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# Towards a covariant model for cosmic self-acceleration

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ABSTRACT: An explicitly covariant formulation is presented of a modified DGP scenario proposed recently [1], to avoid the instability of the self-accelerating branch. It is based on the introduction of a bulk scalar field with appropriate non-minimal coupling to the bulk Einstein-Hilbert term. The method is general and may be applied to other models as well.

KEYWORDS: Large Extra Dimensions, Cosmology of Theories beyond the SM.

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## Contents

## 1. Introduction

The current observational data [2] strongly support Dark Energy domination in the Universe. The nature of the Dark Energy is still a mystery and the "coincidence question" unanswered. So, alternative pictures exist in the literature, among which is the one proposed recently in the framework of Brane-World cosmology [3, 4, 5], based on the observation that due to energy exchange with the bulk, the present accelerating Universe may be a late-time stable attractor of the cosmological evolution equations [6]. This is an effort towards a natural resolution of the "coincidence problem" of cosmology, which in addition leads to several phenomenologically interesting properties at the fixed point. However, to accommodate the correct amount of matter $\Omega_m \simeq 0.3$ at the fixed point, one has to assume that the brane has negative tension, with the exponential expansion on the brane driven by energy influx from the bulk [7].

However, negative tension branes are believed a priori to be unsatisfactory for a realistic physical model, for essentially two reasons. First, it is known that the gravitational force on a negative tension brane in Randall-Sundrum-like scenarios [4] (with AdS bulk) becomes repulsive. Indeed, following [8] one can show that the four-dimensional Newton's constant $G_N$ in this case is proportional to the brane tension $\sigma$ with positive definite coefficient, leading to negative $G_N$ for negative $\sigma$. Additional matter in the bulk, for instance in the form of a minimally coupled scalar field with normal kinetic term and arbitrary potential, does not improve the situation [9]. Second, negative tension RS-II branes are believed to be unstable.

However, the physics changes considerably, if one extends the model to include the Ricci scalar term on the brane, induced by matter quantum loops or by finite brane-thickness

effects. The action of this, so called DGP, model [5] in its simplest version (without bulk cosmological constant or matter) is

$$S = \frac{1}{2} \int d^4x dy \sqrt{-\tilde{g}} \tilde{M}^3 \tilde{R} + \int d^4x \sqrt{-g} \tilde{M}^3 \Delta K + \frac{1}{2} \int d^4x \sqrt{-g} M^2 R + \sigma \int d^4x \sqrt{-g} \quad (1.1)$$

and describes a 3-brane with tension $\sigma$ but no extra matter on it, embedded in Minkowski $4+1$-dimensional bulk. $R$ is the aforementioned intrinsic scalar curvature term on the brane, while $\Delta K = K^+ - K^-$ is the jump of the trace of the extrinsic curvature across the brane. As usual, the purpose of this modified $K$ dependent Gibbons-Hawking (GH) term is to cancel unwanted terms in the variation of the bulk action related to the discontinuities of the derivative of the metric across the brane, so that one ends up with the proper Israel matching conditions[1].

The solution for the metric, relevant to cosmology which is our main interest here, is found to be

$$\tilde{g}_{AB} = \begin{pmatrix} (1 + \varepsilon H|y|)^2 \begin{pmatrix} -1 & 0 \\ 0 & e^{2Ht}\delta_{ab} \end{pmatrix} & 0 \\ 0 & 1 \end{pmatrix} \quad (1.2)$$

where $Z_2$ symmetry across the brane is manifest and $\varepsilon = \pm 1$. The parameter $H$ is given by

$$H^2 = 2\varepsilon \frac{\tilde{M}^3}{M^2} H + \frac{\sigma}{3M^2} \quad \Rightarrow \quad H = \frac{\tilde{M}^3}{M^2} \left( \varepsilon \pm \sqrt{1 + \frac{\sigma M^2}{3\tilde{M}^6}} \right). \quad (1.3)$$

Metrics with opposite $H$ are related by time reversal. So, only two of these four values of $H$ correspond to independent solutions. We choose the ones which correspond to the two values of $\varepsilon = \pm 1$, both with the $+$ sign in front of the square root.

We are interested in solutions with the following properties: (a) $H > 0$, in order to correctly describe the present accelerating expansion of the Universe, (b) $\varepsilon H < 0$ in order to improve the chances for stability, (c) attractive Newton's law on the brane, and (d) all the above consistent even with $\sigma \le 0$. This, may eventually allow for a natural explanation of the cosmic acceleration and a resolution of the "coincidence issue" [6].

Notice, though, that none of the solutions given in (1.2), (1.3) satisfies these requirements. Closest to being satisfactory is the self-accelerating one, corresponding to $\varepsilon = +1$. This has $H > 0$ even with a not too negative $\sigma$, but the metric in this case grows in the bulk and leads, perhaps not surprisingly, to ghost instabilities in the spectrum of perturbations [11, 12, 13, 14, 15, 16]. For a positive tension brane a helicity-0 excitation of the spin-2 graviton is a ghost, while for a negative tension brane the spin-0 mode becomes a ghost. For tensionless brane the ghost field is a linear combination of the spin-0 mode and the helicity-0 excitation of the graviton.

The other independent solution with $\varepsilon = -1$ is stable, but it is unsatisfactory in connection with (d) above, because one needs positive cosmological constant on the brane to explain the accelerating cosmic expansion. Is there a way to satisfy all four requirements (a)-(d)?

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[1] A very explicit derivation of the variation of the GH term can be found in [10].

A step in this direction was outlined in [1][2]. The proposal was to modify the bulk action (1.1) by the multiplication with an appropriate smearing function near the brane as follows

$$S_{\text{bulk}} = \frac{1}{2} \int d^4x dy \sqrt{-\tilde{g}} \tilde{M}^3 \tilde{R} \rightarrow \frac{1}{2} \int d^4x dy \sqrt{-\tilde{g}} \tilde{M}^3 \tilde{F}(x,y) \tilde{R} \qquad (1.4)$$

where in the simplest case $\tilde{F}(x,y) = 1 - m\bar{\delta}(y)$ with $m$ a parameter and $\bar{\delta}(y)$ a $\delta$-like function with a second parameter $\alpha$. A simple choice is $\bar{\delta}(y) = \pi^{-1}\alpha/(\alpha^2 + y^2)$. The extrinsic curvature terms should be modified accordingly. The model still has the solution (1.2) for the metric, with $H$ now given by

$$H = \frac{\tilde{M}^3}{M^2} \left(1 - \frac{m}{\pi\alpha}\right) \left(\varepsilon \pm \sqrt{1 + \frac{\sigma M^2}{3\tilde{M}^6 \left(1 - \frac{m}{\pi\alpha}\right)^2}}\right). \qquad (1.5)$$

The solution with the $+$ sign in front of the square root, $\varepsilon = -1$ and in the limit $\alpha \to 0$ and $m \to 0$ with $m/(\pi\alpha) \sim \text{const} > 1$, has $H > 0$, describes a self-accelerated brane, which satisfies all our requirements, with the exception of the stability issue which cannot be decided at the level of such a non-covariant formulation of the model. Nevertheless, as it was shown in [1] such a modification leads to a flip of the sign in front of the extrinsic curvature terms in the brane equations of motion. Exactly this sign is responsible for the stability of the metric perturbations and the absence of unstable modes in the normal branch. Thus, modifying the action in the above mentioned way we expect to obtain self-acceleration while keeping equations as they are in the normal stable branch. Further nice properties of this modification are considered the relaxation of the bulk gravity scale and the conservation of the number of parameters.

The purpose of the present paper is to develop a manifestly covariant formulation of the above modification, making use of a bulk scalar field instead of the function $\tilde{F}(x,y)$. Naturally, extra complications arise from the fact that one has to satisfy also the scalar field equation of motion, as well as the corresponding additional matching condition on the brane.

## 2. Codimension-1 brane in 5-dimensional bulk

Consider a 3-brane embedded in a 5-dimensional bulk. The coordinates in the bulk are denoted by $x^A$ with capital latin indices running from 0 to 4 and with $x^4 \equiv y$. The coordinates on the brane are denoted by $\xi^\mu$, with Greek indices from the middle of the alphabet taking values from 0 to 3. Occasionally, we shall use $t$ instead of the coordinate with index 0, while spatial indices on the brane will be denoted by lowercase Latin letters $a, b, \ldots$. The position of the brane in the bulk is parameterized as $x^A = X^A(\xi^\mu)$, $X^4 \equiv Y$. Finally, we shall be using tildes to designate quantities referring to the bulk. Thus, the bulk metric is $\tilde{g}_{AB}$ and the induced metric on the brane is $g_{\mu\nu} = \partial_\mu X^A \partial_\nu X^B \tilde{g}_{AB}(X^A(\xi^\mu))$. Here $\partial_\mu \equiv \partial/\partial\xi^\mu$ and our convention for the signature is $(-, +, +, +, +)$.

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[2]For a related approach see also [17].

We shall be interested in the model described by the generic action

$$S_{\text{bulk}} = \int d^4x \, dy \sqrt{-\tilde{g}} \left( \frac{\tilde{M}^3}{2} \tilde{F}(\Phi) \tilde{R} - \frac{1}{2} \tilde{X}(\Phi) \tilde{g}^{AB} \partial_A \Phi \partial_B \Phi - \tilde{V}(\Phi) \right), \qquad (2.1)$$

$$S_{\text{brane}} = \int d^4x \sqrt{-g} \left( \tilde{M}^3 \tilde{F}(\Phi) \Delta K + L_{\text{brane}}(g_{\mu\nu}, \Phi, \psi) \right). \qquad (2.2)$$

The bulk scalar field $\Phi$ is supposed to be a modulus field. This is the field relevant to the hereby proposed modification of the DGP model. $\tilde{V}(\Phi)$ may have a constant term $\tilde{\Lambda}$, being the cosmological constant in the bulk. $\tilde{F}, \tilde{X}$ and $\tilde{V}$ do not depend on $\tilde{g}$. $\Delta K = K^+ - K^-$ is, as in (1.1), the jump of the trace of the extrinsic curvature across the brane. Standard Model fields $\psi$ confined on the brane may also be present. They will not be needed in our discussion. We choose to study the equations of motion (EOM) in the Gauss-Normal (GN) coordinate system, with the brane located at $y = 0$. In GN coordinates it is convenient to identify $X^\mu = \xi^\mu$ so that $x^\mu$ and $\xi^\mu$ are indistinguishable (static gauge). Further, in this coordinate system $\tilde{g} = \text{diag}(\tilde{g}_{\mu\nu}, 1)$ and the induced metric on the brane is $g_{\mu\nu} = \tilde{g}_{\mu\nu}|_{y=0}$. The non-vanishing Christoffel symbols are $\tilde{\Gamma}^y_{\mu\nu} = -\frac{1}{2} \frac{\partial \tilde{g}_{\mu\nu}}{\partial y}$, $\tilde{\Gamma}^\nu_{\mu y} = \frac{1}{2} \tilde{g}^{\nu\rho} \frac{\partial \tilde{g}_{\rho\mu}}{\partial y}$ and $\tilde{\Gamma}^\rho_{\mu\nu}$, where $\tilde{\Gamma}^\rho_{\mu\nu}|_{y=0} = \Gamma^\rho_{\mu\nu}$, the being the Christoffel symbols obtained from the brane metric $g_{\mu\nu}$. With $x^\mu$ and $\xi^\mu$ identified, we have $\partial_\mu = \partial/\partial x^\mu = \partial/\partial \xi^\mu$, while $\partial_A \equiv \partial/\partial x^A$.

As a side remark, notice that if $\tilde{F}(\Phi)$ is not a constant, it can be brought to any other non-constant form by an appropriate redefinition of the field $\Phi$. The two actions are equivalent at the classical level. A similar observation applies to the function $\tilde{X}(\Phi)$. If it is non-zero, it may be transformed to any non-vanishing constant. The two redefinitions cannot, in general, be applied together.

## 2.1 Bulk

Varying the bulk action (2.1) with respect to the metric one obtains in the bulk

$$-\tilde{M}^3 \tilde{F} \tilde{G}_{AB} + \tilde{M}^3 (\tilde{D}_A \tilde{D}_B \tilde{F} - \tilde{g}_{AB} \tilde{\Box} \tilde{F}) + \tilde{X} \partial_A \Phi \partial_B \Phi - \frac{\tilde{X}}{2} \tilde{g}_{AB} \tilde{g}^{CD} \partial_C \Phi \partial_D \Phi - \tilde{g}_{AB} \tilde{V} = 0. \qquad (2.3)$$

Here $\tilde{G}_{AB} = \tilde{R}_{AB} - \frac{1}{2} \tilde{R} \tilde{g}_{AB}$ is the bulk Einstein tensor, $\tilde{D}$ is the covariant derivative for the bulk metric $\tilde{g}_{AB}$ and $\tilde{\Box} = \tilde{g}^{AB} \tilde{D}_A \tilde{D}_B$. Similarly, the EOM of the scalar field $\Phi$ reads

$$\tilde{X} \tilde{\Box} \Phi + \frac{\tilde{X}^{(1)}}{2} \tilde{g}^{AB} \partial_A \Phi \partial_B \Phi + \frac{\tilde{M}^3 \tilde{F}^{(1)} \tilde{R}}{2} - \tilde{V}^{(1)} = 0. \qquad (2.4)$$

Here the superscript $^{(n)}$ denotes the $n$-th derivative with respect to the field $\Phi$.

For simplicity we shall restrict ourselves to diagonal bulk metrics. Furthermore, following the cosmological principle, we shall take space on the brane to be homogeneous and isotropic. In the special case of zero spatial curvature on the brane the most general ansatz for the metric in GN coordinates is then

$$\tilde{g}_{AB} = \begin{pmatrix} -N^2(t,y) & 0 & 0 \\ 0 & A^2(t,y)\delta_{ab} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \qquad (2.5)$$

The non-zero Christoffel symbols for the $(\mu, \nu)$ part are $\tilde{\Gamma}^t_{tt} = \dot{N}/N$, $\tilde{\Gamma}^t_{ab} = A\dot{A}\delta_{ab}/N^2$, $\tilde{\Gamma}^b_{at} = \dot{A}\delta^b_a/A$. In what follows, we will use dot for time derivative and prime for $y$ derivative. Direct calculation of the Einstein tensor gives the following non-zero components

$$\tilde{G}_{tt} = (-N^2)3 \left[ -\frac{\dot{A}^2}{A^2 N^2} + \frac{A'^2}{A^2} + \frac{A''}{A} \right], \tag{2.6}$$

$$\tilde{G}_{ab} = (A^2 \delta_{ab}) \left[ -\frac{1}{N^2}\left( \frac{\dot{A}^2}{A^2} + 2\frac{\ddot{A}}{A} - 2\frac{\dot{A}\dot{N}}{AN} \right) + \frac{A'^2}{A^2} + 2\frac{A''}{A} + 2\frac{A'N'}{AN} + \frac{N''}{N} \right], \tag{2.7}$$

$$\tilde{G}_{yy} = 3 \left[ -\frac{1}{N^2}\left( \frac{\dot{A}^2}{A^2} + \frac{\ddot{A}}{A} - \frac{\dot{A}\dot{N}}{AN} \right) + \frac{A'^2}{A^2} + \frac{A'N'}{AN} \right], \tag{2.8}$$

$$\tilde{G}_{ty} = 3 \left[ \frac{\dot{A}N'}{AN} - \frac{\dot{A}'}{A} \right]. \tag{2.9}$$

## 2.2 Brane

Life on the brane is described by the equations of motion on the brane. They are obtained from the brane lagrangian, supplemented by the GH term. We use the following conventions: the unit vector $n^A$ normal to the brane is taken to point from the region $y < 0$ into the region $y > 0$ in GN coordinates. The same on both sides of the brane. In terms of this vector the induced metric is given by the tangent to the brane components of the projection operator $g_{AB} = \tilde{g}_{AB} - n_A n_B$. The extrinsic curvature (the second fundamental form of the surface) is defined as $K_{AB} = -g^C_A g^D_B \tilde{D}_C n_D$ and its trace is $K = g^{AB} K_{AB}$. Its components in the GN frame are $K_{\mu\nu} = -\frac{1}{2}\partial_y \tilde{g}_{\mu\nu}|_{y=\text{const}}$. For our purposes, we will have to evaluate it at $y = 0+$ and $y = 0-$, since we put our brane at $y = 0$ and allow for discontinuities of the $y$ derivatives of the metric components.

The equations of motion for the metric on the brane become

$$- \tilde{M}^3 \left[ \tilde{F}(K_{\mu\nu} - g_{\mu\nu}K) + g_{\mu\nu}\tilde{F}' \right] - M^2 F G_{\mu\nu} + M^2(D_\mu D_\nu F - g_{\mu\nu}\Box F) +$$
$$+ X\partial_\mu \Phi \partial_\nu \Phi - \frac{X}{2}g_{\mu\nu}g^{\alpha\beta}\partial_\alpha \Phi \partial_\beta \Phi - g_{\mu\nu}V = 0. \tag{2.10}$$

Here $\Box = g^{\mu\nu}D_\mu D_\nu$ and $D_\mu$ is a covariant derivative built upon the induced metric. From now on, we shall be using square brackets to denote the discontinuity across the brane of the quantity inside, i.e. $[W] \equiv W(y = 0+) - W(y = 0-)$, for any quantity $W$. Of course, if one assumes $Z_2$ symmetry across the brane, then $W(y = 0+) = -W(y = 0-)$ and $[W] = 2W(y = 0+)$.

Similarly, the field $\Phi$ obeys the following equation on the brane

$$\left[ \tilde{X}' + \tilde{M}^3 \tilde{F}^{(1)} K \right] + X\Box\Phi + \frac{X^{(1)}}{2}g^{\mu\nu}\partial_\mu \Phi \partial_\nu \Phi + \frac{M^2 F^{(1)} R}{2} - V^{(1)} = 0. \tag{2.11}$$

## 3. Static scalar $\Phi = \Phi(y)$

This is the simplest possibility, since to implement the modification proposed in [1], we need eventually a non-trivial $y$-dependent function $\tilde{F}$. With $\Phi$ a function of $y$ only, the system

simplifies considerably. Namely, $\Phi$-dependent quantities on the brane become constants and all derivatives of $\Phi$ along the brane are zero. In the bulk only $y$-derivatives survive and one is led to the following set of equations.

In the bulk

$$-\tilde{M}^3\tilde{F}\tilde{G}_{\mu\nu} + \tilde{M}^3\left(\frac{1}{2}\tilde{g}'_{\mu\nu}\tilde{F}' - \frac{1}{2}\tilde{g}_{\mu\nu}\bar{g}\tilde{F}' - \tilde{g}_{\mu\nu}\tilde{F}''\right) - \frac{\tilde{X}}{2}\tilde{g}_{\mu\nu}\Phi'^2 - \tilde{g}_{\mu\nu}\tilde{V} = 0, \quad (3.1)$$

$$-\tilde{M}^3\tilde{F}G_{yy} - \frac{\tilde{M}^3}{2}\bar{g}\tilde{F}' + \frac{\tilde{X}}{2}\Phi'^2 - \tilde{V} = 0, \quad (3.2)$$

$$\tilde{X}\left(\Phi'' + \frac{1}{2}\bar{g}\Phi'\right) + \frac{\tilde{X}^{(1)}}{2}\Phi'^2 + \frac{\tilde{M}^3\tilde{F}^{(1)}\tilde{R}}{2} - \tilde{V}^{(1)} = 0, \quad (3.3)$$

$$\frac{\dot{A}N'}{AN} - \frac{\dot{A}'}{A} = 0. \quad (3.4)$$

Where $\bar{g} \equiv \tilde{g}^{\alpha\beta}\tilde{g}'_{\alpha\beta}$ and $\tilde{\Box} = \partial_y^2 + \frac{1}{2}\bar{g}\partial_y$. Contracting the Einstein equations (3.1) and (3.2) with $\tilde{g}^{AB}$ one obtains

$$\frac{3\tilde{M}^3\tilde{F}\tilde{R}}{2} - 2\tilde{M}^3\left(\bar{g}\tilde{F}' + 2\tilde{F}''\right) - \frac{3\tilde{X}}{2}\Phi'^2 - 5\tilde{V} = 0. \quad (3.5)$$

Under our assumption that $\tilde{F}' \neq 0$ we can solve (3.3) and (3.5) for $\tilde{R}$ and $\bar{g}$, provided

$$\tilde{X} \neq -\frac{4}{3}\tilde{M}^3\frac{(\tilde{F}^{(1)})^2}{\tilde{F}}. \quad (3.6)$$

This leads to time-independent $\tilde{R}$ and $\bar{g}$. In what follows we assume that condition (3.6) is valid.

On the brane the field equations are

$$-\tilde{M}^3 g_{\mu\nu}[\tilde{F}'] - \tilde{M}^3\tilde{F}[K_{\mu\nu} - g_{\mu\nu}K] - M^2 FG_{\mu\nu} - g_{\mu\nu}V = 0, \quad (3.7)$$

$$[\tilde{X}'] + \tilde{M}^3\tilde{F}^{(1)}[K] + \frac{M^2 F^{(1)}R}{2} - V^{(1)} = 0. \quad (3.8)$$

All tilded quantities are evaluated on the brane and consequently are constants. Contracting equation (3.7) with $g^{\mu\nu}$ one obtains

$$-4\tilde{M}^3[\tilde{F}'] + 3\tilde{M}^3\tilde{F}[K] + M^2 FR - 4V = 0. \quad (3.9)$$

This, together with (3.8) form a system of two linear algebraic equations for $[K]$ and $R$, which has a solution provided $3\tilde{F}F^{(1)} - 2\tilde{F}^{(1)}F \neq 0$. In this case both $[K]$ and $R$ are constants. If, on the other hand, $3\tilde{F}F^{(1)} - 2\tilde{F}^{(1)}F = 0$, then for consistency we have to require $3\tilde{F}\left(V^{(1)} - [\tilde{X}']\right) = 4\tilde{F}^{(1)}\left(V + \tilde{M}^3[\tilde{F}']\right)$ and $[K]$ and $R$ would not, a priori, have to be time-independent. However, given that in our notation $K(y_0) = -\frac{1}{2}g^{\mu\nu}g'_{\mu\nu} = -\frac{1}{2}\tilde{g}|_{y=y_0}$, which is time-independent, we conclude that $R$ is also necessarily time-independent, a consequence of our assumption that $\Phi = \Phi(y)$.

It is rather straightforward to take into account the remaining equations. One is led to the following conclusions:

- $\tilde{R}$, $\bar{g}$, $R$ and $[K]$ do not depend on time

- $A = \alpha(y)a(t)$ and $N = \nu(y)n(t)$, i.e. the functions $A$ and $N$ are factorized.

- There are two possibilities for these functions: either $N = b(t)\dot{A}$, or $\dot{A} = 0$ and $N$ undetermined.

The second possibility gives Minkowski brane, which is not particularly interesting for our purposes. The solution in this case can be found analytically, reduced to quadratures. The first possibility, to which we concentrate next, looks more interesting for cosmology, because it leads to a time-dependent metric on the brane.

## 4. Construction of the solution for static scalar and $N = b(t)\dot{A}$

### 4.1 Bulk part

In this case $n(t) = b(t)\dot{a}(t)$ and $\alpha(y) = \nu(y)$. The non-zero components of the Einstein tensor become

$$\tilde{G}_{tt} = (-b^2\dot{A}^2)3P, \ \tilde{G}_{ab} = (A^2\delta_{ab})(P+2Q), \ \tilde{G}_{yy} = 3\left(P+Q-2\frac{\nu''}{\nu}\right),$$

$$\text{where } P = -\frac{1}{A^2b^2} + \frac{\nu'^2}{\nu^2} + \frac{\nu''}{\nu}, \ Q = \frac{\dot{b}}{A\dot{A}b^3} + \frac{\nu'^2}{\nu^2} + \frac{\nu''}{\nu}.$$

Let us, now, take a closer look at the $\{tt\}$ component of equation (3.1). All terms in this equation, with the exception of the first one, are of the form $\dot{a} \times$ (a function of y). This is a direct consequence of the factorization property of $\tilde{g}_{tt}$ and the $t-$independence of $\Phi$. Only the first term may a priori depend on time, due to the presence of the $-1/(A^2b^2)$ term in $P$. Thus, the $\{tt\}$ component of equation (3.1) for non-vanishing $\dot{a}$ takes the form $a(t)b(t) = $ (function of y), and leads to $b = 1/(Ha)$ where $H$ is a constant. The Einstein tensor then becomes

$$\tilde{G}_{\mu\nu} = \frac{3}{\nu^2}(-H^2 + \nu'^2 + \nu\nu'')\tilde{g}_{\mu\nu}, \quad \tilde{G}_{yy} = \frac{6}{\nu^2}(-H^2 + \nu'^2). \tag{4.1}$$

The Einstein equations read

$$\frac{3}{\nu^2}(-H^2 + \nu'^2 - \nu\nu'') = +\frac{1}{\tilde{M}^3\tilde{F}}\left(\tilde{X}\Phi'^2 + \tilde{M}^3\tilde{F}'' - \tilde{M}^3\frac{\nu'}{\nu}\tilde{F}'\right), \tag{4.2}$$

$$\frac{6}{\nu^2}(-H^2 + \nu'^2) = -\frac{1}{\tilde{M}^3\tilde{F}}\left(-\frac{1}{2}\tilde{X}\Phi'^2 + \tilde{V} + 4\tilde{M}^3\frac{\nu'}{\nu}\tilde{F}'\right). \tag{4.3}$$

Notice that $a(t)$ remains arbitrary and will not be fixed by the equations of motion. However, any form of $a(t)$ leads to (anti) de Sitter brane metric, because it can be absorbed into the definition of time by $\dot{a}/(HA)dt = d\tau$, in terms of this new time $\tau$ one obtains $a(\tau) = a_0e^{H\tau}$. So, the bulk metric reads

$$\tilde{g}_{AB} = \left( \begin{array}{cc} \nu^2(y)\left(\begin{array}{cc} -1 & 0 \\ 0 & e^{2H\tau} \end{array}\right) & 0 \\ 0 & 1 \end{array} \right) \tag{4.4}$$

where, without loss of generality, $a_0$ was chosen equal to 1.

Given the functions $\tilde{F}(y)$ and $\nu(y)$ one can use (4.2) to obtain $\tilde{X}\Phi'^2$ as a function of $y$

$$\tilde{X}\Phi'^2 = \frac{3h\tilde{M}^3\tilde{F}}{\nu^2} - \tilde{M}^3\left(\tilde{F}'' - \frac{\nu'}{\nu}\tilde{F}'\right) \tag{4.5}$$

where $h = -H^2 + \nu'^2 - \nu\nu''$. Finally, plug this into equation (4.3) and express the potential $\tilde{V}$

$$\tilde{V} = -\frac{(9h + 12\nu\nu'')\tilde{M}^3\tilde{F}}{2\nu^2} - \frac{\tilde{M}^3}{2}\left(\tilde{F}'' + \frac{7\nu'}{\nu}\tilde{F}'\right). \tag{4.6}$$

One can check by direct substitution that equation (2.4) is satisfied.

### 4.2 Life on the brane

The geometry and dynamics on the brane, using the time coordinate $\tau$, are given by

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 \\ 0 & e^{2H\tau}\delta_{ab} \end{pmatrix}, \ R = 12H^2, \ G_{\mu\nu} = -3H^2 g_{\mu\nu},$$

$$[K_{\mu\nu}] = -[\nu']g_{\mu\nu}, \ [K] = -4[\nu'].$$

As usual, $[\nu'] \equiv \nu'_+ - \nu'_-$ is the discontinuity of the function $\nu'(y)$ across the brane. $Z_2$ symmetry would force them to satisfy $\nu'_+ = -\nu'_-$. Also we choose $\nu(0) = 1$.

As discussed above, we have to distinguish two cases. (a) If $3\tilde{F}F^{(1)} - 2\tilde{F}^{(1)}F \neq 0$ we may unambiguously express $[\nu']$ and $H$ through $\tilde{F}$, $F$ and $V$ as follows

$$[\nu'] = \frac{F\left(V^{(1)} - [\tilde{X}']\right) - 2F^{(1)}\left(V + \tilde{M}^3[\tilde{F}']\right)}{2\tilde{M}^3(3\tilde{F}F^{(1)} - 2\tilde{F}^{(1)}F)},$$

$$H = \pm\sqrt{\frac{3\tilde{F}\left(V^{(1)} - [\tilde{X}']\right) - 4\tilde{F}^{(1)}\left(V + \tilde{M}^3[\tilde{F}']\right)}{6M^2(3\tilde{F}F^{(1)} - 2\tilde{F}^{(1)}F)}}. \tag{4.7}$$

(b) If, on the other hand, $3\tilde{F}F^{(1)} - 2\tilde{F}^{(1)}F = 0$, we have to require $3\tilde{F}\left(V^{(1)} - [\tilde{X}']\right) = 4\tilde{F}^{(1)}\left(V + \tilde{M}^3[\tilde{F}']\right)$. In this case, one of the quantities $[\nu']$ and $H$ remains undetermined. However, they are related by

$$H = \pm\sqrt{\frac{\tilde{M}^3\tilde{F}}{M^2F}[\nu'] + \frac{V + \tilde{M}^3[\tilde{F}']}{3M^2F}}. \tag{4.8}$$

As a special case, it is easy to check that taking $\tilde{F} = F = 1$, $\tilde{X} = X = \tilde{V} = 0$, $V = \sigma$ and $\nu'_+ = -\nu'_- = \varepsilon H$ one reproduces the known relation (1.3) between brane-tension and $H$ of the DGP setup.

## 5. Application to the modified DGP model and discussion

Let us recapitulate the steps one has to take to construct a modified model of the type we are proposing here, based on $\Phi$ which depends only on $y$. For our purposes, it is convenient to start with a given $\tilde{F}$, since the modified DGP model [1] gives some idea about the desirable form of $\tilde{F}(y)$. Namely, $\tilde{F} = 1 - m\bar{\delta}(y)$ such that $\tilde{F}(y = 0) = \text{const} < 0$ and $\tilde{F}(y \gg \alpha) = 1$, where $\alpha$ is the width of localization of $\tilde{F}$ near the brane. The metric has the general form (4.4). Given, in addition, the function $\nu(y)$, one uses equations (4.5) and (4.6) to obtain $\tilde{X}$ and $\tilde{V}$ as functions of $y$. Finally, on the brane, one has to satisfy (3.7) and (3.8). It remains to determine the $\Phi$ dependence of the bulk quantities. For that, one needs to specify (or know independently) either $\Phi(y)$ or the dependence on $\Phi$ of one of $\tilde{F}$, $\tilde{X}$ or $\tilde{V}$.

Consider the special case of Minkowski bulk. This fixes $\nu(y) = 1 + \nu_1 y$, with $\nu_1 = \varepsilon H$ and we assume $Z_2$ symmetry. Take $\tilde{F} = 1 - \frac{m}{\alpha \cosh(y^2/\alpha^2)}$ and $\Phi(y) = y$. One is led to the following model

$$\tilde{F} = 1 - \frac{m}{\alpha \cosh\left(\Phi^2/\alpha^2\right)},$$

$$\tilde{X} = -\frac{m\tilde{M}^3}{\alpha^5 \cosh^3\left(\Phi^2/\alpha^2\right)}\left(2\Phi^2\left(3 - \cosh\left(2\Phi^2/\alpha^2\right)\right) + \alpha^2 \frac{\sinh\left(2\Phi^2/\alpha^2\right)}{1 + \nu_1\Phi}\right),$$

$$\tilde{V} = -\frac{m\tilde{M}^3}{\alpha^5 \cosh^3\left(\Phi^2/\alpha^2\right)}\left(\Phi^2\left(3 - \cosh\left(2\Phi^2/\alpha^2\right)\right) + \alpha^2 \frac{1 + 8\nu_1\Phi}{2(1 + \nu_1\Phi)}\sinh\left(2\Phi^2/\alpha^2\right)\right),$$

$$F = 1, \qquad V = \sigma. \tag{5.1}$$

It is easy to check that bulk equations are satisfied. Equation (3.8) on the brane is satisfied trivially, while equation (3.7) reduces to

$$-6\tilde{M}^3\left(1 - \frac{m}{\alpha}\right)\varepsilon H + 3M^2 H^2 - \sigma = 0 \tag{5.2}$$

which determines $H$ as

$$H = \frac{\tilde{M}^3}{M^2}\left(1 - \frac{m}{\alpha}\right)\left(\varepsilon \pm \sqrt{1 + \frac{\sigma M^2}{3\tilde{M}^6\left(1 - \frac{m}{\alpha}\right)^2}}\right) \tag{5.3}$$

which coincides with one given in (1.5) up to the rescaling of $\alpha$ by $\pi$ and thus explicitly realizes the trick outlined in [1].

As another special model, let us take $\tilde{X} = 0$, still with Minkowski bulk. Then expression (4.5) becomes an equation for the function $\tilde{F}$ with the general solution

$$\tilde{F}(y) = C_1(1 + \nu_1 y)^2 + C_2. \tag{5.4}$$

Here we see that $\tilde{F}$ is not localized near the brane and becomes important in the whole range of $y$ up to the Rindler horizon $y = -1/\nu_1$. However, the potentially dangerous domain of $y$ with $\tilde{F} < 0$ can be made arbitrarily narrow, with $|\tilde{F}(y = 0)|$ also infinitesimal. Indeed, adjust $\tilde{F}(y = 0) = C_2 + C_1 = -\alpha < 0$ to be extremely small. Then, the width of

the negative domain becomes $-\alpha/(C_2\nu_1)$. In order to have realistic $H$ we have to adjust $(\alpha\tilde{M}^3) \sim (100\text{MeV})^3$ [18].

Let us briefly summarize the results of the present analysis. A covariant implementation of the modified DGP model [1] was constructed, by means of a bulk scalar field, with appropriate coupling to gravity. Flat metric in the bulk is a possible solution with non-constant scalar field. The parameters of the theory can be adjusted in such a way that one has not growing metric in the bulk (the usual normal-branch behavior) with positive Hubble constant on the brane, leading to accelerating expansion on the brane, even without any extra matter and even with negative brane tension. We explicitly demonstrated two models satisfying our requirements (a)-(d) of the Introduction. (i) In the first one the $\delta$-function-like profile of the function $\tilde{F}$ is localized near the brane and in the limit $\alpha \to 0$ the width of localization goes to zero. Thus, in the $\alpha \to 0$ limit the bulk is modified in a narrow domain near the brane only. However, the presence of the kinetic term for the scalar field in the bulk is necessary. (ii) In the second model, on the other hand, the scalar field does not have a kinetic term, but the profile of $\tilde{F}$ does not go rapidly to unity away from the brane. However, one may adjust integration constants and make the $y$-domain with negative $\tilde{F}$ infinitesimal, if necessary to be consistent with observations.

A detailed stability analysis of the hereby proposed class of models is an open question. Also, time-dependent bulk scalar field solutions, which were not studied here, may give rise to interesting phenomena on the brane. Finally, asymmetric brane-world setups in the spirit of the recent paper [19] may provide further possibilities for model building.

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## References

[1] G. Gabadadze, *A Model for Cosmic Self-Acceleration*, hep-th/0612213; G. Gabadadze, *Cargese Lectures on Brane Induced Gravity*, arXiv:0705.1929.

[2] S.J. Perlmutter et al., *Measurements of Omega and Lambda from 42 High-Redshift Supernovae*, Astroph. J. **517** (1999) 565; astro-ph/9812133; A. Riess et al., *Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant*, Astron. J. **116** (1998) 1009; astro-ph/9805201; A. Riess et al., *Type Ia Supernova Discoveries at $z > 1$ From the Hubble Space Telescope: Evidence for Past Deceleration and Constraints on Dark Energy Evolution*, Astrophys. J. **607** (2004) 665; astro-ph/0402512; R.A. Knop et al., *New constraints on $\omega_m$, $\omega_\lambda$, and w from an independent set of eleven high — redshift supernovae observed with HST*, Astrophys.J. **598** (2003) 102, astro-ph/0309368; M. Tegmark et al., *The 3-d power spectrum of galaxies from the SDSS*, Astroph. J. **606** (2004) 702; astro-ph/0310723; D.N. Spergel et al., *First Year Wilkinson Microwave Anisotropy Probe*

(WMAP) Observations: Determination of Cosmological Parameters, Astroph. J. Suppl. **148** (2003) 175; astro-ph/0302209; D.N. Spergel et al., *Wilkinson microwave anisotropy probe (WMAP) three year results: implications for cosmology*, astro-ph/0603449.

[3] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, *The hierarchy problem and new dimensions at a millimeter*, Phys. Lett. B **429**, 263 (1998), arXiv:hep-ph/9803315.

[4] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, Phys. Rev. Lett. **83**, 3370 (1999), arXiv:hep-ph/9905221; L. Randall and R. Sundrum, *An alternative to compactification*, Phys. Rev. Lett. **83**, 4690 (1999), arXiv:hep-th/9906064.

[5] G. Dvali, G. Gabadadze, M. Porrati, *4D Gravity on a Brane in 5D Minkowski Space*, Phys.Lett. **B485** (2000) 208-214, arXiv:hep-th/0005016.

[6] G. Kofinas, G. Panotopoulos, T.N. Tomaras, *Brane-bulk energy exchange: a model with the present universe as a global attractor*, JHEP **0601** (2006) 107, arXiv:hep-th/0510207.

[7] E. Kiritsis, G. Kofinas, N. Tetradis, T.N. Tomaras and V. Zarikas, *Cosmological evolution with brane-bulk energy exchange*, JHEP **0302** (2003) 035, arXiv:hep-th/0207060.

[8] T. Shiromizu, K. Maeda, M. Sasaki, *The Einstein Equations on the 3-Brane World*, Phys.Rev. D62 (2000) 024012, gr-qc/9910076.

[9] K. Aoyanagi, K. Maeda, *Creation of a brane world with a bulk scalar field*, JCAP **0603** (2006) 012, hep-th/0602149; C. Barcelo, M. Visser, *Moduli fields and brane tensions: generalizing the junction conditions*, Phys.Rev. **D63** (2001) 024004, arXiv:gr-qc/0008008; C. Barcelo, M. Visser, *Braneworld gravity: Influence of the moduli fields*, JHEP **0010** (2000) 019, arXiv:hep-th/0009032.

[10] H.A. Chamblin, H.S. Reall, *Dynamic Dilatonic Domain Walls*, Nucl.Phys. **B562** (1999) 133-157, arXiv:hep-th/9903225.

[11] M. A. Luty, M. Porrati and R. Rattazzi, *Strong interactions and stability in the DGP model*, JHEP **0309** (2003) 029, arXiv:hep-th/0303116.

[12] A. Nicolis and R. Rattazzi, *Classical and quantum consistency of the DGP model*, JHEP **0406** (2004) 059, arXiv:hep-th/0404159.

[13] K. Koyama and K. Koyama, *Brane induced gravity from asymmetric compactification*, Phys. Rev. **D72** (2005) 043511, arXiv:hep-th/0501232; K. Koyama, *Are there ghosts in the self-accelerating brane universe?*, Phys. Rev. **D72** (2005) 123511, arXiv:hep-th/0503191.

[14] D. Gorbunov, K. Koyama and S. Sibiryakov, *More on ghosts in DGP model*, Phys. Rev. **D73** (2006) 044016, arXiv:hep-th/0512097.

[15] M. Carena, J. Lykken, M. Park and J. Santiago, *Self-acclearating warped braneworlds*, Phys. Rev. **D75** (2007) 026009, arXiv:hep-th/0611157.

[16] Ch. Charmousis, R. Gregory, N. Kaloper, A. Padilla, *DGP Specteroscopy*, JHEP 0610 (2006) 066, hep-th/0604086; C. Deffayet, G. Gabadadze, A. Iglesias, *Perturbations of Self-Accelerated Universe*, JCAP **0608** (2006) 012, arXiv:hep-th/0607099; A. Padilla, *A short review of "DGP Specteroscopy"*, arXiv:hep-th/0610093.

[17] V. Sahni, Yu. Shtanov, *Brane world models of dark energy*, JCAP **0311** (2003) 014, astro-ph/0202346.

[18] G. Gabadadze, *Looking At The Cosmological Constant From Infinite-Volume Bulk*, hep-th/0408118.

[19] Ch. Charmousis, R. Gregory, A. Padilla, *Stealth Acceleration and Modified Gravity*, arXiv:0706.0857.