

# Solar System tests of some models of modified gravity proposed to explain galactic rotation curves without dark matter

Lorenzo Iorio

INFN-Sezione di Pisa. Permanent address for correspondence: Viale Unità di Italia 68,  
70125, Bari (BA), Italy. E-mail: lorenzo.iorio@libero.it

and

Matteo Luca Ruggiero

Dipartimento di Fisica del Politecnico di Torino and INFN-Sezione di Torino Corso Duca  
degli Abruzzi 24, 10129, Torino (TO), Italy. E-mail: ruggierom@polito.it

Received \_\_\_\_\_; accepted \_\_\_\_\_

## ABSTRACT

We consider the recently estimated corrections  $\Delta\dot{\varpi}$  to the Newtonian/Einsteinian secular precessions of the longitudes of perihelia  $\varpi$  of several planets of the Solar System in order to evaluate whether they are compatible with the predicted precessions due to models of long-range modified gravity put forth to account for certain features of the rotation curves of galaxies without resorting to dark matter. In particular, we consider a logarithmic-type correction and a  $f(R)$  inspired power-law modification of the Newtonian gravitational potential. The results obtained by taking the ratio of the apsidal rates for different pairs of planets show that the modifications of the Newtonian potentials examined in this paper are not compatible with the secular extra-precessions of the perihelia of the Solar System's planets estimated by E.V. Pitjeva as solve-for parameters processing almost one century of data with the latest EPM ephemerides.

*Subject headings:* Experimental tests of gravitational theories; Modified theories of gravity; Celestial mechanics; Orbit determination and improvement; Ephemerides, almanacs, and calendars

## 1. Introduction

Dark matter and dark energy are, possibly, the most severe theoretical issues that modern astrophysics and cosmology have to face, since the available observations seem to question the model of gravitational interaction on a scale larger than the Solar System. In fact, the data coming from the galactic rotation curves of spiral galaxies (Binney and Tremaine 1987) cannot be explained on the basis of Newtonian gravity or General Relativity (GR) if one does not introduce invisible dark matter compensating for the observed inconsistency of the Newtonian dynamics of stars. Likewise, since 1933, when Zwicky (1933) studied the velocity dispersion in the Coma cluster, there is the common agreement on the fact that the dynamics of cluster of galaxies is poorly understood on the basis of Newtonian gravity or GR (Peebles 1993; Peacock 1999) if the dark matter is not taken into account. In order to reconcile theoretical models with observations, the existence of a peculiar form of matter is postulated, the so called *dark matter*, which is supposed to be a cold and pressureless medium, whose distribution is that of a spherical halo around the galaxies. On the other hand, a lot of observations, such as the light curves of the type Ia supernovae and the cosmic microwave background (CMB) experiments (Riess et al. 1998; Perlmutter et al. 1999; Tonry et al. 2003; Bennet et al. 2003), firmly state that our Universe is now undergoing a phase of accelerated expansion. Actually, the present acceleration of the Universe cannot be explained, within GR, unless the existence of a cosmic fluid having exotic equation of state is postulated: the so-called *dark energy*.

The main problem dark matter and dark energy bring about is understanding their nature, since they are introduced as *ad hoc* gravity sources in a well defined model of gravity, i.e. GR (or Newtonian gravity). Of course, another possibility exists: GR (and its approximation, Newtonian gravity) is unfit to deal with gravitational interaction at galactic, intergalactic and cosmological scales. The latter viewpoint, led to the introduction

of various modifications of gravity (Nojiri and Odintsov 2007; Esposito-Farèse 2008).

However, we must remember that, even though there are problems with galaxies and clusters dynamics and cosmological observations, GR is in excellent agreement with the Solar System experiments (see Will (2006) and Damour (2006)): hence, every theory that aims at explaining the large scale dynamics and the accelerated expansion of the Universe, should reproduce GR at the Solar System scale, i.e. in a suitable weak field limit. So, the viability of these modified theories of gravity at Solar System scale should be studied with great care.

In this paper we wish to quantitatively deal with such a problem by means of the secular (i.e. averaged over one orbital revolution) precessions of the longitudes of the perihelia  $\dot{\varpi}$  of some planets of the Solar System (see also (Iorio 2007a,b)) in the following way. Generally speaking, let LRMOG (Long-Range Modified Model of Gravity) be a given exotic model of modified gravity parameterized in terms of, say,  $K$ , in a such a way that  $K = 0$  would imply no modifications of gravity at all. Let  $\mathcal{P}(\text{LRMOG})$  be the prediction of a certain effect induced by such a model like, e.g., the secular precession of the perihelion of a planet. For all the exotic models considered it turns out that

$$\mathcal{P}(\text{LRMOG}) = Kg(a, e), \tag{1}$$

where  $g$  is a function of the system's orbital parameters  $a$  (semimajor axis) and  $e$  (eccentricity); such  $g$  is a peculiar consequence of the model LRMOG (and of all other models of its class with the same spatial variability). Now, let us take the ratio of  $\mathcal{P}(\text{LRMOG})$  for two different systems A and B, e.g. two Solar System's planets:  $\mathcal{P}_A(\text{LRMOG})/\mathcal{P}_B(\text{LRMOG}) = g_A/g_B$ . The model's parameter  $K$  is now disappeared, but we still have a prediction that retains a peculiar signature of that model, i.e.  $g_A/g_B$ . Of course, such a prediction is valid if we assume  $K$  is not zero, which is just the case both theoretically (LRMOG is such that should  $K$  be zero, no modifications of gravity at

all occurred) and observationally because  $K$  is usually determined by other independent long-range astrophysical/cosmological observations. Otherwise, one would have the meaningless prediction  $0/0$ . The case  $K = 0$  (or  $K \leq \overline{K}$ ) can be, instead, usually tested by taking one perihelion precession at a time, as already done, e.g., by Iorio (2007a,b).

If we have observational determinations  $\mathcal{O}$  for A and B of the effect considered above such that they are affected also<sup>1</sup> by LRMOG (it is just the case for the purely phenomenologically estimated corrections to the standard Newton-Einstein perihelion precessions, since LRMOG has not been included in the dynamical force models of the ephemerides adjusted to the planetary data in the least-square parameters' estimation process by Pitjeva (Pitjeva 2005a,b, 2006b)), we can construct  $\mathcal{O}_A/\mathcal{O}_B$  and compare it with the prediction for it by LRMOG, i.e. with  $g_A/g_B$ . Note that  $\delta\mathcal{O}/\mathcal{O} > 1$  only means that  $\mathcal{O}$  is compatible with zero, being possible a nonzero value smaller than  $\delta\mathcal{O}$ . Thus, it is perfectly meaningful to construct  $\mathcal{O}_A/\mathcal{O}_B$ . Its uncertainty will be conservatively evaluated as  $|1/\mathcal{O}_B|\delta\mathcal{O}_A + |\mathcal{O}_A/\mathcal{O}_B^2|\delta\mathcal{O}_B$ . As a result,  $\mathcal{O}_A/\mathcal{O}_B$  will be compatible with zero. Now, the question is: Is it the same for  $g_A/g_B$  as well? If yes, i.e. if

$$\frac{\mathcal{O}_A}{\mathcal{O}_B} = \frac{\mathcal{P}_A(\text{LRMOG})}{\mathcal{P}_B(\text{LRMOG})} \quad (2)$$

within the errors, or, equivalently, if

$$\left| \frac{\mathcal{O}_A}{\mathcal{O}_B} - \frac{\mathcal{P}_A(\text{LRMOG})}{\mathcal{P}_B(\text{LRMOG})} \right| = 0 \quad (3)$$

within the errors, LRMOG survives (and the use of the single perihelion precessions can be used to put upper bounds on  $K$ ). Otherwise, LRMOG is ruled out.

---

<sup>1</sup>If they are differential quantities constructed by contrasting observations to predictions obtained by analytical force models of canonical effects,  $\mathcal{O}$  are, in principle, affected also by the mismodelling in them.

The paper is organized as follows. In Section 2 we apply this approach to a test particle in motion around a central mass  $M$  whose Newtonian gravitational potential exhibits a logarithmic-type correction. Then, in Section 3, we consider the power-law modification of the gravitational potential inspired by  $f(R)$  extended theories of gravity. Comments and conclusions are outlined in Section 4.

## 2. The perihelion precession due to a $1/r$ force

Logarithmic corrections to the Newtonian gravitational potential have been recently used to phenomenologically tackle the problem of dark matter in galaxies (van Moorsel 1987; Cretton et al. 2000; Kinney and Brisudova 2001; Kirillov 2006; Fabris and Campos 2007; Sobouti 2007). E.g., Fabris and Campos (2007) used

$$V_{\text{ln}} = -\alpha GM \ln \left( \frac{r}{r_0} \right), \quad (4)$$

where  $\alpha$  has the dimension of  $L^{-1}$ , to satisfactorily fit the rotation curves of 10 spiral galaxies getting

$$\alpha \approx -0.1 \text{ kpc}^{-1}. \quad (5)$$

The extra-potential of eq. (4) yields an additional  $1/r$  radial force<sup>2</sup>

$$\mathbf{A}_{\text{ln}} = \frac{\alpha GM}{r} \hat{\mathbf{r}}. \quad (6)$$

Various theoretical justifications have been found for a logarithmic-like extra-potential. E.g., according to Kirillov (2006), it would arise from large-scale discrepancies of the topology of the actual Universe from the Friedmann space; Sobouti (2007) obtained it in

---

<sup>2</sup>For another example of a  $1/r$  extra-force and its connection with galaxy rotation curves see (Sanders 2006).

the framework of the  $f(R)$  modifications of general relativity getting preliminarily flat rotation curves, the Tully-Fisher relation (admittedly with some reservations) and a version of MOND, while Fabris and Campos (2007) pointed out that string-like objects (Soleng 1995; Capozziello et al. 2006) would yield a logarithmic-type potential with  $\alpha$  related to the string tension.

Alternative tests of such proposed correction to the Newtonian potential, independent of the dark matter issues themselves, would be, of course, highly desirable and could be, in principle, conducted in the Solar System. This is, indeed, considered one of the tasks to be implemented in further investigations by Sobouti (2007); Fabris and Campos (2007) argue that, in view of their extreme smallness due to eq. (5), no detectable effects induced by eq. (6) would be possible at the level of Solar System.

E.V. Pitjeva has recently processed almost one century of data of different types for the major bodies of the Solar System in the effort of continuously improving the EPM2004/EPM2006 planetary ephemerides (Pitjeva 2005a, 2006b). Among other things, she also simultaneously estimated corrections  $\Delta\dot{\varpi}$  to the secular rates of the longitudes of perihelia  $\varpi$  of the inner (Pitjeva 2005b) and of some of the outer (Pitjeva 2006a,b) planets of the Solar System as fit-for parameters of a global solution in which she contrasted, in a least-square way, the observations to their predicted values computed with a complete set of dynamical force models including all the known Newtonian and Einsteinian features of motion. As a consequence, any un-modelled exotic force present in nature is, in principle, entirely accounted for by the obtained apsidal extra-precessions<sup>3</sup>  $\Delta\dot{\varpi}$ . See Table 1 for the

---

<sup>3</sup>Of course, since modelling is not perfect, in principle,  $\Delta\dot{\varpi}$  include also the mismodelled parts of the standard Newtonian/Einsteinian effects, but it turns out that their inclusion in the following computation do not alter our conclusions.

inner planets and Table 2 for the gaseous giant ones. In regard to them, we must note that modern data sets cover at least one full orbital revolution only for Jupiter, Saturn and, barely, Uranus; this is why no secular extra-precessions of perihelia for Neptune and Pluto are today available.

In order to make a direct comparison with them, we will now analytically work out the secular effects induced by the extra-acceleration of eq. (6) on the pericentre of a test particle. To this aim, we will treat eq. (6) as a small perturbation of the Newtonian monopole. The Gauss equation for the variation of  $\varpi$  under the action of an entirely radial perturbing acceleration  $A_r$  is

$$\frac{d\varpi}{dt} = -\frac{\sqrt{1-e^2}}{nae}A_r \cos f, \quad (7)$$

in which  $a$  is the semimajor axis,  $e$  is the eccentricity,  $n = \sqrt{GM/a^3}$  is the (unperturbed) Keplerian mean motion related to the orbital period  $P$  by  $n = 2\pi/P$ , and  $f$  is the true anomaly. After being evaluated onto the unperturbed Keplerian ellipse, eq. (6) must be inserted into eq. (7); then, the average over one orbital period  $P$  must be performed. It is useful to use the eccentric anomaly  $E$  by means of the relations

$$\left\{ \begin{array}{l} r = a(1 - e \cos E), \\ dt = \frac{(1-e \cos E)}{n} dE, \\ \cos f = \frac{\cos E - e}{1 - e \cos E}, \\ \sin f = \frac{\sin E \sqrt{1-e^2}}{1 - e \cos E}. \end{array} \right. \quad (8)$$

On using

$$\frac{1}{2\pi} \int_0^{2\pi} \left( \frac{\cos E - e}{1 - e \cos E} \right) dE = \frac{-1 + \sqrt{1 - e^2}}{e}, \quad (9)$$

it is possible to obtain

$$\langle \dot{\varpi} \rangle_{\text{ln}} = -\alpha \sqrt{\frac{GM(1-e^2)}{a}} \left( \frac{-1 + \sqrt{1 - e^2}}{e^2} \right). \quad (10)$$



Table 1: Inner planets. First row: estimated perihelion extra-precessions, from Table 3 of (Pitjeva 2005b). The quoted errors are not the formal ones but are realistic. The units are arcseconds per century ( $" \text{ cy}^{-1}$ ). Second row: semimajor axes, in Astronomical Units (AU). Their formal errors are in Table IV of (Pitjeva 2005a), in m. Third row: eccentricities. Fourth row: orbital periods in years.

	Mercury	Earth	Mars
$\langle \Delta \dot{\omega} \rangle$ ( $" \text{ cy}^{-1}$ )	$-0.0036 \pm 0.0050$	$-0.0002 \pm 0.0004$	$0.0001 \pm 0.0005$
$a$ (AU)	0.387	1.000	1.523
$e$	0.2056	0.0167	0.0934
$P$ (yr)	0.24	1.00	1.88

Table 2: Outer planets. First row: estimated perihelion extra-precessions (Pitjeva 2006a,b). The quoted uncertainties are the formal, statistical errors re-scaled by a factor 10 in order to get the realistic ones. The units are arcseconds per century ( $" \text{ cy}^{-1}$ ). Second row: semimajor axes, in Astronomical Units (AU). Their formal errors are in Table IV of (Pitjeva 2005a), in m. Third row: eccentricities. Fourth row: orbital periods in years.

	Jupiter	Saturn	Uranus
$\langle \Delta \dot{\omega} \rangle$ ( $" \text{ cy}^{-1}$ )	$0.0062 \pm 0.036$	$-0.92 \pm 2.9$	$0.57 \pm 13.0$
$a$ (AU)	5.203	9.537	19.191
$e$	0.0483	0.0541	0.0471
$P$ (yr)	11.86	29.45	84.07

Note that eq. (10) is an exact result with respect to  $e$ . Eq. (10) agrees with the precession obtainable dividing by  $P$  the adimensional perihelion shift per orbit  $\Delta\theta_p(\log)$  worked out by Adkins and McDonnell (2007) within a different perturbative framework; note that for Adkins and McDonnell (2007) the constant  $\alpha$  has the dimensions of<sup>4</sup>  $\text{M L}^2 \text{T}^{-2}$ , so that that the substitution  $\alpha/m \rightarrow -GM\alpha$  must be performed in (50) of (Adkins and McDonnell 2007) to retrieve eq. (10).

It may be interesting to note that for the potential of eq. (4) the rates for the semimajor axis and the eccentricity turn out to be zero; it is not so for the mean anomaly  $\mathcal{M}$ , but no observational determinations exist for its extra-rate.

## 2.1. Comparison with data

According to the general outline of Section 1, we may now consider a pair of planets A and B, take the ratio of their estimated extra-rates of perihelia and compare it to the prediction of eq. (10) in order to see if they are equal within the errors

$$\Psi_{\text{AB}} = \left| \frac{\Delta\dot{\omega}_{\text{A}}}{\Delta\dot{\omega}_{\text{B}}} - \sqrt{\frac{a_{\text{B}}(1-e_{\text{A}}^2)}{a_{\text{A}}(1-e_{\text{B}}^2)}} \left(\frac{e_{\text{B}}}{e_{\text{A}}}\right)^2 \left(\frac{-1 + \sqrt{1-e_{\text{A}}^2}}{-1 + \sqrt{1-e_{\text{B}}^2}}\right) \right| \quad (11)$$

If the modification of the gravitational potential (4), not modelled by Pitjeva in estimating  $\Delta\dot{\omega}$ , accounts for what is unmodelled in the perihelia precessions, i.e. just for  $\Delta\dot{\omega}$ , then (11) must be compatible with zero, within the errors. It must be noted that our approach is able to directly test the hypothesis that the proposed  $1/r$  exotic force is not zero, irrespective of the magnitude of  $\alpha$  because our prediction for the ratio of the extra-rates of perihelia, i.e.

$$\frac{\mathcal{P}_{\text{A}}}{\mathcal{P}_{\text{B}}} = \sqrt{\frac{a_{\text{B}}(1-e_{\text{A}}^2)}{a_{\text{A}}(1-e_{\text{B}}^2)}} \left(\frac{e_{\text{B}}}{e_{\text{A}}}\right)^2 \left(\frac{-1 + \sqrt{1-e_{\text{A}}^2}}{-1 + \sqrt{1-e_{\text{B}}^2}}\right), \quad (12)$$

is just independent of  $\alpha$  itself and is a specific function of the planets' orbital parameters  $a$  and  $e$ . Of course, the ratio of the perihelion rates cannot be used, by definition, to test the zero hypothesis

---

<sup>4</sup>Indeed,  $V(r)$  of (47) in (Adkins and McDonnell 2007) is an additional potential energy.

Table 3: A B denotes the pair of planets used;  $\Pi = \Delta\dot{\omega}_A/\Delta\dot{\omega}_B$ ,  $\mathcal{A} = (a_B/a_A)^{1/2}$  and  $F(e_A, e_B)$  is given by eq. (14). The perihelion extra-rates for the inner planets have been retrieved from (Pitjeva 2005b); their errors are not the formal, statistical ones. The perihelion extra-rates for the outer planets come from (Pitjeva 2006a); their formal errors have been re-scaled by a factor 10. The uncertainties in the semimajor axes have been retrieved from Table IV of (Pitjeva 2005a): they are the formal ones, but, as can be noted, their impact is negligible. While  $\Pi$  is always compatible with zero, this is definitely not the case for  $\mathcal{A}$ . The eccentricity function  $F$  is always close to unity.

A B	$\Pi$	$\mathcal{A}$	$F(e_A, e_B)$
Mars Mercury	$-0.03 \pm 0.2$	$0.504 \pm \mathcal{O}(10^{-12})$	1.008
Mercury Jupiter	$-0.6 \pm 4.1$	$3.666 \pm \mathcal{O}(10^{-9})$	0.989
Earth Jupiter	$-0.03 \pm 0.25$	$2.281 \pm \mathcal{O}(10^{-10})$	1.000
Mars Jupiter	$0.02 \pm 0.17$	$1.847 \pm \mathcal{O}(10^{-10})$	0.998
Mercury Saturn	$0.004 \pm 0.017$	$4.963 \pm \mathcal{O}(10^{-9})$	0.989
Earth Saturn	$0.0002 \pm 0.0011$	$3.088 \pm \mathcal{O}(10^{-9})$	1.000
Mars Saturn	$-0.0001 \pm 0.0009$	$2.501 \pm \mathcal{O}(10^{-9})$	0.998
Jupiter Saturn	$-0.006 \pm 0.060$	$1.353 \pm \mathcal{O}(10^{-9})$	1.000
Mercury Uranus	$-0.006 \pm 0.152$	$7.041 \pm \mathcal{O}(10^{-8})$	0.989
Earth Uranus	$-0.0003 \pm 0.0087$	$4.380 \pm \mathcal{O}(10^{-8})$	1.000
Mars Uranus	$0.0002 \pm 0.0048$	$3.459 \pm \mathcal{O}(10^{-8})$	0.983
Jupiter Uranus	$0.01 \pm 0.31$	$1.920 \pm \mathcal{O}(10^{-8})$	0.999

which, instead, can be checked by considering the apsidal precessions separately; indeed, in this case the prediction for the ratio of the perihelion precessions would be, by definition,  $0/0$ . Note that, being  $\Delta\dot{\varpi}$  observational quantities, their ratio  $\Pi = \Delta\dot{\varpi}_A/\Delta\dot{\varpi}_B$  is a well defined quantity, with an associated uncertainty  $\delta\Pi$  which will be evaluated below.

From eq. (11) and Table 3 it is possible to obtain

$$\left\{ \begin{array}{l} \Psi_{\text{MarMer}} = 0.5 \pm 0.2, \\ \Psi_{\text{MerJup}} = 4.2 \pm 4.1, \\ \Psi_{\text{EarJup}} = 2.3 \pm 0.2, \\ \Psi_{\text{MarJup}} = 1.8 \pm 0.2, \\ \Psi_{\text{MerSat}} = 4.91 \pm 0.02, \\ \Psi_{\text{EarSat}} = 3.090 \pm 0.001, \\ \Psi_{\text{MarSat}} = 2.4984 \pm 0.0008, \\ \Psi_{\text{JupSat}} = 1.36 \pm 0.06, \\ \Psi_{\text{MerUra}} = 6.9 \pm 0.1, \\ \Psi_{\text{EarUra}} = 4.383 \pm 0.009, \\ \Psi_{\text{MarUra}} = 3.543 \pm 0.005, \\ \Psi_{\text{JupUra}} = 1.9 \pm 0.3. \end{array} \right. \quad (13)$$

The exact eccentricity-dependent factor

$$F(e_A, e_B) = \sqrt{\frac{1 - e_A^2}{1 - e_B^2}} \left(\frac{e_B}{e_A}\right)^2 \left(\frac{-1 + \sqrt{1 - e_A^2}}{-1 + \sqrt{1 - e_B^2}}\right) \quad (14)$$

of eq. (11) is always close to unity, so that its impact on the results of eq. (13) is negligible. The errors in  $\Psi_{AB}$  due to  $\delta a$  and  $\delta\Delta\dot{\omega}$  have been conservatively computed as

$$\begin{aligned} \delta\Psi_{AB} \leq & \left| \frac{\Delta\dot{\omega}^A}{\Delta\dot{\omega}^B} \right| \left( \frac{\delta\Delta\dot{\omega}^A}{|\Delta\dot{\omega}^A|} + \frac{\delta\Delta\dot{\omega}^B}{|\Delta\dot{\omega}^B|} \right) + \\ & + \frac{1}{2} \left( \frac{a^B}{a^A} \right)^{\frac{3}{2}} \left( \frac{\delta a^A}{a^A} + \frac{\delta a^B}{a^B} \right) F(e_A; e_B); \end{aligned} \quad (15)$$

we did not optimistically summed the biased terms in a root-sum-square fashion because of the existing correlations among the estimated extra-precessions, although they are low with a maximum of about<sup>5</sup> 20% between Mercury and the Earth. It turns out that by re-scaling the formal errors in the semimajor axes released by Pitjeva (2005a) by a factor 10, 100 or more the results of eq. (13) do not change because the major source of uncertainty is given by far by the errors in the perihelion precessions. In regards to them let us note that the tightest constraints come from the pairs involving Saturn and Uranus<sup>6</sup> for which Pitjeva (2005a) used only optical data, contrary to Jupiter<sup>7</sup>. Now, even by further re-scaling by a factor 10 the errors in their perihelion extra-rates, i.e. by a factor 100 their formal, statistical uncertainties, the results do not change (apart from A=Jupiter B=Uranus). In the case of the pairs A=Earth, B=Uranus and A=Mars, B=Uranus even if the real error in the correction to the perihelion precession of Uranus was 1,000 times larger than the estimated formal one the answer of eq. (13) would still remain negative. For A=Mars and B=Saturn a re-scaling of even a factor 10,000 of the formal error in the perihelion extra-rate of Saturn would be possible without changing the situation. In the case A=Earth, B=Saturn a re-scaling of 1,000 for the Saturn’s perihelion extra-precession would not alter the result.

---

<sup>5</sup>L.I. thanks E.V. Pitjeva for such an information.

<sup>6</sup>But not for the pairs A=Uranus/Saturn, B=Saturn/Uranus yielding values of  $\Psi_{AB}$  compatible with zero.

<sup>7</sup>This is why a re-scaling of 10 of the formal error in its perihelion extra-precession should be adequate.

As a conclusion, the logarithmic correction to the Newtonian potential of eq. (4) is ruled out by the present-day observational determinations of the Solar System’s planetary motions.

### 3. The power-law corrections in the $f(R)$ extended theories of gravity

In recent years there has been a lot of interest in the so called  $f(R)$  theories of gravity (Sotiriou and Faraoni 2008). In them, the gravitational Lagrangian depends on an arbitrary analytic function  $f$  of the Ricci scalar curvature  $R$  (see (Capozziello and Francaviglia 2008) and references therein). These theories are also referred to as “extended theories of gravity”, since they naturally generalize, on a geometric ground, GR, in the sense that when  $f(R) = R$  the action reduces to the usual Einstein-Hilbert one, and Einstein’s theory is obtained. It has been showed (Capozziello and Francaviglia 2008) that these theories provide an alternative approach to solve, without the need of dark energy, some puzzles connected to the current cosmological observations and, furthermore, they can explain the dynamics of the rotation curves of the galaxies, without requiring dark matter. Indeed, for instance Capozziello et al. (2007), starting from  $f(R) = f_0 R^k$ , obtained a power-law correction to the Newtonian gravitational potential of the form

$$V_\beta = -\frac{GM}{r} \left( \frac{r}{r_c} \right)^\beta, \quad (16)$$

(where  $\beta$  is related to the exponent  $k$  of the Ricci scalar  $R$ ), and they applied eq. (16) to a sample of 15 low surface brightness (LSB) galaxies with combined HI and H $\alpha$  measurements of the rotation curve extending in the putative dark matter dominated region. They obtained a very good agreement between the theoretical rotation curves and the data using only stellar disk and interstellar gas when the slope  $k$  of the gravity Lagrangian is set to the value  $k = 3.5$  (giving  $\beta = 0.817$ ) obtained by fitting the SNeIa Hubble diagram with the assumed power-law  $f(R)$  model and no dark matter.

Here we wish to put on the test eq. (16) in the Solar System with the approach previously examined.

First, let us note that an extra-radial acceleration

$$\mathbf{A}_\beta = \frac{(\beta - 1)GM}{r_c^\beta} r^{\beta-2} \hat{\mathbf{r}} \quad (17)$$

can be obtained from eq. (16); let us now work out the secular precession of the pericentre of a test particle induced by eq. (17) in the case  $\beta - 2 < 0$ . By proceeding as in Section 2 we get<sup>8</sup>

$$\langle \dot{\omega} \rangle_\beta = \frac{(\beta - 1)\sqrt{GM}}{2\pi r_c^\beta} a^{\beta-\frac{3}{2}} G(e; \beta), \quad (18)$$

with

$$G(e; \beta) = -\frac{\sqrt{1-e^2}}{e} \int_0^{2\pi} \frac{\cos E - e}{(1 - e \cos E)^{2-\beta}} dE. \quad (19)$$

Since we are interested in taking the ratios of the perihelia, there is no need to exactly compute eq. (19): the eccentricities of the Solar System planets are small and similar for all of them, so that we will reasonably assume that  $G(e_A; \beta)/G(e_B; \beta) \approx 1$ . Note that<sup>9</sup>  $\beta = 0$  would yield a vanishing apsidal precession because  $G(e; 0) = 0$ . In fact, the case  $\beta = 0$  is compatible with all the estimated extra-rates of Table 1 and Table 2;  $\beta = 0$ , within the errors, is also the outcome of different tests performed in the Solar System by Zakharov et al. (2006).

### 3.1. Comparison with data

According to Capozziello et al. (2007), a value of  $\beta = 0.817$  in eq. (16) gives very good agreement between the theoretical rotation curves and the data, without need of dark matter. Again, if we consider a pair of planets A and B and take the ratio of their estimated extra-rates of perihelia, by taking  $\beta = 0.817$ , we may define

$$\Xi_{AB} = \left| \frac{\Delta \dot{\omega}_A}{\Delta \dot{\omega}_B} - \left( \frac{a_B}{a_A} \right)^{0.683} \right|. \quad (20)$$

---

<sup>8</sup>See also (Adkins and McDonnell 2007).

<sup>9</sup>The case  $\beta = 1$  is not relevant because it would yield a constant additional potential and no extra-force.



Of course, such kind of test could not be applied to the  $\beta = 0$  case since, in this case, we would have the meaningless prediction  $\mathcal{P}_A/\mathcal{P}_B = 0/0$ . If the modification of the gravitational potential of eq. (16) exists and is accounted for by the estimated corrections  $\Delta\dot{\varpi}$  to the standard Newton-Einstein perihelion precessions, then the quantities  $\Xi_{AB}$  must be compatible with zero, within the errors. Instead, what we obtain, from Table 1, Table 2, Table 4 and eq. (18), is

Table 4: A B denotes the pair of planets used;  $\Pi = \Delta\dot{\omega}_A/\Delta\dot{\omega}_B$ ,  $\mathcal{B} = (a_B/a_A)^{0.683}$  (corresponding to  $\beta = 0.817$ ). The perihelion extra-rates for the inner planets have been retrieved from (Pitjeva 2005b); their errors are not the formal, statistical ones. The perihelion extra-rates for the outer planets come from (Pitjeva 2006a); their formal errors have been re-scaled by a factor 10. The uncertainties in the semimajor axes have been retrieved from Table IV of (Pitjeva 2005a): they are the formal ones, but, as can be noted, their impact is negligible. While  $\Pi$  is always compatible with zero, this is definitely not the case for  $\mathcal{B}$ .

A B	$\Pi$	$\mathcal{B}$
Earth Mercury	$0.05 \pm 0.19$	$0.523 \pm \mathcal{O}(10^{-13})$
Mars Mercury	$-0.03 \pm 0.2$	$0.392 \pm \mathcal{O}(10^{-13})$
Mercury Jupiter	$-0.6 \pm 4.1$	$5.898 \pm \mathcal{O}(10^{-9})$
Earth Jupiter	$-0.03 \pm 0.25$	$3.084 \pm \mathcal{O}(10^{-9})$
Mars Jupiter	$0.02 \pm 0.17$	$2.313 \pm \mathcal{O}(10^{-10})$
Mercury Saturn	$0.004 \pm 0.017$	$8.921 \pm \mathcal{O}(10^{-8})$
Earth Saturn	$0.0002 \pm 0.0011$	$4.665 \pm \mathcal{O}(10^{-9})$
Mars Saturn	$-0.0001 \pm 0.0009$	$3.499 \pm \mathcal{O}(10^{-9})$
Jupiter Saturn	$-0.006 \pm 0.060$	$1.512 \pm \mathcal{O}(10^{-9})$
Mercury Uranus	$-0.006 \pm 0.152$	$14.383 \pm \mathcal{O}(10^{-8})$
Earth Uranus	$-0.0003 \pm 0.0087$	$7.522 \pm \mathcal{O}(10^{-8})$
Mars Uranus	$0.0002 \pm 0.0048$	$5.642 \pm \mathcal{O}(10^{-8})$
Jupiter Uranus	$0.01 \pm 0.31$	$2.438 \pm \mathcal{O}(10^{-8})$

$$\left\{ \begin{array}{l}
 \Xi_{\text{EarMer}} = 0.5 \pm 0.2, \\
 \Xi_{\text{MarMer}} = 0.4 \pm 0.2, \\
 \Xi_{\text{MerJup}} = 6.5 \pm 4.2, \\
 \Xi_{\text{EarJup}} = 3.1 \pm 0.2, \\
 \Xi_{\text{MarJup}} = 2.3 \pm 0.2, \\
 \Xi_{\text{MerSat}} = 8.92 \pm 0.02, \\
 \Xi_{\text{EarSat}} = 4.666 \pm 0.001, \\
 \Xi_{\text{MarSat}} = 3.4997 \pm 0.0008, \\
 \Xi_{\text{JupSat}} = 1.52 \pm 0.06, \\
 \Xi_{\text{MerUra}} = 14.4 \pm 0.1, \\
 \Xi_{\text{EarUra}} = 7.523 \pm 0.008, \\
 \Xi_{\text{MarUra}} = 5.642 \pm 0.005, \\
 \Xi_{\text{JupUra}} = 2.4 \pm 0.3.
 \end{array} \right. \quad (21)$$

It is remarkable to note that even if the formal error in the Saturn’s apsidal extra-precession was re-scaled by a factor 10,000 instead of 10, as done in this paper, the pairs A=Earth B=Saturn and

A=Mars B=Saturn would still rule out eq. (18). A re-scaling by 1,000 of the Uranus' perihelion extra-precession would still be fatal, as shown by the pairs A=Earth B=Uranus and A=Mars B=Uranus.

Thus, also the power-law correction to the Newtonian potential of eq. (16) with  $\beta = 0.817$  is ruled out. Criticisms to  $R^k$  models of modified gravity were raised on different grounds by Nojiri and Odintsov (2003) who proposed more realistic models in (Cognola et al. 2008). Moreover, it is well-established now that DM shows also particle-like properties. In this respect, the proposal of  $R^k$  gravity as DM (thanks to a change of the Newton potential) is not considered as a realistic one now. A more realistic DM candidate from  $f(R)$  gravity was suggested by Nojiri and Odintsov (2008). They show that not only a correction to the Newton potential appears, but also composite graviton degree of freedom shows particle-like behavior, as requested by DM data.

#### 4. Comments and conclusions

In this paper we have studied the secular precession of the pericentre of a test particle in motion around a central mass  $M$  whose Newtonian gravitational potential exhibits a correction which has a logarithmic and power-law behavior. In order to put on the test the hypothesis that such extra-forces are not zero we devised a suitable test by taking into account the ratios (and not the extra-rates of the perihelia of each planet at a time separately, or a linear combination of them, since their uncertainties would fatally prevent to obtain any useful constraints) of the corrections to the secular precessions of the longitudes of perihelia estimated by E.V. Pitjeva for several pairs of planets in the Solar System. The results obtained, resumed by eq. (13) and eq. (21), show that modifications of the Newtonian potentials like those examined in this paper are not compatible with the currently available apsidal extra-precessions of the Solar System planets. Moreover, the hypothesis that the examined exotic force terms are zero, which cannot be tested by definition with our approach, is compatible with each perihelion extra-rate separately, in

agreement with our results. It must be noted that to give the hypothesized modifications of the Newtonian law the benefit of the doubt, and given that the formal errors in  $a$  and  $\Delta\dot{\omega}$  of the outer planets are probably underestimates of the true uncertainties, we multiplied them by a factor ten or even more, as suggested to one of us (L.I.) by some leading experts in the ephemerides generation field like E.M. Standish, but the answers we obtained were still negative. Another thing that should be pointed out is that, in principle, in assessing  $\Psi_{AB}$  and  $\Xi_{AB}$  one should have used  $\Delta\dot{\omega}^* = \Delta\dot{\omega} - \delta\dot{\omega}_{\text{canonical}}$ , where  $\delta\dot{\omega}_{\text{canonical}}$  represents the mismodelled part of the modelled standard Newton/Einstein precessions. However, neglecting them did not affect our conclusions, as can be easily noted by looking at the  $\Pi$  and  $\mathcal{A}/\mathcal{B}$  columns in Table 3 and Table 4. Indeed, the residual precessions due to the imperfect knowledge of, e.g., the solar quadrupole mass moment (Pitjeva 2005b)  $\delta J_2/J_2 \approx 10\%$  are of the order of  $\leq 10^{-3}'' \text{ cy}^{-1}$  (Iorio 2007c), and even smaller are the mismodelled precessions due to other potential sources of errors like the asteroid ring or the Kuiper Belt objects (Iorio 2007d).

However, caution is in order because, at present, no other teams of astronomers have estimated their own corrections to the Newtonian/Einsteinian planetary perihelion rates, as it would be highly desirable. If and when we will have, say, two independent determinations of the anomalous perihelion rate of a given planet  $x = x_{\text{best}} \pm \delta x$  and  $y = y_{\text{best}} \pm \delta y$  we could see if they are compatible with each other and take their difference  $|x_{\text{best}} - y_{\text{best}}|$  as representative of the real uncertainty affecting the apsidal extra-precession of that planet. Moreover, it would be interesting to see if for different sets of estimated corrections to the perihelion rates the figures for  $\Pi$  in Table 3 and Table 4 change by an extent sufficient to alter the conclusions of eq. (13) and eq. (21).

## Acknowledgments

M.L. Ruggiero acknowledges financial support from the Italian Ministry of University and Research (MIUR) under the national program “Cofin 2005” - *La pulsar doppia e oltre: verso una nuova era della ricerca sulle pulsar*.



## REFERENCES

- Adkins, G.S., McDonnell, J.: *Phys. Rev. D* **75**, 082001 (2007)
- Bennet, C.L. et al.: *Astrophys. J. Suppl.* **148**, 1 (2003)
- Binney, J., Tremaine, S.: *Galactic Dynamics*. Princeton University Press (1987)
- Capozziello, S., Cardone, V.F., Lambiase, G., Troisi, A.: *Int. J. Mod. Phys. D* **15**, 69 (2006)
- Capozziello, S., Cardone, V.F., Troisi, A.: *Mon. Not. Roy. Astron. Soc.* **375**, 1423 (2007)
- Capozziello, S., Francaviglia, M.: *Gen. Relativ. Gravit.* **40**, 357 (2008)
- Cognola, G., Elizalde, E., Nojiri, S., Odintsov, S.D., Sebastiani, L., Zerbini, S.: *Phys. Rev. D* **77**, 046009 (2008)
- Cretton, N., Rix, H.-W., de Zeeuw, P.T.: *Astroph. J.* **536**, 319 (2000)
- Damour, T.: *Albert Einstein Century International Conference* **861**, 135 (2006)
- Esposito-Farèse, G.: *Class. Quantum Grav.* **25**, 114017 (2008)
- Fabris, J.C., Campos, J.P.: *Gen. Relativ. Gravit.* DOI:10.1007/s10714-008-0654-0 (2008)
- Iorio, L.: *J. High En. Phys.* **10**, 41 (2007a)
- Iorio, L.: *Adv. High En. Phys.* **2007**, 90731 (2007b)
- Iorio, L.: *Planet. Space Sci.* **55**, 1290 (2007c)
- Iorio, L.: *Planet. Space Sci.* **55**, 2045 (2007d)
- Kinney, W.H., Brisudova, M.: *Ann. N.Y. Acad. Sci.* **927**, 127 (2001)
- Kirillov, A.A.: *Phys. Lett. B*, **632**, 453 (2006)
- Nojiri, S., Odintsov, S.D.: *Phys. Rev. D*, **68**, 123512 (2003)

- Nojiri, S., Odintsov, S.D.: *Int. J. Geom. Meth. Mod. Phys.*, **4**, 115 (2007)
- Nojiri, S., Odintsov, S.D.: in Epp, V. (ed.) *Problems of Modern Theoretical Physics. A Volume in Honour of Prof. I.L.Buchbinder in the Occasion of His 60th Birthday*. Tomsk State Pedagogical University Press. (2008). pp. 266-285. arXiv:0807.0685v1 [hep-th]
- Peacock, J.: *Cosmological Physics*. Cambridge University Press (1999)
- Peebles, P.J.E.: *Principles of Physical Cosmology*. Princeton University Press (1993)
- Perlmutter, S. et al.: *Astroph. J.* **517**, 565 (1999)
- Pitjeva, E.V.: *Sol. Syst. Res.* **39**, 176 (2005a)
- Pitjeva, E.V.: *Astron. Lett.* **31**, 340 (2005b)
- Pitjeva, E.V.: Private communication to Iorio, L. (2006a)
- Pitjeva, E.V.: The Dynamical Model of the Planet Motions and EPM Ephemerides. Abstract no. 14 presented at Nomenclature, Precession and New Models in Fundamental Astronomy, 26th meeting of the IAU, Joint Discussion 16, Prague, Czech Republic, 22-23 August (2006)
- Riess, A.G. et al.: *Astron. J.* **116**, 1009 (1998)
- Ruggiero, M.L., and Iorio, L., in preparation, (2007)
- Sanders, R.H.: *Mon. Not. Roy. Astron. Soc.* **370**, 1519 (2006)
- Sobouti, Y.: *Astron. Astrophys.* **464**, 921 (2007)
- Soleng, H.H.: *Gen. Relativ. Gravit.* **27**, 367 (1995)
- Sotiriou, T., Faraoni, V.: arXiv:0805.1726 [gr-qc] (2008).
- Tonry, J.L. et al.: *Astroph. J.* **594**, 1 (2003)
- van Moorsel, G.A.: *Astron. Astrophys.* **176**, 13 (1987)



Will, C.M.: Living Rev. Relativity **9** (2006)

Zakharov, A.F., Nucita, A.A., De Paolis, F., Ingrosso, G.: Phys. Rev. D **74**, 107101 (2006)

Zwicky, F.: Helv. Phys. Acta **6**, 110 (1933)