Holographic interacting dark energy in the braneworld cosmology

Kyoung Yee Kim, Hyung Won Lee and Yun Soo Myung[*]

Institute of Mathematical Sciences and School of Computer Aided Science
Inje University, Gimhae 621-749, Korea

Abstract

We investigate a model of brane cosmology to find a unified description of the radiation-matter-dark energy universe. It is of the interacting holographic dark energy with a bulk-holographic matter χ. This is a five-dimensional cold dark matter, which plays a role of radiation on the brane. Using the effective equations of state $\omega_\Lambda^{\text{eff}}$ instead of the native equations of state ω_Λ, we show that this model cannot accommodate any transition from the dark energy with $\omega_\Lambda^{\text{eff}} \geq -1$ to the phantom regime $\omega_\Lambda^{\text{eff}} < -1$. Furthermore, the case of interaction between cold dark matter and five dimensional cold dark matter is considered for completeness. Here we find that the redshift of matter-radiation equality z_{eq} is the same order as $z_{\text{eq}}^{\text{ob}} = 2.4 \times 10^4 \Omega_{\text{m}} h^2$. Finally, we obtain a general decay rate Γ which is suitable for describing all interactions including the interaction between holographic dark energy and cold dark matter.

*e-mail address: ysmyung@inje.ac.kr

1 Introduction

Recent observations from Supernova (SN Ia) [1] and large scale structure [2] imply that our universe is accelerating. Also cosmic microwave background observations [3, 4] provide an evidence for the present acceleration. A combined analysis of cosmological observations shows that the present universe consists of 70% dark energy and 30% dust matter including cold dark matter (CDM) and baryons.

Although there exist a number of dark energy models, a promising candidate is the cosmological constant. However, one has the two famous cosmological constant problems: the fine-tuning and coincidence problems. In order to solve the first problem, we may introduce a dynamical cosmological constant model inspired by the holographic principle. The authors in [5] showed that in quantum field theory, the UV cutoff Λ could be related to the IR cutoff L_Λ due to the limit set by introducing a black hole (the effects of gravity). In other words, if $\rho_\Lambda = \Lambda^4$ is the vacuum energy density caused by the UV cutoff, the total energy of system with the size L_Λ should not exceed the mass of the black hole with the same size L_Λ: $L_\Lambda^3 \rho_\Lambda \leq 2M_p^2 L_\Lambda$. If the largest cutoff L_Λ is chosen to be the one saturating this inequality, the holographic energy density (HDE) is given by $\rho_\Lambda = 3c^2 M_p^2/8\pi L_\Lambda^2$ with a constant c. The lower limit of c is protected as $c \geq 1$ by the entropy bound. Here we regard ρ_Λ as a dynamical cosmological constant. Taking the IR cutoff as the size of the present universe ($L_\Lambda = 1/H$), the resulting energy is close to the present dark energy [6]. However, this approach with $L_\Lambda = 1/H$ is not fully satisfied because it fails to recover the equation of state (EoS) for the dark energy-dominated universe [7]. Further studies in [8, 9, 10, 11] have shown that choosing the future event horizon as the IR cutoff determines an accelerating universe with the native EoS $\omega_\Lambda \equiv -1/3(1 + d\ln\rho_\Lambda/d\ln a) = -1/3 - 2\sqrt{\Omega_\Lambda}/3c$.

Also if the interaction is turned on, the coincidence problem could be resolved [12]. The interacting dark energy models provided a new direction to understand the dark energy [13, 14, 15]. The authors in [16] introduced an interacting holographic dark energy model where an interaction exists between HDE and CDM. They derived the phantom-phase of $\omega_\Lambda < -1$ using ω_Λ. However, it turned out that the interacting holographic dark energy model could not describe a phantom regime when using the effective equation of state $\omega_\Lambda^{\text{eff}}$ [17]. More recently, it was shown that for non-flat universe of $k \neq 0$ [18, 19], the interacting holographic dark energy model could not describe a phantom regime of $\omega_\Lambda^{\text{eff}} < -1$ [20]. In Ref.[21], the authors discussed the cosmological dynamics of interacting holographic dark energy model using the phase-space variables. A key of this system is an interaction between two matters. Their contents are changing due to energy transfer from

HDE to CDM until the two components are comparable. If there exists a source/sink in the right-hand side of the continuity equation, we must be careful to define its EoS. In this case, the effective EoS is the only candidate to represent the state of the mixture of two components arisen from decaying of HDE into CDM. This is clearly different from the non-interacting case which can be described by the native EoS ω_Λ completely.

On the other hand, if the brane cosmology is introduced, one could have interesting interaction between bulk and brane matters. In the low energy limit, the brane cosmology reduces to the Friedmann-Robertson-Walker (FRW) form with a bulk-holographic matter χ. This is just a five dimensional cold dark matter(5DCDM) which play a role of a four-dimensional radiation when using the effective EoS approach. Then a unified description of radiation-matter-dark energy universe could be performed within the brane cosmology. Here we obtain two kinds of interaction: HDE-5DCDM and CDM-5DCDM. The first interaction may be allowed because one may allow the interaction of HDE with radiation. However, the latter seems not to be permitted because we assume that the CDM is not a source of radiation and it does not interact with the radiation. However, we suggest that the two interactions are possible to occur within the brane cosmology.

Concerning the brane-bulk interaction, there were contradictions: if one uses the effective EoS of w_{de}^{eff} [1], a transition occurs between $w_{de}^{eff} > -1$ and $w_{de}^{eff} < -1$ [22]. On the other hand, using w_Λ^{eff}, it was shown that such a transition does not occur [23]. In this work, we wish to address this issue again. We solve three coupled differential equations for density parameters Ω_i numerically by assuming three interactions between them. Furthermore, we introduce three types of the decay rate Γ to find the dark energy-dominated evolution on the brane. We confirm that any phantom-phase is not found on the brane.

2   Brane-bulk interaction model

Generalization of the Randall-Sundrum scenario [24] in cosmology considers the AdS_5 geometry containing the bulk cosmological constant Λ, but explores arbitrary energy densities on the brane and in the bulk. The Binetruy-Deffayet-Langlois (BDL) approach is a genuine extension of the Kaluza-Klein cosmology to account for the local distribution on the brane [25]. In this case, the location of the brane is fixed with respect to the bulk direction. This approach is useful for describing the cosmological evolution of the brane

---

[1]The authors in [22] use a different definition $w_{de}^{eff} = -1 - \frac{1}{3}\frac{d\ln(\delta H^2)}{d\ln a}$ from our definition w_Λ^{eff}. Here $\delta H^2 = H^2/H_0^2 - \Omega_m/a^3$ accounts for all terms in the Friedmann equation not related to the brane matter Ω_m. The $w_{de}^{eff} = -1$ crossing is achieved by considering the brane-bulk interaction without specifying dark energy as holographic dark energy.

when a brane-bulk interaction exists. Hence, we follow the BDL brane cosmology. We introduce the gaussian-normal bulk metric for $(1+3+1)$-dimensional spacetime

$$ds^2_{BDL} = -c^2(t,y)dt^2 + a^2(t,y)\gamma_{ij}dx^i dx^j + b^2(t,y)dy^2, \tag{1}$$

where γ_{ij} is the metric of a three-dimensional space with a constant curvature of $6k$. Let us express the bulk Einstein equation $G_{MN} = \frac{1}{2M^3}T_{MN}$ in terms of the BDL metric [2]. We introduce a $(1+3)$-dimensional brane located at $y=0$. For simplicity, we choose the total stress-energy tensor $T^M{}_N = \text{diag}(-\Lambda, -\Lambda, -\Lambda, -\Lambda, -\Lambda) + \tilde{T}^M{}_N + \tau^\mu{}_\nu$. Here Λ is the bulk cosmological constant and the bulk stress-energy tensor $\tilde{T}^M{}_N$ from $\tilde{\mathcal{L}}^{mat}_B$ is not needed to have a specific form initially. If $\tilde{T}^t{}_y = 0$, it is obvious that there is no brane-bulk interaction. The brane stress-energy tensor from $\mathcal{L}^{mat}_b$ including the brane tension σ and the energy density ρ is assumed to take the form

$$\tau^\mu{}_\nu = \frac{\delta(y)}{b}\text{diag}(-\rho - \sigma, p - \sigma, p - \sigma, p - \sigma, 0). \tag{2}$$

We are interested in solving the Einstein equations at the location of the brane. Initially we indicate by the subscript "0" for the value of various quantities on the brane. Also it is convenient to choose the gaussian-normal gauge with $b_0 = 1$ and the temporal gauge with $c_0 = 1$ on the brane. We obtain from $G_{0y} = \frac{1}{2M^3}T_{0y}$,

$$\dot{\rho} + 3\frac{\dot{a}_0}{a_0}\rho(1+\omega) = -2\tilde{T}^0{}_y. \tag{3}$$

Here we assumed an equation of state $p = \omega\rho$ on the brane.

On the other hand, the average part of yy-component equation is given by

$$\frac{\ddot{a}_0}{a_0} + \left(\frac{\dot{a}_0}{a_0}\right)^2 + \frac{k}{a_0^2} = \frac{1}{6M^3}\left(\Lambda + \frac{\sigma^2}{12M^3}\right) - \frac{1}{144M^6}\left(\sigma(3p-\rho) + \rho(3p+\rho)\right) - \frac{1}{6M^3}\tilde{T}^y{}_y. \tag{4}$$

Then, we rewrite Eq.(4) in the following equivalent form by introducing the two bulk-holographic energy densities $\tilde{\chi}$ and ϕ:

$$H_0^2 = \frac{1}{144M^6}\left(\rho^2 + 2\sigma\rho\right) + \tilde{\chi} + \phi + \frac{1}{12M^3}\left(\Lambda + \frac{\sigma^2}{12M^3}\right) - \frac{k}{a_0^2}, \tag{5}$$

$$\dot{\tilde{\chi}} + 4H_0\tilde{\chi} = \frac{1}{36M^6}(\rho + \sigma)\tilde{T}^0{}_y, \tag{6}$$

$$\dot{\phi} + 4H_0\phi = -\frac{1}{3M^3}H_0\tilde{T}^y{}_y, \tag{7}$$

---

[2]Our action is given by $S_5 = \int d^5x\sqrt{-g}\left(M^3 R - \Lambda + \tilde{\mathcal{L}}^{mat}_B\right) + \int d^4x\sqrt{-\hat{g}}\mathcal{L}^{mat}_b$ with $M^3 = 1/16\pi G_5 = 1/2\kappa_5^2$ and $\mathcal{L}^{mat}_b = -(\sigma + \rho)$ [26].

with $H_0 = \dot{a}_0/a_0$. In the case of $p = \rho = 0$ and $\phi = \tilde{\chi} = 0$, one finds the Randall-Sundrum vacuum state [24]. We choose the cosmological constant $\Lambda = -\sigma^2/12M^3 = -12M^3/\ell^2$ with the brane tension $\sigma = 12M^3/\ell$ to have a critical brane. Hence the cosmological evolution will be determined by four initial parameters $(\rho_i, a_{0i}, \tilde{\chi}_i, \phi_i)$ instead of two (ρ_i, a_{0i}) in the FRW universe. This is so because the generalized Friedmann equation (5) is not a first integral of the Einstein equation. It is mainly due to the energy exchange $\tilde{T}^t{}_y$ between the brane and bulk. In the case of $\phi = 0$ and $\tilde{T}^t{}_y = A\rho > 0$ with $\tilde{T}^y{}_y = 0$, one finds a mirage-radiation term $\tilde{\chi} \sim (1 - e^{-At/2})/a_0^4$ for an energy outflow from the brane [27]. It is a cosmological model that the real matter on the brane decays into the extra dimension. Also for $\phi = 0, \tilde{T}^t{}_y \sim -\frac{1}{a_0^q}$ and $\tilde{T}^y{}_y = 0$, it is shown that the energy influx from the bulk generates a cosmological acceleration on the brane with the acceleration parameter $Q \equiv \frac{1}{H_0^2}\frac{\ddot{a}_0}{a_0} = 1 - \frac{q}{3}$, where $0 \leq q \leq 4$ [28]. However, in general, it will be a formidable task to solve Eqs.(5)-(7) with Eq.(3) because it gives rise to a complicated dynamics between the brane and the bulk. In Ref.[22], they used $\tilde{T}^t{}_y \propto Ha^n$ to derive the super-acceleration using $\omega_{\text{de}}^{\text{eff}}$.

For our purpose, let us imagine a brane universe made of CDM ρ_{m} with $\omega_{\text{m}} = 0$, but obeying the holographic principle. In addition, we propose that the holographic energy density ρ_Λ exists with its native EoS $\omega_\Lambda \geq -1$ on the brane. If one assumes a form of the interaction T with $\phi = \tilde{T}^y{}_y = 0$, their continuity equations take the simple forms[3]

$$\dot{\rho} + 3H(1+\omega)\rho = -T, \quad \rho = \rho_\Lambda + \rho_{\text{m}} \tag{8}$$

$$\dot{\chi} + 4H\chi = T \tag{9}$$

and the generalized Friedmann equation (5) on the critical brane leads to

$$H^2 = \frac{8\pi}{3M_p^2}\Big[\rho + \chi\Big] - \frac{k}{a^2}. \tag{10}$$

Now we consider the case of decaying from HDE to 5DCDM with $T = \Gamma\rho_\Lambda$, while the CDM is conserved by choosing

$$\dot{\rho}_\Lambda + 3H(1+\omega_\Lambda)\rho_\Lambda = -T, \tag{11}$$

$$\dot{\rho_{\text{m}}} + 3H\rho_{\text{m}} = 0. \tag{12}$$

This decaying process impacts their equations of state and particularly, it induces the effective EoS for the 5DCDM. Interestingly, an accelerating phase could arise from a large

---

[3]Hereafter, we focus on the brane. Hence we use the notation without the subscript "0" and $T = 2\tilde{T}^0{}_y$, and $\chi = (72M^6/\sigma)\tilde{\chi}$, and $\sigma/77M^6 = 1/6M^3\ell = 8\pi/3M_p^2$. Also we concentrate on the low-energy region of $\rho \ll \sigma$ and thus ρ^2-term in Eq.(5) is negligible.

effective non-equilibrium pressure Π_χ defined as $\Pi_\chi \equiv -\Gamma\rho_\Lambda/3H(=\Pi_\Lambda)$. Then the two equations (11) and (9) are translated into those of the two dissipatively imperfect fluids

$$\dot{\rho}_\Lambda + 3H\Big[1 + \omega_\Lambda + \frac{\Gamma}{3H}\Big]\rho_\Lambda = \dot{\rho}_\Lambda + 3H\Big[(1 + \omega_\Lambda)\rho_\Lambda + \Pi_\Lambda\Big] = 0, \tag{13}$$

$$\dot{\chi} + 3H\Big[1 + \frac{1}{3} - \frac{\rho_\Lambda}{\chi}\frac{\Gamma}{3H}\Big]\chi = \dot{\chi} + 3H\Big[(1 + \frac{1}{3})\chi - \Pi_\chi\Big] = 0. \tag{14}$$

The positivity of $\Pi_\Lambda > 0$ shows a decaying of HDE via the cosmic frictional force, while $\Pi_\chi < 0$ induces a production of the mixture via the cosmic anti-frictional force simultaneously [29, 30]. This is a sort of the vacuum decay process to generate a particle production within the two-fluid model [31]. As a result, a mixture of two components will be created. When turning on the interaction term, from Eqs.(13) and (14), we read off their effective equations of state as

$$\omega_\Lambda^{\text{eff}} = \omega_\Lambda + \frac{\Gamma}{3H}, \quad \omega_\chi^{\text{eff}} = \frac{1}{3} - \frac{\rho_\Lambda}{\chi}\frac{\Gamma}{3H}. \tag{15}$$

Hence it is clear that the 5DCDM χ plays a role of radiation on the brane, if there is no interaction. Introducing the density parameters defined by $\Omega_{\text{i}} = \rho_{\text{i}}/\rho_{\text{c}}$ as

$$\Omega_{\text{m}} = \frac{8\pi\rho_{\text{m}}}{3M_p^2 H^2}, \quad \Omega_\Lambda = \frac{8\pi\rho_\Lambda}{3M_p^2 H^2}, \quad \Omega_{\text{k}} = \frac{k}{a^2 H^2}, \quad \Omega_\chi = \frac{8\pi\chi}{3M_p^2 H^2}, \tag{16}$$

we can rewrite the Friedmann equation (10) as a simplified form

$$\Omega_{\text{m}} + \Omega_\Lambda + \Omega_\chi = 1 + \Omega_{\text{k}}. \tag{17}$$

Hereafter we use this relation instead of Eq.(10).

For the non-flat universe of $k \neq 0$, we introduce the future event horizon $L_\Lambda = R_{\text{FH}} = a\xi_{\text{FH}}(t) = a\xi_{\text{FH}}^k(t)$ with

$$\xi_{\text{FH}}(t) = \int_t^\infty \frac{dt}{a}. \tag{18}$$

Here the comoving horizon size is given by

$$\xi_{\text{FH}}^k(t) = \int_0^{r(t)} \frac{dr}{\sqrt{1 - kr^2}} = \frac{1}{\sqrt{|k|}}\text{sinn}^{-1}\Big[\sqrt{|k|}r(t)\Big], \tag{19}$$

where leads to $\xi_{\text{FH}}^{k=1}(t) = \sin^{-1}r(t)$, $\xi_{\text{FH}}^{k=0}(t) = r(t)$, and $\xi_{\text{FH}}^{k=-1}(t) = \sinh^{-1}r(t)$. For our purpose, we use a comoving radial coordinate $r(t)$,

$$r(t) = \frac{1}{\sqrt{|k|}}\text{sinn}\Big[\sqrt{|k|}\xi_{\text{FH}}^k(t)\Big]. \tag{20}$$

6

$L_\Lambda = ar(t)$ is a useful length scale for the non-flat universe [18]. Its derivative with respect to time t leads to

$$\dot{L}_\Lambda = HL_\Lambda + a\dot{r} = \frac{c}{\sqrt{\Omega_\Lambda}} - \mathrm{cosn}y, \tag{21}$$

where $\mathrm{cosn}y = \cos y$, y, $\cosh y$ for $k = 1, 0, -1$ with $y = \sqrt{k}R_{\mathrm{FH}}/a$. Hereafter we consider three classes of interactions: HDE-CDM, HDE-5DCDM, and CDM-5DCDM. Using the definition of $\rho_\Lambda = \frac{3c^2 M_p^2}{8\pi L_\Lambda^2}$ and (15), one finds the equation of state for HDE

$$\dot{\rho}_\Lambda + 3H\Big[1 - \frac{1}{3} - \frac{2\sqrt{\Omega_\Lambda}}{3c}\mathrm{cosn}y\Big]\rho_\Lambda = 0. \tag{22}$$

Here we can read off the effective EoS for HDE as

$$\omega_\Lambda^{\mathrm{eff}}(x) = -\frac{1}{3} - \frac{2\sqrt{\Omega_\Lambda(x)}}{3c}\mathrm{cosn}y \tag{23}$$

with $x = \ln a$. At the first sight, the above effective EoS seems not to be relevant to the interaction, but it depends on the decay rate Γ through Ω_Λ.

3 Unified picture for interactions

For the interaction between HDE and CDM on the brane, we assume to have

$$\dot{\rho}_{\mathrm{m}} + 3H\rho_{\mathrm{m}} = \hat{T}, \tag{24}$$

$$\dot{\rho}_\Lambda + 3H(1 + \omega_\Lambda)\rho_\Lambda = -\hat{T}, \tag{25}$$

$$\dot{\chi} + 4H\chi = 0, \tag{26}$$

where $\hat{T}$ is chosen as $\hat{T} = \Gamma\rho_\Lambda$ for decaying from HDE to CDM, while $\hat{T} = -\Gamma\rho_{\mathrm{m}}$ for decaying from CDM to HDE. This case is not realized by the brane cosmology because the interaction $\hat{T}$ is effective on the brane. Hence there is no brane-bulk interaction ($T = 0$). However, we include this type of interaction for completeness.

In the case of interaction between HDE and 5DCDM, their continuity equations are given by

$$\dot{\rho}_{\mathrm{m}} + 3H\rho_{\mathrm{m}} = 0, \tag{27}$$

$$\dot{\rho}_\Lambda + 3H(1 + \omega_\Lambda)\rho_\Lambda = -T, \tag{28}$$

$$\dot{\chi} + 4H\chi = T, \tag{29}$$

Here T is chosen as $T = \Gamma\rho_\Lambda$ for decaying from HDE to 5DCDM, whereas $T = -\Gamma\chi$ for decaying from 5DCDM to HDE.
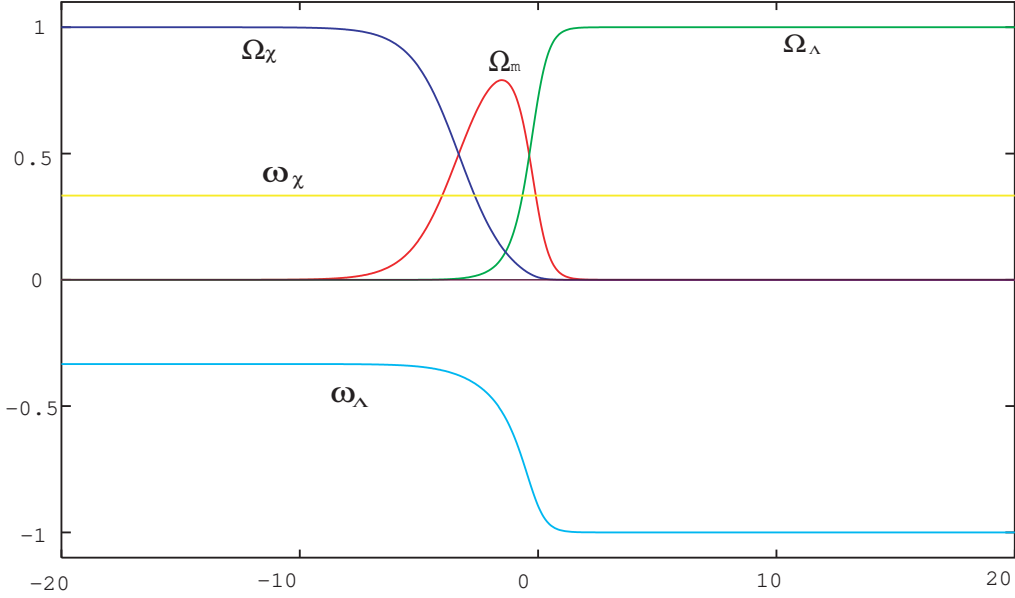
7

Figure 1: (color online) Graph for the noninteracting case. For $b^2 = 0$ and $c = 1$, $k = 1$ evolution of Ω_Λ (green), Ω_m (red), and Ω_χ (blue) and the equations of state, ω_Λ (cyan) and $\omega_\chi^{\text{eff}} = 1/3$ (yellow) with $\omega_m = 0$. Here $x = \ln a$ moves backward direction $(-)$ or forward direction $(+)$, starting at the present time $x = 0(a_0 = 1)$.

Finally, the case of interaction between CDM and 5DCDM takes the form

$$\dot{\rho}_m + 3H\rho_m = -T, \tag{30}$$

$$\dot{\rho}_\Lambda + 3H(1 + \omega_\Lambda)\rho_\Lambda = 0, \tag{31}$$

$$\dot{\chi} + 4H\chi = T, \tag{32}$$

where T is chosen as $T = \Gamma\rho_m$ for decaying from CDM to 5DCDM, while $T = -\Gamma\chi$ for decaying from 5DCDM to CDM.

By choosing appropriate effective equations of state, the above equations for all three cases can be unified as follows:

$$\dot{\rho}_m + 3H(1 + \omega_m^{\text{eff}})\rho_m = 0, \tag{33}$$

$$\dot{\rho}_\Lambda + 3H(1 + \omega_\Lambda^{\text{eff}})\rho_\Lambda = 0, \tag{34}$$

$$\dot{\chi} + 3H(1 + \omega_\chi^{\text{eff}})\chi = 0, \tag{35}$$

All effective EoS are summarized on the Table 1. However, $\omega_\Lambda^{\text{eff}}$ is the same for all cases as is given by Eq.(23). Here we choose three types for the decay rate Γ with $b^2 = 0.2$:

$$(1) - \text{type}: \Gamma = 3Hb^2(1 + \frac{\Omega_i}{\Omega_j}), \tag{36}$$

8

Table 1: Summary of effective equations of state and related information. Here I, Λ, m, and χ represent interaction, HDE, CDM, and 5DCDM, respectively. The redshift factor z_{eq} is determined from the relation $x = -\ln(1+z)$ when $\Omega_{\mathrm{m}} = \Omega_{\chi}$. NA denotes "not available". Finally, yes (no) represent the status of evolution.

IT	$\omega_{\mathrm{m}}^{\mathrm{eff}}$	$\omega_{\chi}^{\mathrm{eff}}$	T/Γ	$\Gamma/3Hb^2$	z_{eq}	status	figure
no	0	$\frac{1}{3}$	0	0	27.1	yes	Fig. 1
$\Lambda \to \mathrm{m}$	$-\frac{\Gamma}{3H}\frac{\Omega_{\Lambda}}{\Omega_{\mathrm{m}}}$	$\frac{1}{3}$	ρ_{Λ}	$(1+\frac{\Omega_{\mathrm{m}}}{\Omega_{\Lambda}})$	2.7	yes	Fig. 2a
				$(1+\frac{\Omega_{\mathrm{m}}}{\Omega_{\Lambda}})\Omega_{\Lambda}$	10.2	yes	Fig. 2c
				$(1+\frac{\Omega_{\mathrm{m}}}{\Omega_{\Lambda}})\Omega_{\Lambda}\Omega_{\mathrm{m}}$	18.9	yes	Fig. 2e
$\mathrm{m} \to \Lambda$	$\frac{\Gamma}{3H}$	$\frac{1}{3}$	$-\rho_{\mathrm{m}}$	$(1+\frac{\Omega_{\Lambda}}{\Omega_{\mathrm{m}}})$	NA	no	Fig. 2b
				$(1+\frac{\Omega_{\Lambda}}{\Omega_{\mathrm{m}}})\Omega_{\mathrm{m}}$	762.3	yes	Fig. 2d
				$(1+\frac{\Omega_{\Lambda}}{\Omega_{\mathrm{m}}})\Omega_{\mathrm{m}}\Omega_{\Lambda}$	36.5	yes	Fig. 2f
$\Lambda \to \chi$	0	$\frac{1}{3}-\frac{\Gamma}{3H}\frac{\Omega_{\Lambda}}{\Omega_{\chi}}$	ρ_{Λ}	$(1+\frac{\Omega_{\chi}}{\Omega_{\Lambda}})$	NA	no	Fig. 3a
				$(1+\frac{\Omega_{\chi}}{\Omega_{\Lambda}})\Omega_{\Lambda}$	NA	no	Fig. 3c
				$(1+\frac{\Omega_{\chi}}{\Omega_{\Lambda}})\Omega_{\Lambda}\Omega_{\chi}$	31.7	yes	Fig. 3e
$\chi \to \Lambda$	0	$\frac{1}{3}+\frac{\Gamma}{3H}$	$-\chi$	$(1+\frac{\Omega_{\Lambda}}{\Omega_{\chi}})$	NA	no	Fig. 3b
				$(1+\frac{\Omega_{\Lambda}}{\Omega_{\chi}})\Omega_{\chi}$	14.1	yes	Fig. 3d
				$(1+\frac{\Omega_{\Lambda}}{\Omega_{\chi}})\Omega_{\chi}\Omega_{\Lambda}$	22.9	yes	Fig. 3f
$\mathrm{m} \to \chi$	$\frac{\Gamma}{3H}$	$\frac{1}{3}-\frac{\Gamma}{3H}\frac{\Omega_{\mathrm{m}}}{\Omega_{\chi}}$	ρ_{m}	$(1+\frac{\Omega_{\chi}}{\Omega_{\mathrm{m}}})$	NA	no	Fig. 4a
				$(1+\frac{\Omega_{\chi}}{\Omega_{\mathrm{m}}})\Omega_{\mathrm{m}}$	NA	no	Fig. 4c
				$(1+\frac{\Omega_{\chi}}{\Omega_{\mathrm{m}}})\Omega_{\mathrm{m}}\Omega_{\chi}$	1109.5	yes	Fig. 4e
$\chi \to \mathrm{m}$	$-\frac{\Gamma}{3H}\frac{\Omega_{\chi}}{\Omega_{\mathrm{m}}}$	$\frac{1}{3}+\frac{\Gamma}{3H}$	$-\chi$	$(1+\frac{\Omega_{\mathrm{m}}}{\Omega_{\chi}})$	NA	no	Fig. 4b
				$(1+\frac{\Omega_{\mathrm{m}}}{\Omega_{\chi}})\Omega_{\chi}$	NA	no	Fig. 4d
				$(1+\frac{\Omega_{\mathrm{m}}}{\Omega_{\chi}})\Omega_{\chi}\Omega_{\mathrm{m}}$	10.0	yes	Fig. 4f

$$(2) - \text{type} : \Gamma = 3Hb^2(1 + \frac{\Omega_i}{\Omega_j})\Omega_j, \qquad (37)$$

$$(3) - \text{type} : \Gamma = 3Hb^2(1 + \frac{\Omega_i}{\Omega_j})\Omega_i\Omega_j. \qquad (38)$$

(1)-type is known as a conventional form for the interaction between HDE and CDM. However, choosing this form leads to an unwanted evolution and thus we have to introduce another interaction (2)-type for the evolution of the dark energy-dominated universe. Finally, (3)-type is chosen because (2)-type is not suitable for describing the interaction between CDM and 5DCDM. Another types are found in Ref.[12].

In order to obtain differential equations for density parameters, Ω_m, Ω_Λ and Ω_χ which govern evolution of the universe, we introduce

$$R_i = \frac{\rho_i}{\rho_c} = \Omega_i, i = m, \Lambda, \chi. \qquad (39)$$

Differentiating R_i with respect to cosmic time t and then using appropriate definitions, we obtain three equations

$$\Omega'_m = \Omega_m \left[2 + (1 + 3\omega_m^{\text{eff}})\Omega_m + (1 + 3\omega_\Lambda^{\text{eff}})\Omega_\Lambda + (1 + 3\omega_\chi^{\text{eff}})\Omega_\chi \right] - 3\Omega_m(1 + \omega_m^{\text{eff}}), (40)$$

$$\Omega'_\Lambda = \Omega_\Lambda \left[2 + (1 + 3\omega_m^{\text{eff}})\Omega_m + (1 + 3\omega_\Lambda^{\text{eff}})\Omega_\Lambda + (1 + 3\omega_\chi^{\text{eff}})\Omega_\chi \right] - 3\Omega_\Lambda(1 + \omega_\Lambda^{\text{eff}}), (41)$$

$$\Omega'_\chi = \Omega_\chi \left[2 + (1 + 3\omega_m^{\text{eff}})\Omega_m + (1 + 3\omega_\Lambda^{\text{eff}})\Omega_\Lambda + (1 + 3\omega_\chi^{\text{eff}})\Omega_\chi \right] - 3\Omega_\chi(1 + \omega_\chi^{\text{eff}}), (42)$$

where $'$ is the differentiation with respect to $x = \ln a$. These equations come from the first and second Friedmann equations combined with their continuity equations. In order to obtain solution, we have to solve the above coupled equations numerically by considering the initial condition at present time[4]: $\Omega'_\Lambda|_{x=0} > 0$, $\Omega^0_\Lambda = 0.72, \Omega^0_{k=1} = 0.01, \Omega^0_m = 0.28, \Omega^0_\chi = 0.01$.

The noninteracting case with $b^2 = 0$ is depicted at Fig. 1, which shows the standard evolution for the HDE. Here the effective EoS reduces to the native EoS because of the absence of interactions except $\omega_\chi^{\text{eff}} = 1/3$ for 5DCDM χ. Each matter satisfies its continuity equation. We find a sequence of dominance in the evolution of the universe: radiation$\rightarrow$ CDM $\rightarrow$dark energy. The redshift factor $z_{\text{eq}} = 27.1$ is determined from the relation of $x = -\ln(1 + z)$ when $\Omega_m = \Omega_\chi$.

Figs. 2a-f show the evolution for the interaction between HDE and CDM on the brane. The left column of HDE$\rightarrow$CDM was already known but the right column shows new results. These all indicate evolutions for dark energy-dominated universe except

---

[4]Here we use the data from the combination of WMAP3 plus the HST key project constraint on H_0 [4].

the case of CDM→HDE with the decay rate $\Gamma = 3Hb^2(1 + \Omega_\Lambda/\Omega_\mathrm{m})$ [16, 17]. This case provides a negative density parameter $\Omega_\mathrm{m} < 0$ for the future evolution and thus induces the unwanted case of $\Omega_\Lambda > 1$.

Figs. 3a-f indicate the evolution for the interaction between HDE and 5DCDM. This corresponds to the case of interaction between HDE and radiation on the brane. The left column is for HDE→ 5DCDM. An evolution for dark energy-dominated universe is possible for only the decay rate of (3)-type: $\Gamma = 3Hb^2(1 + \Omega_\Lambda/\Omega_\mathrm{m})\Omega_\Lambda\Omega_\mathrm{m}$. The right column is for 5DCDM→HDE. Here evolutions come out when choosing (2)and (3)-type. All forward evolutions are possible, whereas backward evolutions are not possible for (1)-type and HDE→ 5DCDM with (2)-type.

Figs. 4a-f show the evolution for the interaction between CDM and 5DCDM. This corresponds to the case of interaction between CDM and radiation on the brane. The left column is for CDM→ 5DCDM and the right column is for 5DCDM→CDM. An evolution for dark energy-dominated universe is possible for only the decay rate of (3)-type. All backward evolutions seem not to be possible for (1) and (2)-types. Especially, we find the unwanted backward evolution of $\Omega_\mathrm{m} < 0$, $\Omega_\chi > 1$ for the 5DCDM→CDM with (1) and (2)-types. In this sense, (3)-type is considered as the general form of decay rate Γ.

4   Discussions

We investigate a unified description of radiation-matter-dark energy universe within the brane cosmology. It is confirmed that there is no phantom phase from brane-bulk interactions (HDE-5DCDM, CDM-5DCDM) and interaction on the brane (HDE-CDM) when using $\omega_\Lambda^\mathrm{eff}$. Thus our results favors Setare's case [23] but disfavors Cai-Gong-Wang's case [22]. This arises mainly because we used a different definition for the effective EoS $\omega_\Lambda^\mathrm{eff}$ from Cai-Gong-Wang's case of $\omega_\mathrm{de}^\mathrm{eff}$ as well as the HDE as dark energy. Recently, the authors in [32] showed that the interacting holographic dark energy with CDM may lead to the phantom phase using the native EoS ω_Λ. Also the authors in [33] showed that the brane-bulk interaction without the holographic dark energy accommodates the $\omega = -1$ crossing when using $\omega_\mathrm{de}^\mathrm{eff}$. Hence, the issue is to choose an appropriate EoS for describing the dark energy universe.

Also, we obtain an additional information from the unified picture of interactions. We suggest a sequence of the evolution: radiation-dominated universe $\rightarrow$ matter-dominated universe $\rightarrow$ dark energy-dominated universe. The 5DCDM plays the same role as a radiation on the brane. As is shown Fig.1 and Table 1 , we have $z_\mathrm{eq} = 27.1$ which is not close to $z_\mathrm{eq}^\mathrm{ob} = 2.4 \times \Omega_\mathrm{m}h^2 \simeq 4.8 \times 10^3$ if there is no interaction. Interestingly, as
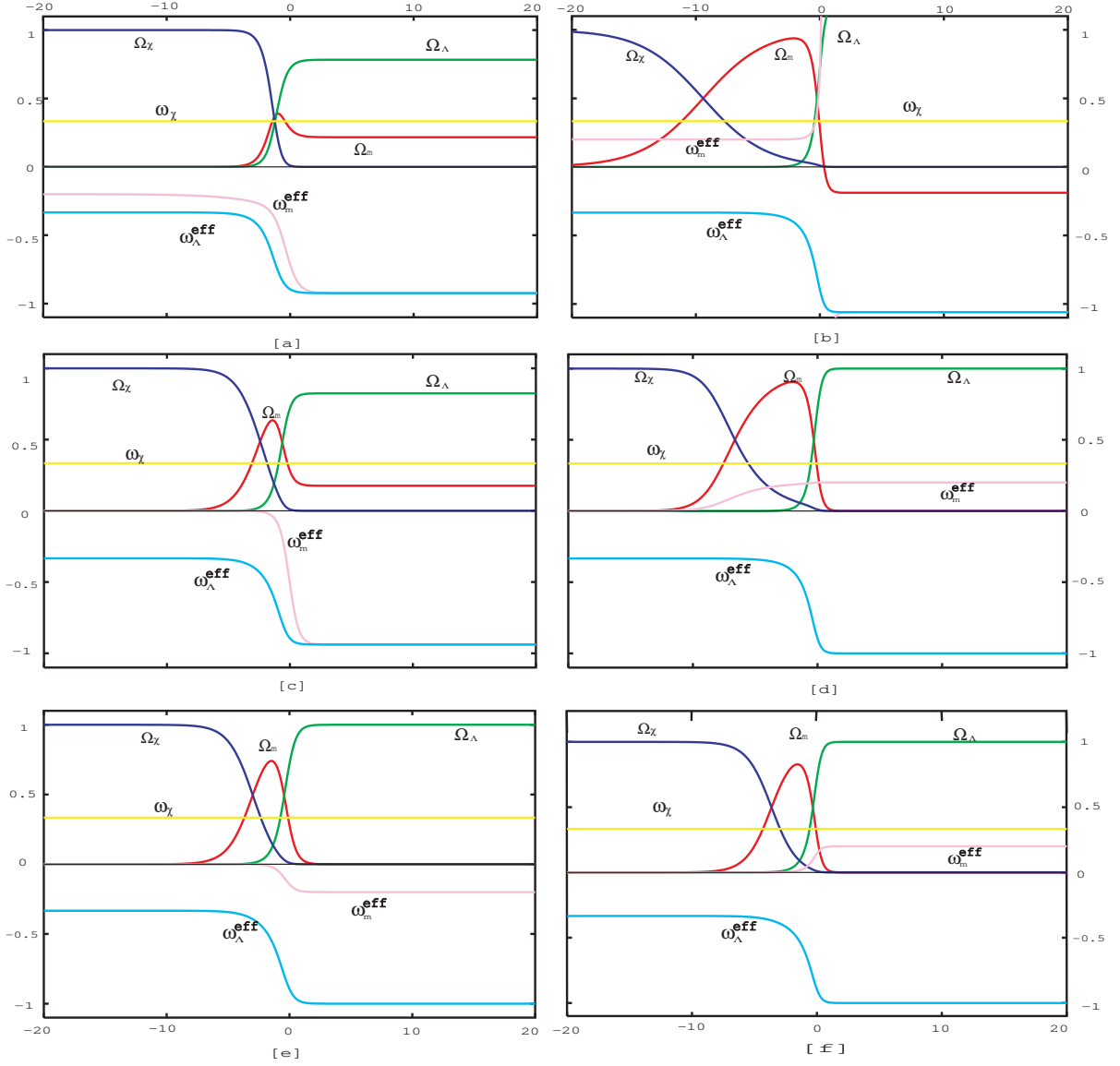
11

Figure 2: (color online) Six graphs for the interaction between HDE and CDM. For $b^2 = 0.2$ and $c = 1$, $k = 1$ evolution of Ω_Λ (green), Ω_m (red), and Ω_χ (blue) and the effective equations of state, $\omega_\Lambda^{\mathrm{eff}}$ (cyan) and $\omega_\chi^{\mathrm{eff}} = 1/3$ (yellow) with ω_m^{eff}(pink). The left column is for HDE→CDM and the right one is for CDM→HDE. Fig. 2a and 2b are for the decay rate of (1)-type, Fig. 2c and 2d for the decay rate of (2)-type, and Fig. 2e and 2f for the decay rate of (3)-type.

is shown Fig. 4e and Table 1, there is a good value of $z_{\mathrm{eq}} = 1.1 \times 10^3$, which is the same order as the observational value $z_{\mathrm{eq}}^{\mathrm{ob}}$ if the interaction between CDM and 5DCDM

12

is included. However, we do not resolve the coincidence problem because there is no interaction between HDE and CDM.

We stress that if one uses 5D CDM χ in the brane cosmology instead of radiation, its late time evolution is not sizably different from the FRW universe with radiation-matter-dark energy.

Concerning the type of decay rate Γ, we find that (3)-type is suitable for all interactions and thus it could be regarded as the general form. (2)-type works for three cases of HDE→CDM, CDM→HDE, and 5DCDM→HDE. Finally, (1)-type works for HDE→CDM only and it belongs to a very restricted decay rate.

Acknowledgment

References

[1] A. G. Riess *et al.*, Astron. J. **116**, 1009 (1998)[astro-ph/9805201]; S. J. Perlmutter *et al.*, Astrophys. J. **517**, 565(1999)[astro-ph/9812133]; A. G. Riess *et al.*, Astrophys. J. **607**, 665(2004)[astro-ph/0402512]; P. Astier *et al.*, Astron. Astrophys. **447**, 31 (2006) [arXiv:astro-ph/0510447].

[2] M. Tegmark *et al.* [SDSS Collaboration], Phys. Rev. D **69**, 103501 (2004) [arXiv:astro-ph/0310723];

K. Abazajian *et al.* [SDSS Collaboration], Astron. J. **128**, 502 (2004) [arXiv:astro-ph/0403325];

K. Abazajian *et al.* [SDSS Collaboration], Astron. J. **129**, 1755 (2005) [arXiv:astro-ph/0410239].

[3] H. V. Peiris *et al.*, Astrophys. J. Suppl. **148** (2003) 213 [astro-ph/0302225]; C. L. Bennett *et al.*, Astrophys. J. Suppl. **148** (2003) 1[astro-ph/0302207]; D. N. Spergel *et al.*, Astrophys. J. Suppl. **148** (2003) 175[astro-ph/0302209].

[4] D. N. Spergel *et al.*, arXiv:astro-ph/0603449.

[5] A. Cohen, D. Kaplan, and A. Nelson, Phys. Rev. Lett. **82**, 4971 (1999)[arXiv:hep-th/9803132].

[6] P. Horava and D. Minic, Phys. Rev. Lett. **85**, 1610 (2000)[arXiv:hep-th/0001145]; S. Thomas, Phys. Rev. Lett. **89**, 081301 (2002).

[7] S. D. Hsu, Phys. Lett. B **594**,13 (2004)[hep-th/0403052].

[8] M. Li, Phys. Lett. B **603**, 1 (2004)[hep-th/0403127].

[9] Q-C. Huang and Y. Gong, JCAP **0408**, 006 (2004)[astro-ph/0403590]; Y. Gong, Phys. Rev. D **70**, 064029 (2004)[hep-th/0404030]; B. Wang, E. Abdalla and Ru-Keng Su, hep-th/0404057; K. Enqvist and M. S. Sloth, Phys. Rev. Lett. **93**, 221302 (2004) [hep-th/0406019]; S. Hsu and A. Zee, hep-th/0406142; K. Ke and M. Li, hep-th/0407056; P. F. Gonzalez-Diaz, hep-th/0411070; H. Kim, H. W. Lee, and Y. S. Myung, hep-th/0501118; Phys. Lett. B **628**,11 (2005)[gr-qc/0507010]; X. Zhang, artro-ph/0504586; X. Zhang and F.-Q. Wu, astro-ph/0506310; H. M. Sadjadi and M. Honardoost, arXiv:gr-qc/0609076; M. R. Setare, J. Zhang and X. Zhang, arXiv:gr-qc/0611084.

[10] Y. S. Myung, Phys. Lett. B **610**, 18 (2005) [hep-th/0412224]; Mod. Phys. Lett. A **27**, 2035 (2005) [hep-th/0501023]; A. J. M. Medved, hep-th/0501100.

[11] D. Pavon and W. Zimdahl, Phys. Lett. B **628**, 206 (2005) [gr-qc/0505020]; W. Zimdahl, gr-qc/0505056;

[12] M. S. Berger and H. Shojaei, Phys. Rev. D **73**, 083528 (2006) [arXiv:gr-qc/0601086].

[13] R. Horvat, Phys. Rev. D **70**, 087301 (2004) [arXiv:astro-ph/0404204].

[14] M. Szydlowski, Phys. Lett. B **632**, 1 (2006) [arXiv:astro-ph/0502034].

[15] E. J. Copeland, M. Sami and S. Tsujikawa, arXiv:hep-th/0603057.

[16] B. Wang, Y. Gong, and E. Abdalla, Phys. Lett. B **624**, 141 (2005)[hep-th/0506069].

[17] H. Kim, H. W. Lee and Y. S. Myung, Phys. Lett. B **632**, 605 (2006) [arXiv:gr-qc/0509040].

[18] Q. G. Huang and M. Li, JCAP **0408**, 013 (2004) [arXiv:astro-ph/0404229].

[19] B. Wang, C. Y. Lin and E. Abdalla, Phys. Lett. B **637** (2006) 357 [arXiv:hep-th/0509107].

[20] M. R. Setare, Phys. Lett. B **642**, 1 (2006) [arXiv:hep-th/0609069]; K. H. Kim, H. W. Lee and Y. S. Myung, Phys. Lett. B **648**, 107 (2007) [arXiv:gr-qc/0612112].

[21] M. R. Setare and E. C. Vagenas, arXiv:0704.2070 [hep-th].

[22] R. G. Cai, Y. g. Gong and B. Wang, JCAP **0603**, 006 (2006) [arXiv:hep-th/0511301].

[23] M. R. Setare, Phys. Lett. B **642**, 421 (2006).

[24] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999) [arXiv:hep-th/9906064].

[25] P. Binetruy, C. Deffayet and D. Langlois, Nucl. Phys. B **565**, 269 (2000) [arXiv:hep-th/9905012]; P. Binetruy, C. Deffayet, U. Ellwanger and D. Langlois, Phys. Lett. B **477**, 285 (2000) [arXiv:hep-th/9910219].

[26] Y. S. Myung and J. Y. Kim, Class. Quant. Grav. **20**, L169 (2003) [arXiv:hep-th/0304033].

[27] E. Kiritsis, G. Kofinas, N. Tetradis, T. N. Tomaras and V. Zarikas, JHEP **0302**, 035 (2003) [arXiv:hep-th/0207060].

[28] N. Tetradis, Phys. Lett. B **569**, 1 (2003) [arXiv:hep-th/0211200].

[29] W. Zimdahl, D. Pavon, and L. P. Cimento, Phys. Lett. B **521**, 133 (2001) [astro-ph/0105479].

[30] Y. S. Myung, Phys. Lett. B **626**, 1 (2005) [hep-th/0502128].

[31] W. Zimdahl, D. J. Schwarz, A. B. Balakin, and D. Pavon, Phys. Rev. D **64**, 063501 (2001) [astro-ph/0009353];

A. B. Balakin, D. Pavon, D. J. Schwarz, and W. Zimdahl, New J. Phys. **5**, 085 (2003) [astro-ph/0302150].

[32] Q. Wu, Y. Gong, A. Wang and J. S. Alcaniz, arXiv:0705.1006 [astro-ph].

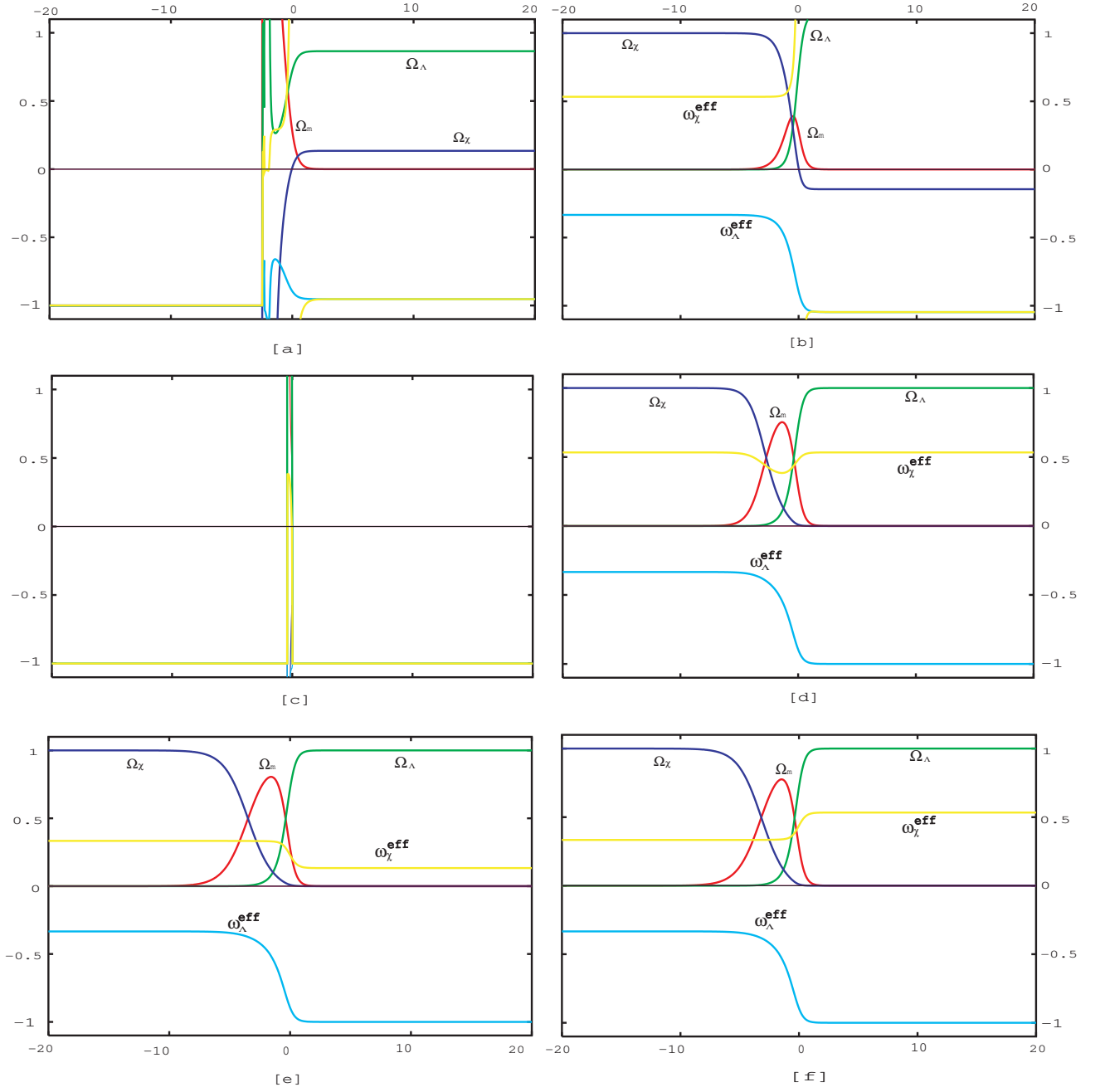[33] C. Bogdanos, S. Nesseris, L. Perivolaropoulos and K. Tamvakis, arXiv:0705.3181 [hep-ph].

Figure 3: (color online) Six graphs for the interaction between HDE and 5DCDM. For $b^2 = 0.2$ and $c = 1$, $k = 1$ evolution of Ω_Λ (green), Ω_m (red), and Ω_χ (blue) and the effective equations of state, $\omega_\Lambda^{\text{eff}}$ (cyan) and ω_χ^{eff} (yellow) with $\omega_m^{\text{eff}} = 0$. The left column is for HDE→5DCDM and the right one is for 5DCDM→HDE. Fig. 3a and 3b are for the decay rate of (1)-type, Fig. 3c and 3d for the decay rate of (2)-type, and Fig. 3e and 3f for the decay rate of (3)-type.
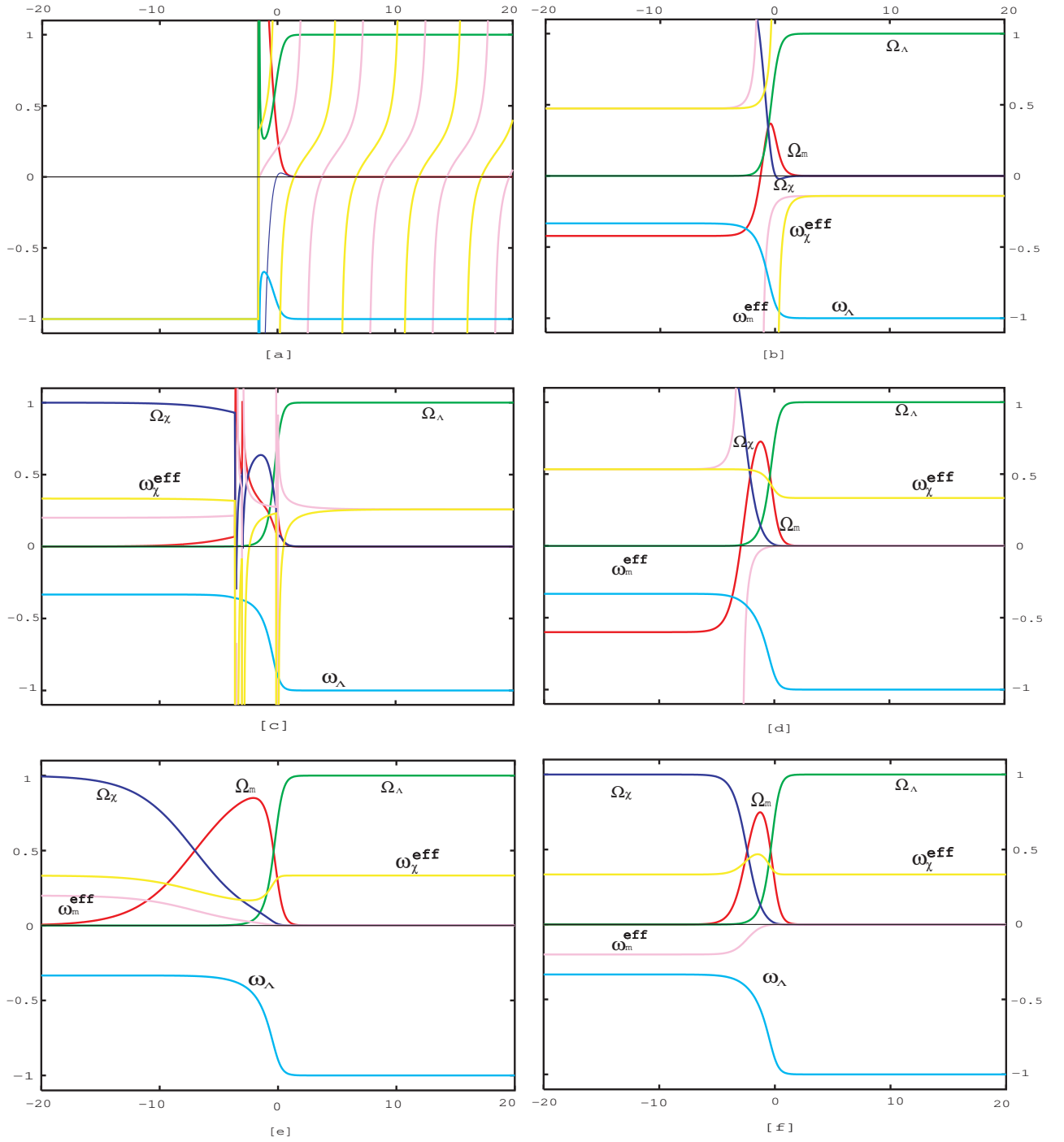
16

Figure 4: (color online) Six graphs for the interaction between CDM and 5DCDM. For $b^2 = 0.2$ and $c = 1$, $k = 1$ evolution of Ω_Λ (green), Ω_m (red), and Ω_χ (blue) and the effective equations of state, $\omega_\Lambda^{\text{eff}}$ (cyan) and ω_χ^{eff} (yellow) with ω_m^{eff} (pink). The left column is for CDM→5DCDM and the right one is for 5DCDM→CDM. Fig. 4a and 4b are for the decay rate of (1)-type, Fig. 4c and 4d for the decay rate of (2)-type, and Fig. 4e and 4f for the decay rate of (3)-type.

17