

Non-Abelian Strings in High Density QCD: Zero Modes and Interactions

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Abstract

The most fundamental strings in high density color superconductivity are the non-Abelian semi-superfluid strings which have color gauge flux tube but behave as superfluid vortices in the energetic point of view. We show that in addition to the usual translational zero modes, these vortices have normalizable orientational zero modes in the internal space, associated with the color-flavor locking symmetry broken in the presence of the strings. The interaction among two parallel non-Abelian semi-superfluid strings is derived for general relative orientational zero modes to show the universal repulsion. This implies that the previously known superfluid vortices, formed by spontaneously broken $U(1)_B$, are unstable to decay. Moreover, our result proves the stability of color superconductors in the presence of external color gauge fields.

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1 Introduction

Quark matter at high density is considered to exhibit color superconductivity [1]. There are lots of phases proposed in the color superconductivity and still there seems no agreement on which is the true ground state [2]. However, when the quark chemical potential becomes much larger than the strange quark mass $\mu \gg m_s$, the color-flavor locking (CFL) phase is expected to be realized [3]. In the CFL phase, the quark-quark pairing breaks the symmetry of QCD such as $G = U(1)_B \times SU(N)_C \times SU(N)_F \rightarrow H = SU(N)_{C+F} \times \mathbf{Z}_N$ with $N = 3$, where $U(1)_B$ is a baryon number symmetry and $SU(N)_C$, $SU(N)_F$ and $SU(N)_{C+F}$ are color, flavor and the color-flavor locked symmetries, respectively. The breaking of $U(1)_B$ produces the superfluid vortices [4, 5] which may play a role in the neutron star physics. It has been, however, shown in Ref. [6] that they are not the fundamental strings in the color superconductivity. The most fundamental strings are then the semi-superfluid strings which are *non-Abelian* strings with color gauge flux tube. A superfluid vortex in three flavor QCD can be topologically (and group-theoretically) decomposed into three non-Abelian strings. It remains as a significant open problem which is really energetically favored, three separated non-Abelian vortices or one superfluid vortex as a bound state of them.

In general, non-Abelian strings are strings which arise for symmetry breaking $G \rightarrow H$ in which the unbroken subgroup H is non-Abelian. This kind of strings themselves attracts lots of attention these several years. Recently the non-Abelian local strings have been found in superstring theory [7] and in supersymmetric QCD [8]. For instance, they have been used to show color confinement and non-Abelian duality in supersymmetric QCD [9]. Since these strings are BPS, namely at critical coupling, there exists no static force between them and therefore the moduli space is admitted. The most generic solutions and their moduli space have been obtained [10] by introducing the method of the moduli matrix [11, 12]. Dynamics of strings such as reconnection (intercommutation) of two strings has been studied using the moduli space approximation [13]. Non-Abelian semi-local strings have also been extensively studied [12, 14]. As for non-Abelian global strings, there has been not so much work, but they may be generated during the chiral phase transition in high temperature QCD [15, 16, 17].

Different from Abelian strings, the distinct character of non-Abelian strings is their internal degrees of freedom which are called *orientation*. In the case of generalized QCD where the number of flavor equals to the number of color, the presence of a string breaks the symmetry H further as $SU(N)_{C+F} \rightarrow SU(N-1)_{C+F} \times U(1)_{C+F}$. Consequently the zero modes corresponding to

$$\frac{SU(N)_{C+F}}{SU(N-1)_{C+F} \times U(1)_{C+F}} \simeq \mathbf{C}P^{N-1} \quad (1)$$

appear along the string. Then we have a continuously infinite number of strings with the same tension, which are parameterized by the *orientation*, namely a point in $\mathbf{C}P^{N-1}$. In the case of local (global) non-Abelian strings, these orientational zero modes are (non-)normalizable because the transformation fixes [8] (changes [16, 17]) the boundary condition of the strings at infinity. In the case of semi-superfluid strings, it is, however, a non-trivial problem whether orientations are normalizable or not, because $U(1)_B$ symmetry is a global symmetry, unlike the case of local strings [8] where $U(1)_B$ is gauged.

In this Letter, we first show that orientational zero modes are in fact normalizable. We then calculate the force among two non-Abelian semi-superfluid strings with general orientations. We find that the static force is always repulsive and is $1/N$ of that between superfluid vortices. It does not depend on the orientations of the strings, which is somewhat surprising because the static force between two global non-Abelian strings *does* depend on the orientations [17]. Our result implies that a superfluid vortex of $U(1)_B$ breaking found in Ref. [4, 5] is actually unstable to decay into N (three) non-Abelian strings by repulsive force between them.

In the case of the usual superconductors, the interaction between two strings were important; the repulsion (attraction) between strings in type II (I) superconductors implies their (in)stability in the presence of an external magnetic field. The universal repulsion found in this Letter ensures the stability of color superconductors in the presence of external color gauge fields, regardless of whether they are of type I or II.

The construction of this Letter is the following. In section 2, we review the Ginzburg-Landau Lagrangian which has generalized QCD symmetry G and the construction of single non-Abelian string. In section 3, the non-Abelian string solution with general orientation is constructed. The interaction among the static two non-Abelian strings with general relative orientations is derived in section 4. We end in section 5 with conclusion and discussion.

2 Non-Abelian semi-superfluid strings

Let us start from constructing the general form of the Ginzburg-Landau Lagrangian [18] on the basis of the generalized QCD symmetry:

$$G = SU(N)_C \times SU(N)_F \times U(1)_B, \quad (2)$$

where we set the number of flavor equals to the number of color and $SU(N)_C$ is the gauge symmetry. For this purpose, we first introduce an N by N matrix field $\Phi_{\alpha i}$ ($\alpha, i = 1, \dots, N$) where $\alpha(i)$ denote the color(flavor) indices. Φ belongs to $[N, N]$ representation

of $SU(N)_C \times SU(N)_F$. The symmetry G transforms the matrix field as

$$\Phi \rightarrow e^{i\alpha} U_C \Phi U_F^t, \quad (3)$$

where U_C and U_F are independent $SU(N)$ matrices and $e^{i\alpha}$ is a global $U(1)_B$ rotation associated with the baryon number conservation.

In the case of color superconductivity, Φ field corresponds to the pairing gap; the spin-zero pairing of the positive energy quarks in antisymmetric combinations of colors and flavors [19]. In this case, we may write Φ as

$$\Phi_{\alpha i} \sim \epsilon_{\alpha\beta\gamma} \epsilon_{ijk} \langle \psi_j^{T\beta} C \gamma_5 \psi_k^\gamma \rangle, \quad (4)$$

where ψ is the quark field and we assumed that the ground state is the positive parity state which would be determined by the instanton effect. Then the most general 3-d Ginzburg-Landau Lagrangian up to $\mathcal{O}(\Phi^4)$ is:

$$\mathcal{L} = \text{tr}(D\Phi)^\dagger(D\Phi) - m^2 \text{tr}(\Phi^\dagger\Phi) - \lambda_1 (\text{tr}\Phi^\dagger\Phi)^2 - \lambda_2 \text{tr}[(\Phi^\dagger\Phi)^2] - \frac{1}{4} F_{ij}^a F^{aij}, \quad (5)$$

where $D \equiv \partial - ig_s A^\alpha T_\alpha$ is the covariant derivative for the color symmetry, and T_a ($a = 1, 2, \dots, N^2 - 1$) are the generators of $SU(N)_C$ in the fundamental representation which we normalize as $\text{Tr}\{T_a T_b\} = \delta_{ab}$. Here g_s is the gauge coupling and $F_{ij}^a = \partial_i A_j^a - \partial_j A_i^a + g_s f_{abc} A_i^b A_j^c$ is the field strength.¹

Stability condition of vacua enforces $\lambda_1 + \lambda_2/N > 0$. When $m^2 < 0$ and $\lambda_2 > 0$, the vacuum expectation value takes

$$\langle \Phi \rangle = v \mathbf{1} \equiv \Phi_0, \quad v = \sqrt{-m^2/2(N\lambda_1 + \lambda_2)}, \quad (6)$$

and the symmetry G is broken to $H = SU(N)_{C+F} \times \mathbf{Z}_N$. This condensation is called the color-flavor locking (CFL) in the case of the color superconductivity. The action of H on $\langle \Phi \rangle$ is $\langle \Phi \rangle \rightarrow e^{i\alpha} U_C \langle \Phi \rangle U_F^t$ with $(e^{i\alpha}, U_C, U_F^t) = (\omega, \omega^{-1}U, U^\dagger) : \omega \in \mathbf{Z}_N, U \in SU(N)$. The coset space $G/H \simeq U(N)$ has the non-trivial first homotopy group $\pi_1[U(N)] \simeq \mathbf{Z}$, which develops non-Abelian as well as Abelian strings. In the vacua, only $U(1)_B$ Nambu-Goldstone boson remains massless with the rests Higgsed. From now on, we will take $v = 1$ for simplicity.

Here we concentrate on the most fundamental string out of which all the other string configurations are made, and it actually has the lowest energy configuration with a non-trivial loop. Since the common element \mathbf{Z}_N of $SU(N)_{C,F}$ and $U(1)_B$ provides the warp

¹ Throughout this Letter we do not consider the electro-magnetic (EM) symmetry for simplicity. The EM group $U(1)_{EM}$ is a subgroup of the flavor symmetry $SU(N)_F$ and explicitly breaks $SU(N)_F$. The zero modes found in this Letter are thus massive through the EM interaction.

points to make the non-trivial loops therein, the fundamental string (non-Abelian string) is generated by *both* $SU(N)_{\text{C,F}}$ and $U(1)_{\text{B}}$ generators.

To make this statement clear, we consider the cylindrically symmetric string configuration along the z -axis. In the polar coordinates (ρ, θ) in the x - y plane, an isolated fundamental string has the form

$$\begin{aligned}\Phi(\theta, \rho) &= \exp\left(i\frac{\theta}{N}\right) \exp\left(-iT_{N^2-1} \frac{\sqrt{N(N-1)}}{N} \theta\right) \text{diag}(f(\rho), g(\rho), \dots, g(\rho)) \\ &= \text{diag}(e^{i\theta} f, g, \dots, g)\end{aligned}\quad (7)$$

We take the basis so that the $(N^2 - 1)$ -th generator is $T_{N^2-1} = \frac{1}{\sqrt{N(N-1)}} \text{diag}(1 - N, 1, \dots, 1)$. f and g are functions of ρ , and $f(0) = 0$ and $f(\infty) = g(\infty) = 1$ at boundaries. At sufficiently large distance from the core, $\rho \gg \lambda$ where $\lambda \equiv m^{-1}$ is the coherence length, the profile of the string is well described by

$$\Phi(\theta, \rho) \simeq \text{diag}(e^{i\theta}, 1, \dots, 1) \quad \text{for } \rho \gg \lambda. \quad (8)$$

From Eq. (7), one can see that the non-trivial loop must be made from both generators of $SU(N)$ and $U(1)_{\text{B}}$. Although from topological reason we could use either $SU(N)_{\text{C}}$ or $SU(N)_{\text{F}}$ or both of them for $SU(N)$, the energy consideration forces us to use the gauge symmetry $SU(N)_{\text{C}}$ to minimize the energy of the covariant derivative term. Therefore, we use both global $U(1)_{\text{B}}$ and local $SU(N)_{\text{C}}$ symmetries to construct the string. The name *semi-superfluid* originates this fact.

The gauge fields associated with the string is determined by minimizing the kinetic energy as much as they can. Note that, different from local strings, the covariant derivative term does not vanish at infinity but it is finite, which makes the energy of the string logarithmically divergent as the system size is infinite, just like superfluid vortices. Given Eq. (8), the form of the gauge field can be deduced as:

$$A_{\theta}^{N^2-1} = \frac{\xi h(\rho)}{g_s \rho}, \quad (9)$$

and the other components of the gauge field vanish. Here the gauge field only has non-zero component for θ direction, which we have denoted as A_{θ} . The function $h(\rho)$ satisfies $h(\infty) = 1$ at boundary. A constant ξ should be determined so as to minimize the kinetic energy density F_{kin} at infinity:

$$F_{\text{kin}} = \text{tr} |D\Phi|^2 = \text{tr} \left| \left(\frac{1}{\rho} \frac{\partial}{\partial \theta} - ig_s A_{\theta}^{N^2-1} T_{N^2-1} \right) \Phi \right|^2 = \frac{1}{\rho^2} \left[\left(\xi + \sqrt{\frac{N-1}{N}} \right)^2 + \frac{1}{N} \right], \quad (10)$$

and its minimization is achieved at

$$\xi = -\sqrt{\frac{N-1}{N}}. \quad (11)$$

The kinetic energy then becomes $F_{\text{kin}} = \frac{1}{N\rho^2}$, which is $1/N$ of that of the global $U(1)_B$ string [4]. Thus, at large distance over the penetration depth $\lambda_v \equiv m_g^{-1} \sim g_s^{-1}v^{-1}$, the gauge field configuration becomes

$$ig_s A_\theta^{N^2-1} T_{N^2-1} \simeq -\frac{i}{N\rho} \begin{pmatrix} 1-N & 0 \\ 0 & \mathbf{1}_{N-1} \end{pmatrix} \quad \text{for } \rho \gg \lambda_v. \quad (12)$$

The numerical solution of f , g , and h for the semi-superfluid string in the color-flavor locked phase is found in Ref. [6], where the color and electro-magnetic fields are mixed and only one of their linear combinations is relevant for flux.

3 Internal space and color gauge transformation

Before going ahead, it is instructive to clarify the internal space of the string. The presence of the string (7) breaks the symmetry H further to $SU(N)_{C+F} \rightarrow SU(N-1)_{C+F} \times U(1)_{C+F}$. The internal space corresponds to the coset space (1). Zero modes parameterizing the space (1) appear along the string, i.e., Eq. (8) denotes just one particular string of a continuously infinite number of strings with the same string tension (flux energy) which are parameterized by the *orientation* in the \mathbf{CP}^{N-1} . Unlike the case of global strings [16, 17] these zero modes are normalizable as seen below.

Here we consider a fundamental string with general orientation in the internal space. We first take the fundamental string Eq. (7) as a reference string ϕ_0 ,

$$\phi_0 = \text{diag}(e^{i\theta}f, g, \dots, g, g). \quad (13)$$

Then the string ϕ with general orientation in \mathbf{CP}^{N-1} relative to the reference string should be obtained by $SU(N)_{C+F}$ transformation to ϕ_0 . However, there are some redundancies in this transformation, i.e., only an $SU(2)_{C+F} (\subset SU(N)_{C+F})$ rotation is enough to be considered for relative orientation to ϕ_0 without loss of generality. This corresponds to a \mathbf{CP}^1 submanifold in the whole \mathbf{CP}^{N-1} . Furthermore, since any regular color-gauge transformation does not change the physical situation, we omit $SU(2)_C$ rotation to ϕ_0 for the moment. Thus we transform ϕ_0 by an element U_F of flavor $SU(2)_F$:

$$\phi = \phi_0 U_F^t = \begin{pmatrix} \begin{pmatrix} e^{i\theta}f & 0 \\ 0 & g \end{pmatrix} u_F^{-1} & 0 \\ 0 & g\mathbf{1}_{N-2} \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} e^{i\theta}af & e^{i\theta}bf \\ -b^*g & a^*g \end{pmatrix} & 0 \\ 0 & g\mathbf{1}_{N-2} \end{pmatrix}, \quad (14)$$

where $u_F \equiv \begin{pmatrix} a^* & -b \\ b^* & a \end{pmatrix}$ (with $|a|^2 + |b|^2 = 1$) is an element of $SU(2)_F$. This is a general expression for the fundamental string. However, as will be shown below, the flavor-rotated string configuration (14) can be transformed either back to the original form $\phi \simeq \text{diag}(e^{i\theta}, 1, \dots, 1)$, or to the form $\phi \simeq \text{diag}(1, e^{i\theta}, 1, \dots, 1)$ with fully opposite orientation, at a region away from the centre of strings, by use of a *twisted* color-gauge transformation.

Any color gauge transformation keeps the physical situation unchanged if they are regular. Here we implement a *twisted* color transformation of $SU(2)_C$, given by

$$u_C(\theta, \rho) = \begin{pmatrix} a^* & -be^{i\theta F(\rho)} \\ b^*e^{-i\theta F(\rho)} & a \end{pmatrix} \quad (15)$$

with $F(\rho)$ being an arbitrary regular function with boundary conditions $F(0) = 0$ and $F(\infty) = 1$. The former condition has been imposed to make the transformation regular at the center of string. This is possible because of $\pi_1[SU(2)_C] = 0$. The upper left 2×2 minor matrix of ϕ in (14) is transformed to

$$\begin{aligned} u_C(\rho, \theta) \begin{pmatrix} e^{i\theta}af & e^{i\theta}bf \\ -b^*g & a^*g \end{pmatrix} &= \begin{pmatrix} |a|^2fe^{i\theta} + |b|^2ge^{i\theta F} & a^*b[-e^{i\theta F} + fe^{i\theta}] \\ ab^*[-1 + fe^{i(1-F)\theta}] & |a|^2g + |b|^2fe^{i(1-F)\theta} \end{pmatrix} \\ &\simeq \begin{pmatrix} e^{i\theta} & 0 \\ 0 & 1 \end{pmatrix} \quad \text{for } \rho \gg \lambda. \end{aligned} \quad (16)$$

This result means $\phi \simeq \phi_0$ for $\rho \gg \lambda$. Also, the fully opposite orientation can be obtained by another color-gauge transformation,

$$\begin{pmatrix} e^{-i\theta F(\rho)} & 0 \\ 0 & e^{i\theta F(\rho)} \end{pmatrix} u_C \begin{pmatrix} fe^{i\theta} & 0 \\ 0 & g \end{pmatrix} u_F^{-1} \simeq \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix} \quad \text{for } \rho \gg \lambda. \quad (17)$$

Note that one cannot change the topological number by use of this kind of regular gauge transformations. We thus have seen that spatial infinity of the string configurations is the same and does not depend on the orientational zero modes. This implies that the orientational zero modes are *normalizable*, unlike the case of global strings [16, 17].

In conclusion, all semi-superfluid strings with general orientation are equivalent to each other away from the core. In other words, strings rotated by the flavor $SU(2)_F$ revert to the first fundamental string given by Eq. (8) via the color gauge transformation at much longer distances than the coherence length. This fact makes the problem of the static long range force between two strings significantly simple.

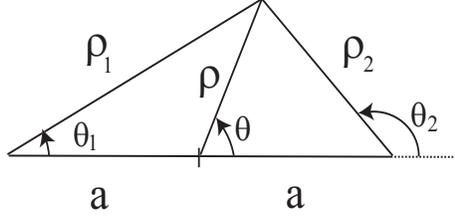


Figure 1: Configuration of two semi-superfluid strings with interval $2a$ in the polar coordinates (ρ, θ) . $\rho_{1,2}$ is distance from string $\phi_{1,2}$, and $\theta_{1,2}$ is angle around it.

4 Interaction between two strings

Here we consider the interaction between arbitrary two strings named ϕ_1 and ϕ_2 . These strings are placed at $(\rho, \theta) = (a, \pi)$ and $(a, 0)$ in parallel along the z -axis, see Fig. 1. We eventually go for the expression of a long range static force between two strings, which is valid if string are sufficiently separated. The situation we consider is sorted out as follows:

- The interval between strings is much larger than both the coherence length and the penetration depth: $a \gg \lambda(= m^{-1}), \lambda_v(= g_s^{-1}v^{-1})$.
- The first string ϕ_1 is approximated everywhere by the asymptotic profile (8): $\phi_1 = \text{diag}(e^{i\theta_1}, 1, \dots, 1)$. The second string ϕ_2 has the profile (14) with general orientation relative to ϕ_1 . At large distance of our interest, however, it is equivalent to the reference string configuration (8): $\phi_2 \simeq \text{diag}(e^{i\theta_2}, 1, \dots, 1)$. $\phi_{1,2}$ becomes an anti-string by changing the signs of $\theta_{1,2}$.
- The total profile of the two string system is given by the Abrikosov ansatz: $\Phi_{\text{tot}} = \phi_1\phi_2$ and $A_{\text{tot}}^\theta = A_1^\theta + A_2^\theta$. The first ansatz does not depend on the ordering of the matrices because the second string transforms to diagonal at large distance as shown in Sec. 3.² $A_{1,2}^\theta$ is the gauge field configuration (12) accompanied with the single string system of $\phi_{1,2}$. For an anti-string, $A_{1,2}^\theta$ changes the signs.

Now we are ready to evaluate the interaction between two parallel non-Abelian strings with general orientations in the internal space. In order to obtain the static force between them, we first calculate the interaction energy density of the two string system, which is obtained by subtracting two individual string energies from the total configuration energy.

² One can easily show that the *twisted* color transformation also works to make a product (13) \times (14) as $\phi_1 \times \phi_2$ diagonalized at large distance.

According to the above situation, the interaction energy density is given as

$$\begin{aligned} F(\rho, \theta, a) &\simeq \text{tr} (|D\Phi_{\text{tot}}|^2 - |D\phi_1|^2 - |D\phi_2|^2) \\ &= \pm \frac{2}{N} \left[\frac{-a^2 + \rho^2}{a^4 + \rho^4 - 2a^2\rho^2 \cos(2\theta)} \right], \end{aligned} \quad (18)$$

where we have used the fact that $V(\Phi_{\text{tot}}) = V(\phi_1) = V(\phi_2) = 0$ and $F_{ij}^a F^{aij} = 0$ at large distance [20]. Here and below, the upper(lower) sign indicates the quantity for string-string(string-anti-string) configuration.

The tension, the energy of the string per unit length, is obtained by integrating the energy density over the x - y plane,

$$E(a, L) = \pm \int_0^L d\rho \int_0^{2\pi} d\theta \rho F(\rho, \theta, a) = \pm \frac{2\pi}{N} [-\ln 4 - 2 \ln a + \ln(a^2 + L^2)], \quad (19)$$

where the IR cutoff L is introduced to make the integral finite. The force between the two strings are then obtained by differentiating E by the interval:

$$f(a, L) = \mp \frac{\partial E}{2\partial a} = \pm \frac{2\pi}{N} \left(\frac{1}{a} - \frac{a}{a^2 + L^2} \right) \simeq \pm \frac{2\pi}{Na}, \quad (20)$$

where the last expression is for $a \ll L \rightarrow \infty$. We can see that the force is *repulsive(attractive)* for string-string(anti-string-string) configuration. The overall factors $1/N$ in Eqs. (18)–(20) are attributed to the fact that the tension of the fundamental non-Abelian string is reduced by $1/N$ compared to the usual Abelian string, then leading to $1/N$ erosion in magnitude of the force.

Note that our result does not depend on whether the superconductivity is of type I or II. This has an important meaning in the case of color superconductivity since although the perturbation theory indicates the color superconductivity is of type I for whole density regime [21], the most fundamental strings, semi-superfluid strings, can be stable at any density regime where CFL realizes. This result also implies that the global $U(1)_B$ superfluid strings $\Phi \simeq \text{diag}(e^{i\theta}, \dots, e^{i\theta})$ found in Ref. [4, 5] as well as the M_2 strings $\Phi \simeq \text{diag}(1, e^{-i\theta}, e^{-i\theta}) \simeq (e^{2i\theta}, 1, 1)$ suggested in Ref. [6] are unstable to decay into N or 2 semi-superfluid strings, respectively. It contrasts to the case of global non-Abelian strings [17], where the $U(1)$ Abelian string is marginally unstable, *i.e.*, no force exists between two strings with opposite orientations.

From all the above arguments we conclude that there exists the long range force between two semi-superfluid strings, which is independent of the orientations in the internal space, and the sign of the force is determined by the difference in their topological charges.

5 Discussion and Outlook

In this Letter, we have considered the interaction between two non-Abelian semi-superfluid strings in the system which has generalized QCD symmetry, $SU(N)_C \times SU(N)_F \times U(1)_B$, using the Abrikosov approximation. This approximation is justified for the case where the strings are far apart. In case of a short interval, however, where the string cores overlaps: $a \sim \lambda, \lambda_v$, the present treatment might not be applicable to describe a fine structure of the force. In such a case the Abrikosov ansatz for Φ_{tot} we have employed might be suspicious, since amplitude functions f and g veer away from the unity near the core, and arbitrary orientation in the internal space made by flavor $SU(2)_F$ rotation generates off-diagonal parts in the string configuration. It means that two strings do not commute: $[\phi_1, \phi_2] \neq 0$. However our conclusion of the instability of $U(1)_B$ strings (or the stability of color superconductors) is unchanged. Even if the short range force is attractive a $U(1)_B$ string will decay by long range repulsion through classical large fluctuations, thermal fluctuations or the quantum tunnelling effect.

As applications of our results to the compact star physics, the universal repulsion implies that the lattice structure of many-string system may be obtained during the rapid cooling of the protoneutron stars or in response to the external electro-magnetic field and/or the rotation. This might have impacts on observables such as the pulsar glitch phenomenon. It is, however, still an open question how the strings terminate at the interface between the color superconductor in the core and the surrounding nuclear matter.

Another interesting issue is how the non-Abelian string releases or interacts with the Nambu-Goldstone bosons corresponding to the global $U(1)_B$ breaking as well as the leptons, quarks, mesons etc which exist in the neutron star. The former may be described using the two index antisymmetric tensor representation in which the Kalb-Ramond action appropriately describes the string [22]. Interaction between strings and other topological solitons is also interesting to be explored. It has been shown in Ref. [23] that $U(1)_A$ domain walls appear when the anomalous $U(1)_A$ is spontaneously broken. Fundamental quarks appear as Skyrmions (called qualitons) in the CFL phase [24]. Interaction of non-Abelian strings and these objects remains an open problem.

When the density is decreased, the various kinds of phases may appear due to the strange quark mass and the electric neutrality conditions; 2SC, dSC, uSC, gluonic phase, meson condensed phases, gapless phases, FFLO etc. There might appear new topological objects in these phases [25, 26]. In particular, it has been shown in Ref. [25] that there appear K -strings, drum vorton and domain walls in one of meson condensed phases, the CFL + K^0 phase. Our work should be applied to the interaction of these strings.

NOTE: The non-Abelian strings discussed in a recent paper [27] are completely differ-

ent from our strings appearing in high density QCD. Their strings are local strings with gauged $U(1)_B$ and are essentially the same with [7, 8].

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