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# Phenomenology of $\Lambda$ -CDM model: a possibility of accelerating Universe with positive pressure

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**Abstract** Among various phenomenological  $\Lambda$  models, a time-dependent model  $\dot{\Lambda} \sim H^3$  is selected here to investigate the  $\Lambda$ -CDM cosmology. The model can follow from dynamics, underlying the origin of  $\Lambda$ . Using this model the expressions for the time-dependent equation of state parameter  $\omega$  and other physical parameters are derived. It is shown that in  $H^3$  model accelerated expansion of the Universe takes place at negative energy density, but with a positive pressure. It has also been possible to obtain the change of sign of the deceleration parameter  $q$  during cosmic evolution.

**Keywords** general relativity · dark energy · variable  $\Lambda$

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## 1 Introduction

Any theoretical model related to cosmology should be supported by observational data. Present cosmological models suggest that the Universe is primarily made of dark matter and dark energy. Various observational evidences, including SN Ia [1,2,3,4,5,6,7] data, support the idea of accelerating Universe and it is supposed that dark energy is responsible for this effect of speeding up. Now-a-days it is accepted that about two third of the total energy density of the Universe is dark energy and the remaining one third consists of visible matter and dark matter [8].

Though dark matter had a significant role during structure formation in the early Universe, its composition is still unknown. It is predicted that the dark matter should be non-baryonic and various particle physics candidates and their mixtures are discussed (see e.g. [9,10] for review). In particular, time-varying forms of dark matter [11,12,13] in Unstable Dark Matter (UDM) scenarios [11,14,15] are still not fully explored and deserve interest, giving simultaneously clustered and unclustered dark matter components.

The standard cold dark matter (SCDM) model introduced in 1980's which assume  $\Omega_{CDM} = 1$  is out of favor today [16,17]. After the emergence of the idea of accelerating Universe, the SCDM model is replaced by  $\Lambda$ -CDM (or LCDM) model. This model includes dark energy as a part of the total energy density of the Universe and is in nice agreement with various sets of observations [18]. In this connection it is to be noted here that according to  $\Lambda$ -CDM model, acceleration of the Universe should be a recent phenomenon. Some recent works [19,20] favor the idea that the present accelerating Universe was preceded by a decelerating one and observational evidence [21] also support this.

Now, in most of the recent cosmological research, the equation of state parameter  $\omega$  has been taken as a constant. However, its seems that for better result  $\omega$  should be taken as time-dependent [22,23,24]. Therefore, in the present work  $\Lambda$ -CDM Universe has been investigated by selecting a specific time-dependent form of  $\Lambda$ , viz.,  $\dot{\Lambda} \sim H^3$  along with  $\omega(t)$ . A kind of this  $\Lambda$ -model was previously studied by Reuter and Wetterich [25] for finding out an explanation of the presently observed small value of  $\Lambda$ . This choice found realization in the approach treating time-variation of  $\Lambda$  as Bose condensate evaporation [26] in the framework of model of self consistent inflation [26,27,28]. Very recently it is also used by Mukhopadhyay et al. [29] for investigating various inherent features of the  $\Lambda$ -CDM Universe.

In this context it is important to note that effects of real and virtual particles can play nontrivial role in the origin of  $\Lambda$  and its time-variation. Indeed,  $R^2$  term was induced by vacuum polarisation and gave rise to inflation in one of the first inflationary models [30]. The back reaction of particle products of Bose-Einstein condensate evaporation slowed down evaporation and damped coherent scalar field oscillations in the approach of self consistent inflation [26,27,28]. It resulted in specific time dependence of  $\Lambda$ , being determined by the cross section of interaction of evaporated particle. The maximal estimation of this cross section gave the time variation of  $\Lambda$  [26,27], leading to possible realization of our model in the form  $\dot{\Lambda} \sim H^3$ .

Therefore, motivated by the time variation of  $\Lambda$  and using the phenomenological model of the kind  $\dot{\Lambda} \sim H^3$  the expression for the time-dependent equation of state parameter  $\omega$  and various physical parameters are derived in the present investigation. The change in sign of the deceleration parameter  $q$  has also been shown in this case which refers to the evolution of the Universe via a phase transition from deceleration to the present acceleration.

Interestingly the expression for energy density obtained in the present model is negative. However, this type of negative density is not at all unavailable in the literature. It can be mentioned here that the negative energy density was first obtained by Casimir [31]. Hawking found the existence of negative energy density at the horizon of a black hole [32]. Davies and Fulling also studied about the negative energy fluxes in the radiation from moving mirrors [33,34].

The scheme of the present investigation is as follows: in Section 2 the field equations are provided whereas their solutions as well as the physical features have been sought for in the Sections 3 and 4. In the Section 5 some concluding remarks have been made.

## 2 Field Equations

The Einstein field equations are given by

$$R^{ij} - \frac{1}{2}Rg^{ij} = -8\pi G \left[ T^{ij} - \frac{\Lambda}{8\pi G}g^{ij} \right] \quad (1)$$

where the cosmological term  $\Lambda$  is time-dependent, i.e.  $\Lambda = \Lambda(t)$  and  $c$ , the velocity of light in vacuum is assumed to be unity.

Let us consider the Robertson-Walker metric

$$ds^2 = -dt^2 + a(t)^2 \left[ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right] \quad (2)$$

where  $k$ , the curvature constant, assumes the values  $-1$ ,  $0$  and  $+1$  for open, flat and closed models of the Universe respectively and  $a = a(t)$  is the scale factor. For the spherically symmetric metric (2), the field equations (1) yield respectively the Friedmann and Raychaudhuri equations which can be given by

$$3H^2 + \frac{3k}{a^2} = 8\pi G\rho + \Lambda, \quad (3)$$

$$3H^2 + 3\dot{H} = -4\pi G(\rho + 3p) + \Lambda \quad (4)$$

where  $G$ ,  $\rho$  and  $p$  are the gravitational constant, matter-energy density and fluid pressure respectively. Here the Hubble parameter  $H$  is related to the scale factor  $a$  by the relation  $H = \dot{a}/a$ . In the present work,  $G$  is assumed to be constant. The generalized energy conservation law for variable  $G$  and  $\Lambda$  is derived by Shapiro et al. [35] using Renormalization Group Theory and also by Vereschagin et al. [36] using a formula of Gurzadyan and Xue [37].

The conservation equation for variable  $\Lambda$  and constant  $G$  is a special case of the above mentioned generalized conservation law and is given by

$$\dot{\rho} + 3(p + \rho)H = -\frac{\dot{\Lambda}}{8\pi G}. \quad (5)$$

### 3 Cosmological model

Let us consider a relationship between the fluid pressure and density of the physical system in the form of the following barotropic equation of state

$$p = \omega\rho \quad (6)$$

where  $\omega$  is the equation of state parameter. In this barotropic equation of state  $\omega$  is assumed to be a time-dependent quantity i.e.  $\omega = \omega(t)$ . Actually, the above equation of state parameter  $\omega$ , instead of being a function of time, may also be function of scale factor or redshift. However, sometimes it is convenient to consider  $\omega$  as a constant quantity because current observational data has limited power to distinguish between a time varying and constant equation of state [38,39]. In this connection it may be useful to stress that equation (6) is assigned to matter content of the Universe (with possible time dependent  $\omega < 0$ , which is not related with  $\Lambda$ ).

Now, using equation (6) in (5) we get

$$8\pi G\dot{\rho} + \dot{\Lambda} = -24\pi G(1 + \omega)\rho H. \quad (7)$$

Again, differentiating (3) with respect to  $t$  we get for a flat Universe ( $k = 0$ )

$$4\pi G\rho = -\frac{\dot{H}}{1 + \omega}. \quad (8)$$

It can be mentioned that equivalence of three phenomenological  $\Lambda$ -models (viz.,  $\Lambda \sim (\dot{a}/a)^2$ ,  $\Lambda \sim \ddot{a}/a$  and  $\Lambda \sim \rho$ ) have been studied in detail by Ray et al. [44] for constant  $\omega$ . So, it is reasonable that similar type of variable- $\Lambda$  model may be investigated with a variable  $\omega$  for a deeper understanding of both the accelerating and decelerating phase of the Universe. Let us, therefore, use the *ansatz*  $\Lambda \propto H^3$ , so that

$$\dot{\Lambda} = AH^3 \quad (9)$$

where  $A$  is a proportional constant. This *ansatz* with negative  $A$  can find realization in the approach of self consistent inflation [26,27,28], in which time-variation of  $\Lambda$  is determined by the rate of Bose condensate evaporation [26] with  $\alpha \sim (m/m_{Pl})^2$  (where  $\alpha$  is the absolute value of negative  $A$  and  $m$  is the mass of scalar field and  $m_{Pl}$  is the Planck mass).

The physical nature of  $\Lambda$  in this approach is related with the energy density of scalar field, which has the amplitude beyond the minimum of scalar field potential. If the mass of scalar field  $m$  strongly exceeds Hubble parameter  $H$ ,  $m \gg H$ , coherent field oscillations should start, being accompanied

by the condensate evaporation. The interaction of evaporated particles with condensate damp coherent oscillations of scalar field and keep the amplitude of this field beyond the minimum of its potential. It makes the rate of dissipation of  $\Lambda$  dependent on the cross section of particle interaction with condensate. The maximal estimation of this cross section leads [26, 27] to the time variation of  $\Lambda$ , given by Eq. (9).

Using equations (6), (8) and (9) we get from (4),

$$\frac{2}{(1+\omega)H^3} \frac{d^2H}{dt^2} + \frac{6}{H^2} \frac{dH}{dt} = A. \quad (10)$$

If we put  $dH/dt = P$ , then equation (10) reduces to

$$\frac{dP}{dH} + 3(1+\omega)H = \frac{A(1+\omega)H^3}{2P}. \quad (11)$$

To arrive at any fruitful conclusion, let us now solve equation (11) under the following specific assumption

$$\omega(t) = -1 + \frac{2\tau P}{H}. \quad (12)$$

where  $\tau$  has dimension of time and is a parameter of our model. Typically, the time scale  $\tau$  has the physical meaning of dissipation time scale for time varying  $\Lambda$ . Here  $\tau$  comes in the picture due to the dimensional requirement. As stated earlier,  $\omega$  is the equation of state parameter which depends upon time i.e.  $\omega = \omega(t)$ . This time-dependence may be represented by functional relationship with cosmic scale factor  $a$  or cosmological redshift  $z$ . In connection to redshift this dependence may be linear as  $\omega(z) = \omega_o + \omega'z$  where  $\omega' = (d\omega/dz)_{z=0}$  [40, 41] or may be of non-linear type as  $\omega(z) = \omega_o + \omega_1 z/(1+z)$  [42, 43]. Now, following these redshift parametrizations of two index pattern we can assume our above equation of state parameter in the form  $\omega(t) = \omega_o + \omega_1[1 + (t/\tau)^2]/[1 + (t/\tau)]$  with  $\omega_o = -1$  and  $\omega_1 = 2$ . The form of this supposition, later on, will be realized from the solution (18).

By the use of above supposition (12), equation (11) becomes

$$\frac{dP}{dH} + 6\tau P = A\tau H^2. \quad (13)$$

Therefore, from the master equation (13) we get the solution set as

$$a(t) = C_1 e^{t/6\tau} \left( \sec \frac{Bt}{\tau} \right)^{1/6B}, \quad (14)$$

$$H(t) = \frac{1}{6\tau} \left( 1 + \tan \frac{Bt}{\tau} \right), \quad (15)$$

$$\Lambda(t) = \frac{1}{6\tau^3} \left[ \frac{\tau}{2} \tan^2 \frac{Bt}{\tau} + 2\tau \log \left( \sec \frac{Bt}{\tau} \right) + 3\tau \tan \frac{Bt}{\tau} - 2Bt \right], \quad (16)$$

$$\rho(t) = -\frac{1}{48\pi G\tau^2} \left( 1 + \tan \frac{Bt}{\tau} \right), \quad (17)$$

$$\omega(t) = -1 + \frac{2B \sec^2 \frac{Bt}{\tau}}{\left( 1 + \tan \frac{Bt}{\tau} \right)}, \quad (18)$$

$$p(t) = \frac{1}{48\pi G\tau^2} \left( 1 + \tan \frac{Bt}{\tau} - 2B \sec^2 \frac{Bt}{\tau} \right) \quad (19)$$

where  $C_1$  is a constant and  $B=A/36$ .

## 4 Physical features of the constants and parameters

### 4.1 Structure of $A$

As evident from the equation (9)  $A$  is, by construction, dimensionless while  $B$  has dimension of inverse time. Therefore, to clarify the units in which one is to measure  $B$  we propose that the equation (9) can be taken in the form  $\dot{\Lambda} = A(t)H^3$  where the constant of proportionality  $A$  is now assumed as time-dependent with the form  $A(t) = A_0 + A_1 t^{-1}$ . In view of this the equation (9) can be considered now as the truncated linear form with constant  $A = A_0$  only.

### 4.2 Nature of $B$

For physically valid  $H$  we should have  $\tan(Bt/\tau) > -1$ . Again, from equation (18) it is clear that  $\omega$  can be greater or less than  $-1$  according as  $B > 0$  or  $B < 0$ . But, the awkward case here is the negativity of  $\rho$ . In this connection it may be mentioned that Ray and Bhadra [45] also obtained negative energy density by introducing a space-varying  $\Lambda$  for a static charged anisotropic fluid source. In fact, according to Cooperstock and Rosen [46], Bonnor and Cooperstock [47] and Herrera and Varela [48], within the framework of general theory of relativity some negative mass density must be possessed by any spherically symmetric distribution of charge. Ray and Bhadra [49] have demonstrated that, model constructed within Einstein-Cartan theory can also contain some negative matter-energy density. In the present work a negative density is obtained for a dynamic, homogeneous and isotropic neutral fluid with time dependent  $\Lambda$ . It is shown here that in  $H^3$  model accelerated expansion of the Universe takes place at negative energy density, but at positive pressure (for negative  $B$ , we have negative energy density and negative  $\omega$ , giving positive pressure). In fact, at negative  $B$  time varying cosmological term disappears at time scale  $t \sim \tau/b = 36\tau/a$  (where  $b$  is the absolute value of negative  $B$ ), therefore addition of dark matter will lead to matter dominance at  $t > \tau/b$ .

This result is quite natural, since negative  $B$  corresponds to negative  $A$ , i.e. to negative time derivative for  $\Lambda$  in equation (9). The physical meaning of

$\tau$  is straightforward for each particular realization of the considered scenario. For example in the approach [26,27,28] it has the meaning of timescale, at which Bose-Einstein condensate, maintaining  $\Lambda$ , evaporates.

#### 4.3 Equation of state parameter $\omega$

It is observed from the equation (18) that unless the second term vanishes  $\omega$  can not be negative as expected from the SN Ia data [3] and SN Ia data with CMB anisotropy and galaxy-cluster statistics [18] in connection to dark energy. In this regards we would like to mention here that the physical significance of the negative density can be realized if one remembers that in the present investigation the dark energy is considered not through the equation of state (6) with negative  $\omega$  rather through the ansatz for  $\Lambda$  where  $\Lambda$  acts as one of the dark energy candidates. With this consideration,  $\Lambda$  makes a definite contribution and ultimately the positive pressure is over-powered by  $\Lambda$  to make it negative and energy density positive so that the Universe becomes accelerating.

#### 4.4 Deceleration parameter $q$

We would like to consider now the equation (15) which yields the expression for the deceleration parameter as  $q = -[1 + 6Bsec^2\frac{Bt}{\tau}/(1 + tan\frac{Bt}{\tau})^2]$ . It is clear from this expression that if  $B < 0$ , then  $q$  can change its sign depending on the value of the time dependent part. So, decelerating-accelerating cosmic evolution can be found from the present  $H^3$  phenomenological  $\Lambda$ -CDM model.

### 5 Discussions and Conclusions

The objective of this work is to observe the effect of a time-dependent equation of state parameter on a dynamic  $\Lambda$  model selected for dark energy investigation. Assuming  $\dot{\Lambda} \sim H^3$ , expressions for time dependent equation of state parameter and matter density have been derived. This assumption can find physical justification through the model of Bose-Einstein condensate evaporation [26,27]. It is also to be noted that the change of sign of the deceleration parameter has been achieved under this special assumption.

Moreover, from the solution set it is clear that the expression for density comes out to be a negative quantity with a positive pressure counter part. Actually, here what is shown is a possible cosmological solution with negative energy density. The mechanism of the accelerated expansion of the present Universe due to this positive pressure and negative density is not yet understood properly. In general, contemporary literature (see e.g. [44] and references therein) suggest that via Cosmological parameter  $\Lambda$ -dark energy acts as the role for the repulsive pressure which is responsible for the accelerated expansion. Here, in our case, negative energy density seems to possess the same role of repulsive gravitation. Perhaps it's role could be understood

in a model with dark matter in the presence of  $\Lambda$  where dark matter will be associated with repulsive nature due to negative density [50].

The main idea of the result is that there is an acceleration due to negative energy density, but at positive pressure. Though physically it is an awkward situation but not at all unavailable in the literature (as mentioned earlier in the introduction) and also quantum field theory admits it. However in cosmology, we don't see negative energy density phenomena very often. This is because there may be some mechanism restricting negative energies or their interactions with ordinary fields. The idea of anti-gravity (i.e. gravitational repulsion between matter and antimatter) originates from this kind of negative energy [51]. Due to symmetry, each gravitating standard model particle corresponds to an anti-gravitating particle. It is thought that this anti-gravitating particle cancels gravitating particle's contribution to the vacuum energy and hence provides a mechanism for smoothing out of the cosmological constant puzzle [52].

We would also like to point out further that-

1. The present work has been done by keeping the gravitational constant  $G$  as a constant. Therefore, a future work can be carried out along the line of this model with  $G$  as a variable.

2. Unless we study *dark matter* +  $\Lambda$ , we can not say anything conclusive about  $\Lambda$ -CDM cosmology.

3. It is also interesting to consider early Universe for  $\Lambda$  + *radiation*, or more general  $\Lambda$  + *radiation* + UDM, and then to study physics of such Universe and possible observational constraint.

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## References

1. Riess, A.G., et al.: *Astron. J.* **116**, 1009 (1998)
2. Perlmutter, S.J., et al.: *Astrophys. J.* **517**, 565 (1999)
3. Knop, R.A., et al.: *Astrophys. J.* **598**, 102 (2003)
4. Riess, A.G., et al.: *Astrophys. J.* **607**, 665 (2004)
5. Tegmark, M., et al.: *Phys. Rev. D* **69**, 103501 (2004a)
6. Astier, P., et al.: *Astron. Astrophys.* **447**, 31 (2005)
7. Spergel, D.N., et al.: *Astrophys. J. Suppl.* **170** 377 (2007)
8. Sahni, V.: *Lec. Notes Phys.* **653**, 141 (2004)
9. Khlopov, M. Yu.: Dark matter in cosmology, clocks and tests of fundamental laws, Eds. Guiderdoni, B., et al. Editions Frontieres, PP. 133-138 (1995)
10. Khlopov, M. Yu.: *Cosmoparticle physics*, World Scientific (1999)
11. Doroshkevich, A.G., Khlopov, M. Yu.: *Yadernaya Fizika* **39**, 869 (1984) [English translation: *Sov. J. Nucl. Phys.* **39**, 551 (1984)]
12. Turner, M. S., Steigman, G., Krauss, L.M.: *Phys. Rev. Lett.* **52** 2090 (1984)
13. Gelmini, G., Schramm, D.N., Valle, J.W.F.: *Phys. Lett. B* **146**, 311 (1984)
14. Doroshkevich, A.G., Khlopov, M. Yu.: *Mon. Not. R. Astron. Soc.* **211**, 279 (1984)

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15. Doroshkevich, A.G., Khlopov, M. Yu., Klypin, A.A.: *Mon. Not. R. Astron. Soc.* **239**, 923 (1989)
  16. Efstathiou, G., Sutherland, W., Madox, S.J.: *Nat.* **348**, 705 (1990)
  17. Pope, A.C., et al.: *Astrophys. J.* **607**, 655 (2004)
  18. Tegmark, M., et al.: *Astrophys. J.* **606**, 702 (2004b)
  19. Padmanabhan, T., Roychowdhury, T.: *Mon. Not. R. Astron. Soc.* **344**, 823 (2003)
  20. Amendola, L.: *Mon. Not. R. Astron. Soc.* **342**, 221 (2003)
  21. Riess, A.G.: *Astrophys. J.* **560**, 49 (2001)
  22. Chervon, S.V., Zhuravlev, V.M.: *Zh. Eksp. Teor. Fiz.* **118**, 259 (2000)
  23. Zhuravlev, V.M.: *Zh. Eksp. Teor. Fiz.* **120**, 1042 (2001)
  24. Peebles, P.J.E., Ratra, B.: *Rev. Mod. Phys.* **75**, 559 (2003)
  25. Reuter, M., Wetterich, C.: *Phys. Lett. B* **188**, 38 (1987)
  26. Dymnikova, I., Khlopov, M.: *Mod. Phys. Lett. A* **15**, 2305 (2000)
  27. Dymnikova, I., Khlopov, M.: *Eur. Phys. J. C* **20**, 139 (2001)
  28. Dymnikova, I., Khlopov, M.: *Grav. Cosmol. Suppl.* **4**, 50 (1998)
  29. Mukhopadhyay, U., Ray, S., Duttachowdhury, S.B.: *Int. J. Mod. Phys. D* **17**, 301 (2008)
  30. Starobinsky, A.A.: *Phys. Lett. B* **91**, 99 (1980)
  31. Casimir, H.B.G.: *Proc. K. Ned. Akad. Wet.* **51**, 793 (1948)
  32. Hawking, S.W.: *Commun. Math. Phys.* **43**, 199 (1975)
  33. Davies, P.C.W., Fulling, S.A.: *Proc. R. Soc. Lond.* **348**, 393 (1976)
  34. Fulling, S.A., Davies, P.C.W.: *Proc. R. Soc. Lond.* **356**, 237 (1977)
  35. Shapiro, I.L.: *J. Solà and H. Stefancic, J. Cosmol. Astroparticle Phys.* **1**, 012 (2005)
  36. Vereschagin, G.V., Yegorian, G.: *Class. Quantum Grav.* **23**, 5049 (2006)
  37. Vereschagin, G.V., Xue, S.-S.: *Mod. Phys. Lett. A* **18**, 561 (2003)
  38. Kujat, J., et al.: *Astrophys. J.* **572**, 1 (2002)
  39. Bartelmann, M., et al.: *New Astron. Rev.* **49**, 19 (2005)
  40. Huterer, D., Turner, M.S.: *Phys. Rev. D* **64**, 123527 (2001)
  41. Weller, J., Albrecht, A.: *Phys. Rev. D* **65**, 103512 (2002)
  42. Polarski, D., Chevallier, M.: *Int. J. Mod. Phys. D* **10**, 213 (2001)
  43. Linder, E.V.: *Phys. Rev. Lett.* **90**, 91301 (2003)
  44. Ray, S., Mukhopadhyay, U., Meng, X.-H.: *Grav. Cosmol.* **13**, 142 (2007)
  45. Ray, S., Bhadra, S.: *Phy. Lett. A* **322**, 150 (2004a)
  46. Cooperstock, F.I., Rosen, N.: *Int. J. Theor. Phys.* **28**, 423 (1989)
  47. Bonnor, W.B., Cooperstock, F.I.: *Phys. Lett. A* **139**, 442 (1989)
  48. Herrera, L., Varela, V.: *Phys. Lett. A* **189**, 11 (1996)
  49. Ray, S., Bhadra, S.: *Int. J. Mod. Phys. D* **13**, 555 (2004b)
  50. Goodman, J.: arXiv:astro-ph/0003018
  51. Hossenfelder, S.: *Phys. Lett. B* **636**, 119 (2006)
  52. Quiros, I.: arXiv:gr-qc/0411064