

Rapid planetesimal formation in turbulent circumstellar discs

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The initial stages of planet formation in circumstellar gas discs proceed via dust grains that collide and build up larger and larger bodies¹. How this process continues from metre-sized boulders to kilometre-scale planetesimals is a major unsolved problem²: boulders stick together poorly³, and spiral into the protostar in a few hundred orbits due to a head wind from the slower rotating gas⁴. Gravitational collapse of the solid component has been suggested to overcome this barrier^{1,5,6}. Even low levels of turbulence, however, inhibit sedimentation of solids to a sufficiently dense midplane layer^{2,7}, but turbulence must be present to explain observed gas accretion in protostellar discs⁸. Here we report the discovery of efficient gravitational collapse of boulders in locally overdense regions in the midplane. The boulders concentrate initially in transient high pressures in the turbulent gas⁹, and these concentra-

tions are augmented a further order of magnitude by a streaming instability^{10–12} driven by the relative flow of gas and solids. We find that gravitationally bound clusters form with masses comparable to dwarf planets and containing a distribution of boulder sizes. Gravitational collapse happens much faster than radial drift, offering a possible path to planetesimal formation in accreting circumstellar discs.

Planet formation models typically treat turbulence as a diffusive process that opposes the gravitational sedimentation of solids to a high density midplane layer in circumstellar discs^{7,13}. Recent models of solids moving in turbulent gas reveal that the turbulent motions not only mix them, but also concentrate metre-sized boulders in the transient gas overdensities⁹ formed in magnetorotational turbulence¹⁴, in giant gaseous vortices^{15,16}, and in spiral arms of self-gravitating discs¹⁷. Short-lived eddies at the dissipation scale of forced turbulence concentrate smaller millimetre-sized solids¹⁸.

Some simulations mentioned above^{9,11,12} were performed with the Pencil Code, which solves the magnetohydrodynamic (MHD) equations on a three-dimensional grid for a gas that interacts through drag forces with boulders. Boulders are represented as superparticles with independent positions and velocities, each having the mass of a huge number of boulders but the aerodynamic behaviour of a single boulder. We have now further developed the Pencil Code to include a fully parallel solver for the gravitational potential of the particles (see Supplementary Information). The particle density is mapped on the grid using the Triangular Shaped Cloud assignment scheme¹⁹ and the gravitational potential of the solids is found using a Fast Fourier Transform method²⁰.

This allows us, for the first time, to simulate the dynamics of self-gravitating solid particles in magnetised, three-dimensional turbulence.

We model a corotating, local box with linearised Keplerian shear that straddles the protoplanetary disc midplane and orbits the young star at a fixed distance. Periodic boundary conditions are applied. An isothermal equation of state is used for the gas, while the induction equation is solved under the ideal MHD assumption of high conductivity. Magnetorotational instability¹⁴ drives turbulence in Keplerian discs with sufficient ionisation²¹, producing in our unstratified models turbulence with Mach number $\text{Ma} \approx 0.05$ and viscosity $\alpha \approx 10^{-3}$, a realistic value to explain observed accretion rates⁸. The ionisation fraction in the dense midplanes of protoplanetary discs may be insufficient for the gas to couple with the magnetic field to drive magnetorotational instability²¹. In the Supplementary Information we therefore describe unmagnetised models as well.

Solid objects orbit the protostar with Keplerian velocity v_K in the absence of gas drag. A radial pressure gradient partly supports the gas, however, so it orbits at sub-Keplerian velocity, with $\Delta v \equiv v_g - v_K < 0$. As a result, large (approximately metre-sized) solid objects feel a strong head wind that causes them to drift radially inwards⁴ with a maximum drift velocity Δv . They also feel gas drag as they fall toward the disc midplane in the effective gravity field of the star. A sedimentary midplane layer forms with a width determined by a balance between settling and turbulent diffusion^{7,13}.

We present three types of models: (1) without self-gravity, with 128^3 zones and 2×10^6 particles, run for 100 orbits, to study the interplay between the streaming instability¹⁰ and concen-

tration by transient high pressures; (2) with self-gravity and boulder collisions, with 256^3 zones and 8×10^6 particles run for 27 orbits, to study gravitational collapse; and (3) models with self-gravity but no magnetorotational turbulence (presented in the Supplementary Information). Magnetorotational turbulence is given 10 orbits to reach steady state before we turn on drag force and vertical gravity, to avoid any influence of the initial conditions on the sedimented midplane layer. We fix the global solids-to-gas bulk density ratio at the canonical galactic value of $\epsilon_0 = 0.01$, but two values of the radial drift are considered: low drift with $\Delta v = -0.02c_s$, where c_s is the isothermal sound speed, and moderate drift with $\Delta v = -0.05c_s$, depending on the assumed radial pressure support (values up to $\Delta v = -[0.2 \dots 0.5]c_s$ are possible¹³, but are not considered here).

For the simulations without self-gravity we consider a fixed particle size parameterised by the dimensionless friction time $\Omega_K \tau_f = 1.0$, where Ω_K is the local Keplerian rotation frequency and τ_f is the time-scale over which gas and solids reach equal velocity. At an orbital distance $r = 5 \text{ AU}$ this corresponds to boulders of approximately one metre in diameter. Figure 1 shows the space-time topography of the sedimented midplane layer. The streaming instability increases the density of boulders in regions where they have already been concentrated by transient high pressures⁹. Increasing radial pressure support from $\Delta v = -0.02c_s$ to $-0.05c_s$ reduces the concentration by streaming instability, although the local solids-to-gas density ratio still reaches 200.

Gravitational collapse of discrete solid objects produces virialised clusters unable to contract further²² in the absence of mechanisms to dynamically cool the cluster—that is, reduce the local rms speed. Two processes that we consider can be important: drag force cooling and collisional

cooling. Drag force cooling occurs because part of the kinetic energy exchanged between the particles and the gas is dissipated. Collisional cooling is produced by the highly inelastic collisions between boulders, transferring kinetic energy to heat and deformation. Collisional cooling occurs generally in simulations of resolved collisions in planetary rings²³. In the Supplementary Information we describe how we treat collisional cooling numerically in the self-gravitating simulations by damping the rms speed of the particles in each grid cell on a collisional time-scale. We have found that in the absence of collisional cooling, gravitational collapse still proceeds if the total surface density (of solids and gas) is augmented by 50%. Collisional cooling is thus not a prerequisite of the collapse, but does allow it to occur in somewhat less massive discs. We ignore all other effects of the collisions, such as coagulation and collisional fragmentation. Collisional cooling and self-gravity are turned on after 20 orbits in the self-gravitating simulations.

Our chosen scale-height-to-radius ratio of $H/r = 0.04$ gives a gas temperature of $T = 80$ K at an orbital radius of $r = 5$ AU. We choose for the 256^3 self-gravitating run the uniform gas volume density to be consistent with the midplane of a disc with surface density of $\Sigma_{\text{gas}} = 300 \text{ g cm}^{-2}$. This corresponds to approximately twice the minimum mass solar nebula (MMSN) at 5 AU from the (proto-)Sun. An alternative theory for giant planet formation, the disc instability hypothesis^{24,25}, requires column densities at least 20 times higher than the MMSN for gravitational fragmentation of the gaseous component of the disc to occur.

We have examined the numerical convergence of our models with resolutions ranging from 64^3 to 256^3 zones (see Supplementary Information). The peak particle density on the grid increases

with increasing resolution, because of less smoothing in the particle-mesh scheme at higher resolution, resulting in a decrease in the column density threshold for gravitational collapse. Although we have not yet fully converged, our results appear to provide good *upper limits* to the column density for which collapse can occur. For the self-gravitating simulation we consider boulders with friction times distributed among $\Omega_K \tau_f = 0.25, 0.50, 0.75, 1.00$. At $r = 5$ AU in our chosen disc model, these correspond to radii of 15–60 cm. Consideration of multiple boulder sizes is vital since differential aerodynamic behaviour could inhibit gravitational instabilities²⁶. The size range covers roughly half of the two orders of magnitude in particle radius produced by coagulation of microscopic grains²⁷. Smaller particles are ignored since they are unlikely to separate from the gas and participate in gravitational collapse. In case of widespread collisional fragmentation, e.g. in the warmer terrestrial planet formation region, up to 80% of the solid material may be bound in small fragments^{27,28}, in which case we must implicitly assume an augmentation in solids-to-gas ratio of up to 5.

In our self-gravitating model we set $\Delta v = -0.02c_s$, but show in the Supplementary Information that gravitationally bound clusters also form for $\Delta v = -0.05c_s$, with a factor of two increase in column density threshold. The Supplementary Information also documents that typical boulder collisions happen at speeds below the expected destruction threshold³. We caution, however, that material properties, and thus destruction thresholds, of the boulders are poorly known. Higher resolution studies, and an improved analytical theory of collision speeds that takes into account epicyclic motion, will be needed to determine whether collision speeds have converged, given an unexplained factor 3 difference for $\Omega_K \tau_f = 1$ particles between typical relative speeds within cells

($\approx 5 \text{ m s}^{-1}$) and the expected collision speed of well-mixed particles.

The development of gravitational instability in the 256^3 run is shown in Figure 2. The four different boulder sizes have accumulated in the same regions (*central panel*) already before self-gravity is turned on, demonstrating that differential drift does not prevent density enhancement by streaming instability. Gravitationally bound boulder clusters form, with a Hill radius—within which the gravity of the cluster dominates over tidal forces from the central star—that increases steadily with time (*inserts*) as the clusters accrete boulders from the surrounding flow.

We show in Figure 3 the peak density and the mass of the most massive gravitationally bound cluster as a function of time. The cluster consists of particles of all four sizes, demonstrating that different boulder sizes can indeed take part in the same gravitational collapse, despite their different aerodynamical properties and drift behaviour. At the end of the simulation the most massive cluster contains 3.5 times the mass of the dwarf planet Ceres. The cluster mass agrees roughly with standard estimates from linear gravitational instability⁵, when applied at $r = 5 \text{ AU}$ to the locally enhanced column densities (see Supplementary Information).

Models lacking magnetic fields, and thus magnetorotational turbulence, are described in the Supplementary Information. Here sedimentation occurs unhindered until the onset of Kelvin-Helmholtz instabilities driven by the vertical shear of gas velocity above the midplane boulder layer. Strong boulder density enhancements in the mid-plane layer nevertheless still form, although for moderate drift an increase of the solids-to-gas ratio from 0.01 to 0.03 (perhaps possible due to radial variation in boulder drift speeds⁶ or photoevaporation of the gas²⁹) was needed to

obtain strong clumping. Magnetorotational turbulence thus has a positive effect on the mid-plane layer's ability to gravitationally collapse, although collapse can occur without it as well.

The Supplementary Information also includes a model with an adiabatic equation of state and explicit gas heating due to energy dissipated by drag and inelastic collisions. We find that gas heating does not prevent collapse. The maximum temperature reached is not even high enough to melt ice, although that may change with the formation of massive bodies with escape velocity near the sound speed.

Our proposed path to planetesimal formation depends crucially on the existence of a dense sedimentary layer of boulders. Future investigations should focus on the formation and survival of such layers in light of processes like coagulation, collisional fragmentation and erosion²⁸. Especially important are higher resolution studies of collision speeds and an improved analytical theory of collisions that includes the epicyclic motion of particles.

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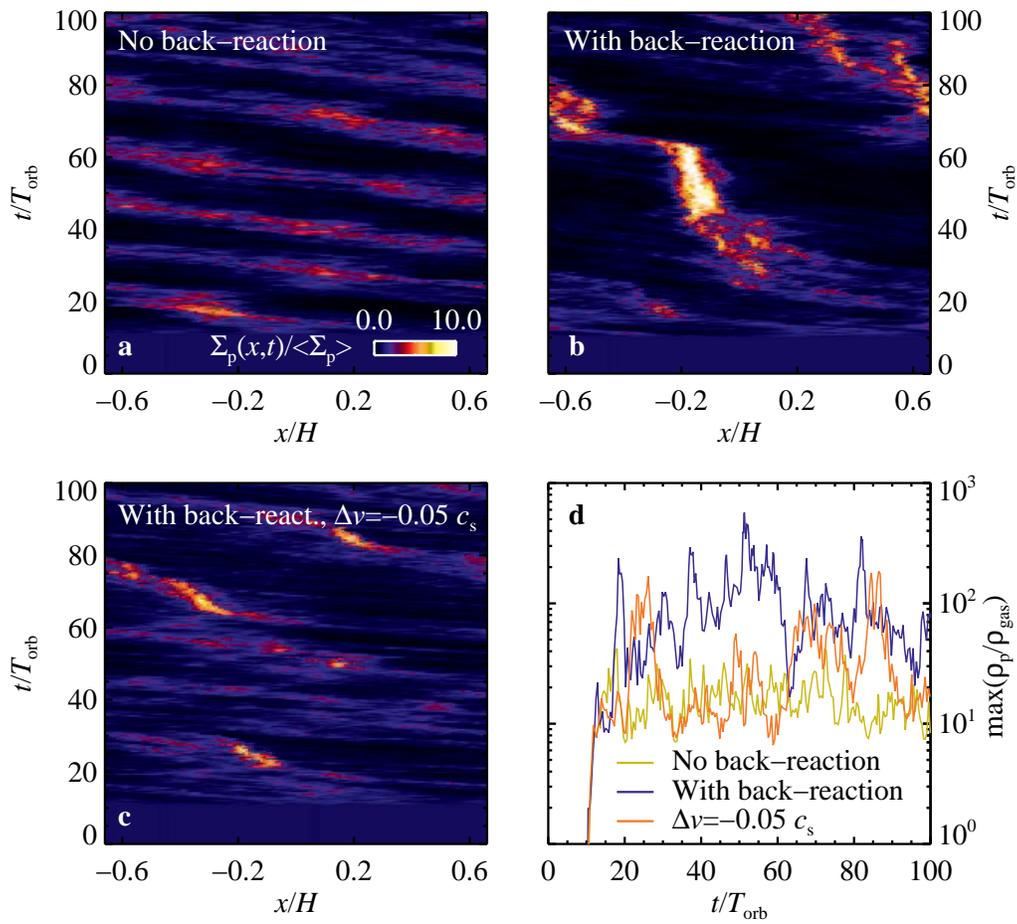
Supplementary Information is linked to the online version of the paper at www.nature.com/nature.

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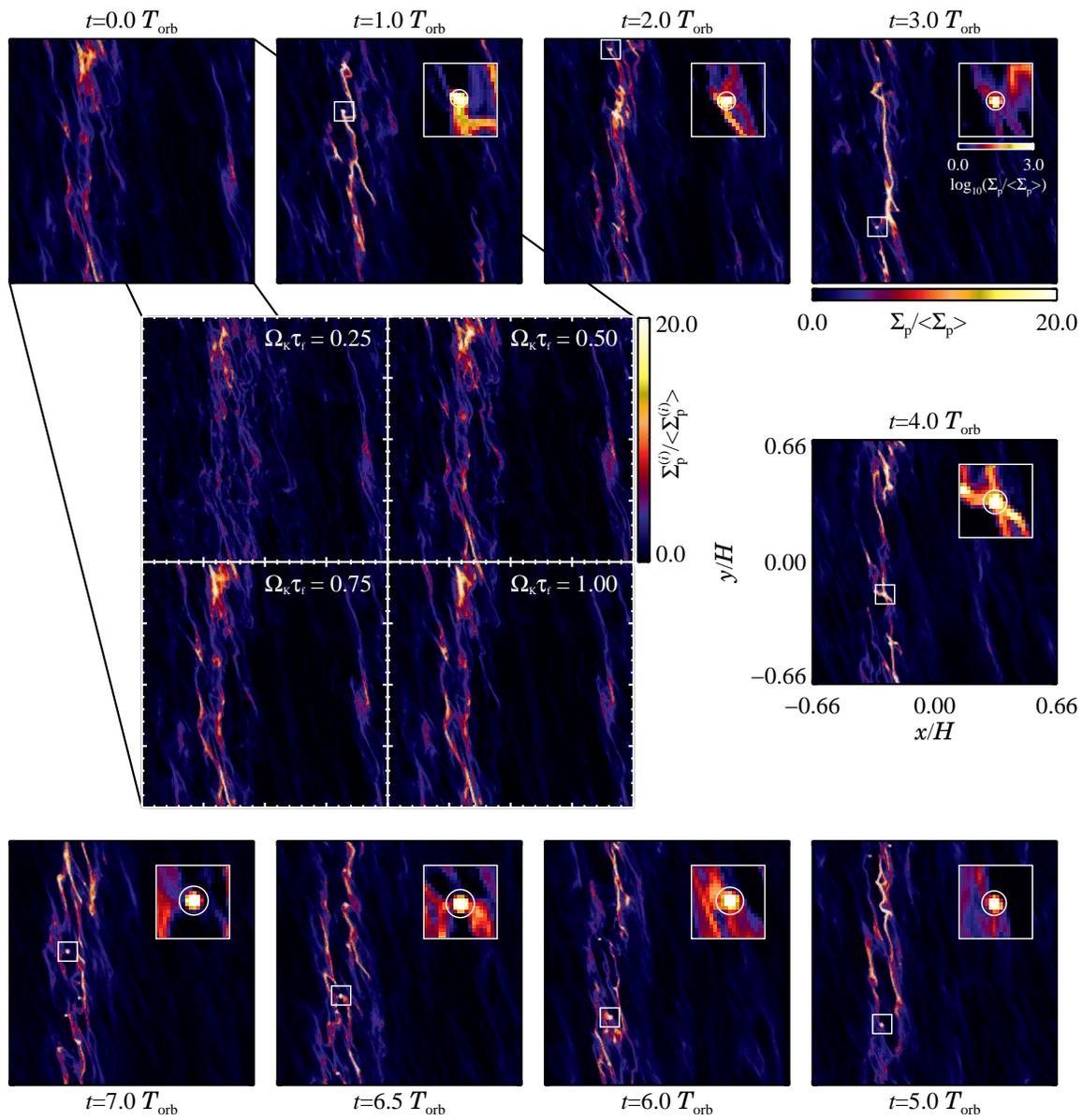
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Figure 1 Topography of the sedimented particle layer in models without self-gravity or collisional cooling. a) The azimuthally averaged vertical column density Σ_p of metre-sized boulders (with $\Omega_K \tau_f = 1$) as a function of radial coordinate x and time t , in a model where the particles feel gas drag, but the gas does not feel drag from the particles. Radial drift is evident from the tilted bands (particles crossing the inner boundary reappear at the outer). Transient regions of mildly increased gas pressure temporarily concentrate boulders. The gas orbits slightly slower on the outer edge of these high pressure regions and slightly faster on the inner edge, resulting in a differential head wind that forces boulders towards their centres^{9,30}. b) Including the drag force from the particles on the gas allows for the development of the streaming instability, seeded by the existing radial density enhancements. The streaming instability occurs where the collective drag force of the solids forces the gas to locally move with an orbital speed that is closer to Keplerian, reducing the gaseous head wind that otherwise causes boulders to drift radially. Solids then drift into already overdense regions from further out, causing runaway growth in the local bulk density of solids. c) The column density when the radial pressure support is increased from $\Delta v = -0.02c_s$ to $-0.05c_s$. Radial density enhancements become narrower and shorter-lived due to downstream erosion of the overdensities by the stronger radial drift. d) The maximum solid-to-gas ratio on the grid as a function of time. The average solids-to-gas ratio in the midplane is 0.5, whereas the maximum reaches well over ten times higher values in transient high pressure regions (yellow) and several hundred times higher values when the streaming instability is active (orange and blue).



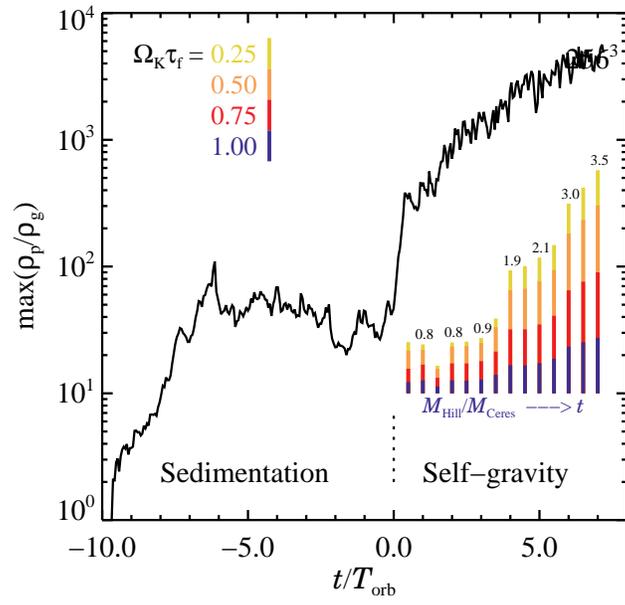
(this is Figure 1)

Figure 2 Time series of the collapse of overdense seeds into gravitationally bound boulder clusters. The central panel shows the column densities of the four different sizes of boulders (in units of the mean column density of each size) plotted independently at a time just before self-gravity is turned on. All four particle sizes have concentrated at similar locations, an important prerequisite for the subsequent gravitational collapse. The surrounding panels show a time series of total column density of solids, in the radial-azimuthal (x - y) plane of the disc, summed over all particle sizes, starting from the upper left and progressing clockwise. Values are normalised to the average value across the grid (see colour bars in upper right panel). Times are given in orbital times T_{orb} after self-gravity is turned on. Inset in each panel is an enlargement of a square region (indicated in the main panel) centred around the Hill sphere of the most massive cluster in the simulation, represented by the white circle. These inserts show the log of the column density ratio (see colour bar in upper right panel) to capture the extreme values reached. Overdense bands initially contract radially, forming thin filaments with densities high enough for a full non-axisymmetric collapse into gravitationally bound clumps to take place. As time progresses, the Hill sphere increases in radius as the clusters grow in mass by accreting boulders from the turbulent flow (see Supplementary Video for an animation of this simulation).



(this is Figure 2)

Figure 3 Mass accretion onto a gravitationally bound cluster. The plot shows the maximum bulk density of solids as a function of time, normalised by the average gas density. Drag force and vertical gravity are turned on at $t = -10$, while self-gravity and collisional cooling are turned on at $t = 0$. The density increases monotonically after the onset of self-gravity because gravitationally bound clusters of boulders form in the mid-plane. After only seven orbits peak densities in these clusters approach $10^4 \rho_g$ or a million times the average boulder density in the disc. The coloured bars show the mass contained within the most massive Hill sphere in the box, in units of the mass of the 970 km radius dwarf planet Ceres ($M_{\text{Ceres}} = 9.5 \times 10^{23}$ g). The most massive cluster accretes about $0.5 M_{\text{Ceres}}$ per orbit (the entire box contains a total boulder mass of $50 M_{\text{Ceres}}$). The cluster consists of approximately equal fractions of the three larger boulder sizes. The smallest size, with $\Omega_K \tau_f = 0.25$, is initially underrepresented with a fraction of only 15% because of the stronger aerodynamic coupling of those particles to the gas, but the fraction of small particles increases with time as the cluster grows massive enough to attract smaller particles as well. The mean free path inside the bound clusters is shorter than the size of the cluster, so any fragments formed in catastrophic collisions between the boulders will be swept up by the remaining boulders before being able to escape the cluster (see Supplementary Information).



(this is Figure 3)