Interactions of Non-Abelian Global Strings

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Abstract

Non-Abelian global strings are expected to form during the chiral phase transition. They have orientational zero modes in the internal space, associated with the vector-like symmetry $SU(N)_{\mathrm{L+R}}$ broken in the presence of strings. The interaction among two parallel non-Abelian global strings is derived for general relative orientational zero modes, giving a non-Abelian generalization of the Magnus force. It is shown that when the orientations of the strings are the same, the repulsive force reaches the maximum, whereas when the relative orientation becomes the maximum, no force exists between the strings. For the Abelian case we find a finite volume correction to the known result. The marginal instability of the previously known Abelian η' strings is discussed.

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1 Introduction

Topological strings play very important roles in physics. Their study ranges from the cosmic strings which are formed in the early universe [1] to the vortices in the condensed matter physics *i.e.* Abrikosov flux tubes in type II superconductors, the superfluid vortices in ^{4}He and cold atoms, and so on. They are accompanied with the spontaneous symmetry breaking at phase transitions. In high energy physics, topological strings appear in the standard model, GUTs and other particle models [2]. In cosmology they had long been the strong candidates for the formation of the galaxies and CMB fluctuations. Although this possibility is now excluded, the study of cosmic strings is under the significant developments recently due to several reasons [3]. Depending on whether a broken symmetry is global or local, strings are called global or local, respectively. In the early stage of developments, global cosmic strings were not focused because their energy is logarithmically divergent. Later it was recognized that the divergence is not a problem because a finite volume system or nearest strings give a natural infrared cutoff. Then their interaction, reconnection (intercommutation) and formation of a network were extensively discussed [4, 5, 6, 1].

One of the important recent developments concerns the non-Abelian strings. Here we use the term "non-Abelian string" for a string which arises at the symmetry breaking $G \to H$ for which the unbroken subgroup H is non-Abelian. Recently the non-Abelian local/semi-local strings have been found in superstring theory [7] and in supersymmetric QCD [8]. Since these strings are BPS *i.e.* at the critical coupling, no static force exists and the so-called moduli matrix approach [9, 10] provides the most generic solutions and their complete moduli space [11]. Their interaction, scattering and reconnection have been studied in the moduli space approximation [12]. Non-Abelian semi-local strings have been further studied [10, 13].

In contrast to these remarkable developments, the non-Abelian *global* strings have not been so much investigated yet, as was so in the case of the Abelian global strings. Despite this, they are interesting for several reasons. First, different from the Abelian global strings, the non-Abelian strings have the internal degrees of freedom which are called *orientation*; the presence of a string breaks the symmetry H further $H \to H'$ and consequently the zero modes corresponding to H/H' appear along the string. Then we have a continuously infinite number of strings with the same tension which are parameterized by this *orientation*, namely a point in H/H'. The interaction among the strings with different orientations is not trivial at all, which is the main issue of the present Letter. Second, it was shown that non-Abelian global strings with domain walls indeed form during the chiral phase transition in QCD [14] where the $SU(N)_{\rm L} \times SU(N)_{\rm R}(\times \mathbf{Z}_{N,\rm A})$ symmetry is broken to its diagonal subgroup $SU(N)_{\rm L+R}$ where N indicates the number of flavors. In

Ref. [14], they explicitly took into account the effect of anomaly.

Before the discovery of the non-Abelian global strings in QCD, it was already shown that the Abelian global strings which are called the η' strings may exist in the early universe [15]. When the temperature becomes very high, the chiral anomaly is not effective [16]. It is because the instantons require both color electric and magnetic fields. But the fluctuation of the electric field is suppressed at high temperature due to the Debye screening. Therefore, if the temperature for the chiral phase transition is so high that the $U(1)_A$ symmetry is effectively restored[1], the Abelian strings arise during the spontaneous symmetry breaking of this effective $U(1)_A$ symmetry due to the chiral condensate $\langle \bar{q}q \rangle$, where $q(\bar{q})$ indicates the (anti)quark fields. The η' strings become unstable as the temperature decreases and the instanton effects become substantial. The authors in Ref. [18] expected that they can be stable if they accompany three domain walls.

In this Letter, we consider non-Abelian global strings which arise during the chiral phase transition when we can neglect the effect of the anomaly, which is just the case considered in Refs. [15, 18]. We derive the interactions among the non-Abelian global strings in the $U(N)_L \times U(N)_R$ linear σ model. There are lots of interesting questions about the formation, evolution of the strings etc. As a first step, however, we consider the interaction among the static two non-Abelian strings with various relative orientations using the Abrikosov approximation. In section 2, the non-Abelian string solution with general orientation is constructed in the $U(N)_L \times U(N)_R$ linear σ model. The interaction among the static two non-Abelian strings is derived in section 3. In the case when the orientations of two strings are the same the calculation reduces to that of two Abelian strings. We find even for this case a finite-volume correction to the known result [4, 5, 6, 1]. We end in section 4 with conclusion and discussion.

2   Non-Abelian global strings and orientations

Let us consider the chiral $U(N)_L \times U(N)_R$ linear σ model. We first introduce the N by N matrix field Φ_{ij} $(i, j = 1 \cdots N)$ in order to parameterize the symmetry breaking. This field belongs to $[N, \bar{N}]$ representation of $SU(N)_L \times SU(N)_R$. Under the global chiral symmetry $G = SU(N)_L \times SU(N)_R \times U(1)_A$, Φ transforms as

$$\Phi \to e^{i\alpha} U_L \Phi U_R^\dagger. \tag{1}$$

where U_L and U_R are independent $SU(N)$ matrices and $e^{i\alpha}$ is the total $U(1)_A$ rotation.

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[1] This point is still controversial and is not settled yet. See Ref. [17], for example.

The Lagrangian which is symmetric under G is

$$\mathcal{L} = \text{tr}(\vec{\partial}\Phi^{\dagger}\vec{\partial}\Phi) - m^2\text{tr}(\Phi^{\dagger}\Phi) - \lambda_1(\text{tr}\Phi^{\dagger}\Phi)^2 - \lambda_2\text{tr}[(\Phi^{\dagger}\Phi)^2] \tag{2}$$

up to quartic order in Φ. When $m^2 < 0$, $\lambda_1 + \lambda_2/N > 0$, and $\lambda_2 > 0$, the vacuum expectation value

$$\langle\Phi\rangle = v\mathbf{1} \equiv \Phi_0, \ \ v = \sqrt{-m^2/2(N\lambda_1 + \lambda_2)} \tag{3}$$

breaks the symmetry G to $H = SU(N)_{\text{L+R}} \times \mathbf{Z}_N$ and corresponding N^2 Nambu-Goldstone bosons appear. The action of H to $\langle\Phi\rangle$ is $\langle\Phi\rangle \rightarrow e^{i\alpha}U_L\langle\Phi\rangle U_R^{\dagger}$ with $(e^{i\alpha}, U_L, U_R) = (\omega, \omega^{-1}U, U) : \omega \in \mathbf{Z}_N, U \in SU(N)$. The coset space has the non-trivial first homotopy group,

$$\frac{G}{H} = \frac{SU(N) \times U(1)}{\mathbf{Z}_N} = U(N) \quad \Rightarrow \quad \pi_1[U(N)] = \mathbf{Z}, \tag{4}$$

which develops both the non-Abelian as well as Abelian strings. Therefore, the non-Abelian vortex strings we are studying here are topological objects contrary to the pion string which is non-topological with $\pi_1[SU(2)] = 0$ [19].

We will consider the cylindrically symmetric string configuration along the z-axis. The most fundamental string is the non-Abelian string which is generated by both $SU(N)$ and $U(1)$ generators of G. At large distance from the core of the fundamental string, the matrix field $\Phi(\theta)$ rotates as:

$$
\begin{aligned}
\Phi(\theta, r) &= \exp\left(i\frac{\theta}{N}\right)\exp\left(-iT_{N^2-1}\frac{\sqrt{N(N-1)}}{N}\theta\right)\Phi(0, r) \\
&= \text{diag}\left(e^{i\theta}f(r), g(r), \cdots, g(r), g(r)\right) \\
&\Rightarrow \text{diag}\left(e^{i\theta}, 1, \cdots, 1, 1\right),
\end{aligned} \tag{5}
$$

where we have already taken $v = 1$ for simplicity, and T_a $(a = 1, 2, \cdots N^2 - 1)$ is the generators of $SU(N)$ in the fundamental representation which we normalize as $\text{Tr}\{T_aT_b\} = \delta_{ab}$. The $(N^2 - 1)$-th generator is $T_{N^2-1} = \frac{1}{\sqrt{N(N-1)}}\text{diag}(1 - N, \cdots, 1, 1)$. Here θ is the angular coordinate in the x-y plane and we set $\Phi(0, r) = \Phi_0$. The full numerical solution of the string with profile functions $f(r)$ and $g(r)$ is given in Ref. [24].

The string configuration breaks the symmetry H further as $SU(N)_{\text{L+R}} \rightarrow SU(N-1)_{\text{L+R}} \times U(1)_{\text{L+R}}$. Consequently the zero modes corresponding to

$$\frac{SU(N)_{\text{L+R}}}{SU(N-1)_{\text{L+R}} \times U(1)_{\text{L+R}}} \simeq \mathbf{C}P^{N-1} \tag{6}$$

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appear along the string. Eq. (5) is in fact just one particular string among a continuously infinite number of strings with the same tension which are parameterized by the *orientation*, namely a point in $\mathbf{C}P^{N-1}$. We will explicitly construct the two string system with general relative orientation in the next section.

Before going to the next section we discuss stability of our solutions here. Regarding dynamical stability of non-Abelian strings, the scaling argument from the Derrick theorem [20] can not be applied to the present case, because they are global strings whose energy diverges in infinite size systems. They usually exist in finite size systems, as $U(1)$ vortex does in Helium superfluid, where the effect from boundary prevents vortex core from collapsing. In this meaning, once a cutoff parameter has been introduced for spatial boundary to make the total energy finite, a modified version of the Derrick theorem makes sense, see e.g., [21]. In contrast to texture-like objects discussed in [21] where only gradient energy terms are taken into consideration, our global string has a non-trivial stable solution where gradient and potential energies are balanced with a finite cutoff Λ. Also, there is an issue whether or not the global non-Abelian string solution is stiff against small perturbations of diagonal elements into which a vortex solution is not embedded. Here we would briefly show this stability: first introduce a small perturbation field $\psi(r,t)$ as $g(r) \rightarrow g(r) + \psi(r,t)$, and suppose that $\psi(r,t) = e^{-i\omega t}\psi(r)$. Plugging this into the equation of motion and linearizing it in $\psi(r)$ lead to a Schrödinger-like equation. After a normalization, we obtain

$$\psi'' + \frac{1}{r}\psi' + \left[1 + \omega^2 - 2\kappa f^2(r) - 2\left(\kappa(N-1) + 1\right)3g^2(r)\right]\psi = 0. \qquad (7)$$

We solve an eigenvalue problem for ω^2 with the boundary condition $\psi'(0) = \psi'(\Lambda) = 0$. If all the eigenvalues of ω^2 are positive for given κ and N, the string solution is stable. 1) In the case of $\kappa = 0$ (the critical coupling), $f(r)$ and $g(r)$ are decoupled and it is immediately found that $g(r) = 1/2$, and then $\psi'' + \frac{1}{r}\psi' + \left[1 + \omega^2 - 6g(r)^2\right]\psi = 0$. The Bessel function gives the solution and only positive ω^2's satisfy the boundary condition. 2) The case of $\kappa \neq 0$ is more complicated. After the substitution of the full solutions for $f(r)$ and $g(r)$ numerically obtained in Ref. [24], for instance for $\kappa = 0.2$ and $N = 3$, we found $\omega^2 = 1.9851$ as the lowest eigen value. We thus see that the non-Abelian string solutions are dynamically stable as expected from the topology arguments. The complete analysis on the stability is beyond the scope of the present paper.

In the situation that the $U(1)_A$ symmetry in our Lagrangian is gauged, our non-Abelian strings become semi-local strings (with finite energy) [22], then dynamical stability mechanism by Hindmarsh [23], which is related to magnitudes between gauge and scalar couplings, might work even in infinite size systems. Although the vortex solution discussed in [23] is not topological, Hindmarsh has also mentioned the instability arising
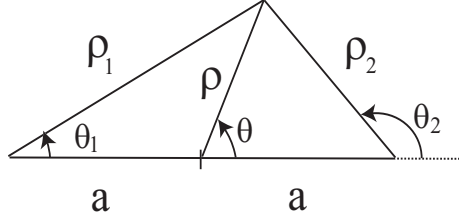
Figure 1: Configuration of two global strings with interval $d = 2a$ in polar coordinate (ρ, θ).

from small fluctuation of the solution. But this is not our present case with global $U(1)_{\text{A}}$.

3   Interaction between two strings

Now we consider the interaction among arbitrary two strings. Let us place two strings $\phi_{1,2}$ parallel along the z-axis with the separation $2a$ in the x-y plane. For definiteness those positions are $(\rho, \theta) = (a, 0)$ and (a, π) as in Fig. 1 where (ρ, θ) are the polar coordinates in the x-y plane. As the orientation in the internal space $\mathbf{C}P^{N-1}$ in (6) is concerned, only the relative orientation matters. Let us take the reference string ϕ_1 as in Eq. (5):

$$\phi_1 = \operatorname{diag}(e^{i\theta_1}, 1, \cdots, 1, 1). \tag{8}$$

Then starting from the same orientation with ϕ_1 in (8), the most general orientation (6) for the second string ϕ_2 is obtained by acting $SU(N)_{\text{L+R}}$ on it. However as far as two string interaction is concerned, only an $SU(2)_{\text{L+R}}(\subset SU(N)_{\text{L+R}})$ rotation is enough to be considered without loss of generality. (This corresponds to considering a $\mathbf{C}P^1$ submanifold inside the whole $\mathbf{C}P^{N-1}$.) We thus have

$$\phi_2 = \begin{pmatrix} g \begin{pmatrix} e^{i\theta_2} & 0 \\ 0 & 1 \end{pmatrix} g^{-1} & 0 \\ 0 & \mathbf{1}_{N-2} \end{pmatrix},$$

where g is an element of $SU(2)$:

$$g = \cos\left(\frac{\alpha}{2}\right) \mathbf{1}_2 + i\vec{n} \cdot \vec{\sigma} \sin\left(\frac{\alpha}{2}\right) \tag{9}$$

with $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ the Pauli matrices and $\vec{n}$ a unit three vector. Since the rotation by σ_3 does not change the relative orientation between ϕ_1 and ϕ_2, n_z is fixed to 0. If we

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define β by

$$e^{i\beta} = n_x - i n_y, \tag{10}$$

then α and β parameterize $\mathbf{C}P^1 \simeq S^2$. Consequently, ϕ_2 is simplified as

$$\phi_2 = \begin{pmatrix} \begin{pmatrix} e^{i\theta_2}\cos^2\left(\frac{\alpha}{2}\right) + \sin^2\left(\frac{\alpha}{2}\right) & \frac{i}{2}(1 - e^{i\theta_2})e^{i\beta}\sin\alpha \\ -\frac{i}{2}(1 - e^{i\theta_2})e^{-i\beta}\sin\alpha & \cos^2\left(\frac{\alpha}{2}\right) + e^{i\theta_2}\sin^2\left(\frac{\alpha}{2}\right) \end{pmatrix} & 0 \\ 0 & \mathbf{1}_{N-2} \end{pmatrix}. \tag{11}$$

For $\alpha = 0$ the orientations of the two strings become the same, and the problem is reduced to the one of the Abelian strings [5, 6]. $\phi_{1,2}$ becomes an anti-string by changing the sign of $\theta_{1,2}$.

Let us now calculate the interaction among two parallel non-Abelian (anti-)strings with general orientations in the internal space. The interaction energy density of the two string system is obtained by subtracting two individual string energies from the total configuration energy:

$$F(\rho, \theta, a, \alpha) = \text{tr}\left(|\partial\Phi_{\text{tot}}|^2 - |\partial\phi_1|^2 - |\partial\phi_2|^2\right), \tag{12}$$

where Φ_{tot} is the total string configuration and we have used the fact that for sufficiently large value of a the potential energies can be approximated by $V(\Phi_{\text{tot}}) = V(\phi_1) = V(\phi_2) = 0$. We employ the Abrikosov ansatz for the configuration where

$$\Phi_{\text{tot}} = \phi_1\phi_2 \quad \text{or} \quad \phi_2\phi_1. \tag{13}$$

We see that either ansatz gives the same result, so we do not have to worry about the ordering of the matrices.[2] Further, for simplicity, $\phi_{1,2}$ and Φ_{tot} are approximated to their values at spatial infinity, Eqs. (8, 11). This approximation is justified when the interval of the strings is much longer than the coherence length (the transverse size of strings [24]):

$$a \gg m^{-1}. \tag{14}$$

Then Eq. (12) is simplified and we get:

$$F(\rho, \theta, a, \alpha) = \pm(1 + \cos\alpha)\left(\frac{-a^2 + \rho^2}{a^4 + \rho^4 - 2a^2\rho^2\cos(2\theta)}\right). \tag{15}$$

---

[2] One can show that $\text{tr}\partial(\phi_1\phi_2)\partial(\phi_1\phi_2) = \text{tr}\partial(\phi_2\phi_1)\partial(\phi_2\phi_1)$ up to reparameterization of g and coordinates.
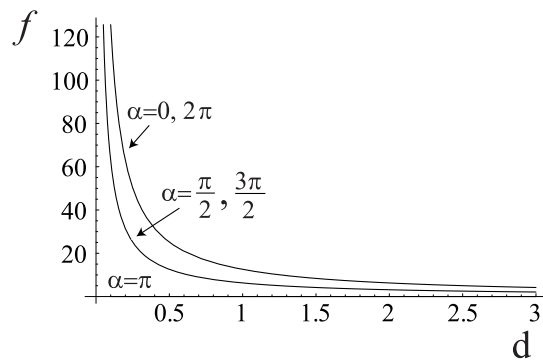
Figure 2: Dependence of the force between two non-Abelian strings on the separation $d = 2a$ for several α.

Here and below, the upper(lower) sign corresponds to the interaction energy density for the string-string (string-anti-string) configuration. For $\alpha = 0$, F reduces to that of Abelian global strings [5, 6]. However in contrast to the results in Refs. [5, 6], we have got the θ dependent interaction energy density which reaches the maximum (minimum) at $\theta = 0, \pi$ when $\rho > a \, (\rho < a)$ for string-string configuration. The θ dependence gives a correction to [5, 6] for the Abelian case ($\alpha = 0$).

The (sum of) tension, the energy of the strings per unit length, is obtained by integrating the energy density over the x-y plane:

$$
\begin{aligned}
E(a, \alpha, L) &= \pm \int_0^L d\rho \int_0^{2\pi} d\theta \rho F(\rho, \theta, a) \\
&= \pm \pi (1 + \cos \alpha) \left[-\ln 4 - 2 \ln a + \ln \left(a^2 + L^2 \right) \right],
\end{aligned} \tag{16}
$$

where the IR cutoff L is introduced to make the integral finite. The force between the two (anti-)strings are then obtained by differentiating E by the interval:

$$
f(a, \alpha, L) = \mp \frac{\partial E}{2 \partial a} = \pm (1 + \cos \alpha) \left(\frac{\pi}{a} - \frac{\pi a}{a^2 + L^2} \right) \simeq \pm (1 + \cos \alpha) \frac{\pi}{a}, \tag{17}
$$

where the last expression is for $a \ll L \to \infty$. This is just the force between two Abelian (anti-)strings known as the Magnus force, multiplied by $(1 + \cos \alpha)/2$. We can see that when the orientation of the strings are the same ($\alpha = 0$) the repulsive(attractive) force reaches the maximum and is the same as that between Abelian global string and (anti-)string [5, 6], where the second term in the middle equation gives a finite-volume (finite

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L) correction to [5, 6]. On the other hand, when the relative orientation becomes the maximum ($\alpha = \pi$), no force exists between the strings. Note that although the most stable configuration is given by the strings with the maximum relative angle ($\alpha = \pi$), it is not possible for the strings to change α because it is non-normalizable and must be fixed by the boundary condition at infinity. This change is possible only if strings emit infinite number of Nambu-Goldstone bosons α in $\mathbf{C}P^{N-1}$.

So far we have considered the case of the strings in infinite region where the relative orientation α is non-normalizable and is fixed. However α becomes a normalizable mode in a finite volume (finite L) which is realistic in experiments such as the heavy-ion collider. In such a case, the force among orientations of two strings can be considered. The interaction energy (16) shows a repulsive force exists between aligned orientations of two strings. The stable configuration is for $\alpha = \pi$ where two orientations are anti-aligned. Therefore we conclude that they behave like antiferromagnet.

4   Discussion

In this Letter, we have considered the interactions among two non-Abelian strings in $U(N)_L \times U(N)_R$ linear σ model. This model also has an Abelian string solution [15, 18], the η' string. However, it is not the fundamental string and is made of N non-Abelian strings:

$$
\begin{aligned}
\Phi(\theta) &= \operatorname{diag}(e^{i\theta}, \cdots, e^{i\theta}, e^{i\theta}) \\
&\sim \operatorname{diag}(e^{i\theta}, \cdots, 1, 1) \times \operatorname{diag}(1, e^{i\theta}, \cdots, 1) \times \cdots \times \operatorname{diag}(1, 1, \cdots, e^{i\theta}).
\end{aligned} \quad (18)
$$

There are no force among any of these non-Abelian strings, which indicates that the η' string is marginally unstable to decay into N non-Abelian strings. No binding energy implies that they decay with arbitrary momentum or by fluctuations. This result holds in the presence of the chiral anomaly at lower temperatures; the Abelian string with N domain walls will decay into N non-Abelian strings, where each is attached by one domain wall. In that case, the instability increases since once the Abelian string decays, the domain wall pulls the string away to infinity. Therefore we do not have a cosmological domain wall problem.

The same type of the non-Abelian strings also appear in the low energy theory of supersymmetric QCD [8] and in the high density QCD (color superconductors) [25] as fundamental strings. In these cases, the strings accompany the gauge fields which may change the interaction among them. The case of strings in color superconductors is reported [26] in which the universal repulsion is found unlike the case of global strings in this Letter.

Another interesting issue is how the non-Abelian strings emit or interact with the Nambu-Goldstone bosons (the pions and the η' meson). In the case of global $U(1)$ strings, this can be described by using the two index antisymmetric tensor fields of the Kalb-Ramond action [27]. The non-Abelian tensor fields [28] may be suitable to describe the non-Abelian case.

Thermal effect was studied for non-Abelian local and semi-local vortices [29]. Finite temperature effect is important to study strings at a collider or in the early universe.

The inclusion of the bare quark mass would be a next step. If the quark mass enters in the theory, the chiral symmetry becomes not intact. Then a new topological object would appear where strings with different orientations are separated by bead-like solitons. Also, the ring-shaped string may appear. We remain the study of these new topological objects as a future work.

Acknowledgements

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