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TASI Lectures on the Cosmological Constant

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ABSTRACT: The energy density of the vacuum, Λ, is at least 60 orders of magnitude smaller than several known contributions to it. Approaches to this problem are tightly constrained by data ranging from elementary observations to precision experiments. Absent overwhelming evidence to the contrary, dark energy can only be interpreted as vacuum energy, so the venerable assumption that $\Lambda = 0$ conflicts with observation. The possibility remains that Λ is fundamentally variable, though constant over large spacetime regions. This can explain the observed value, but only in a theory satisfying a number of restrictive kinematic and dynamical conditions. String theory offers a concrete realization through its landscape of metastable vacua.

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1. Introduction: The cosmological constant problem

When Einstein wrote down the field equation for general relativity,

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{1.1}$$

he had a choice: The cosmological constant Λ was not fixed by the structure of the theory. There was no formal reason to set it to zero, and in fact, Einstein famously tuned it to yield a static cosmological solution—his "greatest blunder".

The universe has turned out not to be static, and Λ was henceforth assumed to vanish. This was never particularly satisfying even from a classical perspective. The situation is similar to a famous shortcoming of Newtonian gravity: Nothing prevents us from equating the gravitational charge with inertial mass, but nothing forces us to do so, either.

Any nonzero value of Λ introduces a length scale and time scale

$$r_\Lambda = ct_\Lambda = \sqrt{3/|\Lambda|} \tag{1.2}$$

into Einstein's theory. An independent, natural length scale arises from the constants of nature: the Planck length[1]

$$l_P = \sqrt{\frac{G\hbar}{c^3}} \approx 1.616 \times 10^{-33}\text{cm} . \tag{1.3}$$

Whether $|\Lambda|$ vanishes or not, it has long been known empirically that it is very small in Planck units (i.e., that r_Λ is large in these natural units). The cosmological constant strongly affects spacetime dynamics at all scales larger than r_Λ and t_Λ. But we see general relativity operate on scales much larger than the Planck length, without any sign of the cosmological constant. In fact, the smallness of Λ can be deduced just from the fact that the universe is large compared to the Planck length, and old compared to the Planck time.

First, consider the case of positive Λ. Assume, for the sake of argument, that no matter is present ($T_{\mu\nu} = 0$). Then the only isotropic solution to Einstein's equation is de Sitter space, which exhibits a cosmological horizon of radius r_Λ [1]. A cosmological horizon is the largest observable distance scale, and the presence of matter will only decrease the horizon radius [2]. We see scales that are large in Planck units ($r \gg 1$), and since r_Λ must be even larger, Eq. (1.2) implies that the cosmological constant is small.

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[1]Here G denotes Newton's constant and c is the speed of light. In this paper Planck units are used unless other units are given explicitly. For example, $t_P = l_P/c \approx .539 \times 10^{-43}$s and $M_P = 2.177 \times 10^{-5}$g.

Negative Λ causes the universe to recollapse independently of spatial curvature, on a timescale t_Λ [3]. The obvious fact that the universe is old compared to the Planck time then implies that $(-\Lambda)$ is small. Summarizing the above arguments, we find

$$-3t^{-2} \lesssim \Lambda \lesssim 3r^{-2} \ , \tag{1.4}$$

where t and r are any time scale and any distance scale that have been observed.

These conclusions did not require cutting-edge experiments: knowing only that the world is older than 5000 years and larger than Belgium would suffice to tell us that $|\Lambda| \ll 1$. For a tighter constraint, note that we can see out to distances of billions of light years, so $r > 10^{60}$; and stars are billions of years old, so $t > 10^{60}$. With these data, known for many decades, Eq. (1.4) implies roughly that

$$|\Lambda| \lesssim 3 \times 10^{-120} \ . \tag{1.5}$$

Hence Λ is very small indeed.

This result makes it tempting to set $\Lambda = 0$ in the Einstein equation and move on. But Λ returns through the back door. The quantum fluctuations in the vacuum of the standard model contribute to the expectation value of the stress tensor in a way that mimics a cosmological constant. It is this effect that turns the cosmological constant from a mere ambiguity into a genuine problem.

In quantum field theory, the vacuum is highly nontrivial. As a harmonic oscillator in the ground state, every mode of every field contributes a zero point energy to the energy density of the vacuum. In a path integral description, this energy arises from virtual particle-antiparticle pairs, or "loops" (Fig. 1a). By Lorentz invariance, the corresponding energy-momentum-stress tensor had better be proportional to the metric,

$$\langle T_{\mu\nu} \rangle = -\rho_{\text{vacuum}} g_{\mu\nu} \ , \tag{1.6}$$

which is confirmed by direct calculation.

Though it appears on the right hand side of Einstein's equation, vacuum energy has the form of a cosmological constant, and we might as well absorb it and redefine Λ via

$$\Lambda = \Lambda_{\text{Einstein}} + 8\pi\rho_{\text{vacuum}} \ . \tag{1.7}$$

Equivalently, we may absorb the "bare" cosmological constant appearing in Einstein's equation, $\Lambda_{\text{Einstein}}$, into the energy density of the vacuum, defining

$$\rho_\Lambda \equiv \rho_{\text{vacuum}} + \frac{\Lambda_{\text{Einstein}}}{8\pi} \ . \tag{1.8}$$

Eqs. (1.2), (1.4), and (1.5) apply to the total cosmological constant, and can be restated
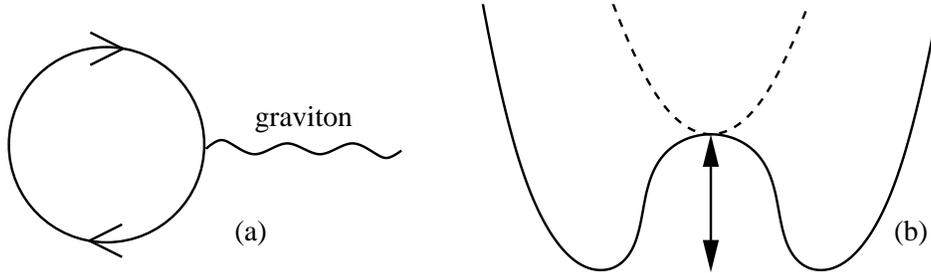
Figure 1: Some contributions to vacuum energy. (a) Virtual particle-antiparticle pairs (loops) gravitate. The vacuum of the standard model abounds with such pairs and hence should gravitate enormously. (b) Symmetry breaking in the early universe (e.g., of the chiral and electroweak symmetries) shifts the vacuum energy by amounts dozens of orders of magnitude larger than the observed value.

as an empirical bound on the total energy density of the vacuum:

$$|\rho_\Lambda| \lesssim 10^{-121} \ . \tag{1.9}$$

But in the standard model, the energy of the vacuum receives many contributions much larger than this bound. Their value depends on the energy scale up to which we trust the theory but is enormous even with a conservative cutoff.

For example, consider the electron, which is well understood at least up to energies of order $M = 100$ GeV [4]. Dimensional analysis implies that electron loops up to this cutoff contribute of order $(100 \text{ GeV})^4$ to the vacuum energy, or 10^{-68} in Planck units. Similar contributions are expected from other fields. The real cutoff is probably of order the supersymmetry breaking scale, giving at least $(1 \text{ TeV})^4 \approx 10^{-64}$. It may be as high as the Planck scale, which would yield $|\rho_\Lambda|$ of order unity. Thus, quantum field theory predicts $|\rho_\Lambda|$ to be some 60 to 120 orders of magnitude larger than the experimental bound, Eq. (1.5).

Additional contributions come from the potentials of scalar fields, such as the potential giving rise to symmetry breaking in the electroweak theory (Fig. 1b). The vacuum energy of the symmetric and the broken phase differ by approximately $(200 \text{ GeV})^4 \approx 10^{-67}$. Any other symmetry breaking mechanisms at higher or lower energy, such as chiral symmetry breaking of QCD with $(300 \text{ MeV})^4 \approx 10^{-79}$, will also contribute.

I have exhibited various known contributions to the vacuum energy. They are uncorrelated with one another and with the (unknown) bare cosmological constant appearing in Einstein's equation, $\Lambda_{\text{Einstein}}$. Each contribution is dozens of orders of magnitude larger than the empirical bound today, Eq. (1.5). In particular, the radiative

correction terms from quantum fields are expected to be at least of order 10^{-64}. They come with different signs, but it would seem overwhelmingly unlikely for such large terms to cancel to better than a part in 10^{120}, in the present era.

This is the cosmological constant problem: *Why is the vacuum energy today so small?*[2] It represents a serious crisis in physics: a discrepancy between theory and experiment, of 60 to 120 orders of magnitude, in a quantity as basic as the weight of empty space.

Outline In Sec. 2-4, I survey a number of general approaches to the cosmological constant problem, without going into detailed theoretical models.[3] Sections 5 and 6 discuss recent experimental and theoretical progress, and Sec. 7 closes with an outlook.

Many tempting ideas can be ruled out quite generally, because they conflict with well-tested physics. In Sec. 2, I discuss a number of examples. Their failure modes illustrate the difficulty of the problem and constitute useful litmus tests for new approaches. I emphasize the physical origin of basic obstructions, rather than the various technical symptoms through which they manifest themselves in specific models.

In Sec. 3, I discuss an example of a class of ideas that predict that $\rho_\Lambda = 0$ today. This strategy did seem viable once, but it never found a concrete theoretical realization, and it is now quite disfavored experimentally.

In Sec. 4, I discuss the idea that ρ_Λ is a dynamical variable that can take on different values in different, large parts of the universe. One can show that galaxies form only in regions where ρ_Λ is not very much larger than the present matter density. Moreover, it is reasonable to suppose that regions without galaxies do not contain any observers. This combination of arguments, first formulated by Weinberg in 1987, predicts that we should observe $\rho_\Lambda \sim \rho_{\mathrm{matter}}$. The approach makes a number of theoretical predictions as well, namely that its stringent kinematic and dynamical requirements could eventually be accommodated in a realistic theory.

In Sec. 5 I argue that cosmological observations since 1998 have discriminated powerfully between approaches to the cosmological constant problem. Weinberg's predic-

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[2]Recent observations have revealed the actual value of the cosmological constant: $\rho_\Lambda \approx 1.5 \times 10^{-123}$. This sharpens the issue, but it does not change the fact that a much larger value is predicted. I will focus first on the question of why ρ_Λ is not large, before considering the implications of its precise value (Sec. 5).

[3]I will not attempt a thorough survey of the literature. A classic review is Ref. [5]. Ref. [6] is comprehensive and includes the discovery of nonzero ρ_Λ. For a particularly clear recent discussion that includes the string landscape, see Ref. [4]. This article is based on lectures at the Theoretical Advanced Study Institute, at the University of Colorado, Boulder, in May/June of 2007, and is being published under the title "The Cosmological Constant" in a special issue on dark energy in the journal *General Relativity and Gravitation*; some material has appeared earlier in Ref. [7].

tion was borne out by the observation of phenomena such as the accelerated expansion and spatial flatness of the universe, which are incompatible with $\rho_\Lambda = 0$ and favor a value $\rho_\Lambda \approx 3.2\,\rho_{\text{matter}}$.

Theory, too, has come down on Weinberg's side. In Sec. 6, I describe a theory of the cosmological constant. It satisfies the theoretical requirements of his approach, and it arises naturally in string theory.

String theory was not invented for the purpose, and it is a rather rigid framework. It is all the more remarkable that some of its most characteristic features, such as extra dimensions and branes, have allowed it to address the cosmological constant problem. They give string theory the vacuum structure of a multidimensional potential land-scape. What we call "the" vacuum is but one of perhaps 10^{500} long-lived, metastable vacua, arising from the combinatorics of branes wrapping handles in compact, extra dimensions. Our "big bang" was actually the decay of a more energetic vacuum. And our own vacuum, too, will decay.

The landscape of string theory opens new fields of inquiry, a few of which I sketch in Sec. 7. Vacuum energy aside, field content, masses, and couplings are all expected to vary from vacuum to vacuum. This raises questions of predictivity. The existence of a multitude of metastable solutions is an essential feature of any theory describing our world, such as the standard model. Effective or statistical descriptions have proven very powerful, and we must learn to develop analogous methods for string theory. This is complicated by the fact that in a gravitational theory, false vacua grow faster than they decay, producing infinite volumes. A cosmological measure or cutoff is needed in order to compute relative probabilities of different vacua.

2. Some ideas, and why they don't work

A discrepancy by a factor of 10^{60} is impressive but, given no other information, might be shrugged off as just another hierarchy problem. However, the cosmological constant problem is far more severe. In order to appreciate its unique features and extraordinary difficulty, it is instructive to consider a few approaches that might come to mind, and to exhibit some of the obstructions they face. (My emphasis on the physical origin of these obstructions, rather than their technical manifestations, is inspired by a similar viewpoint compellingly advocated in Polchinski's review [4].)

In this and the following sections, I will only consider experimental constraints that have long been known. I will not use results from recent precision cosmological experiments, such as the 1998 discovery of accelerated expansion. Their impact will be considered in Sec. 5.

2.1 Quantum gravity

Fundamentally, the problem amounts to a clash between particle physics, which sources vacuum energy through quantum effects, and gravity, which responds to it classically. To describe this situation accurately, perhaps we need quantum gravity, which we do not understand well enough. So we cannot trust the above arguments.

But it is not so easy to sweep the cosmological constant problem under the rug of our ignorance. All matter is quantum mechanical. Yet, its large scale gravitational interactions are accurately described by feeding the expectation value of the stress tensor into classical general relativity. A quantum theory of gravity, like any other extension of our theoretical framework, may help with the cosmological constant problem—string theory certainly does—but it cannot do so merely by failing to reproduce semi-classical gravity in the appropriate limit.

Quantum gravity would be needed to describe loop momenta exceeding 10^{19} GeV, or curvature radii smaller than 10^{-33} cm. To exhibit the cosmological constant problem it is not necessary to appeal to such extreme regimes; it arises well inside the regime of validity of both gravity and quantum field theory. For example, by Eq. (1.2), electron loops with momenta up to just 1 MeV alone should curl up the universe to about a million kilometers—a small world, but well described by general relativity.

2.2 Infrared or ultraviolet modifications of gravity

This last example makes it clear that classical modifications of gravity are of no apparent use either. We can only modify the theory on scales where it has not been tested. But the above example falls into a regime where gravity is well constrained experimentally.

More generally, short-distance modifications are irrelevant since the smallness of the cosmological constant manifests itself through the large size of the universe. The universe is much larger than the smallest distance scale at which gravity has been tested (fractions of a millimeter). At intermediate scales, we can trust general relativity, and we know that it would have responded to the large vacuum energy predicted by the standard model. So we can be sure that the vacuum energy is really unnaturally small, or zero.

Long-distance (infrared) modifications are unhelpful because we know that the universe started out small, and the cosmological constant problem is the prediction that the horizon should have never become larger than Planck size generically, or at most 100 μm in some models. This would have been a true event horizon, so causality would have prevented larger scales from playing any dynamical role. In particular, a

modification of gravity on the present horizon scale would never have come into play.[4]

In summary, the shortest and longest distances are precisely the ones that play no essential role in the cosmological constant problem. We know that vacuum energy is unnaturally small ($\rho_\Lambda \ll 10^{-60}$), assuming only that general relativity is valid on at least one intermediate scale between 100 μm and 1 Gpc. This includes broad regimes where we know that general relativity is very accurate and experimental constraints prevent us from modifying it.

2.3 Violations of the equivalence principle and de-gravitating the vacuum

Perhaps general relativity can be modified selectively, so that only vacuum fluctuations do not couple to gravity? Empirically we know that virtual particles contribute to the inertial mass, for example through the Lamb shift. But perhaps the equivalence principle is violated, and they do not contribute to the gravitational mass?

In fact, free fall experiments show that virtual particles do gravitate in matter, satisfying the equivalence principle at least to 1 part in a million [4]. The only remaining possibility, then, is to arrange that they gravitate in matter, but not in the vacuum. Ref. [4] contains a careful discussion of the difficulties of this approach, which I will not repeat here.

2.4 Initial conditions

Another tempting rug under which to sweep the cosmological constant problem is the beginning of the universe. Singularity theorems suggest that classical spacetime had a beginning. If so, there should be a theory of initial conditions. Perhaps it determines that the universe must start out with zero vacuum energy?

In fact, this would be a disaster. As discussed earlier, the energy of the vacuum dropped sharply during various known phase transitions in the early universe. If the universe had started with $\rho_\Lambda = 0$, the vacuum energy would have decreased to -10^{-67} at the electroweak phase transition, leading to a big crunch. The universe would have ended when it was only 10^{-10} seconds old.

This argument could also be used against the idea that a dynamical mechanism attracted ρ_Λ to 0 in the early universe. But attractor mechanisms are already ruled out by more general arguments, which I will turn to next.

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[4]This problem affects any approach in which the present horizon size, or some other large scale, appears directly as input. This is inevitably circular, since the smallness of the cosmological constant is a necessary condition for the largeness of the universe. If we start by assuming its largeness, there is nothing left to explain.

2.5 Nongravitational dynamical attractor mechanisms

Perhaps there is a local dynamical mechanism that allowed the standard model to adjust coupling constants, masses, and/or effective potentials, until their various contributions to the vacuum energy cancelled out? But nongravitational physics depends only on energy differences, so the standard model cannot respond to the actual value of the cosmological constant it sources. This implies that $\rho_\Lambda = 0$ is not a special value from the particle physics point of view. In particular, it cannot be a dynamical attractor in a nongravitational theory.

2.6 Gravitational dynamical attractor mechanisms

Gravity can see the cosmological constant, and among vacuum solutions of Einstein's equation, Minkowski space ($\rho_\Lambda = 0$) certainly is special for having no curvature. Perhaps, then, a dynamical mechanism operated in the past, which attracted ρ_Λ to zero through gravitational interactions?

The catch with this idea is that the universe is not vacuous. Today's cosmological constant was dynamically irrelevant in the early universe. This is one of the greatest difficulties in solving the cosmological constant problem, and it is frequently overlooked. A mechanism that works only in an empty universe solves nobody's problem.

It is worth going over this point in more detail. Gravity couples to the stress tensor, and vacuum energy is only one of many contributions to the stress tensor. Today, the energy density of the vacuum is comparable to the average density of matter, and its pressure is 10^4 times greater than that of radiation. But while matter and radiation redshift under expansion, vacuum energy does not dilute. Until the recent past, therefore, a vacuum energy density of 10^{-120} would have constituted a negligible contribution to the stress tensor. Gravity was responding to much larger energies and pressures. The notion of a gravitational feedback mechanism adjusting the cosmological constant to precision 10^{-120} in the early universe can be roughly compared to an airplane following a prescribed flight path to atomic precision, in a storm.

Consider, for example, the era of nucleosynthesis, at a temperature of 1 MeV. The energy density of radiation was $\rho_{\mathrm{BBN}} = 1.6 \times 10^{-88}$ and its pressure was $p_{\mathrm{BBN}} = 0.53 \times 10^{-88}$. Hence, no dynamical mechanism could have adjusted the vacuum energy to precision better than $\delta\rho_\Lambda \approx 10^{-88}$ prior to nucleosynthesis. This exceeds the conservative upper bound of Eq. (1.5) by a factor of 10^{33}.

This problem can be restated in a geometric language. Spacetime geometry is the only physical entity that *can* be affected by vacuum energy, but it need not be, because other forms of energy might curve it more. At nucleosynthesis, the curvature scale was $H_{\mathrm{BBN}}^{-1} \approx 3 \times 10^{43}$. *Any* cosmological constant much smaller than $3H_{\mathrm{BBN}}^2/8\pi$ would have

left this geometry unaffected, so no imprint could have distinguished, say, $\rho_\Lambda = 10^{-90}$ from $\rho_\Lambda = 10^{-123}$ at that time, and nothing could have selected for the latter.

One may speculate that a dynamical mechanism was operating continuously, keeping the vacuum energy density comparable to the density of matter or radiation at all times. But this is impossible. The cosmological dilution of a perfect fluid is completely determined by the conservation of the stress tensor (and thus, in particular, by general relativity). Its density must redshift as $\rho = a^{-3(1+w)}$, where a is the scale factor and $p = w\rho$ is the pressure. Hence, vacuum energy, or anything behaving like it ($w \approx -1$) cannot continuously dilute in sync with matter ($w = 0$) or radiation ($w = 1/3$). For an analysis ruling out the continuous transfer of vacuum energy into matter or radiation, see Ref. [5].

2.7 Summary

I make no claim to have represented all approaches one might try, nor have I identified all of their problems. In order to focus on general constraints, I have granted various optimistic assumptions and ignored technical gaps in several of the ideas considered.

The obstructions listed here are powerful, but since some solution must exist, they cannot be insurmountable. It is best to think of them as litmus tests. A serious proposal should be able to state precisely how it satisfies each constraint. We have innumerable "solutions" that work in a world without matter and radiation, are spoiled by symmetry breaking in the early universe, or appeal exclusively to quantum gravity effects. They fail to confront the main difficulties of the problem.

3. An old idea that might have worked but didn't

The entire discussion so far could have been written in 1980. The cosmological constant problem—that ρ_Λ is much smaller than predicted—was well known then. What was not known is whether the cosmological constant is strictly zero or just very small, but either way there was a crisis. Undoubtedly, all of the above ideas were already considered then, and were dismissed, since their flaws are fatal quite independently of the precise value of ρ_Λ.[5]

In this section I will discuss a strategy that would have been viable in 1980 but has since become unattractive. In order to give the idea a fair hearing, let us ignore for a moment the 1998 discovery of a nonzero cosmological constant. We know only that $|\rho_\Lambda| \lesssim 10^{-121}$, and we would like to understand why.

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[5]Happily, this seems not to stand in the way of their continued, enthusiastic rediscovery.

3.1 ρ_Λ vanishes in the asymptotic future

Except in the case of attractor behavior, the state of a dynamical system cannot be determined from its equations of motion alone; one needs some knowledge of boundary conditions. But from Sec. 2 we already know that neither initial conditions, nor attractor behavior in the early universe, are likely to solve the problem.

However, final conditions might work. In the absence of a reliable theory of either initial or final conditions, it is legitimate to speculate that the boundary conditions on the universe are most naturally formulated in the asymptotic future, and that they dictate that $\rho_\Lambda = 0$ in the late time limit. (The final conditions might also set ρ_Λ to a small nonzero value, but this is less attractive, since it introduces an arbitary scale.[6])

What does this imply for the cosmological constant today? All known effective scalars are presently in their vacuum. One can hypothesize additional scalars that are not (such as quintessence), but this is quite difficult to implement without significant fine-tuning, and in any case constitutes a complication of the model. In any natural implementation of our idea, therefore, there will be no difference between the vacuum energy today and the vacuum energy in the infinite future, $\rho_\Lambda = 0$.

3.2 Predictions

Thus, the final-condition approach generically predicts that there is no vacuum energy in the present era. This prediction has been falsified. Experiments began to show around 1998 that the vacuum energy is positive and of order 10^{-123} (see Sec. 5).

The final-condition approach makes a second prediction, which should not be overlooked just because it is theoretical. I made two very strong assumptions:

1. The boundary conditions of the universe are set in the far future.

2. They require $\rho_\Lambda = 0$.

If the idea is right, these assumptions should eventually be vindicated by progress in our understanding of fundamental theory.

To date, at least, this has not happened. Another way to say this is to note that since the time when many theorists began worrying seriously about the cosmological constant problem (the 1980s at the latest), we have not succeeded in making this

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[6]A nonzero value, no matter how small, might even spoil the idea. Negative ρ_Λ leads to a big crunch with high energy densities and perhaps also the restoration of some broken symmetries. Setting $|\rho_\Lambda|$ to a small value at the big crunch thus faces one or more of the problems that frustrated attempts to fix it in the early universe. Positive ρ_Λ at $t = \infty$ implies an eternal de Sitter universe. Under plausible assumptions, our observation of a universe with a long semiclassical history is ruled out in such a cosmology [8]; see, however, Ref. [9].

idea any more precise, or in showing that it can actually be realized in a plausible context such as string theory. In summary, the experimental developments of recent decades strongly disfavor this approach, while theoretical developments have failed to corroborate it.

But in fact, the same can be said for other approaches that predicted that the energy of the vacuum should vanish in the present era. In several decades of theoretical work, we have not identified any concrete reason why it should; and since 1998 we know that it does not.

4. An old idea that could have been falsified but wasn't

I will now consider a different strategy, various versions of which were suggested throughout the 1980s. Its initial status was very similar to the previous strategy: It made an experimental prediction for ρ_Λ, and it optimistically anticipated that theoretical progress would eventually justify a number of implicit assumptions. Unlike the previous strategy, however, this one has since found support on both counts.

4.1 ρ_Λ is an environmental variable

The strategy [10–14] is to posit that the universe is much larger than its presently visible portion, and that ρ_Λ varies from place to place, though it can be constant over very large distances. According to Weinberg [14] (see also Refs. [15,16]), structure such as galaxies will only form in locations where

$$-10^{-123} \lesssim \rho_\Lambda \lesssim 3 \times 10^{-121} \ . \tag{4.1}$$

Assuming that structure is a prerequisite for the existence of observers, we should then not be surprised to find ourselves in such a region.

Why is ρ_Λ related to structure formation? To form galaxies and clusters, the tiny density perturbations visible in the cosmic microwave background radiation had to grow under their own gravity, until they became nonlinear and decoupled from the cosmological expansion. This growth is logarithmic during radiation domination, and linear in the scale factor during matter domination.

Vacuum energy does not get diluted, so it inevitably comes to dominate the dynamics of the universe, at a time of order $t_\Lambda \sim |\rho_\Lambda|^{-1/2}$. If $\rho_\Lambda > 0$, small perturbations will cease to grow at this time. The only structures that will remain are highly overdense regions that have already become gravitationally bound and decoupled from the cosmological expansion.

This means that there would be no structure in the universe if the cosmological constant had been large enough to dominate the energy density before the first galaxies

formed [14], tens or hundreds of millions of years after the big bang. Careful analysis then leads to the upper bound in Eq. (4.1).[7] The lower bound comes about because the universe would have already recollapsed into a big crunch if $(-\rho_\Lambda)$ had been larger than the matter density today [5, 16].

Note that Weinberg's point was not to improve the experimental upper bound on ρ_Λ but to explain its smallness. In fact, the limits on ρ_Λ from galaxy formation are more lenient than bounds derived from other data available at the time, such as constraints on the expansion rate and flatness of the universe. But those data were not in any obvious way required for the existence of observers, so they could not have been used in an anthropic argument.

4.2 Experimental Predictions

If it is true that all observers live in regions satisfying Eq. (4.1), then it is obvious why we do not observe a value of $|\rho_\Lambda|$ greater than 10^{-120}. But what *do* we expect to see?

We assumed that many different values of ρ_Λ are possible, but this does not mean they are all equally likely. However, Eq. (4.1) represents an exceedingly small interval compared with the natural scale of ρ_Λ (the latter being unity, or in any case not less than 10^{-60}). It is plausible that the likelihood of different values of ρ_Λ should not vary significantly over such a tiny interval. (You may think that a divergence at $\rho_\Lambda = 0$ is possible, since 0 looks like a special value, but I argued earlier that it is not.) Technically, this means that we should consider a "flat" prior probability for values in Eq. (4.1):

$$dp/d\rho_\Lambda \approx \text{ const.} \tag{4.2}$$

Then it would be surprising if we should find ourselves in a region with, say, $\rho_\Lambda = 10^{-150}$. This would not be a typical value; it would be much smaller than necessary for observers, an unlikely accident. It is far more likely that the local cosmological constant has a *typical* value in the range compatible with structure formation, which by Eqs. (4.1) and (4.2) is of order 10^{-121}.

Therefore, the "environmental" approach predicts [14] that the vacuum energy should be nonzero and not much smaller than 10^{-121}. In other words, its magnitude should be comparable to, or somewhat larger than, the present matter density, $\rho_{\text{matter}} \approx 4.6 \times 10^{-124}$. But this means that it should be detectable by careful experiments.

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[7]Since this argument was first proposed, dwarf galaxies have been discovered at higher redshift. This raises the upper bound on ρ_Λ obtained by Weinberg's argument, and it can make the observed value seem surprisingly small (though by a factor of 10^{-3}, still much better than 10^{-123}). This discrepancy may grow if parameters other than Λ can also to vary. Its magnitude, however, depends on the manner in which the divergent numbers of observers living in different parts of the universe are regulated and compared. The discrepancy is entirely absent in one natural proposal (see Sec. 7).

Weinberg's prediction was confirmed in 1998, when it was discovered that $\rho_\Lambda \approx 1.5 \times 10^{-123}$. I discuss this development further in Sec. 5.

4.3 Theoretical Predictions

The environmental strategy makes several highly nontrivial assumptions:

1. ρ_Λ is fundamentally not fixed but variable.

2. Its possible values are continuous, or are sufficiently closely spaced that Eq. (4.1) is satisfied by at least one of them.

3. (a) Either, boundary conditions ensure that ρ_Λ will satisfy Eq. (4.1) in the present epoch.

   (b) Or, starting from generic initial conditions, many other values are eventually realized in different spacetime regions by some dynamical mechanism. In particular, at least one value of ρ_Λ satisfying Eq. (4.1) can be dynamically attained somewhere.

4. Regions satisfying Eq. (4.1) can grow larger than the entire observable universe, and can survive for longer than 13 billion of years.

5. Such regions can contain matter and radiation.

Thus, the credibility of the environmental approach rested not only on its (successful) experimental prediction. It also depended on the ability of future theoretical developments to provide a justification for each of the assumptions involved.

These theoretical predictions are not trivial. It was far from clear whether they could be satisfied and explained by a concrete model, even at the level of effective field theory. To see how hard it is, it helps to consider one of the constructions that came closest (see Sec. 6.1).

String theory is particularly rigid in restricting the ingredients it allows us to work with. It might have seemed overwhelmingly unlikely, therefore, that string theory should fit snugly into the complicated mold of constraints described above—constraints that even less fundamental, more flexible frameworks seemed unable to conform to.

Yet, we now have strong evidence that string theory succeeds at this task. The solution, the gist of which I will describe in Sections 6.2 and 6.3, depends crucially on some of string theory's most characteristic, defining elements, such as the existence of extra dimensions and of higher-dimensional objects (branes). This development is perhaps no less remarkable than the experimental confirmation of Weinberg's prediction.

5. Why dark energy is a cosmological constant

We now know that the cosmological constant is not zero. This was discovered in 1998 by measuring the apparent luminosity of distant supernovae [17, 18]. Their dimness indicates that the expansion of the universe has recently begun to accelerate, consistent with a positive cosmological constant [19]

$$\rho_\Lambda = (1.48 \pm 0.11) \times 10^{-123} \; , \tag{5.1}$$

and inconsistent with $\rho_\Lambda = 0$. Cross-checks have corroborated this conclusion. For example, the above value of ρ_Λ also explains the observed spatial flatness of the universe, which cannot be accounted for by baryonic and dark matter alone. (See Ref. [19] for a recent summary of constraints from various experiments.)

What does this observation imply for the cosmological constant problem? It neither creates it nor solves it. But it sharpens it, and so discriminates powerfully between approaches to its solution. The fact that $0 \neq \rho_\Lambda \approx 3.2 \, \rho_{\text{matter}}$ disfavors theories that leads to vanishing ρ_Λ, and it favors any theory that predicts ρ_Λ to be comparable to the present matter density.

5.1 Calling it a duck

I have failed to describe the recent discovery in tones of wonder and stupefaction, as a "mysterious dark energy", a nonclustering fluid, with equation of state $w = p_{\text{DE}}/\rho_{\text{DE}}$ close to -1, which currently makes up 75% of the energy density of the universe. Why obfuscate? If a poet sees something that walks like a duck and swims like a duck and quacks like a duck, we will forgive him for entertaining more fanciful possibilities. It could be a unicorn in a duck suit—who's to say! But we know that more likely, it's a duck.

In science, it can be wrong to keep an open mind, and the expression "dark energy" is an example of misplaced political correctness. Dark energy is the cosmological constant until proven otherwise, for the same reason that the moon is not made of cheese until proven otherwise: It is by far the most economical interpretation of the data, even if it fails to sustain a fondly held preconception—in this case, the prejudice that $\rho_\Lambda = 0$.

Let me make this completely explicit. A conservative, well-tested framework, the standard model coupled to general relativity, has encountered a problem: Why is ρ_Λ much smaller than several known contributions to it? No proposal for its resolution can claim a solid footing.[8] If experiment, rather than theoretical bias, is to be our

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[8] I will argue in the next section that this is no longer entirely true. Still, none is on a footing comparable to the standard model or general relativity.

guide, then we must remain agnostic as to how the cosmological constant problem will be solved.

Thus, ρ_Λ remains an unexplained parameter that we must fit to the data until help arrives. Dark energy is experimentally indistinguishable from vacuum energy, but definitely distinct from any other known form of matter. It is reasonable, then, to consider dark energy to be vacuum energy, and to fit ρ_Λ to its observed density.

Thus, Eq. (5.1) is a straightforward, unbiased interpretation of dark energy. Without interposing theoretical speculations, we have allowed experiment to determine precisely *which problem* physics (properly defined to include only well-established and tested theories) actually faces: Why is ρ_Λ much smaller than many known contributions to it, and why is it comparable to the energy density of matter today?

Only now may we turn to the realm of speculation, and ask *how the problem might be resolved.* It behooves us to use experiment to evaluate our hypotheses, not the other way around. Among hypotheses that made specific predictions for the value of ρ_Λ, we may safely conclude that some (Sec. 4) are favored by the discovery of nonzero ρ_Λ, and others (Sec. 3) are disfavored.

5.2 Two problems for the price of one

In order to motivate a different interpretation of "dark energy", we would need to turn the scientific method on its head, and begin by decreeing that the cosmological constant problem will one day be resolved by a theory that sets $\rho_\Lambda = 0$ precisely. We would have to treat this claim not as a hypothesis among others, to be judged against empirical evidence, but as a dogma that constrains our interpretation of any observation.

As theoretical bets go, this one is daring: No concrete, viable theory predicting $\rho_\Lambda = 0$ was known by 1998, and none has been found since. By contrast, we do have a concrete implementation of Weinberg's approach, which predicts $\rho_\Lambda \sim \rho_{\text{matter}}$. But this will be the topic of Sec. 6.

Here, my argument is not about which bet we should make. We should not be betting at all. By looking at the world through the lens of just one hypothesis, we prevent experiment from discriminating between it and other hypotheses, depriving ourselves of the fruits of considerable labor.

To make matters worse, the assumption that $\rho_\Lambda = 0$ is not just pure speculation. It is pure speculation that turns one problem into two:

1. Why is $\rho_\Lambda = 0$?[9] (This is the old cosmological constant problem of Sec. 1, made more specific not by experiment but by our decree that $\rho_\Lambda = 0$.)

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[9]This problem is sometimes overlooked, as if $\rho_\Lambda = 0$ required no explanation [20]—a potentially expensive fallacy that greatly exaggerates the plausibility of time-dependent dark energy. It creates false expectations and may distort our assessment of important future experiments.

2. If, by our decree, dark energy is not vacuum energy, then it must be something else—but something that looks and acts, in every respect so far observed, just like vacuum energy, and unlike any other known substance. What could it be?

The two problems have very different origins. The cosmological constant problem, at least in its original form, is real, in that it arises within well-tested physics. The dark energy problem is an artifact produced by insisting on the untested hypothesis that $\rho_\Lambda = 0$.

On the upside, much progress is being made on the second problem (if we count fine-tuned scalar fields and *ad hoc* modifications of gravity[10]). Ironically, the very observations that have finally provided a clue about the cosmological constant problem—and which, for that reason alone, would rank among the great triumphs of experimental science—seem to have diverted our attention to the entirely fictitious problem of dark energy.

5.3 The real second problem

By sharpening the cosmological constant problem, the discovery of nonzero vacuum energy did create a new challenge, sometimes called the coincidence problem or why-now problem. Vacuum energy, or anything behaving like it (which includes all options still allowed by current data) does not redshift like matter. In the past, vacuum energy was negligible, and in the far future, matter will be very dilute and vacuum energy will dominate completely. The two can be comparable only in a particular epoch. It is intriguing that this is the same epoch in which we are making the observation.

Note that this apparent coincidence involves us, the observers, in an essential way. If it has any explanation, it will by definition have to be an anthropic explanation.

In fact, Weinberg's approach (Sec. 4) explained the coincidence before it was discovered. It predicts that vacuum energy will be just small enough to allow galaxies to form, which implies that in the epoch immediately following galaxy formation it will be comparable to the matter density. This explains the coincidence if one assumes not only that observers require galaxies, but also that typical observers form not too long after galaxies do. (Both assumptions seem to hold in our universe, but in the context of the string landscape their general validity could be debated. At the end of Sec. 7, I will discuss a natural measure on the multiverse that renders both assumptions unnecessary, explaining the coincidence more directly and more generally.)

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[10]Infrared modifications of gravity do not solve the cosmological constant problem (see Sec. 2), but given enough small parameters, they can mimic dark energy.

6. A theory of the cosmological constant

In this section I review a theory of the cosmological constant that realizes the general idea of Sec. 4. It satisfies all five constraints identified in Sec. 4.3, and it arises naturally in string theory.

I will begin in Sec. 6.1 by reviewing an older idea. The model does not succeed, but it presents a natural starting point for the discussion of a more powerful mechanism in Sec. 6.2. In Sec. 6.3, I argue that this construction arises naturally in string theory.

6.1 The Brown-Teitelboim mechanism

In this subsection, I will discuss an influential construction by Brown and Teitelboim [21, 22] that anticipated a number of features of the landscape of string theory. It will turn out to satisfy a subset of the conditions (1)-(5) of Sec. 4.3, bringing the remaining challenges into sharp profile.

Discretely adjustable vacuum energy from a four-form field Extending an idea of Abbott [23], Brown and Teitelboim [21, 22] introduced a four-form field to make the cosmological constant variable (condition 1). An analogy with electromagnetism helps clarify its role.

The Maxwell field, F_{ab}, is derived from a potential, $F_{ab} = \partial_a A_b - \partial_b A_a$. The potential is sourced by a point particle through a term $\int e\mathbf{A}$ in the action, where the integral is over the worldline of the particle, and e is the charge. Technically, $\mathbf{F}$ is a two-form (a totally antisymmetric tensor of rank 2), and $\mathbf{A}$ is a one-form coupling to a one-dimensional worldvolume (the worldline of the electron).

The field content of string theory and supergravity is completely determined by the structure of the theory. It includes a four-form field, F_{abcd}, which derives from a three-form potential:

$$F_{abcd} = \partial_{[a} A_{bcd]} \ , \tag{6.1}$$

where square brackets denote total antisymmetrization. This potential naturally couples to a two-dimensional object, a membrane, through a term $\int q\mathbf{A}$, where the integral runs over the 2+1 dimensional membrane worldvolume, and q is the membrane charge.

The properties of the four-form field in our 3+1 dimensional world mirror the behavior of Maxwell theory in a 1+1 dimensional system. Consider, for example, an electric field between two parallel capacitor plates of equal and opposite charge. If the plates are very large, then the field strength in the interior is constant both in space and time. Its magnitude depends on how many electrons the negative plate contains; thus it will be an integer multiple of the electron charge: $E = ne$.

The energy density will be one half of the field strength squared:

$$\rho = \frac{F_{ab}F^{ab}}{2} = \frac{n^2 e^2}{2} \tag{6.2}$$

In order to treat this as a system with only one spatial dimension, I have integrated over the directions transverse to the field lines, so ρ is energy per unit length. The pressure is equal to $-\rho$. The corresponding 1+1 dimensional stress tensor has the form of Eq. (1.6), so the electromagnetic stress tensor acts like vacuum energy in 1+1 dimensions.

The same is true for the four-form in our 3+1 dimensional world. First of all, the equation of motion in the absence of sources is $\partial_a(\sqrt{-g}F^{abcd}) = 0$, with solution [24,25]

$$F^{abcd} = c\epsilon^{abcd} , \tag{6.3}$$

where ϵ is the unit totally anti-symmetric tensor and c is an arbitrary constant. In string theory, there are "magnetic" charges (technically, five-branes) dual to the "electric charges" (the membranes) sourcing the four-form field. Then, by an analogue of Dirac quantization of the electric charge, one can show that c is quantized in integer multiples of the membrane charge, q:

$$c = nq . \tag{6.4}$$

Note that the actual value of the four-form field is thus quantized, not only the difference between possible values [26].

The four-form field strength squares to $F_{abcd}F^{abcd} = 24c^2$, and the stress tensor is proportional to the metric, with

$$\rho = \frac{1}{2 \times 4!}F_{abcd}F^{abcd} = \frac{n^2 q^2}{2} \tag{6.5}$$

In summary, the four-form field is nondynamical, and it contributes $n^2 q^2/2$ to the vacuum energy. It is thus indistinguishable from a contribution to the cosmological constant.

Dynamics Classically, the field configurations we studied have no dynamics, but quantum mechanically, they are unstable to nonperturbative tunneling effects. This is readily apparent in the electromagnetic, 1+1 dimensional analogy. The electric field between the plates will be slowly discharged by Schwinger pair creation of field sources. This is a process by which a electron and a positron tunnel out of the vacuum. Since field lines from the plates can now end on these particles, the electric field between the two particles will be lower by one unit $[ne \rightarrow (n-1)e]$. The particles will appear precisely at such a separation that the corresponding decrease in field energy compensates for their

combined rest mass. They are then subjected to constant acceleration by the electric field until they hit the plates. If the plates are far away, they will move practically at the speed of light by that time.

For weak fields, this tunneling process is exponentially suppressed, with a rate of order $\exp(-\pi m^2/ne^2)$, where the exponent arises as the action of a Euclidean-time solution describing the appearance of the particles. Thus, a long time passes between creation events. However, over large enough time scales, the electric field will decrease by discrete steps of size e. Correspondingly, the 1+1 dimensional "vacuum energy", i.e., the energy per unit length in the electric field, will gradually decrease by discrete amounts $[n^2e^2 - (n-1)^2e^2]/2 = (n - \frac{1}{2})e^2$. Note that this step size depends both on the electric charge, e, and on the remaining flux, n. The cascade of decays will only terminate once the electric field has been depleted to the point where not enough energy is left for the creation of another electron-positron pair.

Precisely analogous nonperturbative effects occur for the four-form field in 3+1 dimensions. By an analogue of the Schwinger process, spherical membranes can spontaneously appear. (This is the correct analogue: the two particles above form a zero-sphere, i.e., two points; the membrane forms a two-sphere.) Inside this source, the four-form field strength will be lower by one unit of the membrane charge $[nq \to (n-1)q]$. The process conserves energy: the initial membrane size is such that the membrane mass is balanced against the decreased energy of the four-form field inside the membrane. The membrane quickly grows to convert more space to the lower energy density, accelerating outward and expanding asymptotically at the speed of light.

Membrane creation is a well-understood process described by a Euclidean instanton, and like Schwinger pair creation, is generically exponentially slow. Ultimately, however, it will lead to the step-by-step decay of the four-form field. Inside a new membrane, the vacuum energy will be lower by[11]

$$\delta\Lambda = (n - \frac{1}{2})q^2 \ . \tag{6.6}$$

This suggests a mechanism for cancelling off the cosmological constant. Let us collect all contributions (see Sec. 1), except for the four-form field, in a "bare" cosmological constant λ. Generically, $|\lambda|$ should be of order unity (at least in the absence of supersymmetry), and we will assume without excessive loss of generality that it is negative. With n units of four-form flux turned on, the full cosmological constant will

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[11]Here and below, I am using reduced Planck units, obtained by replacing G with $8\pi G$ in the definition of all Planck units. This avoids annoying factors of 8π (at the expense of consistency with earlier sections). In the new units, $\Lambda = \rho_\Lambda$; Weinberg's upper bound in Eq. (4.1) is 2×10^{-118}; and the observed value corresponding to Eq. (5.1) is 0.9×10^{-120}.

be given by

$$\Lambda = \lambda + \frac{1}{2}n^2 q^2 \tag{6.7}$$

If n starts out large, the cosmological constant will decay by repeated membrane creation, until it is close to zero.

Limitations However, we must verify whether Λ gets close *enough* to zero, i.e., whether condition (2) can be satisfied. The smallest value of $|\Lambda|$ is attained for the flux n_{best}, given by the nearest integer to $\sqrt{2|\lambda|}/q$. The step size near $\Lambda = 0$ is thus given by $(n_{\text{best}} - \frac{1}{2})q^2$. For the Brown-Teitelboim mechanism to produce a value in the Weinberg window, Eq. (4.1), this step size would need to be of order 10^{-118} or smaller. This requires an extremely small membrane charge,

$$q \lesssim 10^{-118}|\lambda|^{-1/2} \ . \tag{6.8}$$

A natural bare cosmological constant λ will be no smaller than of order 10^{-64}, so $q \lesssim 10^{-86}$.

Such a small membrane charge q is unnatural. In particular, it is not known how to realize a sufficiently small charge in string theory. Thus, it is not clear how condition (2) is to be satisfied. This is the *step size problem*.

This still looks like progress: Naively, it would seem that we have succeeded in reducing the cosmological constant problem to a hierarchy problem. All we need is to introduce a small coupling and stabilize it against corrections. At least in principle, this is something we know how to do, and perhaps the details can be worked out later. In fact, however, this example illustrates just how much worse the cosmological constant problem is. To see this, let us assume the small-charge problem solved and take Eq. (6.8) to be satisfied.

In this hypothetical model, the Brown-Teitelboim adjustment mechanism would indeed produce a small cosmological constant, but only in regions containing no matter and radiation. This is not surprising. Rather general arguments in Sec. 2 showed that a primordial dynamical adjustment mechanism for Λ can only be as accurate as the energy density in the early universe, which was much larger than 10^{-118}. Now we encounter the other side of the same coin: the *empty universe problem*. We have found a mechanism which cancels Λ to high precision, but at the price of removing all matter as well.[12]

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[12]Steinhardt and Turok [27] consider an extension of another small-step model [23], aiming to overcome its empty universe problem. Their mechanism requires the universe to pass exponentially many times through an apparent big crunch/big bang transition, and to do so with negligible integrated probability of disturbing the small-step field governing the cosmological constant. (These are strong

In detail the problem shows up as follows.[13] Because membrane nucleation is a slow tunneling process, small values of Λ are approached very gradually from above. While the universe waits for the next membrane to nucleate, it is dominated by positive vacuum energy. Hence, it expands at an exponential rate, rapidly diluting the energy density of any matter or radiation that might have been around. By the time a membrane appears and the flux is reduced by one unit, the universe is completely empty. This is true at every step, and so it will be true in particular when the vacuum energy enters the Weinberg window.

Some energy is liberated when a membrane appears, because the vacuum energy drops by $\delta\Lambda$. Most of this energy goes into accelerating the membrane as it expands outward. (This is the famous graceful-exit problem of old inflation.) But let us be overly optimistic and assume that instead all the energy goes into the production of new matter and radiation. Unfortunately, we were forced earlier to assume that the step size is very small: each membrane nucleation decreases the vacuum energy by $\delta\Lambda \sim 10^{-118}$ or less, or else our downward cascade would miss the Weinberg window. With $\rho \sim T^4$, this freed-up energy could at best reheat the universe to a temperature $T \sim 10^{-30}$, or about 10^{-2}eV. This falls well short of the 10 MeV necessary to make

assumptions whose validity remains to be demonstrated.) With the additional assumption that the spacing of values of ρ_Λ is of order 10^{-123} (the unsolved step size problem), this model would allow for the production of hot regions with small cosmological constant. All other vacua, with larger cosmological constant, will also be produced in different spacetime regions; and every region, in most of its four-volume, is empty. Thus, anthropic localization in spacetime is necessary in any case to explain the observed ρ_Λ. However, small-step single-field models like Refs. [21, 23, 27] differ from a large-step, multi-field model like the string landscape in that nearly every worldline will experience *all* positive values of the cosmological constant, including the smallest one. Steinhardt and Turok argue that this is a desirable feature: "All other things being equal, a theory that predicts that life can exist almost everywhere is overwhelmingly preferred by Bayesian analysis (or common sense) over a theory that predicts it can exist almost nowhere." At present, other things are far from equal, but I would not accept this particular criterion even as a tie-breaker. It would suggest that a theory that predicts a universe densely packed with suns and earths is preferable to one that predicts large voids, where life cannot exist. Put differently, we have already observed that most patches of space do not harbor life, so it seems questionable to demand that a good theory predict the opposite. I find it more reasonable to judge a theory by whether it predicts correctly what observers observe, from economical and compelling assumptions. Both in the observable universe, and in the multiverse, the dominance of empty regions is a dynamical consequence of a simple theory. The fact that we do not live in such a region is not considered a problem, since it is an obvious consequence of the absence of matter.

[13]The empty universe haunts many other attempts to solve the cosmological constant problem, such as Ref. [28] and (I would argue in spite of Ref. [29]) even Coleman's famous wormhole approach [30], which suffers in any case from technical problems. In these cases it takes on a different guise: the amplitude of the wavefunction of the universe diverges for empty universes with vanishing cosmological constant. Anthropic arguments will not help, because the probability for a universe containing *only* observers vastly exceeds the probability for the universe we see.

contact with standard cosmology, a theory we trust at least back to nucleosynthesis.

In summary, the Brown-Teitelboim mechanism, with one four-form field, fails to satisfy conditions (2) and (5) of Sec. 4.3. The exceedingly small step size required for a sufficiently dense spectrum (2) cannot be attained in a natural model, and in any case the associated slow descent would ensure that regions with small cosmological constant are devoid of matter and radiation (5).

6.2 The discretuum

The above problems can be overcome by considering a theory with more than one species of four-form field [26]. In Sec. 6.3, I will explain why this situation arises naturally in string theory. First let us see how multiple four-form fields can produce a dense "discretuum" without requiring small charges, and how this solves the empty universe problem.

Multiple four-form fields   Consider a theory with J four-form fields. Correspondingly there will be J membrane species, with charges $q_1, \ldots, q_J$. Above, I analyzed the case of a single four-form field; essentially the conclusions still apply to each field separately. In particular, each field strength separately will be constant in 3+1 dimensions,

$$F_{(i)}^{abcd} = n_i q_i \epsilon^{abcd} \ , \tag{6.9}$$

and it will contribute a discrete amount of vacuum energy to the stress tensor.

Let us again collect all contributions to vacuum energy, except for those from the J four-form fields, in a bare cosmological constant λ, which I assume to be negative but otherwise generic (i.e., of order unity). Then the total cosmological constant will be given by

$$\Lambda = \lambda + \frac{1}{2} \sum_{i=1}^{J} n_i^2 q_i^2 \ . \tag{6.10}$$

This will include a value in the Weinberg window, Eq. (4.1), if there exists a set of integers n_i such that

$$2|\lambda| < \sum n_i^2 q_i^2 < 2(|\lambda| + \Delta\Lambda) \ , \tag{6.11}$$

where $\Delta\Lambda \approx 10^{-118}$.

A nice way to visualize this problem is to consider a J-dimensional grid, with axes corresponding to the field strengths $n_i q_i$, as shown in Fig. 2.   Every possible configuration of the four-form fields corresponds to a list of integers n_i, and thus to a discrete grid point. The Weinberg window can be represented as a thin shell of radius $\sqrt{2|\lambda|}$ and width $\Delta\Lambda/\sqrt{2|\lambda|}$. The shell has volume

$$V_{\text{shell}} = \Omega_{J-1}(\sqrt{2|\lambda|})^{J-1} \frac{\Delta\Lambda}{\sqrt{2|\lambda|}} = \Omega_{J-1}|2\lambda|^{\frac{J}{2}-1}\Delta\Lambda \ , \tag{6.12}$$
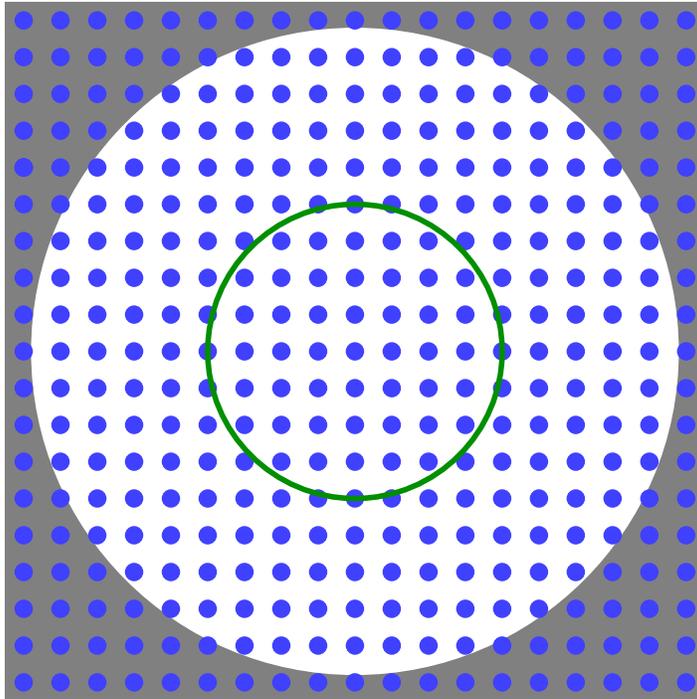
Figure 2: Possible configurations of the four-form fluxes correspond to discrete points in a J-dimensional grid, of which a two-dimensional section is shown. The grid spacing in the i-th direction is the charge q_i of the corresponding membrane species. The Weinberg window corresponds to the thin (green) shell. Inside the shell, $\Lambda < 0$; outside $\Lambda > 0$. The physically relevant regime ($-1 \lesssim \Lambda \lesssim 1$) is shown on white background.

where $\Omega_{J-1} = 2\pi^{J/2}/\Gamma(J/2)$ is the area of a unit $J-1$ dimensional sphere. The volume of a grid cell is

$$V_{\text{cell}} = \prod_{i=1}^{J} q_i \ . \tag{6.13}$$

There will be at least one value of Λ in the Weinberg window, if $V_{\text{cell}} < V_{\text{shell}}$, i.e., if

$$\frac{\prod_{i=1}^{J} q_i}{\Omega_{J-1}|2\lambda|^{\frac{J}{2}-1}} < |\Delta\Lambda| \ . \tag{6.14}$$

The most important consequence of this formula is that charges no longer need to be very small. I will shortly argue that in string theory one naturally expects J to be in the hundreds. With $J = 100$, for example, Eq. (6.14) [and thus, condition (2) of Sec. 4.3] can be satisfied with charges q_i of order $10^{-1.6}$, or $\sqrt{q_i} \approx 1/6$ (the latter has mass dimension 1 and so seems an appropriate variable for the judging naturalness of

this scenario). Interestingly, the large expected value of the bare cosmological constant is actually welcome: it becomes more difficult to satisfy Eq. (6.14) if $|\lambda| \ll 1$.

As it turns out, the largeness of the charges will also allow condition (5) to be satisfied in a model with multiple four-form fields: regions with small cosmological constant can contain matter and radiation.

Dynamics Classically, every flux configuration $(n_1, \ldots, n_J)$ is stable. But quantum-mechanically, fluxes can change if a membrane is spontaneously created. As discussed in Sec. 6.1, this Schwinger-like process is generically exponentially suppressed. Thus, multiple four-forms naturally give a dense discretuum of metastable vacua which can have extremely long life-times.

Starting from generic initial conditions, the universe will grow arbitrarily large. Over time, it will come to contain enormous regions ("bubbles" or "pockets") corresponding to each metastable vacuum (Fig. 3). In particular, our vacuum will be realized somewhere in this "multiverse". Moreover, it can be efficiently reheated, so the empty-universe problem of Sec. 6.1 will not arise. Let us see how this works in more detail.
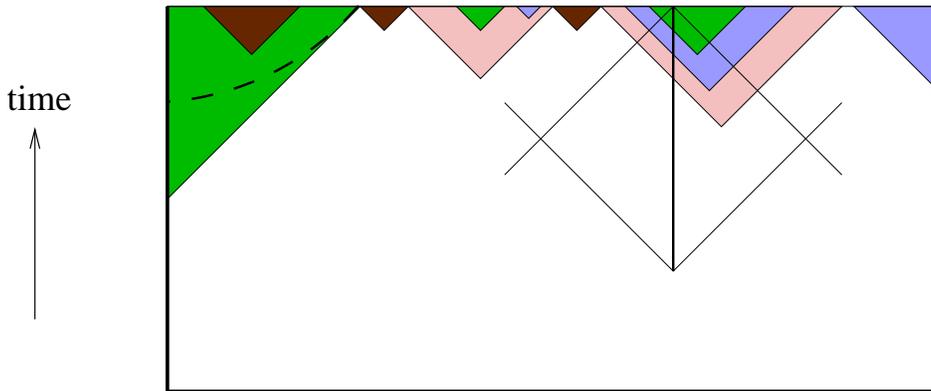
Figure 3: Bird's eye view of the universe. The triangles are pocket universes corresponding to different vacua in the landscape. Each pocket is an infinite, spatially open universe; the dashed line shows an example of an instant of time as picked out by constant density hypersurfaces in the pocket.—This is a conformal diagram. The actual amount of physical time and volume near the top boundary is infinite, and the top boundary is a fractal containing an infinite number of pocket universes. The black diamond is an example of a spacetime region that is causally accessible to a single worldline (see Sec. 7).

By Eq. (6.10), all but a finite number of metastable vacua will have $\Lambda > 0$. Let us assume that the universe begins in one of these vacua. Of course, this means that typically the cosmological constant will be large initially. Since $\Lambda > 0$, the universe will

be well described by de Sitter space. It can be thought of as a homogeneous, isotropic universe expanding exponentially on a characteristic time scale $t_\Lambda \sim \Lambda^{-1/2}$.

Every once in a long while (this time scale being set by the action of a membrane instanton, and thus typically much larger than t_Λ), a membrane will spontaneously appear and the cosmological constant will jump by $(n_i - \frac{1}{2})q_i^2$. But this does not affect the whole universe. Λ will have changed only inside the membrane bubble. This region grows arbitrarily large as the membrane expands at the speed of light.

But crucially, this does *not* imply that the whole universe is converted into the new vacuum [31]. This technical result can be understood intuitively. The ambient, old vacuum is still, in a sense, expanding exponentially fast. The new bubble eats up the old vacuum as fast as possible, at nearly the speed of light. But this is not fast enough to compete with the background expansion.

More and more membranes, of up to J different types, will nucleate in different places in the rapidly expanding old vacuum. Yet, there will always be some of the old vacuum left. One can show that the bubbles do not "percolate", i.e., they will never eat up all of space [32]. Thus different fluxes can change, and different directions in the J-dimensional flux space are explored.

Inside the new bubbles, the game continues. As long as Λ is still positive, there is room for everyone, because the background expands exponentially fast. In this way, all the points in the flux grid $(n_1, \ldots, n_J)$, are realized as actual regions in physical space. The cascade comes to an end wherever a bubble is formed with $\Lambda < 0$, but this affects only the interior of that particular bubble (it will undergo a big crunch). Globally, the cascade continues endlessly (Fig. 3).

Perhaps surprisingly, each bubble interior is an open FRW universe in its own right, and thus infinite in spatial extent.[14] Yet, each bubble is embedded in a bigger universe (sometimes called "multiverse" or "megaverse"), which is extremely inhomogeneous on the largest scales.

An important difference to the model with only one four-form is that the vacua will not be populated in the order of their vacuum energy. Because charges are large, two neighboring points in flux space will differ enormously in cosmological constant. That is, they differ by one unit of flux, and the charges q_i are not much smaller than one, so by Eq. (6.10) this translates into an enormous difference in cosmological constant, of order $\delta\Lambda \sim q^2$ or more. Conversely, vacua with very similar values of the cosmological

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[14]In an open universe, spatial hypersurfaces of constant energy density are three-dimensional hyperboloids. This shape is dictated by the symmetries of the instanton describing the membrane nucleation. It is closely related to the hyperbolic shape of the spacetime paths of accelerating particles, like the electron-positron pair studied above.

constant will be well separated in the flux grid, and will not directly decay into one another.

This feature is crucial for solving the empty universe problem. When our vacuum was produced in the interior of a new membrane, the cosmological constant may have decreased by as much as 1/100 of the Planck density. Hence, the temperature before the jump was enormous (in this example, the Gibbons-Hawking temperature [2] of the corresponding de Sitter universe would have been of order 1/10 of the Planck temperature), and only extremely massive fields will have relaxed to their minima. Most fields will be thermally distributed and can only begin to approach equilibrium after the jump decreases the vacuum energy to near zero.

Thus, the final jump takes on a role analogous to the big bang in standard cosmology. The "universe" (really, just our particular bubble) starts out hot and dense. If the effective theory in the bubble contains scalar fields with suitable potentials, there will be a period of slow-roll inflation as their vacuum energy slowly relaxes. (This was apparently the case in our vacuum.) At the end of this slow-roll inflation process, the universe reheats.

6.3 String theory

It seems *ad hoc* to posit the existence of hundreds of species of membranes, though perhaps a small price to pay for solving the cosmological constant problem. In fact, however, they arise inevitably when string theory is applied to our four-dimensional world [26].

Membrane species and extra dimensions   The origin of the large number of four-form fields lies in the topological complexity of small extra dimensions. String theory is formulated in 9+1 or 10+1 spacetime dimensions. For definiteness, let us work with the latter formulation (also known as M-theory). If it describes our world, then 7 of the spatial dimensions must be compactified on a scale that would have eluded our most careful experiments. Thus one can write the spacetime manifold as a direct product:

$$M = M_{3+1} \times X_7 \ . \tag{6.15}$$

Typically, the compact seven-dimensional manifold X_7 will have considerable topological complexity, in the sense of having large numbers of noncontractable cycles of various dimensions.

To see what this will mean for the 3+1 dimensional description, consider a string wrapped around a one-cycle (a "handle") in the extra dimensions. To a macroscopic observer this will appear as a point particle, since the handle cannot be resolved. Now, recall that M-theory contains five-branes, the magnetic charges dual to membranes.

Like strings on a handle, five-branes can wrap higher-dimensional cycles within the compact extra dimensions. A five-brane wrapping a three-cycle (a kind of noncontractible three-sphere embedded in the compact manifold) will appear as a two-brane, i.e., a membrane, to the macroscopic observer.

Six-dimensional manifolds, such as Calabi-Yau geometries, generically have hundreds of different three-cycles, and adding another dimension will only increase this number. The five-brane—one of a small number of fundamental objects of the theory—can wrap any of these cycles, giving rise to hundreds of apparently different membrane species in 3+1 dimensions, and thus, to $J \sim O(100)$ four-form fields, as required.

The charge q_i is determined by the five-brane charge (which is set by the theory to be of order unity), the volume of X_7, and the volume of the i-th three-cycle. The latter factors can lead to charges that are slightly smaller than 1, which is all that is required. Note also that the volumes of the three-cycles will generically differ from each other, so one would expect the q_i to be mutually incommensurate. This is important to avoid degeneracies in Eq. (6.10).

Vacuum stabilization The model I have presented is an oversimplification. When it was first proposed, it was not yet understood how to stabilize the compact manifold against deformations (technically, how to give a mass to all moduli fields including the dilaton). This is clearly necessary in any case if string theory is to describe our world, since we do not observe massless scalars. But one would expect that in a realistic compactification, the fluxes wrapped on cycles should deform the compact manifold, much like a rubber band wrapping a doughnut-shaped balloon. Yet, I have pretended that X_7 stays exactly the same independently of the fluxes n_i.

Therefore, Eq. (6.10) will not be correct in a realistic model. The charges q_i, and indeed the bare cosmological constant $|\lambda|$, will themselves depend on the integers n_i. Thus the cosmological constant may vary quite unpredictably. But the crucial point remains unchanged: the number of metastable vacua, N, can be extremely large, and the discretuum should have a typical spacing $\Delta\Lambda \approx 1/N$. For example, if there are 500 three-cycles and each can support up to 9 units of flux, there will be of order $N = 10^{500}$ metastable configurations. If their vacuum energy is effectively a random variable with at most the Planck value ($|\Lambda| \lesssim 1$), then there will be 10^{380} vacua in the Weinberg window, Eq. (4.1).

In the meantime, there has been significant progress with stabilizing the compact geometry (e.g., Refs. [33, 34]; see Refs. [35–37] for reviews.). In particular, Kachru, Kallosh, Linde, and Trivedi [38] have shown that metastable de Sitter vacua can be realized in string theory while fixing all moduli. (Constructions in noncritical string theory were proposed earlier [39, 40].) These results confirmed the above argument

that the number of flux vacua can be large enough to solve the cosmological constant problem. More sophisticated counting methods [41] bear out the quantitative estimates obtained from the simple model I have presented.

7. Outlook: The landscape of string theory

The developments described in the previous section have changed the status of the cosmological constant problem: we have, at last, a concrete candidate for a solution. Perhaps it is not the right solution, but its existence makes it less acceptable to ignore the problem, to fine-tune it away, or to indulge approaches that demonstrably conflict with experiment.

They have also changed the status of string theory: the theory has made contact with experiment—not merely in the sense of including a smaller theory such as the standard model (which, arguably, can be constructed from a suitable compactification), but in more exhilarating ways: by being the first theory to explain a mysterious observation that has long haunted us, and by doing so through means completely its own. Branes, fluxes, and extra dimensions are inevitable in string theory. They have turned out to be just what was needed to get the job done.

And they have changed our thinking about how string theory will make other predictions. There are 10^{500} or more metastable vacua, which can be thought of as local minima in a huge, multidimensional potential landscape. They differ not only in the value of their vacuum energy, but in their entire low-energy effective field theory, which is determined by local properties near the foot of a valley and thus only very indirectly by the fundamental building blocks of the landscape. Different vacua will have different matter content, coupling constants, and forces. We will not predict the standard model uniquely. We will have to predict many of the features of our universe statistically, from their relative abundance in the landscape.

But it would be wrong to say that there are now 10^{500} "string theories", suggesting a loss of fundamental simplicity and uniqueness. This is like saying that there are myriads of standard models. The standard model, like string theory, contains countless metastable solutions, such as atoms, molecules, and condensed matter at zero temperature. They are all constructed from just five different particle species (electrons, photons, and quarks). Strictly, the number of possibilities is infinite; even with an energy or volume cutoff, it quickly exceeds 10^{500}. This is not usually considered a problem for the standard model.

Rather, a multitude of solutions is an essential feature of any theory capable of explaining the multitude of complex phenomena that make up the messy, real world. It does not mean that anything goes. There are only a finite number of elements, and

a random combination of atoms is unlikely to form a stable molecule. Even quantities such as material properties ultimately derive from standard model parameters and cannot be arbitrarily dialed.

The complexity of an object need not be an obstacle to its effective description. Suppose we set out to derive the properties of metals from the standard model. Looking at the size of the system, the problem looks daunting, but we know very well that it yields to the laws of large numbers. Moreover, the predictive power of statistical or effective theories is completely deterministic in practice.

The large number of vacua in string theory arises in a very similar way, by combining a small set of fundamental ingredients in different ways. We cannot see the fluxes wrapped on handles in the extra dimensions, but not long ago the same could be said about atoms, not to speak of quarks. We do not expect every aspect of a theory to be testable, we just need to convince ourselves that it gives us more than we put in.

So how do we get predictions from string theory? Getting the cosmological constant right is nice, but to confirm the landscape, we need more. We cannot create other vacua in the laboratory [42]. For now, all we can do is measure the properties of our own, and do what we always do when we compare experiment to theory: see if our observations are likely (i.e., typical [43]), or unlikely, according to the theory.

This means making statistical predictions. Since we cannot repeat experiments in cosmology, the most interesting predictions will be those that can be made with probability extremely close to 1, similar to those in thermodynamics. The problem can be divided into three tasks, each of which are intensively studied at present.

Landscape statistics   What is the relative abundance of stable or long-lived meta-stable vacua with specified low-energy properties? This is the most obvious question to ask in pursuit of statistically dominant features (see, e.g., Refs. [41, 44–46], or Ref. [47] for a review). Our understanding of metastable vacua is still rather qualitative, so many investigations focus on supersymmetric vacua instead, which are under better control. It will be important to develop more powerful techniques for dealing with broken supersymmetry; meanwhile, it would help to understand the extent to which current samples are representative of more realistic vacua.

In particular, it can be useful to proceed by elimination. One would expect that most Lagrangians that make sense to a low-energy effective field theorist cannot arise from the limited set of ingredients supplied by string theory, no matter how elaborately they are combined. This vast "swampland" is not encompassed by the string theory landscape. The challenge is to identify specific predictions that arise from such limitations [48–50].

Dynamical selection effects   Another major challenge is posed by eternal inflation, the cosmological dynamics that produces different vacua in large, widely separated regions of the universe. One would expect that this mechanism favors some vacua over others, and thus enters into statistical predictions.

Divergences in the global structure of the universe make this effect very difficult to quantify. As seen in Fig. 3, each vacuum is realized infinitely many times as a bubble embedded in the global spacetime. Moreover, every bubble is an open universe and thus of infinite spatial extent.

It would seem natural to regulate these infinities by considering the universe at finite time before taking a limit. However, there is an ambiguity as to whether one should compare the volumes, or simply the number of each type of bubble on this time slice (or some intermediate quantity). Either way, results depend strongly on the choice of time variable [51, 52], which is rendered ambiguous by the inhomogeneity of the global spacetime. This is known as the measure problem of eternal inflation.

A small number of relatively simple proposals arise by generating a time variable from a geodesic congruence [51–57], though the fundamental significance of this procedure is not clear. A more radical proposal [58], motivated by black hole complementarity, restricts attention to a spacetime region causally accessible to a single worldline (a "causal diamond"—see Fig. 3). In this case, the global distribution of vacua (which is unobservable in any case [59]) need not be computed and regulated. Instead, one computes the relative probability that the worldline will enter a given vacuum. This is unambiguous and finite.

It is not yet clear how to derive the correct measure from first principles (see Ref. [60] for an interesting approach). But considerable progress has been made by the more pedestrian method of elimination. The number of simple candidate measures is not large, and many make wrong predictions, which go by colorful names such as Q-catastrophe, youngness paradox, Boltzmann brain paradox, and staggering problem [56, 61–69]. But they just come down to an old-fashioned (and usually violent) conflict with observation. This has already forced us to abandon or modify (and complicate) some of the simplest approaches.

Anthropic selection effects   Vacua without observers will not contribute to the statistical ensemble that determines what observations are likely. For example, most vacua in the landscape have a cosmological constant of order unity. They will be about one Planck length in size and contain at most a handful of quantum states [70]. Even without strong assumptions about what observers look like, we can be quite sure that these vacua will not be observed.

Quantifying this selection effect is a challenging task, for two reasons. First, it

inevitably leads to the problematic question of what constitutes an observer. Supposing we can agree on some criterion, we can then ask what a typical observer sees. This involves tallying observers, or observations, across the whole universe. Each vacuum bubble is an infinite open universe in itself, so it contains either zero observers or infinitely many. To define the relative abundance of different observations, a cutoff is required. This entangles us, once more, in the measure problem discussed earlier.

One possibility is to use one measure to compute only the abundance of vacuum bubbles, and a separate regularization scheme inside each bubble to define the abundance of observers. An example of the latter is the number of observers per baryon, or per photon [71–76]. But this quantity seems somewhat arbitrary, and it may not be well-defined throughout the string landscape, since not all vacua will contain the specified reference particles. Another strategy is to calibrate a "unit comoving volume" [54], but it is not clear that this can be rigorously defined.

The problem simplifies if we restrict to vacua that differ from ours only in ρ_Λ. Then all of the above schemes are equivalent. Assuming only that observers require galaxies, they prefer a cosmological constant about three orders of magnitude larger than the observed value. The agreement improves if more detailed assumptions about observers are made. It worsens when additional parameters, in particular the primordial density contrast, are allowed to vary (as seems inevitable in the string landscape) [62, 77, 78].

Another possibility is to use the same measure for counting both vacuum bubbles and the observers in them. For example, the number of observers in a single causally connected region is already finite in any vacuum with nonzero cosmological constant, by Eq. (1.2). (In the string landscape, vacua with $\rho_\Lambda = 0$ have unbroken supersymmetry. This would seem to preclude any form of condensed matter, and thus, presumably, observers.)

The causal diamond measure has another, quantitative advantage: the size of the causally connected region depends on the cosmological constant through Eq. (1.4). The smaller the cosmological constant, the larger the causally connected region, and the more complexity it allows. Thus, small values of ρ_Λ are not just enforced by galaxy formation. They are favored more generally because they allow more room, and more time, for observers.

Let us again restrict to the set of vacua identical to ours except for ρ_Λ, and ask what probability distribution for ρ_Λ the causal diamond measure predicts. One finds that the favored value of ρ_Λ is that in which vacuum energy comes to dominate the energy density around the time when most observations are made. This result generalizes to very different vacua, so the coincidence problem (Sec. 5.3) is actually the primary problem solved by this measure.

The numerical value of ρ_Λ then depends on the low energy physics determining the

epoch of observers. With values larger than of order 10^{-123}, galaxies are expelled from the horizon before stars and observers form; smaller values are simply less frequent in the landscape. Thus, the causal diamond predicts a probability distribution for ρ_Λ that is in remarkable agreement with the observed value [79].

One would not expect this agreement to worsen when the primordial density contrast is allowed to vary as well. Larger density perturbations speed up galaxy formation, undermining the necessity for small ρ_Λ. But in the causal diamond measure, galaxy formation (while still a necessary condition for observers) is no longer the dominant constraint on ρ_Λ.

The problem of characterizing observers, especially in vacua very different from ours, remains challenging. A surprisingly good approximation, in examples studied so far, is to replace the number of observers by the amount of entropy that is produced by a given vacuum in the causal diamond, ΔS_{CD}. Whatever observers are, they must obey the laws of thermodynamics. As they compute, store and retrieve information, they convert free energy into entropy. Of course, not all entropy is produced by observers. But ΔS_{CD} can be thought of as an upper bound on the complexity of a spacetime region. At least on average, it might be proportional to the number of observations made.

In vacua similar to ours, where we do have ideas about what observers look like, this *Causal Entropic Principle* agrees with conventional weighting by the observers present in the causal diamond [79]. Remarkably, this allows us to explain ρ_Λ without even assuming that observers require galaxies. One finds that ΔS_{CD} is dominated by infrared radiation from dust heated by starlight. Much less entropy would have been produced in the absence of heavy elements, or stars, or galaxies. Thus, we may have identified a primitive and universal criterion capable of capturing conditions often assumed (by hand) as necessary for complex life. The causal diamond and ΔS_{CD} are well-defined in any vacuum, and it may be possible to estimate them, at least on average, even in distant regions of the landscape.

Tools like this will be crucial if we want to progress from conditional predictions, which correlate various features of our own vacuum, to a fundamental understanding of their origin. In the string landscape, a scale like 10^{-123} ultimately must arise from the density of its discretuum and the range of complexity of its particle physics.

Acknowledgments

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