5-dimensional solution with acceleration and small variation of G

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Abstract

A 5-dimensional cosmological solution in the model with two 2-forms and two "phantom" scalar fields is considered. The model contains two dilatonic coupling vectors obeying certain restrictions. It is shown that there exists a time interval where accelerating expansion of "our" 3-dimensional space is compatible with a small value of effective gravitational "constant" variation.

Here we study the problem of G-dot in a simple 5-dimensional cosmological model with two forms of rank 2 and two "phantom" scalar fields. (For variation of G in multi-dimensional models see [1]-[7] and refs. therein).

The main problem here is to find an interval of synchronous time variable τ where the scale factor of our 3-dimensional space exhibits an accelerated expansion according to observational data [8, 9] while the relative variation of the effective 4-dimensional gravitational constant is small enough in comparison with the Hubble parameter, see [10, 11, 12] and refs. therein.

Recently it was shown in [5] that in the model with two non-zero curvatures there exists a time interval of τ where an accelerating expansion of "our" 3-dimensional space is compatible with small enough value of G-dot. In this paper we suggest an analogous mechanism for the model with two 2-forms and two scalar fields (e.g. phantom ones) when proper restrictions on dilatonic coupling vectors are obeyed.

We deal here with 5-dimensional cosmological solution in the model with two 2-forms and two scalar "phantom" fields. The model is governed by the action

$$S = \int d^5 z \sqrt{|g|} \{ R[g] + \delta_{\alpha\beta} g^{MN} \partial_M \varphi^\alpha \partial_N \varphi^\beta - \sum_{a=1,2} \frac{1}{2} \exp[2\vec{\lambda}_a \vec{\varphi}] (F^a)^2 \}.$$
(1)

Here $g = g_{MN}(z)dz^M \otimes dz^N$ is 5-dimensional metric of pseudo-Euclidean signature $(-, +, +, +, +), F^a = dA^a$ is a 2-form, $\vec{\varphi} = (\varphi^1, \varphi^2) \in \mathbb{R}^2$ is a vector of two "phantom" scalar fields, $\vec{\lambda}_a \in \mathbb{R}^2$ is dilatonic coupling vector corresponding to the form F^a , a = 1, 2. In (1) $|g| = |\det(g_{MN})|$ and $\alpha, \beta = 1, 2$.

Here we consider a cosmological solution for the following choice of vector couplings

$$\vec{\lambda}_1^2 = \vec{\lambda}_2^2 = \lambda^2 > \frac{2}{3},\tag{2}$$

$$\vec{\lambda}_1 \vec{\lambda}_2 = 1 - \frac{1}{2} \lambda^2. \tag{3}$$

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The solution reads

$$ds^2 = g_{MN}(z)dz^M dz^N =$$
(4)

$$X^{2A} \Big\{ -dt^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2 + X^{-8h}t^2(dy)^2 \Big\},$$

$$\varphi^{\alpha} = -2h(\lambda_{1\alpha} + \lambda_{2\alpha}) \ln X.$$
(5)

$$\varphi^{\alpha} = -2h(\lambda_{1\alpha} + \lambda_{2\alpha})\ln X, \qquad (5)$$

$$F^{1} = F^{2} = qX^{-2}tdt \wedge dy \tag{6}$$

Here q is constant,

$$X = 1 + Pt^2, \qquad P = \frac{1}{8h}q^2$$
 (7)

t is a time variable and

$$h = \frac{3}{2 - 3\lambda^2}, \qquad A = \frac{4h}{3}.$$
 (8)

This solution may be verified just by a substitution into equations of motion. In what follows we use a "synchronous" time variable $\tau = \tau(t)$

$$\tau = \int_0^t d\bar{t} [X(\bar{t})]^A \tag{9}$$

Since $\lambda^2 > \frac{2}{3}$ we get P < 0, h < 0 and A < 0. Let us consider two intervals of the parameter A:

(a)
$$A < -1$$
, or $2/3 < \lambda^2 < 2$ (10)

and

(b)
$$-1 < A < 0$$
, or $\lambda^2 > 2$. (11)

For the first case (a) the function $\tau = \tau(t)$ is monotonically increasing from 0 to $+\infty$, for $t \in (0, t_1)$, where $t_1 = |P|^{-1/2}$, while for the second case (b) it is monotonically increasing from 0 to finite value $\tau_1 = \tau(t_1)$.

The scale factor of 3-dimensional space is

$$a = X^A. (12)$$

For the first branch (a) we get an asymptotical relation

$$a \sim \operatorname{const} \tau^{\nu},$$
 (13)

for $\tau \to +\infty$, where

$$\nu = \frac{A}{A+1} = \frac{4}{6-3\lambda^2}$$
(14)

and, due to (10), $\nu > 1$. For the second branch (b) we obtain

$$a \sim \operatorname{const} (\tau_1 - \tau)^{\nu},$$
 (15)

for $\tau \to \tau_1 - 0$, where $\nu < 0$ due to (11).

Thus, we are led to an asymptotical accelerated expansion of 3-dimensional factor space in both cases a) and b) and $a \to +\infty$.

This accelerated expansion takes place for all $\tau > 0$, i.e.

$$\dot{a} > 0, \qquad \ddot{a} > 0. \tag{16}$$

Here and in what follows we denote $f = df/d\tau$.

Let us prove relations (16). Using the relation $d\tau/dt = X^A$ (see (9)) we get

$$\dot{a} = \frac{dt}{d\tau}\frac{da}{dt} = \frac{2|A||P|t}{X} > 0,$$
(17)

and

$$\ddot{a} = \frac{dt}{d\tau} \frac{d}{dt} \frac{da}{d\tau} = \frac{2|A||P|}{X^{2+A}} (1+|P|t^2) > 0.$$
(18)

Now, let us consider the variation of effective gravitational constant G. For our model the 4-dimensional gravitational "constant" (in Jordan frame) is

$$G = \text{const } b^{-1} = X^{2A} t^{-1}, \tag{19}$$

where

$$b = X^{-2A}t \tag{20}$$

is the scale factor of the "internal" 1-dimensional space with the metric dy^2 .

The function $G(\tau)$ has a minimum at the point τ_0 corresponding to

$$t_0 = \frac{|P|^{-1}}{1+4|A|}.$$
(21)

At this point the variation of G is zero. This follows from explicit relation for dimensionless variation of G

$$\delta = \dot{G}/(GH) = 2 + \frac{1 - |P|t^2}{2A|P|t^2}.$$
(22)

Here

$$H = \frac{\dot{a}}{a} \tag{23}$$

is the Hubble parameter.

The function $G(\tau)$ is monotonically decreasing from $+\infty$ to $G_0 = G(\tau_0)$ for $\tau \in (0, \tau_0)$ and monotonically increasing from G_0 to $+\infty$ for $\tau \in (\tau_0, \bar{\tau}_1)$. Here $\bar{\tau}_1 = +\infty$ for the case (a) and $\bar{\tau}_1 = \tau_1$ for the case (b).

The function $b(\tau)$ is monotonically increasing from zero to $b(\tau_0)$ for $\tau \in (0, \tau_0)$ and monotonically decreasing from $b(\tau_0)$ to zero for $\tau \in (\tau_0, \overline{\tau}_1)$.

We should treat only solutions with accelerated expansion of our space and small enough variations of the gravitational constant obeying the present experimental constraint [10, 11]

$$|\delta| < 0.1. \tag{24}$$

Here like in the case of the model with two curvatures [5] the τ is restricted by the interval containing τ_0 . It follows from (22) that in the asymptotical regions (13) and (15) $\delta \to 2$ that is not acceptable by experimental bounds (24). This restriction is satisfied for the interval containing the point τ_0 where $\delta = 0$.

For small $\tau - \tau_0$ we get the following approximate relation

$$\delta \approx (7 + \frac{3}{2}\lambda^2)H_0(\tau - \tau_0), \qquad (25)$$

where $H_0 = H(\tau_0)$. This relation gives approximate bounds on values of time variable τ allowed by the restriction on G-dot.

Conclusions. Thus, here we have considered 5D cosmological solution with two Abelian gauge fields and two phantom scalar fields. The solution contains two factor spaces corresponding to "our" 3-dimensional flat space and to 1-dimensional "internal" space. This solution takes place for special choice of dilatonic coupling vectors. We have found that there exists a time interval of τ where an accelerating expansion of "our" 3dimensional space is compatible with small enough value of \dot{G}/G obeying the experimental bounds.

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References

- [1] V.D. Ivashchuk and V.N. Melnikov, *Nuovo Cim.* B 102, 131 (1988).
- [2] K.A. Bronnikov, V.D. Ivashchuk and V.N. Melnikov, *Nuovo Cim.* B 102, 209 (1988).
- [3] S.B. Fadeev, V.D. Ivashchuk and V.N. Melnikov, Variations of Constants and Exact Solutions in Multidimensional Gravity, In Gravitation and Modern Cosmology, Plenum, N.-Y., 1991, pp. 37-49.
- [4] V.D. Ivashchuk and V.N. Melnikov. Problems of G and Multidimensional Models. In Proc. JGRG11, Eds. J. Koga et al., Waseda Univ., Tokyo, 2002, pp. 405-409.
- [5] H. Dehnen, V.D. Ivashchuk, S.A. Kononogov and V.N. Melnikov, *Grav. Cosmol.*, 11 340 (2005).
- [6] J.-M. Alimi, V.D. Ivashchuk, S.A. Kononogov and V.N. Melnikov, *Grav. Cosmol.* 12, No. 2-3 (46-47), 173-178 (2006); gr-qc/0611015.
- [7] V.D. Ivashchuk, S.A. Kononogov, V.N. Melnikov and M. Novello, *Grav. Cosmol.* 12, No. 4 (48), 273-278 (2006); hep-th/0610167.
- [8] A.G. Riess *et al*, *AJ*, **116**, 1009 (1998).
- [9] S. Perlmutter *et al*, ApJ, **517**, 565 (1999).
- [10] R. Hellings, *Phys. Rev. Lett.* **51**, 1609 (1983).
- [11] J.O. Dickey et al, Science **265**, 482 (1994).
- [12] V. Baukh and A. Zhuk, *Phys. Rev.* **D 73** 104016 (2006).