WMAPping the Inflationary Universe

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Abstract

An epoch of accelerated expansion, or inflation, in the early universe solves several cosmological problems. While there are many models of inflation only recently has it become possible to discriminate between some of the models using observations of the cosmic microwave background radiation and large-scale structure. In this talk, we discuss inflation and its observational consequences, and then the status of current cosmological observations and their implications for different models of inflation.

1 Introduction

Inflation is a period of rapid expansion in the early universe that resolves several cosmological problems. While many different models of inflation such as new inflation, chaotic inflation, natural inflation and hybrid inflation have been proposed over the past two and a half decades, we are now entering the era where some models can be ruled out or shown to be consistent with observations. In this talk we shall first discuss inflation and its observational consequences. We shall then discuss what the current observations of the cosmic microwave background radiation (CMBR) and large-scale structure (LSS) in the universe imply for models of inflation.

2 Inflation and its observational consequences

Formally, inflation can be divided into two eras - the inflationary era and the reheating era. In the inflationary era the universe undergoes accelerated expansion because of the dominant potential energy of a slowly moving scalar field ϕ (the inflaton). In the subsequent reheating era, the inflaton decays and reheats the universe. Inflation provides an explanation as to why the CMBR is (nearly) isotropic (the horizon problem) and why the universe today is spatially flat (the flatness problem). Furthermore it dilutes away unwanted relics such as GUT monopoles.

In addition to the above effects, quantum fluctuations of the inflaton during inflation give rise to fluctuations in the energy density of particle species after the inflaton decays. Furthermore, quantum (tensor) fluctuations of the metric during inflation give rise to a cosmic gravitational wave background.

Fluctuations in the density of non-relativistic particles, or matter, are the seed for large-scale structure observed in the universe today. If δ_k is the Fourier transform of $\delta\rho_m/\rho_m$ then the matter power spectrum is given as $P(k) = |\delta_k|^2$. Inflation predicts that $P(k) = Ak^n$ with the spectral index $n \sim 1$. [If $n = 1$, $P(k)$ is called a scale-invariant or Harrison-Zeldovich spectrum.] A related quantity is the scalar power spectrum, $P_S(k) = A_S k^{n-1}$, which is associated with the scalar curvature. More generally, inflation gives

$$n(k) = n(k_0) + \frac{1}{2} dn/d(\ln k_0)\ln(k/k_0)\,, \tag{1}$$

which is referred to as running of the spectral index. Different models of inflation give slightly different behaviour for $n(k)$. Simulations of large-scale structure for an inflationary universe in the context of any particular inflation model can be compared with large-scale structure data from the Sloan Digital Sky Survey (SDSS), 2 Degree Field Galaxy Redshift Survey (2dFGRS), etc.

Fluctuations in the photon density and interactions of the photons with the fluctuations in the matter at decoupling give the anisotropies in the CMBR temperature (first detected by the Cosmic Background Explorer (COBE) in 1992). The 2-point correlation function for photons is

$$\frac{\langle T(\hat{n}_1)T(\hat{n}_2)\rangle_\theta - T_0^2}{T_0^2}, \tag{2}$$

where $T(\hat{n})$ is the temperature in direction $\hat{n}$, T_0 is the mean temperature and $\langle..\rangle_\theta$ indicates averaging over all points in the sky separated by an angle θ. The shape of the 2-point function depends on $P(k)$ which is provided by inflation. The matter power spectrum in inflation models imply a flat 2-point temperature correlation function at large angles and peaks at smaller angles.

In addition to the above form of the 2-point correlation function, inflation predicts that the fluctuations in the CMBR will be largely Gaussian. The n-point temperature correlation function $\langle T(\hat{n}_1)T(\hat{n}_2)...T(\hat{n}_n)\rangle \propto \langle \phi(x_1)\phi(x_2)...\phi(x_n)\rangle$. The small value of the 2-point temperature correlation function ($\sim O(10^{-5})$ at large angles) implies that the inflaton is a very weakly coupled field [1]. Hence the n-point correlation in ϕ is approximately 0 for odd n and $\sim \langle \phi(x_1)\phi(x_2)\rangle ... \langle \phi(x_{n-1})\phi(x_n)\rangle$ for even n. Thus higher point correlations in the temperature are 0 or powers of the 2-point correlation function, indicating Gaussian fluctuations.

Tensor fluctuations of the metric generated during inflation are associated with the gravitational wave background. The tensor power spectrum is given by $P_T(k) = A_T k^{n_T}$. The ratio of the tensor and scalar power spectra is denoted as r. Now the scale of inflation is equal to $(r/0.07)^{1/4}\, 1.8 \times 10^{16}\,\mathrm{GeV}$, and so a detection of gravitational waves can give the scale of inflation.

The gravitational wave background affects the CMBR and contributes to the 2-point correlation function. Furthermore, the interaction of gravitational waves with photons gives a quadrupole anisotropy in the photon distribution which leads to polarisation in the CMBR after Thomson scattering off electrons at decoupling.

The current observations of the CMBR and LSS broadly imply the following in relation to inflation:

- The matter density fluctuations are nearly scale-invariant with $P(k) = Ak^n$ with $n \sim 1$
- The presence of peaks in the CMBR fluctuations are consistent with inflation and not with other mechanisms of producing primordial fluctuations such as cosmic strings, etc.
- There is no evidence for non-Gaussianity in the temperature fluctuations [2]
- There are correlations on super-horizon scales, i.e., on scales larger than the horizon at decoupling, in the polarisation spectrum indicating the presence of fluctuations on scales larger than the horizon size at decoupling. The presence of such seemingly acausal fluctuations is a prediction of inflation. (Correlations on super-horizon scales are also seen in the temperature spectrum. However these can be generated causally after decoupling from sub-horizon scale fluctautions via the integrated Sachs-Wolfe effect.)

Thus the inflationary paradigm is consistent with current observations. With regards to gravitational waves generated during inflation, there has been no direct detection yet. Furthermore, there are other sources of the quadrupole anisotropy at decoupling and the polarisation detected so far does not provide the value of the gravitational wave contribution. We now look at specific values of cosmological parameters inferred from the CMBR and LSS data to discriminate between different models of inflation.

3 Distinguishing between different inflation models

Inflation models can be parametrised by the following slow roll parameters,

$$\epsilon_V = \frac{M_{\rm Pl}^2}{16\pi}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 \quad \eta_V = \frac{M_{\rm Pl}^2}{8\pi}\left(\frac{V''(\phi)}{V(\phi)}\right) \quad \xi_V = \left(\frac{M_{\rm Pl}^2}{8\pi}\right)^2\left(\frac{V'V'''}{V^2}\right) \tag{3}$$

and higher derivatives. Above $V(\phi)$ is the inflaton potential and $'$ refers to a derivative with respect to ϕ. Note that different authors use different definitions of the slow roll parameters; ours are taken from Ref. [3]. The assumption that the inflaton rolls slowly during the inflationary epoch is equivalent to $\epsilon_V, \eta_V, \xi_V \ll 1$. Now, $n = 1 - 6\epsilon_V + 2\eta_V$, $r = 16\epsilon_V$ and $dn/d(\ln k) = 16\epsilon_V\eta_V - 24\epsilon_V^2 - 2\xi_V$. Therefore constraints on $n, dn/d(\ln\, k)$ and r from CMBR and LSS data give limits on ϵ_V, η_V and ξ_V. Below we first present constraints on $n, dn/d(\ln\, k)$ and r.

WMAP3 alone is consistent with [4]

$$0.94 < n < 1.04 \quad dn/d(\ln k) = 0 \quad \text{and} \quad r < 0.60 \tag{4}$$

For a running spectral index [4]

$$1.02 < n < 1.38 \quad -0.17 < dn/d(\ln k) < -0.02 \quad \text{and} \quad r < 1.09 \tag{5}$$

The parameters are estimated at a pivot scale $k_* = 0.002\text{Mpc}^{-1}$ in Ref. [4]. Note that $dn/d(\ln k) < -0.02$ rules out all single field slow roll models as it leads to an insufficient duration of inflation [5].

Combining results from WMAP3 and SDSS gives [4]

$$0.93 < n < 1.01 \quad dn/d(\ln k) = 0 \quad \text{and} \quad r < 0.31 \tag{6}$$

or

$$0.97 < n < 1.21 \quad -0.13 < dn/d(\ln k) < 0.007 \quad \text{and} \quad r < 0.38 \tag{7}$$

Note that the upper limit on r has decreased. This is because as r increases, the scalar amplitude decreases and for $r \geq 0.3$ this then adversely affects the LSS.

Including small angle CMBR data from CBI, ACBAR, VSA and B2K with WMAP3 and 2dFGRS LSS data gives [6]

$$0.95 < n < 0.98 \quad dn/d(\ln k) = 0 \tag{8}$$

or

$$0.94 < n < 1.09 \qquad -0.14 < dn/d(\ln k) < -0.013 \tag{9}$$

The corresponding upper limit on r of 0.26 gives an upper limit on the scale of inflation of 2×10^{16} GeV. In Ref. [6] the pivot scale $k_* = 0.01\text{Mpc}^{-1}$.

Implications for inflation models

Different models of inflation are distinguished by the form of the inflaton potential and therefore correspond to different ranges of values for the slow roll parameters [4]. Thus the above constraints on spectral parameters provide constraints on inflation models.

New inflation: In new inflation the inflaton potential has the form

$$V = V_0 \left[1 - \left(\frac{\phi}{\mu}\right)^p\right] \tag{10}$$

with the initial position of the inflaton at small values of ϕ. An example of such a potential is the Coleman-Weinberg potential which is approximately given by $V = V_0 - \lambda\phi^4$ in the small ϕ region. New inflation models with $p \geq 3$ are consistent with current observations [7].

Chaotic inflation: Chaotic inflation models have a potential of the form $\sim A\phi^p$ with the inflaton field initially displaced far from the minimum of its potential. The initial value of the field is greater than $M_{\rm Pl}$ (though $V(\phi) < M_{\rm Pl}^4$). The field rolls slowly till $\phi \sim 0.1\, M_{\rm Pl}$ during the inflationary era, and then oscillates in its potential and decays. Models with $V(\phi) \sim m^2\phi^2$ are consistent with the data while models with $V(\phi) \sim \lambda\phi^4$ are ruled out by WMAP3 and LSS data [4]. However, recently it has been pointed out that chaotic inflation models with $V(\phi) \sim \lambda\phi^4$ are still allowed if the neutrino fraction $f_\nu \equiv \Omega_\nu/\Omega_c = 0.03 - 0.05$. This further implies that $\sum_i m_{\nu i} = 0.3 - 0.5\,\mathrm{eV}$, as in quasi-degenerate nuetrino mass models [8]. This is an interesting consistency relation between a model of inflation and a model of neutrino masses.

Natural inflation: In natural inflation models the inflaton is a pseudo-Nambu-Goldstone boson associated with spontaneous symmetry breaking at a scale f and small explicit (dynamical) symmetry breaking of order Λ. In these models the flat potential of a Nambu-Goldstone boson ϕ is tilted because of explicit symmetry breaking and

$$V(\phi) = \Lambda^4[1 + \cos(\phi/f)]\,. \qquad (11)$$

The tilt of the potential $\sim$ height/width $\sim (\Lambda/f)^4$ and for $\Lambda \sim 10^{15}$ GeV and $f \sim 10^{19}$ GeV the potential is flat enough to satisfy constraints on the inflaton potential. Because the flatness of the potential is associated with a naturally flat Nambu-Goldstone boson potential and small dynamical symmetry breaking, rather than an unnaturally small coupling, this scenario is called natural inflation. Natural inflation is consistent with the WMAP3 data [9].

Hybrid inflation: These models involve a potential with 2 fields ϕ and χ. The potential has the form

$$V(\phi,\chi) = \lambda(\chi^2 - \chi_0^2) + \frac{1}{2}m^2\phi^2 + \frac{1}{2}\lambda'\chi^2\phi^2\,. \qquad (12)$$

ϕ is initially displaced from the minimum of its potential and rolls slowly in its potential. For $\phi > \phi_1$, χ is localised near the origin and inflation is driven by the energy density of ϕ or the false vacuum energy of χ (in the latter stages). When ϕ crosses the threshhold value ϕ_1 the potential for χ turns over and now χ rolls down towards the new minimum χ_0. This ends the inflationary era and ϕ and χ then oscillate in their potential and decay. The original non-SUSY version of this model implies that $n \geq 1$ [10] and is ruled out if indeed $n < 1$. However hybrid inflation models in the context of SUSY and SUGRA, referred to as D-term and F-term inflation, can give $n < 1$ and there does exist a region of parameter space consistent with the data [11].

4 Conclusion

In summary, the inflationary paradigm is consistent with the CMBR and LSS data. However, while current data on CMBR and LSS provide information on the scalar and tensor power spectra making it possible to discriminate between some models of inflation, it is still not possible to rule out many specific models of inflation. In the future, the European Space Agency's Planck mission will give better data on the spectral index and its running which will help in constraining inflation models. For example, a detection of large negative running of the spectral index will rule out single field slow roll inflation models. Furthermore, Planck and the ground-based Clover experiment will measure r upto 10^{-2}. This will help to constrain the scale of inflation.

References

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