Typicality, Freak Observers and the Anthropic Principle of Existence

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Abstract

We propose an alternative anthropic probability for calculating the probabilities in eternal inflation. This anthropic probability follows naturally from the weak anthropic principle, and does not suffer the freak observer or the typicality problems. The problem that our observed cosmological constant is not at the peak of the usual anthropic probability distribution is also solved using this proposal.

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1 Introduction

Recent work in string theory indicates that there may be a vast landscape of metastable vacua in the string theory [1]. Some constants of nature are used to label these vacua thus can not be uniquely determined from the first principle. This string theory landscape, just like any multitude of vacua, can be realized in cosmology through the eternal inflation scenario [2, 3, 4]. If this is to be the case, we can not expect to have a deterministic explanation of nature, we can only calculate probability for each vacuum. The measure or the probability distribution in eternal inflation have become one of the central problems in cosmology [5, 6, 7, 8, 9, 10, 11]. Whether the eternal inflation scenario can make any useful prediction, even less precise in the eyes of traditional science, is a matter of debate.

Unfortunately, if we do not use extra selectional effects in calculation of probability, vacua with large probability never look like the vacuum we live in. To take a selectional effect into account one usually starts with Bayes' formula

$$P(\text{theory } x|\text{selection}) = \frac{P(\text{selection}|\text{theory } x)P(\text{theory } x)}{\sum_{y} P(\text{selection}|\text{theory } y)P(\text{theory } y)} , \qquad (1)$$

where the word selection stands for a set of selectional conditions.

The anthropic principle [12] is used as a kind of selectional conditions in this calculation of probability, although it betrays the usual concept of scientific selectional principles-such as the lowest energy principle. In addition to the anthropic principle, there are other possible selectional effects such as those discussed in [13]. There is also the possibility that no such selectional effect really works and some constants of nature are determined by pure chance.

One of the authors of the present note (ML) is fully aware of the fact that in the history of science many false leads appeared because scientists are blinded by their limited experimental knowledge as well as limited theoretical understanding at the time, chasing after aether is one of such examples. To the 19th century theoretical physicists, aether seemed to be inevitable. Anthropic selectional effects, as applied to some of nature constants, may be another false lead. Nevertheless, so long as it appears inevitable at the present time, it is a legitimate scientific topic to study.

There have been several versions of the anthropic principle [12], for example, the strong, the weak, and the weakest anthropic principle. The weakest anthropic principle states that "our observations about the universe should be in accord with the fact that we are observing it". In this weakest version, the selectional effect is no more than a set of experimental data. It does not try to explain the coincidence problems for the existence of observers [14], unless the solution of coincidence is contained in the theory itself. Recently, Hartle an Srednicki [15] discussed the typicality problem and method of calculation using the weakest anthropic principle. While other discussions on this issue can be found in [16].

In this paper we mainly discuss the weak anthropic principle, all it says is "what we can observe is restricted by the conditions needed for the presence of observers." The weak anthropic principle is the weakest one which can explain the coincidences for the existence of observers, so it provides a necessary and modest starting point for our discussion. The recipe we propose can help to avoid several puzzles concerning the current study in this direction, the main result of our proposal is that among those theories (or vacua) which can accommodate a large number of galaxies, the preferred one (ones) is largely determined by the a priori probability.

When we want to calculate probabilities using the anthropic principle, we need to transcribe this principle into formulas. Several kinds of anthropic probability are proposed, such as the volume weighted, galaxy weighted [5, 8] and entropy weighted [17] probabilities. A common attribute of these probabilities is that they are proportional to the number of the observers. So in the following discussion, we call them number weighted probability uniformly. In the number weighted method, the probability is expressed as [5]

$$P_{\rm obs}(X) \sim P(X) n_{\rm obs}(X), \tag{2}$$

where P(X) is the volume fraction of interest, for example, the volume for thermalized regions, with the given values of the constants X. And $n_{obs}(X)$ is the number of observers (or the amount of entropy production) in such regions per unit volume. In such a number weighted prescription, the freak observer problem [18] often arises, and the typicality of our human observer is debated [19, 15, 16].

In this paper, we propose an alternative formula to calculate the anthropic probability. In Section 2, we give our recipe from anthropic reasoning. It is shown that this probability follows directly from the anthropic principle, and does not suffer the typicality problem or the freak observer problem. In Section 3, we discuss the derivation-from-peak problem of the observed cosmological constant as an application of this probability. We also use a simple model to illustrate the use of this probability combined with the a priori probability. We conclude in Section 4.

2 The anthropic principle of existence

In this section, we propose an alternative anthropic probability. The anthropic principle claims that "what we can expect to observe must be restricted by the conditions necessary for our presence as observers" [20]. So it is natural that the probability should not be proportional to the number of observers, rather, it is just the probability for the existence of observers. For example, if the probability for observers to appear per galaxy (or space volume in a given unit) is p, and there are n galaxies (or space volumes) in our pocket universe, then instead of the number weighted probability np, the anthropic probability should be

$$P(p,n) = 1 - (1-p)^n .$$
(3)

It is clear that when $np \ll 1$, P(p, n) is approximated by np. If observers are not rare in galaxies, this probability becomes almost one, and a further increase of number of galaxies does not significantly change the probability. This property is very different from that of the number weighted probability. To get the full probability using Bayes' formula, one also needs to multiply a priori probability for a given theory.

Besides reasoning from the definition of anthropic principle, there are also several other reasons that the existence-or-not weight is more natural then the weight given by the number of the observer.

First, the main support for the anthropic principle is the coincidence that we just live in the universe suitable for us to live in. The existence of the observer is needed in this argument, while the number of the observers does not figure in.

Second, there is "experimental" evidence that we are not selected by the number of observers. For example, anthropic reasoning can be used to explain why we live on the earth where there is liquid water. But the earth does not seem to be the largest planet in the universe suitable for human to live, where more human beings can be developed. On the other hand, we do not seem to be the smallest possible intelligent beings in size suitable to live on the earth [21], which consumes less resources, so can develop a larger population. In conclusion, the anthropic selectional effect does not become stronger just because there can be more observers.

Third, there are paradoxes in the number weighted probability, which do not arise in our probability.

One paradox is the freak observer problem (also known as the Boltzmann brain problem). If the universe is asymptotically de Sitter, there should be an infinite number of observers developing from the thermal fluctuations. This is more than the finite number of observers like us. So the question is, why we are human observers, not freak observers.

Calculation using the number weighted measure suffers this problem in two or three ways. First, it is often assumed in such calculation that we are typical observers, because the probability for a species of observer to do observation is assumed to be proportional to the number of this species of observers. But if freak observers are infinite, we can not be typical. Second, if there are both a finite number of humans and a finite number of freak observers (or without freak observers) in our universe, then the anthropic probability for our universe should be infinitely small compared with some other universe with infinite number of freak observers, which can be selfconsistently realized by a pure de Sitter phase, or something like that.

The anthropic probability we propose suffers none of these two problems. The first problem is bypassed because we never assume whether we are typical or not in our reasoning. The selectional effect is always a selection of vacua, but not a selection of observers. The second problem is avoided because if there are a large number of human observers in the universe, then the corresponding anthropic probability is nearly one. So it is not much smaller than the probability for a universe filled by a infinite number of freak observers.

There is potentially a third kind of the freak observer problem that if the freak observers really exist and require less meticulous arrangements (coincidences) than the human observer, then the anthropic explanation for the coincidences is pointless. The anthropic probability we propose is not completely free of this kind of freak observer problem. But it still fares better than the number weighted anthropic probability in this regard. On the other hand, it is not clear whether or not the assumption that the freak observer requires less coincidences than the human observer is correct. So we do not discuss this problem in detail in this paper.

Another paradox is the prediction of our fate. As the population is now growing on the earth, why do we live at the present time, while do not live in the future? Does it predict that the population will stop growing in the near future? It seems that such prediction is absurd, and the above questions should be answered by other branches of science such as sociology. Again, using our anthropic probability, without assuming typicality or counting the number of observers, this problem does not exist either.

If one is supposed to calculate the probability of a given theory based on the data set D we collect, using Bayes' theorem:

$$P(T_i|D) = \frac{P(D|T_i)P(T_i)}{\sum_i P(D|T_i)P(T_i)},\tag{4}$$

where $P(T_i)$ is a priori probability of theory T_i , then in computing $P(D|T_i)$, our proposal eq.(3) will play a part. For a given theory, the number of galaxies and the probability p for intelligence beings like us to develop are supposed to be calculable. We need to try to avoid assuming typicality of us humans in such a calculation, namely there is no reason to assume that we are typical among all possible intelligent beings, thus p must be the probability for human beings to occur, not the probability for just any kind of intelligence. As we shall discuss in the next section, for most of applications, p(p, n) is either 1 or 0, thus according to Bayes' formula, as long as the data set boils down to the minimum: only the existence of humans, then the posterior probability of a given theory is either 0 or largely determined by its a priori probability $P(T_i)$. Thus, for those theories which can accommodate a large number of galaxies, anthropic reasoning alone does not help to discriminate among them, only $P(T_i)$ figure in the probability distribution, implying that theoretical selectional principle is cried for if we are to be able to select one or a few theories.

3 Applications and examples

In addition to explaining paradoxes mentioned above, our anthropic probability can be used to solve the problem that the observed cosmological constant is not at the peak of the anthropic probability distribution. As is shown in Fig. 1, the cosmological constant we observe is not the most probable value if we use the baryon number weighted probability. However this problem does not exist with our probability, it is because if the probability p for an observer to exist in one galaxy is not too small, and the number of galaxies n is large enough, according to eq.(3) the anthropic probability $1-(1-p)^n$ can be very close to 1. So the distribution is more flat and we find ourselves live in our meta-stable vacuum as it is with a large anthropic probability. Note that although the probability distribution is sometimes more flat in our probability than in the number weighted one, it is still good enough to explain the coincidences required for observers' existence, because the anthropic probability distribution usually falls exponentially when the deviation is large from the most probable value, while our patch of the universe can not be infinitely large [22]. Other solutions to this peak problem can be found in [17].

As another example of the utility of our proposal, consider the combined probability of the a priori probability and the anthropic one. We calculate the probability in the "ABZ" model proposed by Bousso [6]. The reason for us to use this model is that it is simple to calculate in this model, and the probability is not number weighted. Also, this application can easily be generalized to other models, where other a priori probabilities are assigned.

Assume there are only three vacua in the landscape, namely, the de Sitter vacua A, B, and the AdS terminal vacuum Z. The decay probabilities are shown in Fig. 2. Suppose we start from the vacuum A, i.e. the left figure in Fig. 2, then the a priori probability for the vacua are [6]

$$q_A = \epsilon/2$$
, $q_B = 1/2$, $q_Z = (1 - \epsilon)/(1 + \epsilon)$, (5)

where we label the a priori probability by q to to distinguish it from the anthropic probability.

Note that the anthropic probability (3) jumps from zero to one very fast if n is



Figure 1: Anthropic selection of the cosmological constant using the baryon number weighted measure [17]. Three curves correspond to difference choices for the minimum required mass for a galaxy: $M_* = (10^7, 10^9, 10^{12}) M_{\odot}$, respectively. The vertical bar corresponds to the observed value of the cosmological constant. We find from this figure that the cosmological constant we observe is not at the peak of the probability distribution.



Figure 2: A toy model of the landscape [6]. Assume that there are two de Sitter vacua A, B and one terminal AdS vacuum Z in the landscape. A can only tunnel to B with unit probability, B can tunnel to A with probability ϵ , or tunnel to Z with probability $1 - \epsilon$, and Z can not tunnel back to A or B.

exponentially large, where the exponential is naturally produced by inflation. So we can approximate $P_X(p,n)$ (X = A, B, Z) by

$$P_X(p,n) = \begin{cases} 0 & \text{when } np_X \ll 1, \text{ i.e. not suitable for observers,} \\ 1 & \text{when } np_X \gg 1, \text{ i.e. suitable for observers.} \end{cases}$$
(6)

The full probabilities for vacua A, B and Z are proportional to $q_A P_A$, $q_B P_B$, $q_Z P_Z$ respectively. We see explicitly in this example that the probability defined in this way is well-behaved, finite, and does not suffer the problems listed above ¹.

4 Conclusion

In this paper, we have argued that the exist-or-not anthropic probability is a natural choice for eternal inflation. This probability is free of the freak observer problem and the typicality problem, it offers a better anthropic explanation of the cosmological constant problem. A simple example is discussed to demonstrate the utility of this probability.

The anthropic probability we propose can also be used in other realizations for the landscape, or other kinds of the multiverses, for a discussion of these realizations and multiverses, see [14, 23].

It should be emphasized that although the anthropic probability proposed in this paper is different from the number weighted probability, we do not claim one of them is correct and others are wrong. Different proposals come by due to different understanding of the anthropic principle. As correctness of the anthropic principle itself is not believed by everyone, it is still too early to judge which version of the anthropic principle is correct. We only suggest that our anthropic probability is natural, and do not suffer the problems listed in this paper.

¹At the first sight, it appears that problems such as the freak observer problem do not arise here just because the ABZ model itself does not suffer these problems. But it should be noticed that the ABZ model itself only assigns a priori probability, and it must be multipled by the anthropic probability to get the full probability. If we do not use the anthropic probability properly, the problems are still present

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