

Gauge k -vortices

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(Dated: October 24, 2018)

We consider gauge vortices in symmetry breaking models with a non-canonical kinetic term. This work extends our previous study on global topological k -defects (hep-th/0608071), including a gauge field. The model consists of a scalar field with a non-canonical kinetic term, while for the gauge field the standard form of its kinetic term is preserved. Topological defects arising in such models, k -vortices, may have quite different properties as compared to “standard” vortices. This happens because an additional dimensional parameter enters the Lagrangian for the considered model — a “kinetic” mass. We briefly discuss possible consequences for cosmology, in particular, the formation of cosmic strings during phase transitions in the early universe and their properties.

Keywords: topological defects, non-linear field theories

I. INTRODUCTION

Vortices are a class of topological defects which may form as a result of symmetry-breaking phase transitions. In condensed matter physics linear defects arise rather commonly. Well-known examples are flux tubes in superconductors [1] and vortices in superfluid helium-4. In cosmology topological defects attract much interest because they might appear in a rather natural way during phase transitions in the early universe. The breaking of discrete symmetries leads to the appearance of domain walls, while the breaking of a global or a local $U(1)$ -symmetry is associated with global [2] and local [3] cosmic strings, respectively. Localized defects or monopoles may form in gauge models possessing a $SO(3)$ symmetry which is spontaneously broken to $U(1)$ [4, 5].

Many properties of topological defects arising in symmetry-breaking models with a canonical kinetic term are well-known, see, e.g. [6, 7]. Adding non-linear terms to the kinetic part of the Lagrangian has interesting consequences for topological defects. For example, defects can exist without a symmetry-breaking potential term [8]. Non-standard kinetic terms in the form of some non-linear function of the canonical term may arise in string theory, due to the presence of higher-order corrections to the effective action for the scalar field. Non-canonical kinetic structures appear also commonly in effective field theories.

During the last years Lagrangians with non-canonical fields were intensively studied in the cosmological context. So-called k -fields were first introduced in the context of inflation [9] and then k -essence models were suggested as solution to the cosmic coincidence problem [10, 11]. Tachyon matter [12] and ghost condensates [13] are other examples of non-canonical fields in cosmology. An interesting application of k -fields is the explanation of

dark matter as a self-gravitating coherent state of k -field matter [14]. The production of strong gravitational waves in models of inflation with nontrivial kinetic term was considered in [15]. The effects of scalar fields with non-canonical kinetic terms in the neighborhood of a black hole were investigated in [16].

Recently, symmetry-breaking models with k -essence-like terms have been discussed in literature. General properties of global topological defects appearing in such models were studied in [17]. It was shown that the properties of such defects (dubbed k -defects) are quite different from “standard” global domain walls, vortices and monopoles. In particular, depending on the concrete form of the kinetic term, the typical size of such a defect can be either much larger or much smaller than the size of a standard defect with the same potential term. A detailed study of global defect solutions for one space dimension was carried out in [18]. A self-gravitating k -monopole was considered in [19]. In [20] the authors argued that a special type of k -defects may be viewed as “compactons”, i.e. solutions with a compact support. Global strings with a Dirac-Born-Infeld (DBI) term were considered in [21].

In this paper we study properties of gauge vortices arising in a model with a k -essence-like kinetic term and a symmetry breaking potential. We dubb such defects “gauge k -vortices”, in analogy to global k -defects. We extend our previous investigation on k -defects [17] including a gauge field into the model. As for the global k -defects, the scalar field has a non-canonical structure of the kinetic term, while for the gauge field we keep the canonical form of the kinetic term. The existence of non-trivial configurations is ensured by the symmetry-breaking potential term. The generic feature of the model with a non-canonical kinetic term is the appearance of a new scale — the kinetic “mass”. The presence of a new mass scale in the model radically changes basic properties of vortices.

We show that generally the size of the scalar core of the gauge vortex solution is almost independent on the presence of the gauge field. Its value can be approxi-

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mated by the core' size of the global k -vortex with the same kinetic structure [17]. With an additional, natural assumption we find that the vector core has roughly the same size as a standard vortex. A particularly interesting result is that the mass of a vortex radically vary depending on the choice of kinetic term. As the concrete examples we study numerically the vortex solutions for the models with DBI and with a power-law kinetic terms.

The paper is organized as follows. In Sec. II, we describe our model and derive its equations of motion and the energy functional for a vortex solution. General properties of k -vortices are studied in Sec. III. In Sec. IV we find constraints on the parameters of the model. Numerical solutions for particular choices of the non-canonical kinetic term are presented in Sec. V. We summarize and discuss results and cosmological applications in the concluding Sec. VI.

II. MODEL

We consider the action

$$S = \int d^4x \left[M^4 K(X/M^4) - U(f) - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \right], \quad (1)$$

with

$$X = (D_\mu \phi)(D^\mu \phi)^*, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

and

$$D_\mu \equiv \partial_\mu - ieA_\mu.$$

The potential term which provides the symmetry breaking is given by

$$U(\phi) = \frac{\lambda}{4} (|\phi|^2 - \eta^2)^2,$$

where η has dimension of a mass, while λ is a dimensionless constant. Note that throughout this paper we use a metric with signature $(+, -, -, -)$. The kinetic term $K(X)$ in (1) is in general some non-linear function of X . The action (1) contains three mass scales: The "usual" scalar and vector masses, $\sqrt{\lambda}\eta$ and $e\eta$ correspondingly, and the "kinetic" mass M . It is worth to note that a kinetic term that is non-linear in X unavoidably leads to a new scale in the action. In the standard case $K = X/M^4$ and the kinetic mass M drops out from the action. For non-trivial choices of the kinetic term, the kinetic mass enters the action and changes the properties of the resulting topological defects.

In what follows it is convenient to make the following redefinition of variables to dimensionless units,

$$x \rightarrow \frac{x}{M}, \quad \phi \rightarrow M\phi, \quad A_\mu \rightarrow MA_\mu.$$

In terms of the new variables the energy density ϵ is also dimensionless: $\epsilon \rightarrow M^4\epsilon$. It is easy to see that

$D_\mu \rightarrow MD_\mu$, $X \rightarrow M^4X$ and the action (1) becomes

$$S = \int d^4x \left[K(X) - V(\phi) - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \right], \quad (2)$$

where

$$V(\phi) = \frac{\lambda}{4} (|\phi|^2 - v^2)^2, \quad (3)$$

with $v \equiv \eta/M$ being a dimensionless quantity. One can calculate the energy-momentum tensor from the action (2),

$$T_{\mu\nu} = 2K_X |D_\mu \phi|^2 - g_{\mu\nu} [K(X) - V(f)] - F_{\mu\alpha} F_\nu^\alpha + \frac{1}{4} g_{\mu\nu} F^{\alpha\beta} F_{\alpha\beta},$$

where we denoted $K_X \equiv dK/dX$, $K_{XX} \equiv d^2K/dX^2$ etc. In the gauge $A_0 = 0$, the energy density for a static configuration, $\dot{\phi} = 0$, $\partial A_i = 0$, is

$$T_0^0 = -K(X) + V(\phi) + \frac{1}{4} F_{ij}^2. \quad (4)$$

Note also that for static configurations $X = -D_i \phi (D_i \phi)^*$. The mass per unit length of a vortex, E , can be expressed as:

$$E = \int \left[-K(-|D_i \phi|^2) + V(\phi) + \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \right] d^2x. \quad (5)$$

From the variation of the action (2) with respect to ϕ^* and A^μ we obtain as equations of motion (EoM)

$$K_X D_\mu D^\mu \phi + K_{XX} X_{,\mu} D^\mu \phi + \frac{dV}{d\phi^*} = 0, \quad (6)$$

$$\partial_\mu F^{\mu\nu} = e j^\nu, \quad (7)$$

where the current j_μ is given by

$$j_\mu = -iK_X [\phi^* D_\mu \phi - \phi (D_\mu \phi)^*].$$

(Note the additional K_X in the above expression as compared to the standard case.) One can check that the current j_μ is conserved,

$$\partial_\mu j^\mu = 0,$$

similar to the standard case. To obtain the solution describing a vortex we use the following ansatz,

$$\phi(x) = e^{i\theta} f(r), \quad (8)$$

$$A_i(x) = -\frac{1}{er^2} \epsilon_{ij} r_j \alpha(r),$$

where $r = (x^2 + y^2)^{1/2}$. It is worth to note that we use the same ansatz (8) as in the standard case. Substituting (8) into (6) and (7) we obtain the EoM for the functions $f(r)$ and $\alpha(r)$,

$$-K_X \left[\frac{1}{r} \frac{d(rf')}{dr} - \frac{f(1-\alpha)^2}{r^2} \right] - K_{XX} X' f' + \frac{\lambda}{2} (f^2 - v^2) f = 0, \quad (9)$$

$$-\frac{d}{dr} \left(\frac{\alpha'}{r} \right) - \frac{2e^2 f^2}{r} (1-\alpha) K_X = 0. \quad (10)$$

One can check that in the standard case, $K(X) = X$, Eqs. (9) and (10) take the familiar form of EoMs for “usual” vortices.

We assume that the kinetic term $K(X)$ has the standard asymptotic behavior at small X . This means that in the perturbative regime in trivial backgrounds the dynamics of the considered system is the same as with a canonical kinetic term. This requirement is introduced to avoid troubles at $X = 0$: In the case of X^δ with $\delta < 1$ there is a singularity at $X = 0$, while for $\delta > 1$ the system becomes non-dynamical at $X = 0$ [17]. One can understand this better in terms of an “emergent geometry”: Because of the non-linearity of the EoMs, small fluctuations of the scalar field feel an “effective” metric, which in general differs from the gravitational one (in our case — from the Minkowski metric). As it was shown in [23], in the case when a kinetic term does not coincide with the canonical one (probably, up to some constant) in the limit of small X , the effective metric for small perturbations diverges as $X \rightarrow 0$. This means that such models are physically meaningless.

In the opposite limit, $X \gg 1$, we restrict our attention to the modifications of the kinetic term having the following asymptotic,

$$K(X) = -(-X)^n.$$

Note that for a static configuration $X < 0$ and a minus sign in the expression above for $K(X)$ provides a positive contribution to the energy density (4). Below we find the criteria for the Lagrangians to have the desired asymptotic $X \gg 1$ in the core of a vortex. Summarizing, we will consider kinetic terms with the following asymptotic behavior,

$$K(X) = \begin{cases} X, & X \ll 1, \\ -(-X)^n, & X \gg 1. \end{cases} \quad (11)$$

Assuming $X \gg 1$, one can easily obtain from (11), (9) and (10) EoMs in this regime,

$$\left[\frac{1}{r} \frac{d(rf')}{dr} - \frac{f(1-\alpha)^2}{r^2} \right] + (n-1)(\ln X)' f' - (-X)^{1-n} \frac{\lambda}{2n} (f^2 - v^2) f = 0, \quad (12)$$

$$\frac{d}{dr} \left(\frac{\alpha'}{r} \right) + \frac{2ne^2 f^2}{r} (1-\alpha) (-X)^{n-1} = 0. \quad (13)$$

As a particular example we choose a DBI-like kinetic term for the scalar field,

$$K(X) = 1 - \sqrt{1 - 2X}. \quad (14)$$

It is easy to see that for this choice in the limit $X \gg 1$ the kinetic term is of form (11) with $n = 1/2$. Another particular example we will study is a power-law form of the Lagrangian:

$$K(X) = X + X^3. \quad (15)$$

III. GENERAL PROPERTIES

The EoMs (12), (13) for arbitrary $K(X)$ are highly non-linear and cannot be solved analytically. We restrict our attention to the study of vortices arising from Lagrangians having the asymptotic behavior (11) for the kinetic term. Although the EoMs can not be integrated even in this case, some general features can be extracted without the knowledge of explicit solutions.

At some point of our estimations we will use an additional simplifying assumption, which makes the understanding of the results more transparent. We will assume that the parameters of the Lagrangian satisfy the following natural relation,

$$e \sim \sqrt{\lambda}, \quad (16)$$

which means that in the linear regime for the kinetic term, $K(X) = X$, the “scalar” and “vector” masses are of the same order. Thus the assumption (16) reduces the number of different scales in the model from 3 to 2: one is the usual “scalar” or “vector” mass, $e\eta \sim \sqrt{\lambda}\eta$, and the other is a new kinetic mass M .

A. The region $r \rightarrow 0$

We start our study from the region close to the center of a vortex, $r \rightarrow 0$. As we assume that in the core of a defect the kinetic term can be approximated by (11), we must require that for models (14) and (15), $X \gg 1$. In the opposite case, $X \lesssim 1$, we end up with a solution which does not deviate much from the standard one. For $r \rightarrow 0$ we search a solution in the following form,

$$\begin{aligned} f(r) &= A_f r + B_f r^3 + O(r^5), \\ \alpha(r) &= A_\alpha r^2 + B_\alpha r^4 + O(r^6) \end{aligned}$$

with unknown constants A_f , B_f , A_α , B_α . Substituting the above expressions into (12) and (13) we find that A_f and A_α are arbitrary, while the others are

$$\begin{aligned} B_\alpha &= -2^{n-3} e^2 n A_f^{2n}, \\ B_f &= -\frac{2^{n-5}}{n^2} \lambda v^2 A_f^{2n-1} + \frac{n(n-2)}{4} A_f A_\alpha. \end{aligned}$$

The standard asymptotics for $K(X) = X$ are recovered from the above expression by setting $n = 1$. Note that the constants A_f and A_α are left undetermined, which means that the size of a defect and its mass are undetermined too. It is possible, however, to estimate these quantities without solving explicitly EoMs, as we will see in III B and III C.

B. Structure of a vortex

The model (1) contains a complex scalar field and a gauge vector field. In accordance to this, there are two

distinct cores for a vortex solutions: One is associated with the scalar field and the other with the vector field. The typical sizes of the cores depend on the parameters of the Lagrangian. In this subsection we will estimate the typical sizes of cores without explicitly solving EoMs. In what follows it will be helpful to use an additional rescaling:

$$f \rightarrow v f, \quad r \rightarrow r L_H, \quad (17)$$

where

$$L_H = v \epsilon^{-1/2n}, \quad (18)$$

and ϵ was defined as

$$\epsilon \equiv \lambda v^4, \quad (19)$$

in analogy with global defects [17]. Later we will see that the quantity ϵ (19) corresponds to the energy density inside the scalar core of a vortex. The rescaling (17) brings the EoMs (12), (13) to the following form,

$$\left[\frac{1}{r} \frac{d(rf')}{dr} - \frac{f(1-\alpha)^2}{r^2} \right] + (n-1) (\ln X)' f' - \frac{1}{2n} (-X)^{1-n} (f^2 - 1) f = 0, \quad (20)$$

$$\frac{d}{dr} \left(\frac{\alpha'}{r} \right) + \frac{2n}{\gamma} (-X)^{n-1} \frac{f^2(1-\alpha)}{r} = 0, \quad (21)$$

where

$$\gamma = \frac{\lambda}{e^2} \epsilon^{-2(n-1)/n}. \quad (22)$$

Notice that in the standard case, $n = 1$, the parameter γ coincides with the usual parameter, defined as the ratio of the scalar and vector masses. The EoM for $f(r)$, Eq. (20), contains parameters of order of 1 as well as the function α , going from 0 to 1. Therefore one can guess that the typical scale on which the function $f(r)$ varies is of the order L_H . Thus the typical size of the scalar core, l_H , is given by

$$l_H \sim L_H, \quad (23)$$

and is almost independent on γ and e . In fact, the value l_H (23) coincides (up to an irrelevant numerical factor of order of 1) with the size of the core in the case of a global k -string, found in [17]. Very roughly speaking, the scalar core remains unaffected by the gauge field. Intuitively it can be understood as follows. Topological defects exist due to the presence of a potential, which provides symmetry breaking for the scalar field ϕ ; the gauge field is in a sense merely an auxiliary component. Therefore, the presence of the gauge field should not radically change the size of the scalar core.

To estimate the size of the vector core is a more tricky task. First of all we note that when $\gamma \sim 1$, i.e.

$$\frac{e}{\sqrt{\lambda}} \epsilon^{(n-1)/n} \sim 1, \quad (24)$$

the size of the vector core, l_V , is of order of the size of the scalar core,

$$l_V \sim l_H, \quad (25)$$

since in this case the EoMs (20), (21) do not contain any large or small parameters.

To consider other cases, when the vector core is much larger/smaller than the scalar core, let us turn back to Eq. (13). The inflection point for the function $\alpha(r)$ is at the point $r \sim l_V$. Then, taking also into account that $\alpha'(r) \sim 1/l_V$ at $r \sim l_V$, we obtain from (13),

$$\frac{1}{l_V^2} \sim e^2 f^2 (-X)^{n-1}. \quad (26)$$

Let us now assume that the vector core is much smaller than the scalar one, $l_V \ll l_H$. From (26) using the estimates, $f \sim v l_V/l_H$ and $(-X)^n \sim \lambda v^4$ at $r \sim l_V$, we find

$$l_V \sim \frac{1}{(e^2 \epsilon)^{1/4}}. \quad (27)$$

Note, that the above result is valid if $l_V \ll l_H$, which can be recasted as follows, using (27):

$$\frac{e}{\sqrt{\lambda}} \epsilon^{(n-1)/n} \gg 1. \quad (28)$$

In the opposite case, $l_V \gg l_H$, the non-linearity in X is negligible, i.e. one has to set $n = 1$ in (26). Then we immediately find the size of the vector core as

$$l_V \sim \frac{1}{ev}. \quad (29)$$

Eq. (29) is valid for $l_V \gg l_H$, or

$$\frac{e}{\sqrt{\lambda}} \epsilon^{(n-1)/2n} \ll 1. \quad (30)$$

For us the most interesting case is $\epsilon \gg 1$, as we will see later, this corresponds to the regime when the non-linearity in X become important. Taking into account our assumption (16) we may summarize our results for $\epsilon \gg 1$ as follows. The size of the scalar core (with the restored physical units) is given by

$$l_H = \frac{\eta}{M^2} \left[\lambda (\eta/M)^4 \right]^{-1/2n}, \quad (31)$$

and the size of the vector core is

$$l_V \sim \frac{1}{e\eta}. \quad (32)$$

It is worth to note that the vector core in our model is roughly as large as in the standard case. This is what one can naively expect from the action (1): The kinetic term for the vector field is unchanged as compared with the standard Lagrangian, so it is unlikely that the vector core varies much.

C. Vortex' mass

Another way to see how the parameter γ (22) appears in the model is to make the change of variables in the action (2) as follows:

$$\phi = vf, \quad x = L_H y, \quad A_\mu = \frac{B_\mu}{eL_H}, \quad (33)$$

with L_H given by (18). Substituting the rescaling (33) into (2) we immediately find the functional of the energy density (with the restored physical units):

$$E = \eta^2 \left[\lambda (\eta/M)^4 \right]^{(n-1)/n} \mathfrak{F}(n, \gamma), \quad (34)$$

where

$$\mathfrak{F}(n, \gamma) = \int d^2y \left[\frac{\gamma}{4} F_{\mu\nu} F^{\mu\nu} + (D_i f)^{2n} + (|f|^2 - 1)^2 \right]. \quad (35)$$

It is important to note that the above expression is only valid in the non-linear regime in X , i.e. when $X \gg 1$ inside the scalar core. In addition, we have to require that the vector core is not larger than the scalar one, $l_V \ll l_H$; otherwise the vector core is partly outside the scalar one and Eq. (34) is inapplicable.

To find the energy density of the vortex for particular parameters of the Lagrangian, one needs to calculate the functional of the energy density (34) applied to a solution. An alternative way is to minimize this functional. All these methods require numerical methods to involve. It is possible, however, to roughly estimate the energy density of a vortex, based on the results of the previous subsection III B.

There are three different contributions to the energy density of the vortex, each associated with different terms in the action (2): The kinetic energy of the scalar,

$$\epsilon_s \equiv -K(X), \quad (36)$$

the potential energy,

$$\epsilon_{\text{pot}} \equiv V(\phi), \quad (37)$$

and the kinetic energy of the gauge field,

$$\epsilon_V \equiv \frac{1}{4} F_{ij}^2. \quad (38)$$

First we note that the energy density inside the scalar core associated with the kinetic term $K(X)$ is approximately equal to the potential energy:

$$\epsilon_s \sim \epsilon_{\text{pot}} \sim \epsilon, \quad (39)$$

while the energy density of the gauge field is given by

$$\epsilon_V \sim F_{ij}^2 \sim \frac{1}{e^2 l_V^4}. \quad (40)$$

Using (39), (40) and taking into account (23), (27) and (29) it is easy to estimate the energy density of the vortex for different forms of the kinetic term $K(X)$:

$$E \sim \begin{cases} \eta^2, & n \leq 1, \\ \eta^2 (\lambda v^4)^{1-1/n}, & n > 1. \end{cases} \quad (41)$$

Notice that Eq. (41) is in agreement with Eq. (34). For $n > 1$ the non-linearity in X is important for both the scalar and the vector fields, thus (34) is directly applicable. For $n < 1$ the vector core spreads wider than the scalar one, so in the region $r \gtrsim l_H$ the kinetic term takes the standard form, therefore we set $n = 1$ in (34) and arrive at Eq. (41).

An important consequence of Eq. (41) is that for a particular choice of the non-canonical kinetic term (namely, $n > 1$), the energy per unit length of a vortex can be (much) larger than that for the standard vortex. The opposite is impossible: There is no Lagrangian that leads to vortices with small energy per unit length. One can understand this as follows: Although the scalar core can be adjusted to have a small size, (exactly as in the case of global defects [17]), the vector core nevertheless spreads widely, with the configuration close to the standard case. Thus the contribution of the vector field to the energy is roughly the same as for an usual vortex, as Eq. (41) shows.

IV. CONSTRAINTS ON THE PARAMETERS OF THE ACTION

Let us now discuss constraints on the parameters of the model. In this section we will closely follow the similar consideration for the case of global k -defects [17] with necessary adjustments.

First of all we must satisfy the hyperbolicity condition. Physically it means that small perturbations on the background solution do not grow exponentially. As applied to our problem, we have to check that the perturbed Eqs. (6), (7) give hyperbolic EoMs for the propagation of small perturbations. Note that for small enough wavelengths the gauge derivative D_μ is replaced by the partial derivative ∂_μ . The Eq. (6) for high wave-numbers becomes the EoM for a global scalar k -field. The hyperbolicity condition for perturbations for k -essence field reads [14, 22]

$$\frac{K_{,X}(X)}{2XK_{,XX}(X) + K_{,X}(X)} > 0. \quad (42)$$

It is easy to check that for the Born-Infeld-like kinetic term (14) the hyperbolicity condition (42) is met for $X < 1/2$, while for the second example we consider, Eq. (15), inequality Eq. (42) is always true.

Meantime the EoM for the gauge field (7) in the limit of small wavelengths coincides with the standard EoM for the normal electromagnetic field, since the r.h.s of (7) can be neglected in this limit.

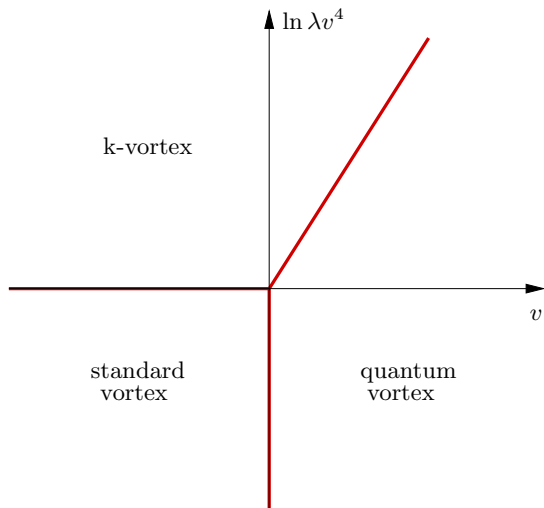


FIG. 1: Constraints on the parameters λ and v for the model (11) are shown. There are three regions in the plane of parameters: i) standard vortex solution, when the non-linearity in X inside the core is small and the standard solution is restored; ii) quantum defect, when the classical picture is not valid; iii) k -vortex, when the non-linearity in X is large and the properties of a vortex are considerably different from the standard case.

Thus we have proved that the system of equations (6), (7) is hyperbolic provided that the inequality (42) holds, and therefore there are no instabilities for small wavelengths. It is worth to note that with the above argumentation we have not proved the stability of the system for long wavelengths. This problem, however, deserves a separate investigation and is not addressed in this paper.

As the second constraint on the parameters of the model we demand that the nonlinear part of $K(X)$ dominates inside the core of the defect. Otherwise we end up with a “standard” solution arising in the model with the canonical kinetic term. Thus we require $X \gtrsim 1$, which can be brought to

$$\lambda \left(\frac{\eta}{M} \right)^4 \gtrsim 1. \quad (43)$$

Finally, the third restriction comes from the validity of the classical description. We consider vortices as classical objects, neglecting quantum effects. This picture is valid if the Compton wave length of the cube with the edge l_H is smaller than the size of a scalar core l_H , and similar must be true for the vector core. This gives

$$l_H^4 \epsilon_s \gtrsim 1, \quad (44)$$

and

$$l_V^4 \epsilon_V \gtrsim 1. \quad (45)$$

Eq. (44) can be rewritten as follows:

$$\lambda \lesssim \epsilon^{2-2/n}, \quad (46)$$

while (45) gives simply

$$e \lesssim 1. \quad (47)$$

It is interesting to note that the only additional constraint, as compared to the global k -vortices [17], is a natural inequality Eq. (47). We summarize the requirements (43) and (44) in Fig. 1.

V. NUMERICAL SOLUTIONS

In this section we present the numerical solutions for the vortices in the model (1) [or, equivalently, (2)] with different choices of the non-canonical kinetic term $K(X)$. We compare the obtained solutions to the standard ones. With the help of these explicit solutions we verify our general results on the properties of the gauge k -vortices, presented in Sec. III.

We solve numerically the system of ordinary differential equation (9), (10) for the model (2) with the following kinetic terms, $K(X)$:

- canonical term, $K(X) = X$;
- DBI-like term, $K(X) = 1 - \sqrt{1 - 2X}$;
- the power-law kinetic term, $K(X) = X + X^3$.

In Fig. 2 the functions $f(r)$ and $\alpha(r)$ are shown for the vortex solution in the case of canonical, DBI and power-law kinetic terms. We have chosen the parameters of the Lagrangian as $\lambda = e = 1/4$ and $v = 5$, thus providing the non-linear in X regime for the model with non-canonical terms (14) and (15), since for these parameters $X \sim 10^2$. One can see that the results of our general consideration, Sec. III, are in a perfect agreement with the numerical results [compare Eqs. (31), (32) with the numerical values for the sizes of the scalar and vector cores]. The properties of k -vortices are indeed quite different from those for a standard vortex in the regime when the non-linearity in $K(X)$ is important. We also have found the functions $f(r)$, $\alpha(r)$ for such parameters of the model, that $X \leq 1$ inside the core of the defect. As it was expected on general grounds, the obtained solutions do not deviate much from the standard vortex solutions, since in this regime the kinetic terms (14), (15) have almost the canonical form.

VI. SUMMARY AND DISCUSSION

We have studied topological linear gauge defects (gauge vortices), in the model with a non-canonical kinetic term. The action for the model (1) contains kinetic terms for the scalar and gauge vector fields and a symmetry-breaking potential. The principal difference of the studied model from the standard one is the presence of a non-standard kinetic part for the scalar field. The

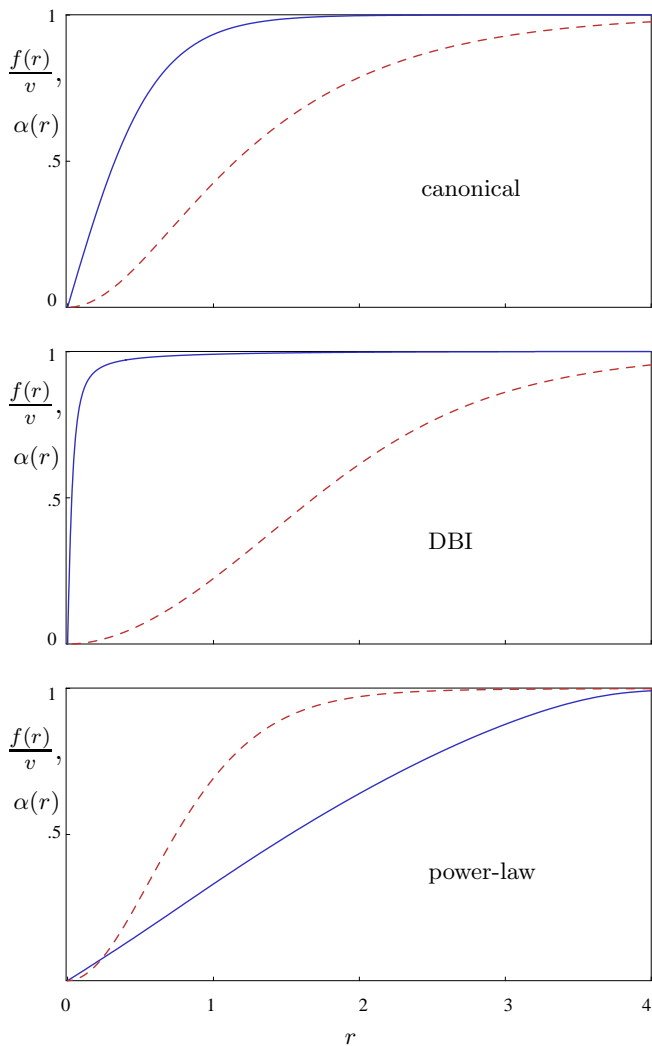


FIG. 2: The numerical solutions for the field profiles $f(r)/v$ (solid), $\alpha(r)$ (dashed) are shown for different choice of the kinetic term $K(X)$. From the up to bottom: the standard case, $K(X) = X$; DBI term, $K(X) = 1 - \sqrt{1 - 2X}$; power-law term, $K(X) = X + X^3$. The parameters of the model are chosen such that the non-linearity in X inside the core of a vortex is large, $\lambda = e = 1/4$, $v = 5$. The field profile for the vector part α , is roughly the same for the different kinetic terms, in accordance with (32). While one can notice a strong dependence of the scalar field profile $f(r)$ on the choice of $K(X)$. The size of scalar core is in a good agreement with our estimations (31).

term $K(X)$ in the action is in general some non-linear function of canonical kinetic term X .

A remarkable feature of the model is that the non-linearity of the kinetic term inevitably leads to the appearance of a new scale in the action, a “kinetic” mass, in addition to the “usual” scalar and vector masses. The presence of another mass scale in the model changes radically the basic properties of a vortex: the size of the

scalar core and the energy of a vortex per unit length vary considerably as compared to the standard case.

We have investigated general properties of k -vortices and found restrictions on the parameters of the model, having in mind a rather general form of a kinetic term with the asymptotic behavior $K(X) \sim X^n$. Also, for the sake of simplicity and clarity of results we assumed that the scalar and vector masses are of the same order, $e\eta \sim \sqrt{\lambda}\eta$. We can summarize our general estimations as follows. The size of the scalar core, l_H , depends on the coupling λ , and mass scales η and M , Eq. (31). A remarkable point is that l_H roughly coincides with the characteristic size of the core in the case of a global k -vortex [17]. The size of the vector core l_V does not depend on the kinetic mass M and is roughly the same as in the standard case, Eq. (32).

Having the values for the core’ sizes one can estimate the energy of k -vortex per unit length, see Eq. (41). An important result is that the mass of a vortex radically vary depending on the choice of kinetic term. In the case $n > 1$ and the limit $\lambda v^4 \gg 1$, we have found a simple exact expression for the energy functional of k -vortex, Eqs. (34), (35).

As particular examples, we studied numerically two concrete models having non-canonical kinetic terms: A DBI-like term, Eq. (14), and a power-law term, Eq. (15). The field profiles of domain walls for different choices of $K(X)$ are shown in Fig. 2: The numerical solutions are in agreement with our general estimations.

As we already discussed in our previous work [17], interesting properties of k -defects may have important consequences for cosmological applications. Standard cosmic strings which might have been formed during phase transitions in the early universe have a mass scale directly connected to the temperature of a phase transition T_c , $\mu \sim \eta^2 \sim T_c^2$. By contrast, the mass scale of a resulting k -string depends both on T_c and the kinetic mass M . This means that the tension of k -strings may not be close to T_c^2 , thus helping to avoid constrains on cosmic strings $G\mu \lesssim 10^{-7}$ [24, 25] (or even stronger, $G\mu < 3 \times 10^{-8}$, see [26]). Meanwhile theoretical predictions give $G\mu \sim 10^{-6} - 10^{-7}$ for GUT strings. If, however, physics at the GUT scale involves non-standard kinetic terms, then the GUT phase transition may have lead to the formation of cosmic strings with smaller tension, $G\mu \ll 10^{-6}$, thereby evading conflicts with the present observations.

Acknowledgments

It is a pleasure to thank M. Kachelriess for critical reading of the manuscript. This work was supported by an INFN fellowship grant.

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