Phenomenology of Λ-CDM model: a possibility of accelerating Universe with positive pressure

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Abstract

Among various phenomenological Λ models, a time-dependent model $\dot{\Lambda} \sim H^3$ is selected here to investigate the Λ-CDM cosmology. Using this model the expressions for the time-dependent equation of state parameter ω and other physical parameters are derived. It is shown that in H^3 model accelerated expansion of the Universe takes place at negative energy density, but with a positive pressure. It has also been possible to obtain the change of sign of the deceleration parameter q during cosmic evolution.

Key words: dark energy, variable Λ, antigravity

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1 Introduction

Any theoretical model related to cosmology should be supported by observational data. Present cosmological models suggest that the Universe is primarily made of dark matter and dark energy. Various observational evidences including SN Ia [1,2,3,4,5,6,7,8] data support the idea of accelerating Universe and it is supposed that dark energy is responsible for this. Now-a-days it is accepted that about two third of the total energy density of the Universe is dark energy and the remaining one third consists of visible matter and dark matter [9].

Though dark matter had a significant role during structure formation in the early Universe, its composition is still unknown. It is predicted that the dark matter should be non-baryonic and various particle physics candidates and their mixtures are discussed (see e.g. [10,11] for review). In particular, time-varying forms of dark matter [12,13,14] in Unstable Dark Matter (UDM) scenarios [12,15,16] are still not fully explored and deserve interest, giving simultaneously clustered and unclustered dark matter components.

The standard cold dark matter (SCDM) model introduced in 1980's which assume $\Omega_{CDM} = 1$ is out of favor today [17,18]. After the emergence of the idea of accelerating Universe, the SCDM model is replaced by Λ-CDM or LCDM model. This model includes dark energy as a part of the total energy density of the Universe and is in nice agreement with various sets of observations [19]. In this connection it is to be noted here that according to Λ-CDM model, acceleration of the Universe should be a recent phenomenon. Some recent works [20,21] favor the idea that the present accelerating Universe was preceded by a decelerating one and observational evidence [22] also support this.

Now, in most of the recent cosmological research, the equation of state parameter ω has been taken as a constant. However for better result ω should be taken as time-dependent [23,24,25]. In the present work Λ-CDM Universe has been investigated by selecting a specific time-dependent form of Λ viz. $\dot{\Lambda} \sim H^3$ along with $\omega(t)$. This Λ model was previously studied by Reuter and Wetterich [26] for finding out an explanation of the presently observed small value of Λ. This choice found realization in the approach treating time-variation of Λ as Bose condensate evaporation [39] in the framework of model of self consistent inflation [39,40,41]. Very recently it is used by Mukhopadhyay et al. [27] for investigating the Λ-CDM Universe. Using this model the expressions for the time-dependent equation of state parameter ω and various physical parameters are derived. The change of sign of the deceleration parameter q has also been shown in this case.

Interestingly the expression for energy density obtained in the present model is negative. However, this type of negative density is not at all unavailable in the literature. It can be mentioned here that the negative energy density was first obtained by Casimir [28]. Hawking found the existence of negative energy density at the horizon of a black hole [29]. Davies and Fulling also studied about the negative energy fluxes in the radiation from moving mirrors [30,31].

The scheme of the present investigation is as follows: in Section 2 the field equations are provided whereas their solutions as well as the physical features have been sought for in the Sections 3 and 4. In the Section 4 some concluding remarks have been made.

2 Field Equations

The Einstein field equations are given by

$$R^{ij} - \frac{1}{2}Rg^{ij} = -8\pi G\left[T^{ij} - \frac{\Lambda}{8\pi G}g^{ij}\right] \tag{1}$$

where the cosmological term Λ is time-dependent, i.e. $\Lambda = \Lambda(t)$ and c, the velocity of light in vacuum is assumed to be unity.

Let us consider the Robertson-Walker metric

$$ds^2 = -dt^2 + a(t)^2\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)\right] \tag{2}$$

where k, the curvature constant, assumes the values -1, 0 and $+1$ for open, flat and closed models of the Universe respectively and $a = a(t)$ is the scale factor. For the spherically symmetric metric (2), field equations (1) yield Friedmann and Raychaudhuri equations respectively given by

$$3H^2 + \frac{k}{a^2} = 8\pi G\rho + \Lambda \tag{3}$$

$$3H^2 + 3\dot{H} = -4\pi G(\rho + 3p) + \Lambda \tag{4}$$

where G, ρ and p are the gravitational constant, matter energy density and pressure respectively and the Hubble parameter H is related to the scale factor by $H = \dot{a}/a$. In the present work, G is assumed to be constant. The generalized energy conservation law for variable G and Λ is derived by Shapiro et al. [32]

using Renormalization Group Theory and also by Vereschagin et al. [33] using a formula of Gurzadyan and Xue [34].

The conservation equation for variable Λ and constant G is a special case of that generalized conservation law and is given by

$$\dot{\rho} + 3(p + \rho)H = -\frac{\dot{\Lambda}}{8\pi G} \tag{5}$$

3 Cosmological model

Let us consider a relationship between the pressure and density of the physical system in the form of the following barotropic equation of state

$$p = \omega\rho \tag{6}$$

where the barotropic index ω is assumed here as a time-dependent quantity i.e. $\omega = \omega(t)$. Actually, the above equation of state parameter ω, instead of being a function of time, may also be function of scale factor or redshift. However, sometimes it is convenient to consider ω as a constant quantity because current observational data has limited power to distinguish between a time varying and constant equation of state [35,36]. Here some useful limits on ω as appeared from SNIa data are $-1.67 < \omega < -0.62$ [3] whereas refined values were indicated by the combined SNIa data (with CMB anisotropy) + galaxy clustering statistics which is $-1.33 < \omega < -0.79$ [37].

Using equation (6) we get from (5),

$$8\pi G\dot{\rho} + \dot{\Lambda} = -24\pi G(1 + \omega)\rho H \tag{7}$$

Differentiating (3) with respect to t we get for a flat Universe ($k = 0$),

$$4\pi G\rho = -\frac{\dot{H}}{1 + \omega} \tag{8}$$

It can be mentioned that equivalence of three phenomenological Λ-models (viz., $\Lambda \sim (\dot{a}/a)^2$, $\Lambda \sim \ddot{a}/a$ and $\Lambda \sim \rho$) have been studied in detail by Ray et al. [38] for constant ω. So, it is reasonable that similar type of variable-Λ model may be investigated with a variable ω for a deeper understanding of both the accelerating and decelerating phase of the Universe. Let us, therefore, use the *ansatz* $\dot{\Lambda} \propto H^3$, so that

$$\dot{\Lambda} = AH^3 \tag{9}$$

where A is a proportional constant. This *ansatz* with negative A can find realization in the framework of model of self consistent inflation [39,40,41], in which time-variation of Λ is determined by the rate of Bose condensate evaporation [39] with $a \sim (m/m_{Pl})^2$ (where a is the absolute value of negative A and m is the mass of bosons and m_{Pl} is the Planck mass).

Using equations (6), (8) and (9) we get from (4),

$$\frac{2}{(1+\omega)H^3}\frac{d^2H}{dt^2} + \frac{6}{H^2}\frac{dH}{dt} = A. \tag{10}$$

If we put $dH/dt = P$, then equation (10) reduces to

$$\frac{dP}{dH} + 3(1+\omega)H = \frac{A(1+\omega)H^3}{2P}. \tag{11}$$

To arrive at any fruitful conclusion, let us now solve equation (11) under the following specific assumption

$$\omega = \frac{2\tau P}{H} - 1. \tag{12}$$

where τ has dimension of time and is a parameter of our model. Typically, the timescale τ has the physical meaning of dissipation time scale for time varying Λ. As stated earlier, ω is the equation of state parameter which depends upon time. However, the mode of dependence of $\omega = \omega(t)$ can be sought for suitably after finding out the solutions of the parameters.

By the use of above substitution, equation (11) becomes

$$\frac{dP}{dH} + 6\tau P = A\tau H^2 \tag{13}$$

Therefore, from the above equation (13) we get the solution set as

$$a(t) = C_1 e^{t/6\tau}\left(sec\frac{Bt}{\tau}\right)^{1/6B} \tag{14}$$

$$H(t) = \frac{1}{6\tau}\left(1 + tan\frac{Bt}{\tau}\right) \tag{15}$$

$$\Lambda(t) = \frac{1}{6\tau^3}\left[\frac{\tau}{2}tan^2\frac{Bt}{\tau} + 2\tau log\left(sec\frac{Bt}{\tau}\right) + 3\tau tan\frac{Bt}{\tau} - 2Bt\right] \tag{16}$$

$$\rho(t) = -\frac{1}{48\pi G\tau^2}\left(1 + tan\frac{Bt}{\tau}\right) \tag{17}$$

$$\omega(t) = -1 + \frac{2Bsec^2\frac{Bt}{\tau}}{\left(1 + tan\frac{Bt}{\tau}\right)} \tag{18}$$

$$p(t) = \frac{1}{48\pi G\tau^2}\left(1 + tan\frac{Bt}{\tau} - 2Bsec^2\frac{Bt}{\tau}\right) \tag{19}$$

where C_1 is a constant and B=A/36.

4 Physical features and discussions

4.1 Nature of B

For physically valid H we should have $tan(Bt/\tau) > -1$. Again, from equation (18) it is clear that ω can be greater or less than -1 according as $B > 0$ or $B < 0$. But, the awkward case here is the negativity of ρ. In this connection it may be mentioned that Ray and Bhadra [42] also obtained negative energy density by introducing a space-varying Λ for a static charged anisotropic fluid source. In fact, according to Cooperstock and Rosen [43], Bonnor and Cooperstock [44] and Herrera and Varela [45], within the framework of general theory of relativity some negative mass density must be possessed by any spherically symmetric distribution of charge. Ray and Bhadra [46] have demonstrated that, model constructed within Einstein-Cartan theory can also contain some negative matter-energy density. In the present work a negative density is obtained for a dynamic, homogeneous and isotropic neutral fluid with time dependent Λ. It is shown here that in H^3 model accelerated expansion of the Universe takes place at negative energy density, but at positive pressure (for negative B, we have negative energy density and negative ω, giving positive pressure). In fact, at negative B time varying cosmological term disappears at time scale $t \sim \tau/b = 36\tau/a$ (where b is the absolute value of negative B), therefore addition of dark matter will lead to matter dominance at $t > \tau/b$.

This result is quite natural, since negative B corresponds to negative A, i.e. to negative time derivative for Λ in equation (9). The physical meaning of τ for each particular realization of the considered scenario. For example in the approach [39,40,41] it has the meaning of timescale, at which Bose-Einstein condensate, maintaining Λ, evaporates.

4.2 Structure of A

As evident from the equation (9) A is, by construction, dimensionless while B has dimension of inverse time. Therefore, to clarify the units in which one is to measure B we propose that the equation (9) can be taken in the form $\dot{\Lambda} = A(t)H^3$ where the constant of proportionality A is now assumed as time-dependent with the form $A(t) = A_0 + A_1 t^{-1}$. In view of this the equation (9) can be considered now as the truncated linear form with constant $A = A_0$ only.

4.3 Structure of ω

It is already mentioned that the equation of state parameter is dependent on time i.e. $\omega = \omega(t)$. Also it is known that it need not be only time-dependent even may have functional relationship with cosmic scale factor a or cosmological redshift z. In connection to redshift this dependence may be linear as $\omega(z) = \omega_o + \omega' z$ where $\omega' = (d\omega/dz)z = 0$ [47,48] or may be of non-linear type as $\omega(z) = \omega_o + \omega_1 z/(1+z)$ [49,50]. Now, following these redshift parametrizations of two index pattern and looking at the structure of our solution for $\omega(t)$ in equation (18) we can generalize it in the form $\omega(t) = \omega_o + \omega_1 t^{-1}$. This seems identical in form with that of $A(t)$ as proposed in the previous paragraph in connection to the equation (9).

4.4 Deceleration parameter q

We would like to consider now the equation (15) which yields the expression for the deceleration parameter as $q = -[1 + 6Bsec^2\frac{Bt}{\tau}/(1 + tan\frac{Bt}{\tau})^2]$. It is clear from this expression that if $B < 0$, then q can change its sign depending on the value of the time dependent part. So, decelerating-accelerating cosmic evolution can be found from the present H^3 phenomenological Λ-CDM model.

4.5 Role of negative energy density

From the solution set it is clear that the expression for density comes out to be a negative quantity with a positive pressure counter part. Actually, here what is shown is a possible cosmological solution with negative energy density. The mechanism of the accelerated expansion of the present Universe due to this positive pressure and negative density is not yet understood properly. In general, contemporary literatures (see e.g. [38] and references therein) suggest that via Cosmological parameter Λ-dark energy acts as the role for the repulsive pressure which is responsible for the accelerated expansion. Here, in our case, negative energy density seems to possess the same role of repulsive gravitation. Perhaps it's role could be understood in a model with dark matter in the presence of Λ where dark matter will be associated with repulsive nature due to negative density [51]. The main idea of the result is that there is an acceleration due to negative energy density, but at positive pressure. Though physically it is an awkward situation but not at all unavailable in the literature (as mentioned earlier in the introduction) and also quantum field theory admits it. However in cosmology, we don't see negative energy density phenomena very often. This is because there may be some mechanism restricting negative energies or their interactions with ordinary fields. The idea of anti-gravity (i.e. gravitational repulsion between matter and antimatter) originates from this kind of negative energy [52]. Due to symmetry, each gravitating standard model particle corresponds to an anti-gravitating particle. It is thought that this anti-gravitating particle cancels gravitating particle's contribution to the vacuum energy and hence provides a mechanism for smoothing out of the cosmological constant puzzle [53].

5 Concluding remarks

The objective of this work is to observe the effect of a time-dependent equation of state parameter on a dynamic Λ model selected for dark energy investigation. Assuming $\dot{\Lambda} \sim H^3$, expressions for time dependent equation of state parameter and matter density have been derived. Moreover, the change of sign of the deceleration parameter has been achieved under this special assumption. It has also been shown that ω can be less than -1 which is in agreement with the SN Ia data [3] and SN Ia data with CMB anisotropy and galaxy-cluster statistics [19].

We would also like to point out further that-

1. The present work has been done by keeping the gravitational constant G as a constant. Therefore, a future work can be carried out along the line of this

model with G as a variable.

2. Unless we study *dark matter* + Λ, we can not say anything conclusive about Λ-CDM cosmology.

3. It is also interesting to consider early Universe for Λ + *radiation*, or more general Λ + *radiation* + UDM, and then to study physics of such Universe and possible observational constraints.

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