Generalized Cosmic Chaplygin Gas Model with or without Interaction

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Recently developed Generalized Cosmic Chaplygin gas (GCCG) is studied as an unified model of dark matter and dark energy. To explain the recent accelerating phase, the Universe is assumed to have a mixture of radiation and GCCG. The mixture is considered for without or with interaction. Solutions are obtained for various choices of the parameters and trajectories in the plane of the statefinder parameters and presented graphically. For particular choice of interaction parameter, we have shown the role of statefinder parameters in various cases for the evolution of the Universe.

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I. INTRODUCTION

Recent observations of type Ia Supernovae indicate that the expansion of the Universe is accelerating [1-5] and lead to the search for a new type of matter which violates the strong energy condition, i.e., $\rho + 3p < 0$. The matter responsible for this condition to be satisfied at some stage of evolution of the universe is referred to as *dark energy* [6 - 8]. Several candidates to present dark energy have been suggested with observations: the cosmological constant [7, 9], quintessence [10, 11], phantom [12, 13], braneworld models [14], pure Chaplygin gas model [15], generalized Chaplygin gas (GCG) model [16, 17], modified Chaplygin gas (MCG) model [19, 20]. In the GCG and MCG approach dark energy and dark matter can be unified by using an exotic equation of state (EOS). Interesting feature of MCG (or GCG) EOS is that it shows radiation era (or dust era) in the past while a ΛCDM model in the future.

In 2003, P. F. González-Diaz [21] have introduced the generalized cosmic Chaplygin gas (GCCG) model in such a way that the resulting models can be made stable and free from unphysical behaviours even when the vacuum fluid satisfies the phantom energy condition. The EOS of this model is

$$p = -\rho^{-\alpha} \left[C + (\rho^{1+\alpha} - C)^{-w} \right] \tag{1}$$

where $C = \frac{A}{1+w} - 1$ with A a constant which can take on both positive and negative values and $-l < w < 0$, l being a positive definite constant which can take on values larger than unity. The EOS reduces to that of current Chaplygin unified models for dark matter and dark energy in the limit $w \to 0$ and satisfies the conditions: (i) it becomes a de Sitter fluid at late time and when $w = -1$, (ii) it reduces to $p = w\rho$ in the limit that the Chaplygin parameter $A \to 0$, (iii) it also reduces to the EOS of current Chaplygin unified dark matter models at high energy density and (iv) the evolution of density perturbations derived from the chosen EOS becomes free from the pathological behaviour of the matter power spectrum for physically reasonable values of the involved parameters at late time. This EOS shows dust era in the past and ΛCDM in the future.

Since models trying to provide a description of the cosmic acceleration are proliferating, there exists the problem of discriminating between the various contenders. To this aim Sahni et al [22] proposed a pair of parameters $\{r, s\}$, called *statefinder* parameters. In fact trajectories in the $\{r, s\}$ plane corresponding to different cosmological models demonstrate qualitatively different behaviour. The above statefinder diagnostic pair has the following form:

$$r = \frac{\dddot{a}}{aH^3} \quad \text{and} \quad s = \frac{r-1}{3\left(q - \frac{1}{2}\right)} \tag{2}$$

where $H\left(= \frac{\dot{a}}{a}\right)$ and $q\left(= -\frac{a\ddot{a}}{\dot{a}^2}\right)$ are the Hubble parameter and the deceleration parameter respectively. The new feature of the statefinder is that it involves the third derivative of the cosmological radius. These parameters are dimensionless and allow us to characterize the properties of dark energy. Trajectories in the $\{r, s\}$ plane corresponding to different cosmological models, for example ΛCDM model diagrams correspond to the fixed point $s = 0$, $r = 1$.

In this paper, we consider the Universe is filled with the mixture of radiation and GCCG in section II. We perform a statefinder diagnostic to this model without and with interaction in different cases in sections III and IV respectively. From statefinder parameters we have shown graphically that the universe starts from radiation era instead of dust era. Different phases of the evolution of the universe have been shown graphically. With interaction case, the model goes from radiation to ΛCDM era only and without interaction case the model goes from radiation to ΛCDM and further from ΛCDM to phantom era and then back to ΛCDM. The paper ends with a short discussion in section V.

II.   MIXTURE OF GCCG AND RADIATION

The metric of a spatially flat isotropic and homogeneous Universe in FRW model is,

$$ds^2 = dt^2 - a^2(t)\left[dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2)\right] \tag{3}$$

where $a(t)$ is the scale factor.

The Einstein field equations are (choosing $8\pi G = c = 1$)

$$3\frac{\dot{a}^2}{a^2} = \rho_{tot} \tag{4}$$

and

$$6\frac{\ddot{a}}{a} = -(\rho_{tot} + 3p_{tot}) \tag{5}$$

The energy conservation equation $(T^\nu_{\mu;\nu} = 0)$ is

$$\dot{\rho}_{tot} + 3\frac{\dot{a}}{a}(\rho_{tot} + p_{tot}) = 0 \tag{6}$$

where, ρ_{tot} and p_{tot} are the total energy density and the pressure of the Universe, given by,

$$\rho_{tot} = \rho + \rho_r \tag{7}$$

and

$$p_{tot} = p + p_r \tag{8}$$

with ρ and p are respectively the energy density and pressure due to the GCCG satisfying the EOS (1) and ρ_r and p_r are the energy density and the pressure corresponding to the radiation fluid with EOS,

$$p_r = \gamma\rho_r \tag{9}$$

where $\gamma = \frac{1}{3}$.

Since GCCG can explain the evolution of the Universe starting from dust era to ΛCDM, considering the mixture of GCCG with radiation would make it possible to explain the evolution of the Universe from radiation to ΛCDM.

III. WITHOUT INTERACTION

In this case GCCG and the radiation fluid are conserved separately. Conservation equation (6) yields,

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0 \tag{10}$$

and

$$\dot{\rho}_r + 3\frac{\dot{a}}{a}(\rho_r + p_r) = 0 \tag{11}$$

From equations (1), (9), (10), (11) we have

$$\rho = \left[C + \left(1 + \frac{B}{a^{3(1+\alpha)(1+w)}}\right)^{\frac{1}{1+w}}\right]^{\frac{1}{1+\alpha}} \tag{12}$$

and

$$\rho_r = \rho_0 \, a^{-3(1+\gamma)} \tag{13}$$

For the two component fluids, equation (2) takes the following forms:

$$r = 1 + \frac{9}{2(\rho + \rho_r)}\left[\frac{\partial p}{\partial \rho}(\rho + p) + \frac{\partial p_r}{\partial \rho_r}(\rho_r + p_r)\right] \tag{14}$$

and

$$s = \frac{1}{(p + p_r)}\left[\frac{\partial p}{\partial \rho}(\rho + p) + \frac{\partial p_r}{\partial \rho_r}(\rho_r + p_r)\right] \tag{15}$$

Also the deceleration parameter q has the form:

$$q = -\frac{\ddot{a}}{aH^2} = \frac{1}{2} + \frac{3}{2}\left(\frac{p + p_r}{\rho + \rho_r}\right) \tag{16}$$

Now substituting $u = \rho^{1+\alpha}$, $y = \frac{\rho_r}{\rho}$, equation (14) and (15) can be written as,

$$r = 1 + \frac{9}{2(1+y)}\left[\left(1 - \frac{C}{u} - \frac{(u-C)^{-w}}{u}\right)\left\{\frac{\alpha C}{u} + \frac{\alpha}{u}(u-C)^{-w} + w(1+\alpha)(u-C)^{-w-1}\right\} + \gamma(1+\gamma)y\right] \tag{17}$$

and

$$s = \frac{2(r-1)(1+y)}{9\left[\gamma y - \frac{C}{u} - \frac{(u-C)^{-w}}{u}\right]} \tag{18}$$

Normalizing the parameters we have shown the graphical representation of the $\{r, s\}$ parameters in figure 1. From the figure we have seen that the universe starts from radiation era ($r = 3$, $s > 0$) via dust stage ($2.3 < r < 2.4$, $s \to \pm\infty$) to ΛCDM ($r = 1$, $s = 0$) model for choices of $C = 1, B = 1, \alpha = 1, w = -2, \rho_0 = 1$.

IV. WITH INTERACTION

We consider the GCCG interacting with radiation fluid through an energy exchange between them. The equations of motion can be written as,

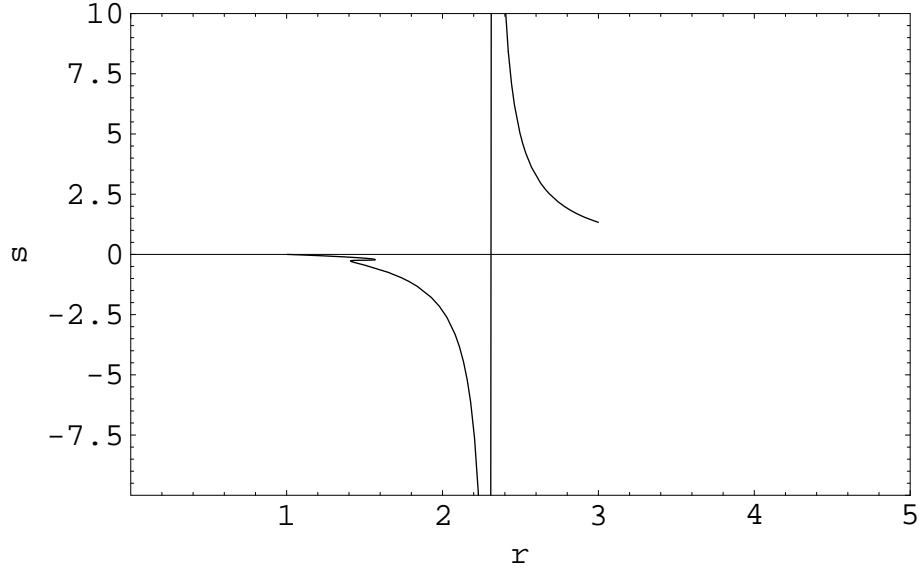$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = -3H\delta \tag{19}$$

Fig.1

Fig. 1 shows the variation of s against r (eqs. (17) and (18)) for $C = 1, B = 1, \alpha = 1, w = -2, \rho_0 = 1$.

and

$$\dot{\rho}_r + 3\frac{\dot{a}}{a}(\rho_r + p_r) = 3H\delta \tag{20}$$

where δ is a coupling function.

Let us choose,

$$\delta = \epsilon \, \frac{(\rho^{1+\alpha} - C)^{-w}}{\rho^\alpha} \tag{21}$$

Now equation (19) together with equation (1) gives,

$$\rho = \left[C + \left(1 - \epsilon + Ba^{3 - (1+\alpha)(1+w)}\right)^{\frac{1}{1+w}}\right]^{\frac{1}{1+\alpha}} \tag{22}$$

Also equations (9), (20) and (22) give

$$\rho_r = \rho_0 \, a^{-3(1+\gamma)} + 3 \, \epsilon \, a^{-3(1+\gamma)} I \tag{23}$$

with

$$I = -\frac{1}{3B(1+\alpha)} \int \frac{dx}{(C+x)^{\frac{\alpha}{1+\alpha}}} \left\{\frac{x^{1+w} + \epsilon - 1}{B}\right\}^{-\frac{1+\gamma}{(1+w)(1+\alpha)} - 1} \tag{24}$$

and

$$x = \left[1 - \epsilon + Ba^{-3(1+w)(1+\alpha)}\right]^{\frac{1}{1+w}} \tag{25}$$

From (22), we see that if $\epsilon = 0$, i.e., $\delta = 0$, then the expression (22) reduces to the expression (12).

Now for the two component interacting fluids with equations of motion (19) and (20), the $\{r, s\}$ parameters read:

$$r = 1 + \frac{9}{2(\rho + \rho_r)} \left[\frac{\partial p}{\partial \rho}(\rho + p + \delta) + \frac{\partial p_r}{\partial \rho_r}(\rho_r + p_r - \delta) \right] \qquad (26)$$

and

$$s = \frac{2(r-1)(\rho + \rho_r)}{9(p + p_r)} \qquad (27)$$

Also the deceleration parameter q has the form:

$$q = \frac{1}{2} \left(1 + 3\frac{p + p_r}{\rho + \rho_r} \right) \qquad (28)$$

Now substituting $u = \rho^{1+\alpha}$, $y = \frac{\rho_r}{\rho}$, equation (14) and (15) can be written as,

$$r = 1 + \frac{9}{2(1+y)} \left[\frac{\partial p}{\partial \rho} \left(1 + \frac{p}{\rho} + \frac{\delta}{\rho} \right) + \gamma \left\{ (1+\gamma)y - \frac{\delta}{\rho} \right\} \right] \qquad (29)$$

and

$$s = \frac{2(r-1)(1+y)}{9 \left(\frac{p}{\rho} + \gamma y \right)} \qquad (30)$$

where,

$$u = \left[C + \left(1 - \epsilon + Ba^{3-(1+\alpha)(1+w)} \right)^{\frac{1}{1+w}} \right]$$

$$y = \frac{\rho_0}{\rho} \, a^{-3(1+\gamma)} + 3 \, \frac{\epsilon}{\rho} \, a^{-3(1+\gamma)} I$$

$$\frac{p}{\rho} = -\frac{1}{u} \left\{ C + (u - C)^{-w} \right\}$$

$$\frac{\delta}{\rho} = \epsilon \frac{(u - C)^{-w}}{u}$$

and

$$\frac{\partial p}{\partial \rho} = \frac{\alpha C}{u} + \frac{\alpha}{u}(u - C)^{-w} + w(1 + \alpha)(u - C)^{-w-1}$$

Now we find the exact solution for the $\{r, s\}$ parameters for the following particular choices of w:

(i) If $-\frac{(1+\gamma)}{(1+w)(1+\alpha)} - 1 = 0$, i.e., $w = \frac{-2-\gamma-\alpha}{1+\alpha}$, equation (23) can be written as

$$\rho_r = \rho_0 \, a^{-3(1+\gamma)} - \frac{\epsilon}{B} a^{-3(1+\gamma)} \rho \qquad (31)$$

as $I = -\frac{1}{3B}(c + x)^{\frac{1}{1+\alpha}}$. Normalizing the parameters, the corresponding statefinder parameters are given in figure 2. From the figure we have seen that the universe starts from radiation era ($r = 3$, $s > 0$) via dust stage ($2.5 < r < 2.6$, $s \to \pm\infty$) to ΛCDM ($r = 1$, $s = 0$) model and further from ΛCDM to phantom era ($r < 1$, $s > 0$) and then back to ΛCDM for choices of $C = 1$, $B = 1$, $\alpha = 1$, $w = -\frac{5}{3}$, $\rho_0 = 1$, $\epsilon = \frac{1}{2}$.
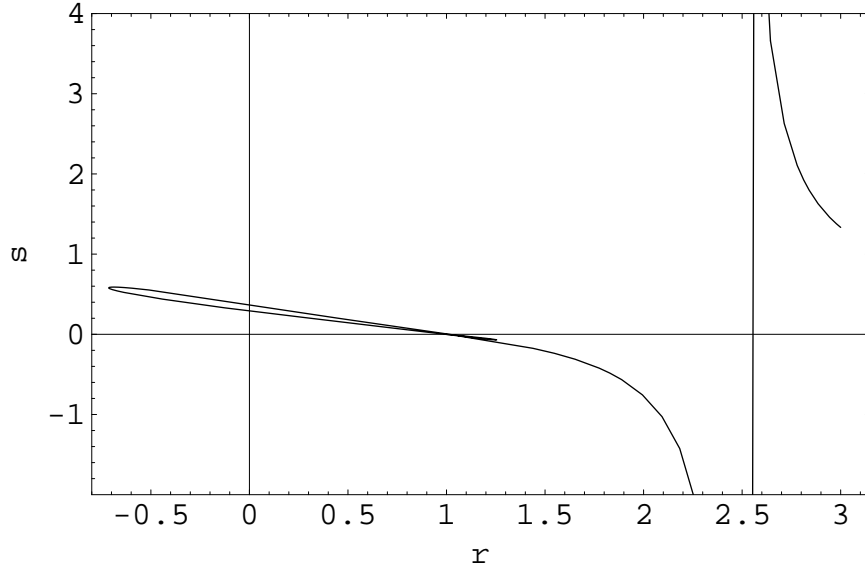
Fig.2

Fig. 2 shows the variation of s against r (case (i)) for $C = 1$, $B = 1, \alpha = 1, w = -\frac{5}{3}, \rho_0 = 1, \epsilon = \frac{1}{2}$.
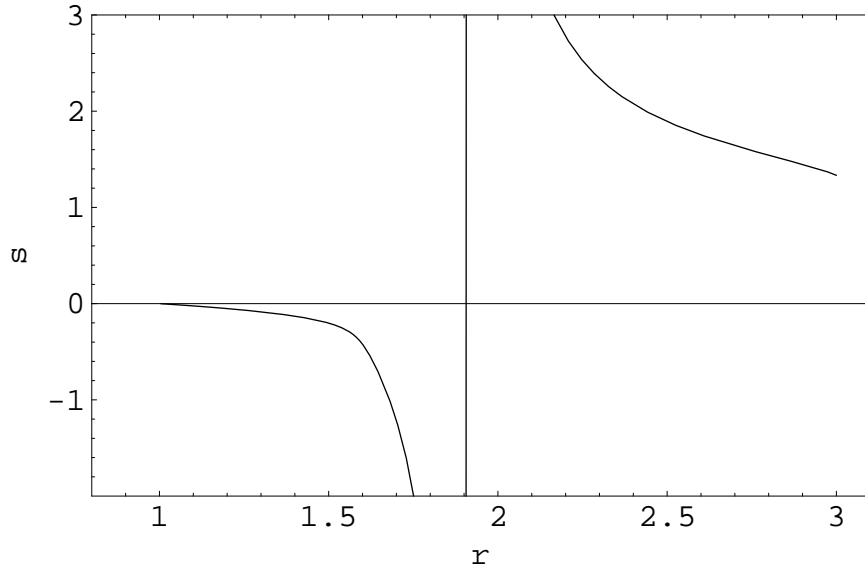

Fig.3

Fig. 3 shows the variation of s against r (case (ii)) for $C = 1$, $B = 1, \alpha = 1, w = -\frac{4}{3}, \rho_0 = 1, \epsilon = \frac{1}{2}$.

(ii) If $-\frac{(1+\gamma)}{(1+w)(1+\alpha)} - 1 = 1$, i.e., $w = \frac{-3-\gamma-2\alpha}{2(1+\alpha)}$, equation (23) can be written as

$$\rho_r = \rho_0 \ a^{-3(1+\gamma)} - \frac{\epsilon(\epsilon-1)}{B^2}a^{-3(1+\gamma)} - \frac{\epsilon a^{-3(1+\gamma)}}{B^2(1+\alpha)(2+w)C^{\frac{\alpha}{1+\alpha}}}x^{2+w} \ _2F_1[2+w, \frac{\alpha}{1+\alpha}, 3+w, -\frac{x}{C}] \ (32)$$

Normalizing the parameters, the corresponding statefinder parameters are given in figure 3.

Fig.4

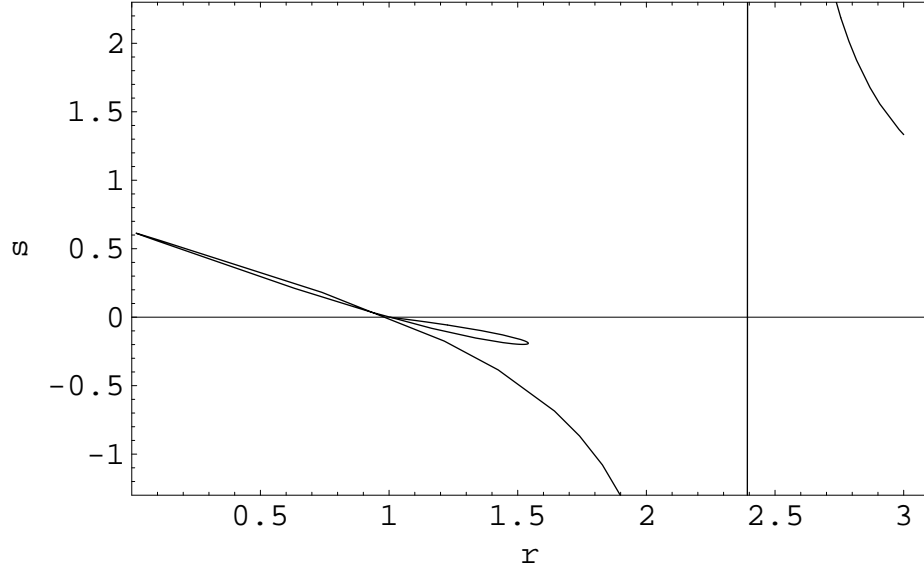Fig. 4 shows the variation of s against r (case (iii)) for $C = 1$, $B = 1, \alpha = 1, \rho_0 = 1, \epsilon = \frac{1}{2}$.

From the figure we have seen that the universe starts from radiation era ($r = 3$, $s > 0$) via dust stage ($1.9 < r < 2$, $s \to \pm\infty$) to ΛCDM ($r = 1$, $s = 0$) model for choices of $C = 1$, $B = 1, \alpha = 1, w = -\frac{4}{3}, \rho_0 = 1, \epsilon = \frac{1}{2}$.

(iii) If $w = -2$, equation (23) can be written as

$$\rho_r = \rho_0\, a^{-3(1+\gamma)} - \frac{\epsilon}{(1 + 2\alpha - \gamma)} \frac{a^{-3(1+\gamma)}}{x^{\frac{1+2\alpha-\gamma}{(1+\alpha)}}} \frac{B^{-\frac{1+\gamma}{(1+\alpha)}}}{C^{\frac{\alpha}{1+\alpha}}} AppellF_1\left[\frac{1 + 2\alpha - \gamma}{(1+\alpha)}, \frac{\alpha}{1+\alpha}, \frac{\alpha - \gamma}{(1+\alpha)}, \frac{2 + 3\alpha - \gamma}{(1+\alpha)}, -\frac{x}{C}, x - x\epsilon\right]$$

(33)

Normalizing the parameters, the corresponding statefinder parameters are given in figure 4. From the figure we have seen that the universe starts from radiation era ($r = 3$, $s > 0$) via dust stage ($2.3 < r < 2.5$, $s \to \pm\infty$) to ΛCDM ($r = 1$, $s = 0$) model and further from ΛCDM to phantom era ($r < 1$, $s > 0$) and then back to ΛCDM for choices of $C = 1$, $B = 1, \alpha = 1, \rho_0 = 1, \epsilon = \frac{1}{2}$.

V. DISCUSSION

In this work, we have considered the matter in our Universe as a mixture of the GCCG and radiation as GCCG can explain the evolution of the Universe from dust era to ΛCDM. These gases are taken both as non-interacting and interacting mixture. In the first case we have considered a non-interacting model and plotted the $\{r, s\}$ parameters. As expected this model represents the evolution of the Universe from radiation era to ΛCDM with a discontinuity at dust. In the second case the interaction term is chosen in a very typical form to solve the corresponding conservation equations analytically. Also the statefinder parameters are evaluated for various choices of parameters and the trajectories in the $\{r, s\}$ plane are plotted to characterize different phases of the Universe. These trajectories show discontinuity at same r in the neighbourhood of $r = 2$ and have peculiar behaviour around $r = 1$. The $\{r, s\}$ curves have two branches on two sides of the asymptote. The branch on the right hand side of the asymptote corresponds to decelerating phase before (or up to) dust era, while the left hand side branch has a transition from decelerating phase upto ΛCDM era. Some peculiarity has been shown in figures 2 and 4 around $r = 1$. In these two cases, the model goes further from ΛCDM to phantom era and then back to ΛCDM. Moreover, in figure 4, there is further transition from ΛCDM to decelerating phase and then then again

back to ΛCDM. Thus we can conclude that the present model describes a number of transitions from decelerating to accelerating phase and vice-versa.

Acknowledgement:

References:

[1] S. J. Perlmutter et al, *Bull. Am. Astron. Soc.* **29** 1351 (1997).
[2] S. J. Perlmutter et al, *Astrophys. J.* **517** 565 (1999).
[3] A. G. Riess et al, *Astron. J.* **116** 1009 (1998).
[4] P. Garnavich et al, *Astrophys. J.* **493** L53 (1998).
[5] B. P. Schmidt et al, *Astrophys. J.* **507** 46 (1998).
[6] V. Sahni and A. A. Starobinsky, *Int. J. Mod. Phys. A* **9** 373 (2000).
[7] P. J. E. Peebles and B. Ratra, *Rev. Mod. Phys.* **75** 559 (2003).
[8] T. Padmanabhan, *Phys. Rept.* **380** 235 (2003).
[9] S. M. Carrol, *Living Rev. Rel.* **4** 1 (2001).
[10] R. R. Caldwell, R. Dave and P. J. Steinhardt, *Phys. Rev. Lett.* **80** 1582 (1998).
[11] V. Sahni and L. Wang, *Phys. Rev. D* **62** 103507 (2000).
[12] R. R. Caldwell, *Phys. Lett. B* **545** 2 (2002).
[13] S. M. Carrol, M. Hoffman and M. Trodden, *Phys. Rev. D* **68** 023509 (2003).
[14] U. Alam, V. Sahni, *astro-ph/* 0209443.
[15] A. Kamenshchik, U. Moschella and V. Pasquier, *Phys. Lett. B* **511** 265 (2001).
[16] V. Gorini, A. Kamenshchik and U. Moschella, *Phys. Rev. D* **67** 063509 (2003).
[17] U. Alam, V. Sahni , T. D. Saini and A.A. Starobinsky, *Mon. Not. Roy. Astron. Soc.* **344**, 1057 (2003).
[18] M. C. Bento, O. Bertolami and A. A. Sen, *Phys. Rev. D* 66 043507 (2002).
[19] H. B. Benaoum, *hep-th*/0205140.
[20] U. Debnath, A. Banerjee and S. Chakraborty, *Class. Quantum Grav.* **21** 5609 (2004).
[21] P. F. González-Diaz, *Phys. Rev. D* **68** 021303 (R), (2003).
[22] V. Sahni, T. D. Saini, A. A. Starobinsky and U. Alam, *JETP Lett.* **77** 201 (2003).