

# Impact of the Neutrino Magnetic Moment on the Neutrino Fluxes and the Electron Fraction in core-collapse Supernovae

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We explore the effect of the neutrino magnetic moment on neutrino scattering with matter in a core-collapse Supernova. We study the impact both on the neutrino fluxes and on the electron fraction. We find that sizeable modifications require very large magnetic moments both for Dirac and Majorana neutrinos.

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## I. INTRODUCTION

Core collapse supernovae are powerful neutrino sources of all flavours, since their explosion produces a very intense burst of neutrinos with an energy of tens of MeV on a timescale of several seconds. These expectations have been roughly confirmed by the explosion of the Supernova 1987A in the Large Magellanic Cloud. Besides, their observation has helped to better constrain various neutrino properties that still remain unknown. The measurement of a future galactic supernova explosion or of the diffuse supernova neutrino background would be of great interest both for our knowledge of neutrino properties and for unraveling the supernova explosion mechanism since in present simulations the shock wave fails to eject the mantle. While this keeps being an open issue, convection or extra deposit of energy might be the key to solve this question. Because 99 % of the energy is emitted by neutrinos, it is likely that they might drive the shock wave out the star by some still unknown mechanism.

Neutrinos also play a role in the rapid neutron capture (r-process) nucleosynthesis scenario [1], where a good fraction of the heavier nuclei were formed. Although an astrophysical site of the r-process is not yet identified, one expects such sites to be associated with explosive phenomena since a large number of interactions are required to take place during a rather short time interval. Time-scale arguments based on meteoritic data imply that r-process nuclei may come from diverse sources [2]. Neutrino-driven wind models of neutron-rich material ejection following core-collapse supernovae indicate a possible site [3, 4, 5]. A key quantity for determining the r-process yields is the neutron-to-seed nucleus ratio, which in turn, is determined by the neutron-to-proton ratio at freeze-out. The neutrino-induced process  $\nu_e + n \rightarrow p + e^-$ , operating during or immediately after the freeze-out, could significantly alter neutron-to-proton ratio. During the epoch of alpha-particle formation almost all the protons and an equal amount of neutrons combine into alpha particles which have a large binding energy. This “alpha effect” reduces the number of free neutrons taking place in the r-process, pushing the electron fraction close to  $Y_e = 0.5$  [6, 7]. One possible scheme to reduce the impact of the alpha effect is to reduce the electron neutrino flux. This should happen at relatively far away from the vicinity of the neutron star so that neutrino heating already can have taken place. Oscillations between active flavors will only increase the conversion of neutrons to protons since mu and tau neutrinos are likely to have higher energies than electron neutrinos to begin with (The exact hierarchy of neutrino energies depends on the details of microphysics [8, 9]). However, a reduction of the electron neutrino flux can be achieved by oscillation of electron neutrinos into sterile neutrinos [10, 11, 12].

In this article we study the impact of the neutrino magnetic moment on both the neutrino fluxes and the electron fraction in a core-collapse supernova environment. The cases of Dirac and Majorana neutrinos are analyzed, inducing active-sterile and active-active conversions respectively. In our considerations we ignore the possibility of the presence of large magnetic fields near the supernova core. Such magnetic fields could cause additional transformations between

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neutrino flavors via spin-flavor precession scenarios [13, 14]. We also ignore neutrino-neutrino interactions [15, 16, 17]. Note that the impact of the neutrino magnetic moment in astrophysical and cosmological contexts has been discussed in various works, e.g. [18] for red-giant cooling, in [19] for big-bang nucleosynthesis, for solar neutrinos [20] and in [21, 22, 23] for core-collapse Supernovae.

The plan of this paper is as follows. In the next section we summarize properties of the post core-bounce supernova in the nucleosynthesis epoch, discuss neutrino magnetic moment scattering in the pertinent plasma. In sections 3 and 4 we analyze the case of Dirac and Majorana neutrinos, respectively. We summarize our results in Section 5 where we present our conclusions.

## II. NEUTRINO ELASTIC SCATTERING VIA MAGNETIC MOMENT INTERACTION IN A CORE-COLLAPSE SUPERNOVA

A heuristic description of the conditions of the neutrino-driven wind in post-core bounce supernova environment is outlined in Refs. [10] and [24]. Neutrino magnetic moment effects could be present inside<sup>1</sup> and just above the proto-neutron star. The medium immediately above the neutron star is a degenerate and relativistic plasma (we have  $T_{Fermi} \gg T \approx 10^{10}\text{K}$ ). The effective photon mass is then [26]

$$m_\gamma^2(N_e, T) = \frac{2\alpha}{\pi} \left( \mu^2 + \frac{1}{3}\pi^2 T^2 \right) \quad (1)$$

with  $\mu$  the electronic chemical potential :

$$\mu = \left( \sqrt{\frac{p_F^6}{4} + \frac{\pi^6 T^6}{27}} + \frac{p_F^3}{2} \right)^{1/3} - \left( \sqrt{\frac{p_F^6}{4} + \frac{\pi^6 T^6}{27}} - \frac{p_F^3}{2} \right)^{1/3}, \quad (2)$$

where the Fermi momentum is given by

$$p_F^3 = 3\pi^2 N_e(r) \quad (3)$$

In these equations the electron number density,  $N_e$  is related to the matter density,  $\rho$ , as

$$N_e(r) = Y_e(r) \times \rho(r)/m_N, \quad (4)$$

where  $Y_e$  is the electron fraction and  $m_N$  is the nucleon mass. At the surface of the proto-neutron star, the density profile falls off steeply over few kms. For regions sufficiently removed from the proto-neutron star, density goes over to the neutrino-driven wind solution ( $\sim 1/r^3$ ). In our calculations we adopted the density profile of Ref.[10] (with entropy  $S = 70k_B$ ).

Magnetic contribution to the differential cross section for elastic neutrino scattering on electron is

$$\frac{d\sigma}{dt} = \left( \sum_f \mu_{if}^2 \right) \frac{\pi\alpha^2}{m_e^2} \frac{s+t-m_e^2}{(t-m_\gamma^2)(s-m_e^2)}. \quad (5)$$

We ignore the contributions from the weak neutral-current scattering which preserves both the neutrino flavor and chirality. Note that, in the most general case, we sum over the contributions coming from both diagonal and transition magnetic moments since the magnetic scattering can produce any neutrino flavor. Hence for an electron neutrino in the initial state one has  $\sum_f \mu_{if}^2 = \mu_{ee}^2 + \mu_{e\mu}^2 + \mu_{e\tau}^2$ . (For Majorana neutrinos there are no diagonal  $\mu_{ii}$  magnetic

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<sup>1</sup> In fact a sufficiently large neutrino magnetic moment can cause significant energy losses during the core-collapse and neutron-star formation epochs. Observational limits on the reduction of the trapped lepton number were also used to constrain the neutrino magnetic moments [21, 22, 23, 25]. Our analysis and limits we obtain deal with later epochs when nucleosynthesis may take place.

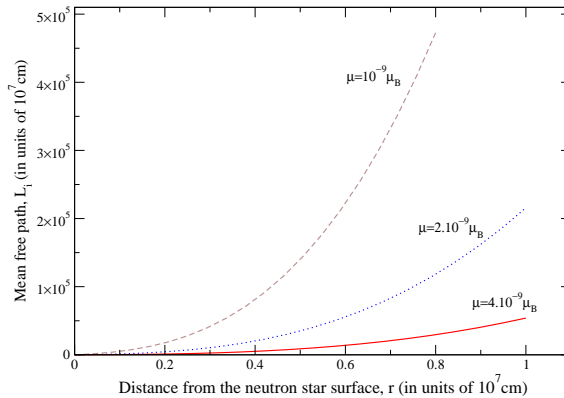


FIG. 1: Neutrino mean free path  $L_e$  as a function of the distance from the neutron star  $r$ , both in units of  $10^7$  cm. From top to bottom,  $\mu_\nu = 1 \times 10^{-9} \mu_B$ ,  $2 \times 10^{-9} \mu_B$ ,  $4 \times 10^{-9} \mu_B$  respectively.

moments). Integrating Eq. (5) we obtain the total cross section <sup>2</sup>

$$\sigma = \left( \sum_f \mu_{if}^2 \right) \frac{\pi \alpha^2}{m_e^2} \left[ \left( 1 + \frac{m_\gamma^2}{2m_e E_\nu} \right) \times \log \left( \frac{2m_e E_\nu + m_\gamma^2}{m_\gamma^2} \right) - 1 \right]. \quad (8)$$

Since the effective photon mass can be large in our case, we keep the constant term in Eq. (8), usually ignored in the literature. The neutrino mean free path,  $L_i$  is then

$$L_i = \frac{1}{\sigma(r, E_\nu, \sum_f \mu_{if}^2) N_e(r)}. \quad (9)$$

In Figure 1 we display the behavior of the neutrino mean free path as a function of the distance  $r$  from the neutron star surface, for various magnetic moment values. It can be seen that  $L_e$  is very large, and therefore the magnetic moment interactions will be significant only very close to the proto-neutron star surface.

The presence of the neutrino magnetic moment modifies the fluxes:

$$\tilde{\phi}(E_\nu, r) = \phi(E_\nu) \mathcal{N}_{\nu_e, \bar{\nu}_e}(E_\nu, r) \quad (10)$$

with  $\phi(E_\nu)$  the neutrino fluxes at the neutrinosphere that we take as Fermi-Dirac distributions.<sup>3</sup> The electron (anti)neutrino fraction  $\mathcal{N}_{\nu_e, \bar{\nu}_e}$  is determined by solving the evolution of the neutrino amplitudes or probabilities in matter including the extra terms due to the magnetic interaction. We give these equations in the next sections both for Dirac and Majorana neutrinos. As far as the equilibrium electron fraction in the supernova is concerned, it is given by

$$Y_e(r) \sim \frac{1}{1 + \frac{\lambda_{\bar{\nu}_e p}(r)}{\lambda_{\nu_e n}(r)}}. \quad (11)$$

<sup>2</sup> We note that the following series expansion is useful for understanding the convergence behavior of this cross section:

$$\sigma = \left( \sum_f \mu_{if}^2 \right) \left( \frac{\pi \alpha^2}{m_e^2} \right) \sum_{n=1}^{\infty} \frac{x^n}{n+1}, \quad (6)$$

where

$$x = \frac{2m_e E_\nu}{2m_e E_\nu + m_\gamma^2}. \quad (7)$$

<sup>3</sup> Note that different neutrino flux shapes (power law) have been pointed out recently [27].

in the absence of a significant number of alpha particles, as the magnetic moment acts at early times, very close to the neutron star surface (Figure 1). In Eq. (11)  $\lambda_{\bar{\nu}_e p}$  is the rate of the reaction  $\bar{\nu}_e + p \rightarrow n + e^+$  producing neutrons and  $\lambda_{\nu_e n}$  is the rate of the reaction  $\nu_e + n \rightarrow p + e^-$  destroying neutrons. These rates are given by

$$\lambda_{\nu_e n, \bar{\nu}_e p}(r) = \int \sigma_{weak}(E_\nu) \phi(E_\nu, r) \mathcal{N}_{\nu_e, \bar{\nu}_e}(E_\nu, r) dE_\nu \quad (12)$$

where the cross section is  $\sigma_{weak}(E_\nu) = 9.6 \times 10^{-44} (E_\nu \pm 1.293)^2 \text{ cm}^2$  for neutrinos (minus for anti-neutrinos).

### III. DIRAC NEUTRINOS

Let us first discuss Dirac neutrinos in the case of two flavors to illustrate the salient features of the evolution. The evolution equation of the neutrino amplitudes, including both the standard matter (MSW) effect and the magnetic moment interaction, is given by

$$i \frac{\partial}{\partial r} \begin{bmatrix} \Psi_{\nu_e}(E_\nu, r) \\ \Psi_{\nu_\mu}(E_\nu, r) \end{bmatrix} = \begin{bmatrix} \varphi(r) - \frac{i}{2L_e} & \frac{\delta m^2}{4E_\nu} \sin 2\theta_v \\ \frac{\delta m^2}{4E_\nu} \sin 2\theta_v & -\varphi(r) - \frac{i}{2L_\mu} \end{bmatrix} \begin{bmatrix} \Psi_{\nu_e}(E_\nu, r) \\ \Psi_{\nu_\mu}(E_\nu, r) \end{bmatrix}, \quad (13)$$

with  $\theta_v$  being the neutrino vacuum mixing angle,  $\delta m^2$  the square mass difference,  $L_i$  from Eq.(9) and where

$$\varphi(r) = \frac{1}{4E_\nu} \left( 2\sqrt{2} G_F N_e(r) E_\nu - \delta m^2 \cos 2\theta_v \right). \quad (14)$$

In Eq.(13) the term due to the neutrino magnetic moment is such that the electron survival probability is suppressed by a  $1/e$  factor at a distance in the star equal to one mean free path. Indeed, since the magnetic scattering produces wrong-chirality (sterile) states, such an equation produces a net loss of flux from all the channels.

Rewriting Eq.(13) for three flavors we have

$$i \frac{\partial}{\partial r} \begin{bmatrix} \Psi_{\nu_e}(E_\nu, r) \\ \Psi_{\nu_\mu}(E_\nu, r) \\ \Psi_{\nu_\tau}(E_\nu, r) \end{bmatrix} = \left( \hat{H}_{\text{MSW}} + \begin{bmatrix} -\frac{i}{2L_e} & 0 & 0 \\ 0 & -\frac{i}{2L_\mu} & 0 \\ 0 & 0 & -\frac{i}{2L_\tau} \end{bmatrix} \right) \begin{bmatrix} \Psi_{\nu_e}(E_\nu, r) \\ \Psi_{\nu_\mu}(E_\nu, r) \\ \Psi_{\nu_\tau}(E_\nu, r) \end{bmatrix}. \quad (15)$$

The region where the density is large enough to render the interaction due to the magnetic moment effective is the high density region very close to the neutron star surface, far from the MSW resonances<sup>4</sup>. In this region the term  $H_{\text{MSW}}$  in Eq. (15) contributes very little, which we verified numerically.

We have solved equation (15), calculated the neutrino fluxes Eq.(10) and the reaction rates Eqs.(11-12) both for neutrinos and antineutrinos<sup>5</sup>. We find that the effect on the neutrino fluxes is not significant even for very large values of the neutrino magnetic moment. Figure 2 presents the variation of the electron fraction, i.e.  $(Y_e(r) - Y_e(r=0))/Y_e(r=0)$ , in percentage at a distance of  $r = 4 \text{ km}$  from the neutron star surface, where the magnetic moment interaction become ineffective. The results are given for different hierarchies of neutrinos temperatures<sup>6</sup>, i.e. different electron fraction  $Y_e(r=0)$ , given in table I. One can see that the electron fraction increases as the magnetic moment  $\mu^2 = \sum_f \mu_{ef}^2$  gets larger. For Dirac neutrinos it is clear that the presence of the neutrino magnetic moment converts both electron neutrino and electron antineutrino fluxes into sterile states. Hence it lowers both of these rates; but, since the electron anti-neutrinos are more energetic, it lowers the neutron production rate more because of the cross section behavior Eq.(8). Then, the ratio  $\lambda_{\bar{\nu}_e p}(r)/\lambda_{\nu_e n}(r)$  decreases and  $Y_e(r)$  increases. A magnetic moment as large as  $\mu = 10^{-9} \mu_B$  (Bohr magnetons) induces an increase of  $Y_e$  of 1%. However the variation  $Y_e$  strongly depends on the  $\mu_{ef}$ . For example a value of  $3 \times 10^{-10} \mu_B$  produces an increase up to at most 0.1%, while for the present experimental upper-limit of  $(\sum_f \mu_{ef}^2)^{1/2} \leq 0.74 \times 10^{-10} \mu_B$  [28] one gets a small effect of less than 0.01%<sup>7</sup>. One also see that, for a  $Y_e(r=0)$  closer to the critical value 0.5, the effect becomes smaller.

<sup>4</sup> For this reason, the neutrino mass hierarchy will not influence our results.

<sup>5</sup> Anti-neutrinos obey the same type of equations but with  $H_{\text{MSW}}$  modified by a minus sign in front of the MSW potential.

<sup>6</sup> Note that different sets of neutrinos temperatures can lead to the same value of  $Y_e(0)$ , cf. Eq 11-12. However, for a given  $Y_e(0)$ , we checked that our results depend only a little on the corresponding hierarchies. Moreover, the dependence of our results on  $\nu_\mu, \nu_\tau$  temperatures is not significant. They have been fixed at the value  $T_{\nu_x, \bar{\nu}_x} = 7.5 \text{ MeV}$ .

<sup>7</sup> One should also note that such large values of magnetic moment would cause the neutron star lose its energy too fast to begin with.

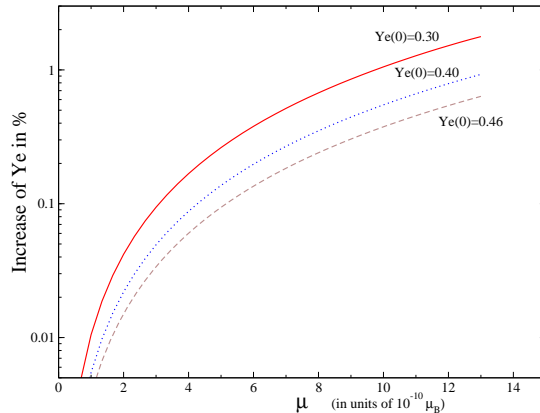


FIG. 2: CASE OF DIRAC NEUTRINOS : Increase of the electron fraction in percentage,  $(Y_e(r) - Y_e(r=0))/Y_e(r=0)$ , evaluated at a distance of  $r = 4$  km from the neutron star surface, as a function of the neutrino magnetic moment  $\mu = (\sum_f \mu_{ef}^2)^{1/2}$ .

Electron fraction $Y_e(0)$	$T_{\nu_e}$ (MeV)	$T_{\bar{\nu}_e}$ (MeV)	$\langle E_{\nu_e} \rangle$ (MeV)	$\langle E_{\bar{\nu}_e} \rangle$ (MeV)
0.30	2.1	7.1	6.6	22.4
0.40	3.0	6.0	9.4	18.9
0.46	3.5	5.7	11	18

TABLE I: Electron fraction at the proto-neutron star surface and one possible set of associated neutrino temperature hierarchies.

#### IV. MAJORANA NEUTRINOS

In the Majorana case, the evolution equations including the neutrino magnetic moment effects involve both neutrinos and anti-neutrinos (the  $\nu_R$ 's are no more sterile). Since in the Dirac case we have checked that standard matter effects modify little the neutrino evolution at the region where the magnetic field effects are important, it is sufficient to consider the evolution equation for all neutrino number fraction  $\mathcal{N}_{\nu_x}$  including the neutrino magnetic moment effect only :

$$\frac{\partial}{\partial r} \begin{bmatrix} \mathcal{N}_{\nu_{eL}} \\ \mathcal{N}_{\nu_{\mu L}} \\ \mathcal{N}_{\nu_{\tau L}} \\ \mathcal{N}_{\nu_{eR}} \\ \mathcal{N}_{\nu_{\mu R}} \\ \mathcal{N}_{\nu_{\tau R}} \end{bmatrix} = \begin{bmatrix} -\lambda_1 - \lambda_2 & 0 & 0 & 0 & \lambda_1 & \lambda_2 \\ 0 & -\lambda_1 - \lambda_3 & 0 & \lambda_1 & 0 & \lambda_3 \\ 0 & 0 & -\lambda_2 - \lambda_3 & \lambda_2 & \lambda_3 & 0 \\ 0 & \lambda_1 & \lambda_2 & -\lambda_1 - \lambda_2 & 0 & 0 \\ \lambda_1 & 0 & \lambda_3 & 0 & -\lambda_1 - \lambda_3 & 0 \\ \lambda_2 & \lambda_3 & 0 & 0 & 0 & -\lambda_2 - \lambda_3 \end{bmatrix} \begin{bmatrix} \mathcal{N}_{\nu_{eL}} \\ \mathcal{N}_{\nu_{\mu L}} \\ \mathcal{N}_{\nu_{\tau L}} \\ \mathcal{N}_{\nu_{eR}} \\ \mathcal{N}_{\nu_{\mu R}} \\ \mathcal{N}_{\nu_{\tau R}} \end{bmatrix} \quad (16)$$

with  $\lambda_1 = 1/L_{e\mu}$ ,  $\lambda_2 = 1/L_{e\tau}$ ,  $\lambda_3 = 1/L_{\mu\tau}$ . The mean free path  $L_{if}$  include the effect of the transition magnetic moments  $\mu_{if}$ . For each species, there are four conversions : two contribute positively (gain), two negatively (loss). For example, the left-handed electron neutrino number gains from  $\nu_{\mu R}, \nu_{\tau R} \rightarrow \nu_{eL}$  and loses from  $\nu_{eL} \rightarrow \nu_{\mu R}, \nu_{\tau R}$ .

Figure 3 shows the effect on the electron neutrino and anti-neutrino fluxes for different values of the neutrino transition magnetic moment  $\mu_M$ , defined such that  $\mu_{e\mu} = \mu_{e\tau} = \mu_M$ . The transition magnetic moment  $\mu_{\mu\tau}$  has been fixed to its experimental upper-limit [28],  $2\mu_{\mu\tau} = 6.8 \times 10^{-10} \mu_B$ , as its influence on  $Y_e(r)$  is very small compared to  $\mu_{e\mu}$  and  $\mu_{e\tau}$ . The initial neutrino temperatures correspond to  $Y_e(0) = 0.30$  (*cf.* Table I). From Figure 3 one can see that the high energy electron (anti-) neutrino flux tail is enhanced (reduced) by the active-active conversion with increasing  $\mu_M$  while less (more) neutrinos are peaked at low energy.

Fig 4 shows the results for the electron fraction obtained by solving Eq.(16), as a function of  $\mu_M$ . One can see on both graphs that  $Y_e$  increases as  $\mu_M$  gets larger and that the Majorana case shows larger effect on  $Y_e$  than the Dirac case. For transition magnetic moments  $\mu_{e\mu}$  and  $\mu_{e\tau}$  between  $1.5 \times 10^{-9} \mu_B$  and  $2 \times 10^{-9} \mu_B$ , the electron fraction meets the critical value of 0.5 for all  $Y_e(0)$  (and then all neutrino energy hierarchies). The effect on  $Y_e$  depends again strongly on the value of magnetic moments and for  $2\mu_M = 0.74 \times 10^{-10} \mu_B$  (*i.e.* for experimental upper-limits),  $Y_e$

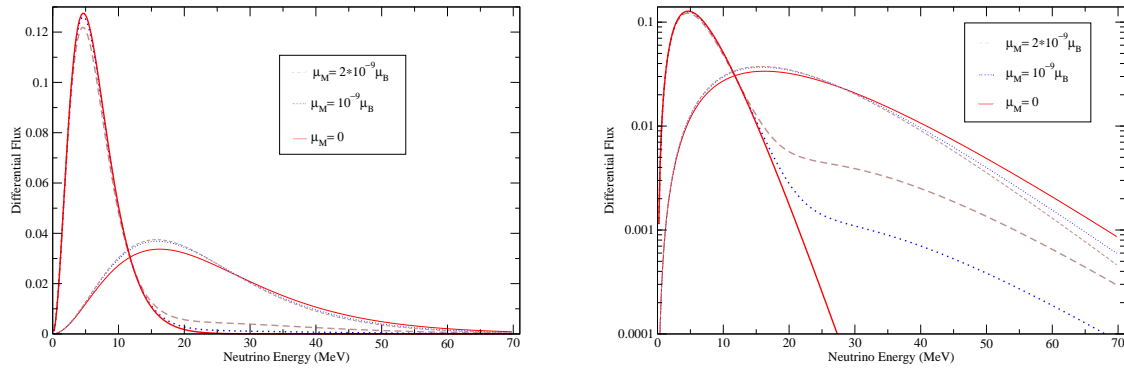


FIG. 3: Electron neutrino (thick) and anti-neutrino fluxes (thin) as a function of neutrino energy for the Majorana case. The curves correspond to  $\mu_M = 0$  (full),  $1 \times 10^{-9}$  (dotted) and  $2 \times 10^{-9} \mu_B$  (dashed). The case  $\mu_M = 10^{-10} \mu_B$  is indistinguishable from  $\mu_M = 0$ .

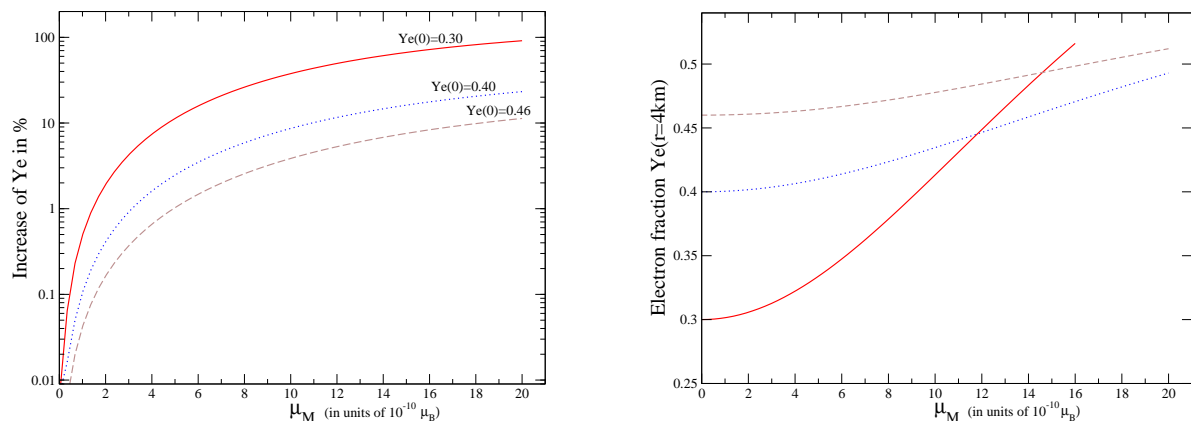


FIG. 4: CASE OF MAJORANA NEUTRINOS : (Left) Increase of the electron fraction in percentage,  $(Y_e(r) - Y_e(r = 0))/Y_e(r = 0)$ , evaluated at a distance of  $r = 4$  km from the neutron star surface, as a function of the magnetic moment  $\mu_M = \mu_{e\mu} = \mu_{e\tau}$  (see text). (Right)  $Y_e(r = 4\text{km})$  as a function of  $\mu_M$ .

increases by less than 0.5%.

## V. CONCLUSIONS

We pointed out that a non-zero magnetic moment suppresses both the electron neutrino and antineutrino fluxes for Dirac neutrinos and slightly increases the electron fraction in a core-collapse Supernova. In the Majorana case, the high (low) energy neutrino flux tail is enhanced (suppressed) for a large neutrino magnetic moment. Very large values of neutrino magnetic moment also increase the initial electron fraction. However such modifications of the fluxes cannot help reheating the shock wave since magnetic moments larger (but uncomfortably close) than the experimental limits are required to have sizeable effects.

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