int. J. Mod. Phys. A 14 (1999) 4061 [arXiv:hep-th/9902105]; J. Majumder and
 A. Sen, JHEP 0006 (2000) 010 [arXiv:hep-th/0003124]; A. Sen, JHEP 0210 (2002) 003

[arXiv:hep-th/0207105]; F. Larsen, A. Naqvi and S. Terashima, JHEP **0302** (2003) 039 [arXiv:hep-th/0212248].

- [22] P. Mukhopadhyay and A. Sen, JHEP 0211 (2002) 047 [arXiv:hep-th/0208142]; S. J. Rey and S. Sugimoto, Phys. Rev. D 67 (2003) 086008 [arXiv:hep-th/0301049]; K. Nagami, JHEP 0401 (2004) 005 [arXiv:hep-th/0309017]; M. Gutperle and P. Yi, JHEP 0501 (2005) 015 [arXiv:hep-th/0409050].
- [23] A. Sen, JHEP 0207 (2002) 065 [arXiv:hep-th/0203265]; A. Sen, Mod. Phys. Lett. A 17 (2002) 1797 [arXiv:hep-th/0204143]; A. Sen, Phys. Scripta T117 (2005) 70 [arXiv:hep-th/0312153].
- [24] J. Polchinski, Phys. Lett. B 209 (1988) 252; D. A. Lowe and L. Thorlacius, Phys. Rev. D 51 (1995) 665 [arXiv:hep-th/9408134]; S. Lee and L. Thorlacius, Phys. Lett. B 413 (1997) 303 [arXiv:hep-th/9707167].
- [25] R. Danos, A. R. Frey and A. Mazumdar, Phys. Rev. D 70 (2004) 106010 [arXiv:hep-th/0409162].
- [26] R. Easther, B. R. Greene, M. G. Jackson and D. Kabat, JCAP 0502 (2005) 009 [arXiv:hep-th/0409121].
- [27] M. Sakellariadou, Nucl. Phys. B 468, 319 (1996) [arXiv:hep-th/9511075].
- [28] T. Prokopec, Class. Quant. Grav. 10, 2295 (1993); R. H. Brandenberger, V. F. Mukhanov and T. Prokopec, Phys. Rev. Lett. 69, 3606 (1992) [arXiv:astro-ph/9206005]; A. Albrecht, P. Ferreira, M. Joyce and T. Prokopec, Phys. Rev. D 50, 4807 (1994) [arXiv:astro-ph/9303001].