Multipolar analysis of spinning binaries

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Abstract. We present a preliminary study of the multipolar structure of gravitational radiation from spinning black hole binary mergers. We consider three different spinning binary configurations: (1) one "hang-up" run, where the black holes have equal masses and large spins initially aligned with the orbital angular momentum; (2) seven "spin-flip" runs, where the holes have a mass ratio $q \equiv M_1/M_2 = 4$, the spins are anti-aligned with the orbital angular momentum, and the initial Kerr parameters of the holes $j_1 = j_2 = j_i$ (where $j \equiv J/M^2$) are fine-tuned to produce a Schwarzschild remnant after merger; (3) three "super-kick" runs where the mass ratio $q = 1$, 2, 4 and the spins of the two holes are initially located on the orbital plane, pointing in opposite directions. For all of these simulations we compute the multipolar energy distribution and the Kerr parameter of the final hole. For the hang-up run, we show that including leading-order spin-orbit and spin-spin terms in a multipolar decomposition of the post-Newtonian waveforms improves agreement with the numerical simulation.

These are exciting times for gravitational wave (GW) research. Earth-based laser-interferometric detectors are collecting data at design sensitivity, and LIGO [1] just completed the longest scientific run to date. The space-based interferometer LISA is expected to open an observational window at low frequencies ($\sim 10^{-4} - 10^{-1}$ Hz) within the next decade [2]. Following remarkable breakthroughs in the simulation of the strongest expected GW sources, the inspiral and coalescence of black hole binaries [3, 4, 5], several groups have now explored various aspects of this problem, including spin-precession and spin-flips [6, 7], comparisons of numerical results with post-Newtonian (PN) predictions [8, 9, 10, 11], multipolar analyses of the emitted radiation [8, 9, 12] and the use of numerical waveforms in data analysis [13, 14, 15, 16].

In Ref. [9] we studied the multipolar distribution of radiation and the final spin resulting from the merger of unequal-mass, non-spinning black holes with mass ratios $q = M_1/M_2$ in the range 1 to 4. The main purpose of this paper is to show preliminary results from our attempt to extend the analysis to spinning binaries.

A second purpose of this study is to test recent predictions for the spin of the black hole resulting from a generic merger. Buonanno, Kidder and Lehner [17] recently introduced a surprisingly accurate model, based on the extrapolation of point-particle results, that was shown to be in good agreement with existing numerical simulations (see also [18]). An interesting question explored in [17] concerns spin-flip configurations. Suppose that initially both black holes have equal Kerr parameters ($j_i = j_1 = j_2$), and spins *antialigned* with respect to the orbital angular momentum. For a given mass ratio q, which value of j_i will produce a Schwarzschild black hole? These "critical" configurations could be very interesting, since mild variations of the parameters around the critical values may produce interesting orbital dynamics (eg. spin flips) and complex gravitational waveforms. As argued in [17], one needs unequal masses to be able to produce a Schwarzschild remnant at all. For $q = 4$ and zero eccentricity, Ref. [17] predicts that a Schwarzschild black hole should be formed when $j_i = -0.815$. A semi-analytical fitting formula [18] predicts a critical spin $j_i = -0.823$. Here we provide a numerical benchmark against which to test these analytical models, and possibly other models that may be developed in the future, by computing the final Kerr parameter $j_{\rm fin}$ from a sequence of spinning binaries with $q = 4$ and values of $j_i \in [-0.75, -0.87]$. The numerical results are well fitted by a linear relation of the form $j_{\rm fin} = -0.570(j_i - 0.842)$. From this fit, our best estimate for the initial spin leading to the formation of a Schwarzschild remnant is $j_i \simeq -0.842 \pm 0.003$.

The plan of the paper is as follows. In Sec. 1 we introduce the numerical code and we list the new simulations considered in this paper. In Sec. 2 we compare the multipolar energy distribution of spinning and non-spinning binaries. In Sec. 3 we generalize our multipolar decomposition of PN waveforms to include leading-order spin contributions in the special case of spins aligned (or anti-aligned) with the orbital angular momentum, and we show (in a special case) that the inclusion of spin terms improves the agreement with numerical results [19]. In Sec. 4 we study in detail "spin-flip" configurations designed to produce a Schwarzschild remnant. Finally, in Sec. 5 we discuss the fraction of energy radiated in ringdown in the different simulations.

1. Numerical simulations

In this work we compare two sequences of numerical black hole binary simulations.

Sequence 1 is a series of simulations of non-spinning black hole binaries with mass ratio q ranging from 1 to 4. These simulations were performed with the BAM code [20], and they were used in [21] to study gravitational recoil and in [9] to investigate the multipolar structure of the emitted gravitational radiation. Sequence 2 consists of simulations of binary systems with mass-ratios in the same range, but with non-vanishing spins. In particular, we study two families of spinning binaries. For the first family, the spin of both holes is either aligned (run uu1) or anti-aligned (runs dd1–dd7) with the orbital angular momentum. For the second family, the spins of the two holes are initially located on the orbital plane, and they point in opposite directions. The latter configuration produces surprisingly large recoil velocities [7, 22], and it is sometimes referred to as a "super-kick" configuration (runs sk1, sk2, sk4).

Sequence 2 binaries have been evolved with an advanced version of the LEAN code described in [23]. The key improvement over the original code is the implementation of sixth-order accurate stencils for spatial derivatives, as introduced in [24]. LEAN is based on the CACTUS computational toolkit and uses CARPET [25] for mesh-refinement, TWOPUNCTURES [26] for puncture initial data and AHFINDERDIRECT

Table 1. Details of the spinning binary simulations of sequence 2. The holes start on the x axis with a coordinate separation of $6M$ for models uu1 and sk1–4 and $8M$ for models dd1–7. Black hole 1 is located at $x > 0$ and hole 2 at $x < 0$. For the spin-kick (sk) runs $\vec{S}_1 = -\vec{S}_2$ and the spins are aligned along the x-axis, while for all other models $\vec{S}_1 = +\vec{S}_2$ and spins are aligned along the z-axis. A common horizon forms at t_{cah} and the number of gravitational wave cycles from $t_0 = r_{\mathrm{ex}} + 50M$ to the peak in the wave amplitude is N_{cyc}. E_{EMOP} is the total ringdown energy radiated in $l = m = 2$ and $(l = 2, m = -2)$; the number in parenthesis is the percentage of energy in the $(l = 2, m = -2)$ mode.

model	$\frac{M_1}{M_2}$	$\frac{S_1}{M_1^2}$	$\frac{P}{M}$	$\frac{t_{\mathrm{cah}}}{M_{\mathrm{ADM}}}$	N_{cyc}	$10^2 \frac{E_{\mathrm{rad}}}{M_{\mathrm{ADM}}}$	$10^2 \frac{E_{\mathrm{EMOP}}}{M_{\mathrm{ADM}}}$	$\frac{J_{\mathrm{rad}}}{M_{\mathrm{ADM}}^2}$	$\frac{J_{\mathrm{fin}}}{M_{\mathrm{fin}}^2}$
uu1	1.0	0.926	0.126	269.8	7.68	8.220 ± 0.330	2.928 (50)	0.4273 ± 0.0299	0.9505
dd1	4.0	-0.75	0.0774	129.7	2.23	0.705 ± 0.028	0.338 (50)	0.0567 ± 0.0040	0.0533
dd2	4.0	-0.80	0.0778	122.5	2.09	0.677 ± 0.027	0.362 (50)	0.0546 ± 0.0038	0.0237
dd3	4.0	-0.82	0.0779	124.0	2.07	0.664 ± 0.027	0.354 (50)	0.0539 ± 0.0038	0.0122
dd4	4.0	-0.83	0.0780	121.4	2.02	0.654 ± 0.026	0.342 (50)	0.0529 ± 0.0037	0.0068
dd5	4.0	-0.84	0.0781	118.3	1.95	0.646 ± 0.026	0.326 (50)	0.0521 ± 0.0036	0.0012
dd6	4.0	-0.85	0.0781	112.8	1.86	0.630 ± 0.025	0.302 (50)	0.0504 ± 0.0035	-0.0038
dd7	4.0	-0.87	0.0783	111.8	1.82	0.626 ± 0.025	0.287 (50)	0.0484 ± 0.0034	-0.0154
sk1	1.0	0.727	0.140	109.5	2.52	3.453 ± 0.138	2.028 (67)	0.2030 ± 0.0142	0.6821
sk2	2.0	0.727	0.125	129.0	2.93	2.665 ± 0.107	1.159 (71)	0.1591 ± 0.0111	-
sk4	4.0	0.727	0.0903	119.0	2.53	1.405 ± 0.056	0.381 (73)	0.0727 ± 0.0051	-

[27, 28] for horizon finding. We have used the advanced version of LEAN for all spinning configurations, except for the model labelled uu1 in Table 1, which has first been reported in Ref. [19].

In Table 1 we list the initial physical parameters of all binaries of sequence 2. We also list: the time t_{cah} of formation of a common apparent horizon; the number of wave cycles N_{cyc} derived from the phase‡ of the $(l = 2, m = 2)$ mode; the total radiated energy E_{rad} and the total radiated angular momentum J_{rad}, excluding again the initial data burst; the energy radiated in ringdown E_{EMOP}, as estimated using the energy-maximized orthogonal projection [9], for the dominant modes $(l = 2, m = 2)$ and when the symmetry is broken (i.e., for the sk runs) also for $(l = 2, m = -2)$; and the final black hole spin $j_{\mathrm{fin}} = J_{\mathrm{fin}}/M_{\mathrm{fin}}^2$. For models sk2 and sk4, some angular momentum is radiated in the x- and y-directions. This results in a realignment of the final spin, and computing j_{fin} becomes more difficult. For this reason, the corresponding entries in the table are empty. Following the convention of Ref. [29], initial data are normalized to $M = M_1 + M_2$, while all radiated quantities are normalized to the ADM mass M_{ADM}. The corresponding details for the non-spinning models can be found in Refs. [9, 21].

In the notation of Sec. II E of Ref. [23], the grid setup is $\{(256, 128, 64, 32, 16, 8) \times (2, 1, 0.5),\ h = 1/80\}$ for the uu1 run, $\{(384, 192, 96, 56, 24, 12) \times (3, 1.5, 0.75),\ h = 1/48\}$ for the sk1 and sk2 runs, and $\{(384, 192, 96, 56, 24) \times (6, 3, 1.5, 0.75),\ h = 1/48\}$ for all other simulations. In addition, we have performed higher-resolution simulations with $h = 1/52$ and $h = 1/56$ of the two models labelled sk1 and sk2 in Table 1.

The resulting convergence plot for the $(l = 2, m = 2)$ mode Ψ_4 of the Newman-Penrose scalar Ψ_4 extracted at $r_{\mathrm{ex}} = 60 M_{\mathrm{ADM}}$ is shown in Fig. 1. The differences between the high-resolution runs have been rescaled by a factor of 1.72, as expected

‡ To remove the initial radiation burst the phase was integrated from $t = 50M + r_{\mathrm{ex}}$, where r_{ex} is the extraction radius, up to the maximum in the wave amplitude.

Figure 1. Convergence analysis of the $(l = 2, m = 2)$ mode of $M_{\mathrm{ADM}}r\Psi_4$ for simulations sk1 and sk2 of Table 1. The difference between the high-resolution runs has been rescaled by 1.72, as expected for sixth-order convergence.

for sixth-order convergence.

To estimate uncertainties arising from finite resolution, we use the differences obtained for these convergence runs. In order to allow for the fact that the LEAN code also contains ingredients of lower than sixth-order accuracy, we follow the approach of Ref. [9] and apply a Richardson extrapolation assuming second-order accuracy. Using these conservative estimates, and the observed $1/r_{\mathrm{ex}}$ fall-off of the errors on radiated energy and momenta, we obtain very similar uncertainty estimates for simulations sk1 and sk2. At resolution $h = 1/48$ and extraction radius $r_{\mathrm{ex}} = 60M_{\mathrm{ADM}}$ these uncertainties are $\sim 4\%$ for the radiated energy and $\sim 7\%$ for the radiated angular momenta. For the discussion below, it is important to remark that our numerical results *underestimate* the radiated quantities in all cases.

2. Energy distribution for non-spinning binaries

The decomposition of gravitational radiation from non-spinning binaries onto spin-weighted spherical harmonics (SWSHs) $_{-2}Y_{l,m}$ can be found in [9], where it was obtained by projecting the 2.5PN gravitational waveforms§ derived in [31, 32]. The most notable result of Ref. [9] is that the leading-order term contributing to each multipolar component $\psi_{l,m}$ of the Weyl scalar Ψ_4 is proportional to the symmetric mass ratio $\eta = q/(1+q)^2$ when m is even, and to $\eta\,\delta M/M = \eta(M_1 - M_2)/M$ when m is odd. More explicitly:

$$e^{i\tilde{\phi}} \, M r \, \psi_{l,m} = \eta \sum_{n=0}^{5} g_{l,m}^{(n)}(\eta)(M\Omega)^{(8+n)/3} \,, \qquad m \text{ even}\,; \qquad (1)$$

§ The SWSH components of $h_+, h_\times$ are related to the corresponding components of Ψ_4 by $(h_+ - ih_\times)_{l,m} = -\psi_{l,m}/(m\Omega)^2$, with the exception of 2.5PN contributions to the $l = m = 2$ component. It has recently been pointed out that by using the known expressions of the radiative multipoles, instead of the waveforms, more information is available and the contribution of each multipole can be computed to higher PN order [30].

$$e^{i\tilde{\phi}} \, M r \, \psi_{l,m} = \eta \frac{\delta M}{M} \sum_{n=1}^{5} k_{l,m}^{(n)}(\eta)(M\Omega)^{(8+n)/3} \,, \qquad m \text{ odd} \,.$$

Here $\tilde{\phi}$ is an orbital phase, including logarithmic corrections in the orbital frequency Ω. The precise definition of this phase, as well as the functional form of the coefficients $g_{l,m}^{(n)}(\eta)$ and $k_{l,m}^{(n)}(\eta)$, can be found in Appendix A of [9]. Each coefficient in the series represents a PN contribution of order $n/2$ to the leading-order (Newtonian) prediction.

Figure 2. Left: Energy E_l in different multipoles for the unequal-mass binaries of sequence 1 as a function of q (from [9]). Right: E_l for some spinning binary configurations belonging to sequence 2, as a function of the multipole index l. Continuous (black) lines refer to equal-mass binaries, the dotted (red) line to a binary with $q = 2$, and dashed (blue) lines to binaries with $q = 4$.

From Eq. (1) it is clear that, for non-spinning binaries, odd-m multipoles are suppressed in the equal-mass limit. Since $l = m$ components typically dominate the radiation for each given l, this also implies that odd-l multipoles are suppressed as $q \to 1$. This is confirmed by numerical simulations of the merger. In addition, higher multipoles are found to carry a larger fraction of the total energy as q deviates from unity. Both of these features are clear from a glance at the left panel of Fig. 2. In [9] we also showed that, to leading order, the total energy radiated in the merger scales like η^2 and the Kerr parameter of the final hole scales like η, providing phenomenological fits of these quantities. More general fitting formulas for the final Kerr parameter, encompassing also binaries with aligned or anti-aligned spins, can be found in [18].

In the right panel of Fig. 2 we show the energy distribution for some of our sequence 2 binaries, as a function of the multipole index l. The relative uncertainties of the mode energies have been determined in analogy to those of the total radiated energy for runs sk1 and sk2 and strongly depend on the energy content (the "signal strength"). We find relative uncertainties of about 5% for values of E_l/M_{ADM} down to approximately $5 \, 10^{-4}$. These grow up to 30% for low signals of $10^{-5}...10^{-4}$. The figure reveals that even in the presence of spin, odd-l multipoles are suppressed when $q = 1$. As first observed in [33], the hang-up (uu) configuration stays in orbit for a longer time and radiates more energy before merging. On the contrary, our spin-flip (dd) simulations with $q = 4$ merge very rapidly (compare the number of cycles N_{cyc} in Table 1). All dd simulations radiate roughly the same amount of energy, so we only show run dd1 in the plot. By comparing the sk1, sk2 and sk4 runs we confirm that our conclusion in [9], that large-q binaries radiate more energy in higher multipoles,

holds true also for these spinning binaries. This is clear from the slopes of E_l as a function of l for the different sk runs, and it is nicely illustrated by a comparison of the sk2 and sk4 runs. Even though sk4 radiates roughly half the energy radiated by sk2, the energy radiated in each $l > 2$ multipole by the sk4 run is larger.

3. Leading-order spin-orbit and spin-spin contributions

For spins aligned or anti-aligned with the orbital angular momentum, the leading-order spin contributions to the various multipolar components are most easily derived by projecting Eqs. (F24) and (F25) of Ref. [34] onto SWSHs, according to the procedure described in [9]. Let S_i be the projection of the spin of body i on the axis orthogonal to the orbital plane. S_i is positive (negative) if the spins are aligned (anti-aligned) with the orbital angular momentum. Define the dimensionless spin parameter $j_i = S_i/M_i^2$ ($i = 1,\ 2$) and the spin combinations $\chi_s = (j_1+j_2)/2$, $\chi_a = (j_1-j_2)/2$. Including only the dominant spin-orbit (1.5PN order) and spin-spin (2PN) terms, in addition to the non-spinning part of Eq. (1) we find three spin-dependent multipolar contributions:

$$Mr\,\psi_{2,2}^{\rm spin}e^{i\tilde\phi} = 32\sqrt{\frac{\pi}{5}}\eta(M\Omega)^{8/3} \tag{2}$$

$$\times\left[\frac{4}{3}\left(\chi_s(\eta-1) - \chi_a\frac{\delta M}{M}\right)M\Omega + 2\eta(\chi_s^2 - \chi_a^2)(M\Omega)^{4/3}\right],$$

$$Mr\,\psi_{2,1}^{\rm spin}e^{i\tilde\phi} = -\frac{8}{3}\sqrt{\frac{\pi}{5}}\eta(M\Omega)^3\left[\frac{3}{2}\left(\chi_s\frac{\delta M}{M} + \chi_a\right)(M\Omega)^{1/3}\right]. \tag{3}$$

$$Mr\,\psi_{3,2}^{\rm spin}e^{i\tilde\phi} = \frac{32}{3}\sqrt{\frac{\pi}{7}}\eta(M\Omega)^{10/3}\left[4\chi_s\eta(M\Omega)^{1/3}\right]. \tag{4}$$

These equations demonstrate that odd-m multipoles do not always vanish for equal-mass systems. For example, $\psi_{2,1}^{\rm spin}$ contains a term which is *not* proportional to the mass difference $\delta M/M$: for equal masses and $j_1 \neq j_2$, the *dominant* contribution to $\psi_{2,1}$ comes from spin terms. Therefore, by simulating binaries with equal masses and unequal spins and by looking at the $(l = 2, m = 1)$ component we can disentangle subleading spin effects from the leading-order non-spin contributions, and thus facilitate the comparison of spin definitions in PN theory and in numerical simulations. Unfortunately, for systems with equal mass *and equal spins*, such as our uu1 model, $\psi_{2,1}^{\rm spin} = 0$. However, we can still study the effect of spin terms on the convergence of the PN approximation by considering the $(l = 2, m = 2)$ component.

To see how this is possible, consider first the non-spinning case. By taking the modulus of (1) we get a PN series relating the orbital frequency and the gravitational wave amplitude $|\psi_{l,m}|$. The convergence rate of the series to the numerical results can be studied as follows.

First, we observe that the frequency of the gravitational waves $\omega_{\rm GW}$ in a multipolar component $\psi_{l,m}$ is related to the orbital frequency Ω by $\omega_{\rm GW} = m\Omega$. Therefore, given a time-dependent component $\psi_{l,m}$ of the Weyl scalar, the numerical value of the binary's orbital frequency can be estimated as $\Omega \simeq \omega_{Dm} = -{\rm Im}(\dot\psi_{l,m}/\psi_{l,m})/m$ [8, 9]. Consider now the modulus of Eq. (1), possibly with the addition of spin terms such as Eq. (3). Given $|\psi_{l,m}(t)|$ on the left-hand side (as obtained from the simulation), at any give PN order we can (at least in principle) numerically invert the PN expansion on the right-hand side. Since this expansion is

only valid for quasi-circular binaries, in this way we get a post-Newtonian quasi-circular (PNQC) estimate of the orbital frequency: $\Omega \simeq \omega_{\mathrm{PNQC}}$. If the PNQC approximation works well, ω_{PNQC} should be close to ω_{Dm}. Furthermore, if the PN approximation is converging, the agreement should get better as we increase the PN order, i.e. the number of terms in the sum on the right-hand side of Eq. (1).

Figure 3. Convergence of the PNQC expansion. Left: non-spinning binaries; right: spinning binaries. In the right panel, thick lines estimate the PNQC frequency including the spin terms of Eq. (3), and thin lines omit the spin terms.

In Fig. 3 we show the relative deviation between ω_{PNQC} and ω_{D2}, considering the dominant ($l = m = 2$) component of the radiation. Let us first consider the left panel. There we show the relative deviation between ω_{PNQC} and ω_{D2} for the longest non-spinning, equal-mass simulation considered in [9], at different PN orders. At early-times we see oscillations in the relative deviation, that damp away as the binary evolves. The magnitude of the relative deviation $|(\omega_{\mathrm{PNQC}} - \omega_{D2})/\omega_{D2}|$ can be taken as an indicator of the accuracy of the PN approximation. The plot shows that the convergence of the PN series is not monotonic. The transition from inspiral to plunge is roughly marked by the vertical lines, that correspond to the point where the orbital frequency equals the innermost stable circular orbit or ISCO (as defined in [35], computed at 2PN and 3PN to bracket uncertainties). At this point the PNQC frequency, which only makes sense in the inspiral phase, decouples from ω_{D2}.

The right panel shows the relative deviation between ω_{PNQC} and ω_{D2} for our uu1 run. Vertical lines mark again the 2PN and 3PN ISCO, that for such large aligned spins corresponds to much higher orbital frequencies: in our case $M\Omega_{\mathrm{ISCO}}^{3\mathrm{PN,spin}} = 0.247$, while for zero spins we would get $M\Omega_{\mathrm{ISCO}}^{3\mathrm{PN,no\ spin}} = 0.129$. Here we also indicate the formation of a common apparent horizon (CAH). The fact that the PNQC estimate again deviates from ω_{D2} at the ISCO seems to indicate that the PN notion of orbital instability makes sense even for such large values of the spin. The relatively short duration of the simulation, and the large wiggles induced by numerical noise, clearly illustrate the need to start simulations of spinning binaries at larger separation. It is also clear that including spin-terms improves the agreement between the numerics and the PNQC approximation at 1.5PN and 2PN orders. The trend is reversed at 2.5PN, possibly because we are not including 2.5PN spin contributions. Higher-order calculations of spin contributions in PN theory and longer simulations will be necessary for more accurate comparisons.

4. Producing a Schwarzschild remnant

A question of particular astrophysical importance concerns the final spin resulting from the inspiral and merger of black-hole binaries with arbitrary initial parameters [17, 18]. An intriguing special case is that where two black holes with initial spins anti-aligned with the orbital angular momentum merge forming a non-spinning hole.

Our sequence of runs with anti-aligned initial spins and mass ratio $q = 4$ (symmetric mass ratio $\eta = 0.16$) has been designed to bracket the critical point of formation of a Schwarzschild black hole, as predicted in Refs. [17, 18]. We calculate the final Kerr parameter using energy and angular momentum balance arguments, i.e. we compute the final black-hole mass as $M_{\rm fin} = M_{\rm ADM} - E_{\rm rad}$ and the final angular momentum as $J_{\rm fin} = J_{\rm ini} - J_{\rm rad}$. The resulting dimensionless Kerr parameter $j_{\rm fin}$ is given in the rightmost column of Table 1. Applying a linear regression analysis to these results and the associated error estimates leads to the fitting formula $j_{\rm fin} = (-0.570 \pm 0.040)[j_i - (0.842 \pm 0.003)]$. A Schwarzschild remnant is thus produced when the initial spin has the "critical" value $j_{\rm ini} \simeq -0.842 \pm 0.003$. As mentioned above, we generally find the uncertainties due to finite differencing and extraction radius to underestimate the radiated angular momentum, so that we expect the correct value to be > -0.842. We can compare our result to that of Ref. [18] by applying standard error propagation to the uncertainties listed in their Eq. (7). Specifically, we solve their Eq. (6) for a, set $a_{\rm fin} = 0$ and calculate the uncertainty $\Delta a^2 = \sum_i (\partial a / \partial v_i)^2 \Delta v_i^2$, where $v_i = s_4$, s_5, t_0, t_2 and t_3. The resulting critical angular momentum of 0.824 ± 0.019 agrees very well with our value. Both results also agree well with that of -0.815 predicted in Ref. [17].

Finally, a systematic uncertainty in our numerical results is due to the relatively small initial separation of the holes. However, a comparison of non-spinning simulations starting at different initial separations shows that most of the angular momentum is radiated during the last orbit prior to formation of a common apparent horizon [9]. Therefore this systematic error should not significantly affect our results. Simulations starting at larger separation are required to verify this expectation, and we plan to investigate this effect in future work.

5. Ringdown energy

Present ringdown searches in LIGO data are based on matched filtering. For this reason, a practical criterion to define the ringdown content of a given waveform is the energy-maximized orthogonal projection (EMOP) discussed in [9], which is basically matched filtering in white noise. As shown in [9], the EMOP estimate of the energy radiated in ringdown is a lower bound on the energy that can be detected by matched filtering. Table 1 lists the sum of the ringdown energies radiated in the dominant $(l = 2, m = 2)$ and $(l = 2, m = -2)$ components of the radiation. For the sk runs the radiation is not symmetric with respect to the z-axis [36], therefore we also list (in parentheses) the percentage of $E_{\rm EMOP}$ radiated in $(l = 2, m = -2)$. From the data we see that as much as $\sim 3\%$ of the rest energy of the system is radiated in the ringdown phase for equal-mass binaries. From our previous study of non-spinning binaries [9] it is reasonable to assume that, for fixed initial Kerr parameters, $E_{\rm EMOP} \sim \eta^2$. Such high ringdown efficiencies are good news for the detection of ringdown waves and for their use in parameter estimation [37, 38].

EMOP estimates for runs sk2 and sk4 should be taken with caution. For these

runs, the spin axis of the final black hole is tilted with respect to the z-axis of the coordinate frame used for the evolutions and for wave extraction, and our chosen reference frame is not appropriate to describe the symmetries of the final hole. The tilt in the final spin angle produces mode mixing in the ringdown phase: a pure (l, m) mode in the frame chosen for numerical computations is a sum of modes with different (l', m')'s in the frame whose z-axis coincides with the symmetry axis of the final hole, and vice versa. Because of this mode mixing, an estimate of the Kerr parameter of the final hole using quasinormal mode fits is not trivial. A detailed treatment of this problem will be presented in future work.

Acknowledgments

References

[1] A. Abramovici *et al. Science*, 256:325–333, 1992.
[2] K. Danzmann *et al. Pre-Phase A Report, 2nd ed.*, 1998.
[3] F. Pretorius. *Phys. Rev. Lett.*, 95:121101, 2005. gr-qc/0507014.
[4] M. Campanelli, C. O. Lousto, P. Marronetti, and Y. Zlochower. *Phys. Rev. Lett.*, 96:111101, 2006. gr-qc/0511048.
[5] J. G. Baker, J. Centrella, D.-I. Choi, M. Koppitz, and J. van Meter. *Phys. Rev. Lett.*, 96:111102, 2006. gr-qc/0511103.
[6] M. Campanelli, C. O. Lousto, Y. Zlochower, B. Krishnan, and D. Merritt. *Phys. Rev.*, D75:064030, 2007. gr-qc/0612076.
[7] M. Campanelli, C. O. Lousto, Y. Zlochower, and D. Merritt. *Astrophys.J.*, 659:L5–L8, 2007. gr-qc/0701164v2.
[8] A. Buonanno, G. B. Cook, and F. Pretorius. *Phys. Rev.*, D75:124018, 2007. gr-qc/0610122.
[9] E. Berti *et al. Phys. Rev.*, D76:064034, 2007. gr-qc/0703053.
[10] M. Hannam, S. Husa, J. A. González, U. Sperhake, and B. Brügmann. 2007. arXiv:0706.1305.
[11] M. Boyle *et al.* 2007. arXiv:0710.0158 [gr-qc].
[12] J. D. Schnittman *et al.* 2007. arXiv:0707.0301 [gr-qc].
[13] T. Baumgarte *et al.* 2006. gr-qc/0612100.
[14] Y. Pan, A. Buonanno, J. G. Baker, J. Centrella, B. J. Kelly, S. T. McWilliams, F. Pretorius, and J. R. van Meter. 2007. gr-qc/0704.1964.
[15] P. Ajith *et al. Class. Quant. Grav.*, 24:S689–S700, 2007. arXiv:0704.3764 [gr-qc].
[16] P. Ajith *et al.* 2007. arXiv:0710.2335 [gr-qc].
[17] A. Buonanno, L. E. Kidder, and L. Lehner. 2007. arXiv:0709.3839 [astro-ph].
[18] L. Rezzolla et al. 2007. arXiv:0710.3345 [gr-qc].
[19] P. Marronetti, W. Tichy, B. Brugmann, J. A. Gonzalez, and U. Sperhake. 2007. arXiv:0709.2160 [gr-qc].
[20] B. Brügmann, M. González, Hannam, S. Husa, U. Sperhake, and W. Tichy. 2006. gr-qc/0610128.

[21] J. A. González, U. Sperhake, B. Brügmann, M. D. Hannam, and S. Husa. *Phys. Rev. Lett.*, 98:091101, 2007. gr-qc/0610154.

[22] J. A. González, M. D. Hannam, U. Sperhake, B. Brügmann, and S. Husa. *Phys. Rev. Lett.*, 98:231101, 2007. gr-qc/0702052.

[23] U. Sperhake. *Phys. Rev.*, D76:104015, 2007. gr-qc/0606079.

[24] S. Husa, J. A. González, M. D. Hannam, B. Brügmann, and U. Sperhake. 2007. arXiv:0706.0740 [gr-qc].

[25] E. Schnetter, S. H. Hawley, and I. Hawke. *Class. Quant. Grav.*, 21:1465–1488, 2004. gr-qc/0310042.

[26] M. Ansorg, B. Brügmann, and W. Tichy. *Phys. Rev.*, D70:064011, 2004. gr-qc/0404056.

[27] J. Thornburg. Phys. Rev., D54:4899–4918, 1996. gr-qc/9508014.

[28] J. Thornburg. Class. Quant. Grav., 21:743–766, 2004. gr-qc/0306056.

[29] U. Sperhake et al. 2007. arXiv:0710.3823 [gr-qc].

[30] L. E. Kidder. 2007. arXiv:0710.0614 [gr-qc].

[31] L. Blanchet, B. R. Iyer, C. M. Will, and A. G. Wiseman. *Class. Quant. Grav.*, 13:575–584, 1996. gr-qc/9602024.

[32] K. G. Arun, L. Blanchet, B. R. Iyer, and M. S. S. Qusailah. *Class. Quant. Grav.*, 21:3771–3802, 2004. gr-qc/0404085.

[33] M. Campanelli, C. O. Lousto, and Y. Zlochower. *Phys. Rev.*, D74:041501, 2006. gr-qc/0604012.

[34] C. M. Will and A. G. Wiseman. *Phys. Rev.*, D54:4813–4848, 1996. gr-qc/9608012.

[35] A. Buonanno, Y.-b. Chen, and M. Vallisneri. *Phys. Rev.*, D67:104025, 2003. gr-qc/0211087.

[36] B. Brügmann, J. A. Gonzalez, M. Hannam, S. Husa, and U. Sperhake. 2007. arXiv:0707.0135 [gr-qc].

[37] E. Berti, V. Cardoso, and C. M. Will. *Phys. Rev.*, D73:064030, 2006. gr-qc/0512160.

[38] E. Berti, J. Cardoso, V. Cardoso, and M. Cavaglia. *Phys. Rev.*, D76:104044, 2007. arXiv:0707.1202 [gr-qc].