Charged Rotating Black Holes on DGP Brane

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ABSTRACT

We consider charged rotating black holes localized on a three-brane in the DGP model. Assuming a Z_2 -symmetry across the brane and with a stationary and axisymmetric metric ansatz on the brane, a particular solution is obtained in the Kerr-Schild form. This solution belongs to the accelerated branch of the DGP model and has the characteristic of the Kerr-Newman-de Sitter type solution in general relativity. Using a modified version of Boyer-Lindquist coordinates we examine the structures of the horizon and ergosphere.

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1 Introduction

Recent astronomical observations indicate that our universe is in the phase of accelerated expansion [1]. There has been much recent interest in the idea that our universe may be a brane embedded in some higher dimensional space. One of the models along this line proposed by Dvali, Gabadadze and Porrati (DGP) [2] is known to contain a branch of solutions exhibiting self accelerated expansion of the universe [3].

The brane world black holes in the Randall-Sundrum(RS) model [4] have been studied by many authors. Firstly, Chamblin *et al.* [5] presented evidence that a non-rotating uncharged black hole on the brane is described by a "black cigar" solution in five dimensions. Then, Dadhich *et al.* [6] showed that the Reissner-Nördstrom metric is an exact solution of the effective Einstein equations on the brane, and Shiromizu *et al.* [7] derived the effective gravitational equations on the brane. A solution for charged brane world black holes in the RS model was obtained in [8] and the charged rotating case was obtained in [9]. In particular, Aliev *et al.* [9] found exact solutions of charged rotating black holes in the Kerr-Schild form [10] using a stationary and axisymmetric metric ansatz on the brane.

In the case of the DGP model, approximate Schwarzschild solutions had been obtained in [11, 12, 13, 14]. An exact Schwarzschild solution on the brane was obtained in [15]. In the DGP model, the sources are assumed to be localized on a brane by a certain mechanism not related to gravity itself. In [16], it was discussed that if the sources were not localized, the brane with the induced graviton kinetic term has effectively repulsive gravity and it would push any source off the brane. As a result, ordinary black holes cannot be held on the brane. However, authors of [15] commented that charged black holes could still be quasilocalized if the corresponding gauge fields are localized. Recently, motivated by the above suggestion, an exact solution of charged black holes on the brane in the DGP model was obtained in [17]. However, up to now no solution of charged rotating black holes on the DGP brane is obtained.

Here, we intend to improve this situation a bit. We try to obtain an exact solution of charged rotating black holes on the brane in the DGP model by noting a particular set of conditions that satisfies the constraint equation. We first obtain the solution in the KerrSchild form [10]. Then, by using a modified Boyer-Lindquist coordinate transformation we find the horizon and ergosphere. In doing this, we use a stationary and axisymmetric metric ansatz on the brane and the solution exhibits the characteristics of self accelerated expansion of the brane world universe.

This paper is organized as follows. In section 2, we set the action and equations of motion of the DGP model following the approach of Ref. [18]. In section 3, we get a rotating black hole solution on the brane in the absence of Maxwell field and examine the properties of the solution. In section 4, we extend the result of section 3 and find a solution for the charged rotating case. In section 5, we conclude with discussion. In this last section, we discuss a possible bulk solution consistent with our on-brane solution.

2 Action and field equations

The DGP gravitational action in the presence of sources takes the form [2]

$$S = M_*^3 \int d^5 x \sqrt{-g} \,^{(5)}R + \int d^4 x \sqrt{-h} \left(M_P^2 R + L_{matter} \right), \tag{1}$$

where R and ⁽⁵⁾R are the 4D and 5D Ricci scalars, respectively and L_{matter} is the Lagrangian of the matter fields trapped on the brane. Here, the (4 + 1) coordinates are $x^A = (x^{\mu}, y(=x^5))$, $\mu = 0, 1, 2, 3$, and g is the determinant of the five-dimensional metric g_{AB} , while h is the determinant of the four-dimensional metric $h_{\mu\nu} = g_{\mu\nu}(x^{\mu}, y = 0)$. A cross-over scale is defined by $r_c = m_c^{-1} = M_P^2/2M_*^3$. There is a boundary(a brane) at y = 0 and Z_2 symmetry across the boundary is assumed. The field equations derived from the action (1) have the form

$$^{(5)}G_{AB} = ^{(5)}R_{AB} - \frac{1}{2}g_{AB} \ ^{(5)}R = \kappa_5^2 \sqrt{\frac{h}{g}} \left(X_{AB} + T_{AB} \right) \delta(y), \tag{2}$$

where $\kappa_4^2 = M_P^{-2}$ and $\kappa_5^2 = M_*^{-3}$, while $X_{AB} = -\delta_A^{\mu} \delta_B^{\nu} G_{\mu\nu} / \kappa_4^2$ and $T_{AB} = \delta_A^{\mu} \delta_B^{\nu} T_{\mu\nu}$ is the energy-momentum tensor in the braneworld.

Now, we consider the metric of the following form [18, 19],

$$ds^{2} = g_{AB}dx^{A}dx^{B} = g_{\mu\nu}(x,y)dx^{\mu}dx^{\nu} + 2N_{\mu}dx^{\mu}dy + (N^{2} + g_{\mu\nu}N^{\mu}N^{\nu})dy^{2}.$$
 (3)

The $(\mu 5)$, (55) components of the field equations (2) are called as the momentum and Hamiltonian constraint equations, respectively, and are given by [18, 19]

$$\nabla_{\nu}K^{\nu}_{\ \mu} - \nabla_{\mu}K = 0, \tag{4}$$

$$R - K^2 + K_{\mu\nu}K^{\mu\nu} = 0, (5)$$

where $K_{\mu\nu}$ is the extrinsic curvature tensor defined by

$$K_{\mu\nu} = \frac{1}{2N} (\partial_y g_{\mu\nu} - \nabla_\mu N_\nu - \nabla_\nu N_\mu), \qquad (6)$$

and ∇_{μ} is the covariant derivative operator associated with the metric $g_{\mu\nu}$.

Integrating both sides of the field equation (2) along the y direction and taking the limit of y = 0 on the both sides of the brane we arrive at the Israel's junction condition [20] on the Z_2 symmetric brane in the relation [17]

$$G_{\mu\nu} = \kappa_4^2 T_{\mu\nu} + m_c (K_{\mu\nu} - h_{\mu\nu} K).$$
(7)

In this paper, we take the electro-magnetic field as the matter source on the brane. Using (7) in the constraint (4) and (5) we find that the momentum constraint equation is satisfied identically, while the Hamiltonian constraint equation is written as

$$R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2 + m_c^2 R + \kappa_4^4 T_{\mu\nu}T^{\mu\nu} - 2\kappa_4^2 R_{\mu\nu}T^{\mu\nu} = 0, \qquad (8)$$

where we used $T = T^{\mu}_{\ \mu} = 0.$

Finally, combining the Einstein Equations in the bulk $(y \neq 0)$

$$^{(5)}G_{AB} = {}^{(5)}R_{AB} - \frac{1}{2}g_{AB} \,{}^{(5)}R = 0 \tag{9}$$

with (7) we arrive at the gravitational field equations on the brane [17]

$$G_{\mu\nu} = -E_{\mu\nu} - \frac{\kappa_4^4}{m_c^2} (T^{\rho}_{\ \mu} T_{\rho\nu} - \frac{1}{2} h_{\mu\nu} T_{\rho\sigma} T^{\rho\sigma}) - \frac{1}{m_c^2} (R^{\rho}_{\ \mu} R_{\rho\nu} - \frac{2}{3} R R_{\mu\nu} + \frac{1}{4} h_{\mu\nu} R^2 - \frac{1}{2} h_{\mu\nu} R_{\rho\sigma} R^{\rho\sigma}) + \frac{\kappa_4^2}{m_c^2} (R^{\rho}_{\ \mu} T_{\rho\nu} + T^{\rho}_{\ \mu} R_{\rho\nu} - \frac{2}{3} R T_{\mu\nu} - h_{\mu\nu} R_{\rho\sigma} T^{\rho\sigma}),$$
(10)

where $E_{\mu\nu}$ is the traceless "electric part" of the 5-dimensional Weyl tensor ⁽⁵⁾ C_{ABCD} [7] and $m_c^{-1} = \kappa_5^2/2\kappa_4^2$. In what follows we shall set $\kappa_4^2 = 8\pi$.

In general, the field equations on the brane are not closed and one needs to solve the evolution equations into the bulk. However, by assuming a special ansatz for the induced metric on the brane, one can make the system of equations on the brane closed.

3 Rotating black hole solution

We start with a stationary and axisymmetric metric describing a rotating black hole localized on a 3-brane in the DGP model. We write it as the Kerr-Schild form [10] in which the metric is expressed in a linear approximation around the flat metric:

$$ds^{2} = (ds^{2})_{\text{flat}} + f(k_{\mu}dx^{\mu})^{2}, \qquad (11)$$

where f is an arbitrary scalar function and k_{μ} is a null, geodesic vector field in both the flat and full metrics with

$$k_{\mu}k^{\mu} = 0, k^{\nu}D_{\nu}k_{\mu} = 0.$$
⁽¹²⁾

Introducing the Kerr-Schild coordinates $x^{\mu} = \{u, r, \theta, \varphi\}$, we write the metric as [9]

$$ds^{2} = h_{\mu\nu}dx^{\mu}dx^{\nu} = \left[-(du+dr)^{2} + dr^{2} + \Sigma d\theta^{2} + (r^{2}+a^{2})\sin^{2}\theta d\varphi^{2} + 2a\sin^{2}\theta dr d\varphi\right] + H(r,\theta)(du-a\sin^{2}\theta d\varphi)^{2}, \quad (13)$$

where

$$\Sigma(r,\theta) = r^2 + a^2 \cos^2 \theta, \tag{14}$$

and a is the angular momentum per unit mass of the black hole.

For the uncharged case we can set $T_{\mu\nu} = 0$, and the Hamiltonian constraint equation (8) is reduced to

$$R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2 + m_c^2 R = 0.$$
(15)

Note that the above equation is satisfied with the following two sets of conditions,

$$R = 0, \quad R_{\mu\nu}R^{\mu\nu} = 0, \tag{16}$$

and

$$R = 12m_c^2, \quad R_{\mu\nu}R^{\mu\nu} = 36m_c^4. \tag{17}$$

The first set (16) is satisfied with the metric function $H(r, \theta) = 2Mr/\Sigma$ in the metric (13), which is the usual Kerr solution in general relativity. The second set (17) corresponds to a non-flat(de-Sitter) case, in which the conditions (17) in terms of the metric function $H(r, \theta)$ in (13) are given by the following:

$$12m_c^2 = \frac{\partial^2 H}{\partial r^2} + \frac{4r}{\Sigma}\frac{\partial H}{\partial r} + \frac{2H}{\Sigma},\tag{18}$$

$$36m_c^4 = \frac{4H}{\Sigma^2} \left(r\frac{\partial H}{\partial r} + a^2 \cos^2 \theta \frac{\partial^2 H}{\partial r^2} \right) + \frac{4r^2}{\Sigma^2} \left(\frac{\partial H}{\partial r} \right)^2 + \frac{2r}{\Sigma} \frac{\partial H}{\partial r} \frac{\partial^2 H}{\partial r^2} + \frac{1}{2} \left(\frac{\partial^2 H}{\partial r^2} \right)^2 + \frac{2}{\Sigma^4} (r^4 - 2a^2 r^2 \cos^2 \theta + 5a^4 \cos^4 \theta) H^2.$$
(19)

The metric function H satisfying the above two equations is given by

$$H = \frac{2Mr + m_c^2(r^4 + 6r^2a^2\cos^2\theta - 3a^4\cos^4\theta)}{\Sigma},$$
(20)

where the parameter M is an arbitrary constant of integration. One can easily check that the metric (13) with (20) satisfies the equation (8) with $T_{\mu\nu} = 0$. In the limit $a \to 0$, the metric (13) with (20) is reduced to the Schwarzschild-de Sitter black hole solution with the cosmological constant $\Lambda = 3m_c^2$ in general relativity. This corresponds to the solution for the U(r) = -2 case in Ref. [15], and belongs to the accelerated branch of Kerr-de Sitter type solution [21, 22].

In order to check the physical properties of the metric given by (13) with (20) we want to transform the Kerr-Schild form to the Boyer-Lindquist coordinates. However, the equations (18) and (19) are not preserved under the transformation for the usual Boyer-Lindquist coordinates. Thus in order to preserve the equations (18) and (19) under coordinate transformation, we use the following modified transformation of Boyer-Lindquist type:

$$du = dt - \frac{r^2 + a^2}{\Delta} dr - X d\theta, \ d\varphi = d\phi - \frac{a}{\Delta} dr - Y d\theta,$$
(21)

where $\Delta = r^2 + a^2 - H(r, \theta)\Sigma(r, \theta)$, X and Y are functions of r and θ only and satisfy the relation

$$\frac{\partial}{\partial r}X(r,\theta) = \frac{\partial}{\partial\theta}\left(\frac{r^2 + a^2}{\Delta}\right), \ \frac{\partial}{\partial r}Y(r,\theta) = \frac{\partial}{\partial\theta}\left(\frac{a}{\Delta}\right).$$
(22)

The newly added terms X and Y can be analytically integrated from the transformation (21) for the function H given in (20). In fact, the modified transformation (21) that satisfy (22) leaves the Hamiltonian constraint (15) invariant.

Under the transformation (21), the metric (13) takes the form

$$ds^{2} = -(1-H)dt^{2} + \frac{\Sigma}{\Delta}dr^{2} + 2[X - H(X - Ya\sin^{2}\theta)]dtd\theta + [\Sigma - (1-H)X^{2} - 2HXYa\sin^{2}\theta + Y^{2}(r^{2} + a^{2} + Ha^{2}\sin^{2}\theta)]d\theta^{2} - 2[(r^{2} + a^{2})Y - aH(X - Ya\sin^{2}\theta)]\sin^{2}\theta d\theta d\phi + (r^{2} + a^{2} + Ha^{2}\sin^{2}\theta)\sin^{2}\theta d\phi^{2} - 2Ha\sin^{2}\theta dtd\phi.$$
(23)

Using "MATHEMATICA" we check that the equations (18) and (19) remain unchanged with the metric (23) for any metric function $H(r, \theta)$. Hence, we can use (20) as a solution for the metric in (23).

For $r \ll r_c$, X and Y with (20) can be approximately written as

$$X \approx -6m_c^2 a^2 \sin 2\theta \left[r - \frac{(r_1^2 + a^2)(r_1^2 - a^2 \cos^2 \theta)}{(r - r_1)(r_1 - r_2)^2} + \frac{2T_1 \ln(\frac{r}{r_1} - 1)}{(r_1 - r_2)^3} + (r_1 \leftrightarrow r_2) \right] + \eta_1(\theta),$$

$$Y \approx 6m_c^2 a^3 \sin 2\theta \left[\frac{(r_1^2 - a^2 \cos^2 \theta)}{(r - r_1)(r_1 - r_2)^2} + \frac{2T_2 \ln(\frac{r}{r_1} - 1)}{(r_1 - r_2)^3} + (r_1 \leftrightarrow r_2) \right] + \eta_2(\theta),$$
(24)

where $T_1 = r_1^4 - 2r_1^3r_2 - r_1r_2a^2\sin^2\theta + a^4\cos^2\theta$, $T_2 = r_1r_2 - a^2\cos^2\theta$ and r_1 , r_2 are two roots of the equation $r^2 - 2Mr + a^2 = 0$. When the crossover scale r_c is infinite (or $m_c = 0$) and both $\eta_1(\theta)$ and $\eta_2(\theta)$ are set to be zero, then the equations (18) and (19) are preserved with the metric function $H = \frac{2Mr}{\Sigma}$. This corresponds to the exact Kerr solution in general relativity once we identify the parameter M as the mass of the black hole.

The governing equation for horizon radius is given by

$$\Delta = r^2 + a^2 - 2Mr - m_c^2 (r^4 + 6r^2 a^2 \cos^2 \theta - 3a^4 \cos^4 \theta) = 0.$$
⁽²⁵⁾

The metric (23) with (20) has three horizons located at r_{\pm} and r_{CH} , provided the total mass M lies in the range $M_{1e}|_{\theta=\pi/2} \leq M \leq M_{2e}|_{\theta=0,\pi}$ where M_{1e} and M_{2e} are given by

$$M_{1e} = \frac{1}{3\sqrt{6}m_c}\sqrt{\alpha - A^{3/2}}, \quad M_{2e} = \frac{1}{3\sqrt{6}m_c}\sqrt{\alpha + A^{3/2}}$$
(26)

with

$$\alpha = 1 + 36m_c^2 a^2 - 18m_c^2 a^2 \cos^2\theta (1 + 12m_c^2 a^2) + 216m_c^4 a^4 \cos^4\theta (1 - 4m_c^2 a^2 \cos^2\theta)$$
(27)

and

$$A = 1 - 12m_c^2 a^2 (1 + \cos^2 \theta).$$
⁽²⁸⁾

Here r_{CH} , which is smaller than the crossover scale r_c , is a cosmological horizon, r_+ and r_- are outer and inner horizon, respectively.

The horizons can be expressed explicitly as follows:

$$r_{\pm} = \frac{1}{2m_c} \left(D_{-}^{1/2} \pm \sqrt{D_{+} - 4Mm_c D_{-}^{-1/2}} \right), \ r_{CH} = \frac{1}{2m_c} \left(-D_{-}^{1/2} + \sqrt{D_{+} + 4Mm_c D_{-}^{-1/2}} \right),$$
(29)

where

$$D_{\pm} = C \pm \frac{1}{3} \left[C + A \left(\frac{2}{B + \sqrt{B^2 - 4A^3}} \right)^{1/3} + \left(\frac{2}{B + \sqrt{B^2 - 4A^3}} \right)^{-1/3} \right]$$
(30)

with $C = 1 - 6m_c^2 a^2 \cos^2 \theta$ and $B = 2C[C^2 + 36m_c^2 a^2(1 + 3m_c^2 a^2 \cos^4 \theta)] - 108m_c^2 M^2$. Note that the horizons r_{\pm} and r_{CH} always have real positive values if the total mass lies between the masses $M_{1e}|_{\theta=\pi/2}$ and $M_{2e}|_{\theta=0,\pi}$.

For $r \ll r_c$, the outer and inner horizons can be approximated as

$$r_{\pm} \approx \frac{M \pm \sqrt{M^2 - a^2(1 - 3m_c^2 a^2 \cos^2 \theta)}}{1 - 6m_c^2 a^2 \cos^2 \theta}.$$
(31)

In the limit of $r_c \to \infty$, we get $M_{1e} \to a$, $M_{2e} \to \infty$, $r_{\pm} \to M \pm \sqrt{M^2 - a^2}$, and $r_{CH} \to \infty$ independent of the angle θ as one can expect from (25).

For the ergosphere we calculate the condition

$$g_{tt} = r^2 + a^2 \cos^2 \theta - 2Mr - m_c^2 (r^4 + 6r^2 a^2 \cos^2 \theta - 3a^4 \cos^4 \theta) = 0,$$
(32)

which has three boundaries of the ergosphere known as the "static limit" surfaces located at r_E^{\pm} and r_E^{CH} .

Setting
$$r_E^{\pm}$$
 and r_E^{CH} as

$$r_E^{\pm} = \frac{1}{2m_c} \left(\tilde{D}_{-}^{1/2} \pm \sqrt{\tilde{D}_{+} - 4Mm_c \tilde{D}_{-}^{-1/2}} \right), \ r_E^{CH} = \frac{1}{2m_c} \left(-\tilde{D}_{-}^{1/2} + \sqrt{\tilde{D}_{+} + 4Mm_c \tilde{D}_{-}^{-1/2}} \right),$$
(33)

where

$$\tilde{D}_{\pm} = C \pm \frac{1}{3} \left[C + \tilde{A} \left(\frac{2}{\tilde{B} + \sqrt{\tilde{B}^2 - 4\tilde{A}^3}} \right)^{1/3} + \left(\frac{2}{\tilde{B} + \sqrt{\tilde{B}^2 - 4\tilde{A}^3}} \right)^{-1/3} \right]$$
(34)

with $\tilde{A} = 1 - 24m_c^2 a^2 \cos^2 \theta$ and $\tilde{B} = 2C[C^2 + 36m_c^2 a^2 \cos^2 \theta (1 + 3m_c^2 a^2 \cos^2 \theta)] - 108m_c^2 M^2$, we obtain

$$r_E^- \le r_- < r_+ \le r_E^+ < r_E^{CH} \le r_{CH}, \tag{35}$$

with the equalities holding at $\theta = 0, \pi$.

Comparing this with the event horizons (29), we see that the ergosphere lies in the region $r_+ < r < r_E^+$ and $r_E^{CH} < r < r_{CH}$, which coincide with the horizon at $\theta = 0, \pi$.

4 Charged rotating black hole solution

In this section, we consider the case in which the brane contains a Maxwell field with an electric charge. Outside the black hole, the Maxwell field can be described by a source-free Maxwell equations. Thus, we have to solve simultaneously the constraint equation (8) and the Maxwell equations:

$$g^{\mu\nu}D_{\mu}F_{\nu\sigma} = 0, \tag{36}$$

$$D_{[\mu}F_{\nu\sigma]} = 0, \tag{37}$$

where D_{μ} is the covariant derivative operator associated with the brane metric $h_{\mu\nu}$. However, we only need to solve Eqs. (8) and (36), since Eq. (37) is satisfied identically.

Hinted from the characteristic of the Kerr-Newman solution in general relativity which yields $R_{\mu\nu}R^{\mu\nu} = 4Q^4/\Sigma^4$, $R_{\mu\nu}T^{\mu\nu} = Q^4/2\pi\Sigma^4$, $T_{\mu\nu}T^{\mu\nu} = Q^4/16\pi^2\Sigma^4$, we note that the Hamiltonian constraint (8) on the brane is satisfied with the following two set of conditions:

$$R = 0, \quad R_{\mu\nu}R^{\mu\nu} = \frac{4Q^4}{\Sigma^4}, \quad R_{\mu\nu}T^{\mu\nu} = \frac{Q^4}{2\pi\Sigma^4}, \quad T_{\mu\nu}T^{\mu\nu} = \frac{Q^4}{16\pi^2\Sigma^4}$$
(38)

and

$$R = 12m_c^2, \quad R_{\mu\nu}R^{\mu\nu} = 36m_c^4 + \frac{4Q^4}{\Sigma^4}, \quad R_{\mu\nu}T^{\mu\nu} = \frac{Q^4}{2\pi\Sigma^4}, \quad T_{\mu\nu}T^{\mu\nu} = \frac{Q^4}{16\pi^2\Sigma^4}.$$
 (39)

The first set (38) is satisfied with the conventional Kerr-Newman solution which is given by the following potential one-form A_{μ} and the metric function H:

$$A_{\mu}dx^{\mu} = -\frac{Qr}{\Sigma}(du - a\sin^2\theta d\varphi), \qquad (40)$$

$$H = \frac{2Mr - Q^2}{\Sigma},\tag{41}$$

where the parameter Q is the electric charge of the black hole. The second set (39) is satisfied with the following solution:

$$A_{\mu}dx^{\mu} = -\frac{Qr}{\Sigma}(du - a\sin^2\theta d\varphi), \qquad (42)$$

$$H = \frac{2Mr - Q^2 + m_c^2(r^4 + 6r^2a^2\cos^2\theta - 3a^4\cos^4\theta)}{\Sigma}.$$
(43)

In the non-rotating limit, $a \to 0$, the second solution, (42) and (43), reduces to the conventional charged de Sitter solution with the cosmological constant $\Lambda = 3m_c^2$ [25]. This case corresponds to the U(r) = -2 case of Ref. [17], in which it was shown to belong to the accelerated branch.

In order to check the physical properties of the solution, we again make a transformation of the Boyer-Lindquist type (21). Since the Maxwell equation (36) should transform covariantly under (21), the potential one-form (42) should also transform covariantly:

$$A_{\mu}dx^{\mu} = -\frac{Qr}{\Sigma} \left[dt - a\sin^2\theta d\phi - (X - Ya\sin^2\theta)d\theta \right] + \frac{Qr}{\Delta}dr.$$
 (44)

The nonvanishing components of the electromagnetic field tensor $F_{\mu\nu}$ are given by

$$F_{r\theta} = -\frac{Q(r^2 - a^2 \cos^2 \theta)(X - Ya \sin^2 \theta)}{\Sigma^2}, \quad F_{rt} = \frac{Q(r^2 - a^2 \cos^2 \theta)}{\Sigma^2},$$
$$F_{t\theta} = \frac{Qra \sin 2\theta}{\Sigma^2}, \quad F_{\phi r} = \frac{Qa(r^2 - a^2 \cos^2 \theta) \sin^2 \theta}{\Sigma^2}, \quad F_{\theta \phi} = \frac{Qar(r^2 + a^2) \sin 2\theta}{\Sigma^2}. \quad (45)$$

Since the Hamiltonian constraint (8) is invariant under (21) with the transformed potential one-form (44), we only need to check the Maxwell equation (36). Indeed, the above potential one-form (44) satisfies the Maxwell equation (36) with the metric (23) and (43).

Now, we would like to examine the gravitational effect on the brane due to the extra dimension. To do that we will calculate the projected Weyl tensor $E_{\mu\nu}$ in (10) using our potential one-form (44) and the metric (23) with (43). In the charged rotating case the tensor $E_{\mu\nu}$ is quite complicated to tell anything definite. For instance, E_r^r component is given by

$$E_{r}^{r} = -\frac{m_{c}^{2}}{\Sigma^{2}\Delta} \left[6a^{2} (r^{4}(1-3\cos^{2}\theta) - 2r^{2}a^{2}\cos^{2}\theta\sin^{2}\theta + a^{4}\cos^{4}\theta(21-19\cos^{2}\theta)) + Q^{2}(r^{4}+6r^{2}a^{2}\cos^{2}\theta - 3a^{4}\cos^{4}\theta) \right] + \frac{Q^{2}}{\Sigma^{2}\Delta} (r^{2}-2Mr+Q^{2}).$$
(46)

In the non-rotating limit $(a \rightarrow 0)$, with (43) and (44) the gravitational field equation (10) becomes

$$\kappa_4^2 T_{\mu\nu} = -E_{\mu\nu},\tag{47}$$

where $T_{\mu\nu}$ is calculated to be the same energy-momentum tensor as in the conventional four dimensional charged black hole case. This tells us that there is no gravitational effect on the brane due to the extra dimension in the non-rotating charged case.

To examine the horizon structure of the metric given by (23) and (43), we write the governing equation for the radius of horizon

$$\Delta = r^2 + a^2 + Q^2 - 2Mr - m_c^2 (r^4 + 6r^2 a^2 \cos^2 \theta - 3a^4 \cos^4 \theta) = 0.$$
(48)

The solution for the above equation provides three horizons located at r'_{\pm} and r'_{CH} when the total mass M lies in the range $M'_{1e}|_{\theta=\pi/2} \leq M \leq M'_{2e}|_{\theta=0,\pi}$ where M'_{1e} and M'_{2e} are given by

$$M'_{1e} = \frac{1}{3\sqrt{6}m_c}\sqrt{\alpha' - A'^{3/2}}, \quad M'_{2e} = \frac{1}{3\sqrt{6}m_c}\sqrt{\alpha' + A'^{3/2}}.$$
(49)

Here,

$$\alpha' = 1 + 36m_c^2(a^2 + Q^2) - 18m_c^2a^2\cos^2\theta(1 + 12m_c^2a^2 + 12m_c^2Q^2) + 216m_c^4a^4\cos^4\theta(1 - 4m_c^2a^2\cos^2\theta)$$
(50)

and

$$A' = 1 - 12m_c^2(a^2 + Q^2 + a^2\cos^2\theta).$$
(51)

The explicit expressions of the horizons are as follows:

$$r'_{\pm} = \frac{1}{2m_c} \left(D_{-}^{\prime 1/2} \pm \sqrt{D_{+}^{\prime} - 4Mm_c D_{-}^{\prime - 1/2}} \right), \ r'_{CH} = \frac{1}{2m_c} \left(-D_{-}^{\prime 1/2} + \sqrt{D_{+}^{\prime} + 4Mm_c D_{-}^{\prime - 1/2}} \right),$$
(52)

where

$$D'_{\pm} = C \pm \frac{1}{3} \left[C + A' \left(\frac{2}{B' + \sqrt{B'^2 - 4A'^3}} \right)^{1/3} + \left(\frac{2}{B' + \sqrt{B'^2 - 4'A^3}} \right)^{-1/3} \right], \quad (53)$$

and $B' = 2C^3 + 72m_c^2C(a^2 + Q^2 + 3m_c^2a^4\cos^4\theta) - 108m_c^2M^2.$

Note that the horizons r'_{\pm} and r'_{CH} always have a real positive value if the total mass lies between the masses $M'_{1e}|_{\theta=\pi/2}$ and $M'_{2e}|_{\theta=0,\pi}$. In the limit $r_c \to \infty$, we get $M'_{1e} \to a$, $M'_{2e} \to \infty, r'_{\pm} \to M \pm \sqrt{M^2 - a^2 - Q^2}$, and $r'_{CH} \to \infty$ independent of the angle θ as in the rotating case.

The defining condition $g_{tt} = 0$ for the ergosphere in this case is given by

$$r^{2} + a^{2}\cos^{2}\theta + Q^{2} - 2Mr - m_{c}^{2}(r^{4} + 6r^{2}a^{2}\cos^{2}\theta - 3a^{4}\cos^{4}\theta) = 0.$$
 (54)

Setting $r_E^{'\pm}$ and $r_E^{'CH}$ as $r_E^{'\pm} = \frac{1}{2m_c} \left(\tilde{D'}_{-}^{1/2} \pm \sqrt{\tilde{D'}_{+} - 4Mm_c \tilde{D'}_{-}^{-1/2}} \right), \ r_E^{'CH} = \frac{1}{2m_c} \left(-\tilde{D'}_{-}^{1/2} + \sqrt{\tilde{D'}_{+} + 4Mm_c \tilde{D'}_{-}^{-1/2}} \right),$ (55)

where

$$\tilde{D'}_{\pm} = C \pm \frac{1}{3} \left[C + \tilde{A'} \left(\frac{2}{\tilde{B'} + \sqrt{\tilde{B'}^2 - 4\tilde{A'}^3}} \right)^{1/3} + \left(\frac{2}{\tilde{B'} + \sqrt{\tilde{B'}^2 - 4\tilde{A'}^3}} \right)^{-1/3} \right], \quad (56)$$

with

$$\begin{split} \tilde{A}' &= 1 - 12m_c^2 Q^2 - 24m_c^2 a^2 \cos^2 \theta, \\ \tilde{B}' &= 2C^3 + 72m_c^2 C(Q^2 + a^2 \cos^2 \theta + 3m_c^2 a^4 \cos^4 \theta) - 108m_c^2 M^2, \end{split}$$

we get the same relation as in the non-charged rotating case

$$r'_{E} \leq r'_{-} < r'_{+} \leq r'_{E} + < r'_{E} \leq r'_{CH} \leq r'_{CH},$$
(57)

and the ergosphere lies in the region $r'_{+} < r < r'_{E}$ and $r'_{E}CH < r < r'_{CH}$ coinciding with the horizon at $\theta = 0, \pi$.

5 Discussion

In this paper we considered charged rotating black holes on a 3-brane in the DGP model. Assuming a Z_2 -symmetry across the brane and with a stationary and axisymmetric metric ansatz on the brane, we solved the constraint equations of (4+1)-dimensional gravity to find a metric for charged rotating black hole on the brane.

First, we obtain a particular solution of the Kerr-Newman-de Sitter type in the Kerr-Schild form, which corresponds to the so-called accelerated branch of the DGP model.

Then, in order to find the event horizon of the black hole, we introduce a modified version of Boyer-Lindquist coordinates. The Hamiltonian constraint equation is quite complicated to solve, even compared with the RS model case [9], and not preserved under the conventional Boyer-Lindquist transformation. Thus in order to use the obtained Kerr-Schild type solution, we have to introduce a modified transformation which preserves the constraint equation.

In the case of the RS model, the authors of [9] devised a transformation for a given fixed angle θ , and showed that the equations are preserved under their transformation thereby the metric function H remains as a solution of the constraint equation. However, with this type of transformation the coordinates patches for different θ angles belong to differently transformed coordinates, and it makes hard to view the obtained event horizon in a single consistent picture. In order to avoid this kind of problem, we use a slightly modified version of Boyer-Lindquist coordinate transformation which covers the entire θ angle while the solution obtained in the Kerr-Schild form can still be used. In this solution, the structure of the horizon is very similar to that of the Kerr-Newman-de Sitter black hole in general relativity except for the θ -angle dependence. When the crossover scale r_c approaches infinity, the θ -angle dependence of the horizon disappears and the solution reduces to that of the Kerr-Newman black hole in general relativity.

Finally, we discuss a possible bulk solution consistent with our on-brane solution. For this, here we limit ourselves to the non-rotating limit to make our discussion tractable. Rather than following the strategy of extending the on-brane solution to the bulk, we try directly to find a bulk solution consistent with our on-brane solution. For the most simple uncharged case, we find that the following 5D metric satisfies the 5D field equations, (2):

$$ds^{2} = e^{-2m_{c}|y|}(dy^{2} + h_{\mu\nu}dx^{\mu}dx^{\nu}), \qquad (58)$$

where

$$h_{\mu\nu}dx^{\mu}dx^{\nu} = -\left(1 - \frac{2Mr + m_{c}^{2}r^{4}}{r^{2}}\right)dt^{2} + \left(1 - \frac{2Mr + m_{c}^{2}r^{4}}{r^{2}}\right)^{-1}dr^{2} + r^{2}(d\theta^{2} + \sin^{2}\theta d\phi^{2}).$$
(59)

Namely, the above metric satisfies the 5D Einstein equation in the bulk, ${}^{(5)}G_{AB} = 0$, as well as the on-brane field equation, (10). The projected Weyl tensor $E_{\mu\nu}$ obtained from the 5D metric (58) vanishes, and this is consistent with the previously obtained relation (47) since the energy-momentum tensor vanishes in this case. Therefore, in the uncharged case we can say that our on-brane solution is consistent with the above given bulk solution.

For the charged case, the electro-magnetic field vanishes off the brane (in the bulk) by the set-up. So far we could not find a bulk solution which smoothly matches the metric on the brane with the metric off the brane while reflects the discontinuity of electro-magnetic field at the boundary which is non-zero on the brane and suddenly vanishes off the brane. We now leave this challenging problem of finding consistent bulk solutions for the charged and rotating case as an open project and welcome anyone who is interested in.

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