The Arrow Of Time In The Landscape

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ABSTRACT

The future is unlike the past. This is an absolutely basic observation, as basic as the observation that "things fall". We expect that a real understanding of the Arrow of Time — the temporal "direction" defined by the physical differences between the future and the past — will arise only in the context of some deep theory, such as string theory. Conversely, we argue here that such an understanding is urgently required if recent stringtheoretic ideas about cosmology are to be made to function.

In this work, we explain in detail why it is so difficult to find a satisfactory theory of the Arrow of Time. There are several difficulties, but the key problems arise from Roger Penrose's observation that the Arrow is ultimately connected with the extremely "non-generic" character of the spatial geometry of the earliest Universe. We argue that the explanations of the Arrow proposed hitherto are unsatisfactory, because they do not address this basic point. In any case, none of them applies to the string Landscape, in which the nucleation of baby universes is postulated to "populate" the multiverse. Here we argue that baby universes can only have an Arrow if they *inherit* one; the problem of explaining the Arrow is thus reduced to explaining it in the case of the original universe. Motivated by a recent formulation of "creation from nothing" in the context of string theory, we propose that the original universe was created along a spacelike surface with the topology of a torus. Using deep results in global differential geometry, we are able to show that the geometry of this surface had to be non-generic. This geometric "specialness" is communicated to matter through the inflaton, the basic physical field postulated by the theory of Inflation. Thus we have a theory of the Arrow which is intrinsically geometric, which incorporates Inflation, and which allows universes in the Landscape to begin with physically acceptable initial conditions.

1. 10^{500} — You Call That "Large"?

As is well known — some would say "notorious" — string theory admits a "very large" number of internally consistent basic models of the Universe: this is the string Landscape [\[1\]](#page-35-0)[\[2\]](#page-35-1)[\[3\]](#page-35-2). The number usually cited, simply for definiteness, is 10^{500} . Here we shall argue that this number is not at all "large": it is in fact perilously small.

Leaving this claim to one side for the moment, having a "large" number like 10^{500} emerge naturally from a physical theory is of the utmost interest. For Nature herself has recently seen fit to present us with an exceedingly *small* number. Since 1998 [\[4\]](#page-35-3)[\[5\]](#page-36-0), evidence has been mounting that an invisible negative-pressure energy component is currently dominating the evolution of our Universe. The evidence for this "dark energy" is strictly gravitational in nature, so, in stating its properties, we use units appropriate to gravitation. These are the Planck Units. An example of such units is the Planck density, given in terms of ordinary units by 5.15500×10^{96} kilogrammes per cubic metre. The dark energy mass density is given in these units by

$$
\rho_{DE} \approx 10^{-120}.\tag{1}
$$

The first point to note with regard to this number is that all attempts to compute a unique value for it, or for the related quantity known as the cosmological constant, have failed. In string theory, one has a likely candidate, the vacuum energy density, but the theory does not fix this quantity uniquely: it varies across the Landscape. The second point to note is that this number does appear to be very small indeed; but perhaps it is not so small when we discover that we may have something like 10^{500} opportunities to realise it. This may possibly be the case if we identify the observed dark energy with the Landscape vacuum energy.

The situation is analogous to the following one. Suppose that you have made it your life's goal to throw a dart at a dartboard in such a manner that the dart strikes the "bullseye". There are various ways in which this ambition might be realised. You might cheat, using perhaps a system of powerful concealed magnets to ensure the desired outcome. A marginally less ignominious scheme would be to throw handfuls of darts at the board, without bothering to aim; with a few thousand darts, a bullseye will probably be scored; the superfluous darts can then be removed.

Now suppose that a game official enters the room and finds a single dart in the bullseye. The question as to which of our suggested techniques has actually been used can only be settled by examining the circumstances. If, for example, a scrupulous search fails to reveal any magnetic devices, but does uncover a crate containing several thousand darts, then it is reasonable to assume that the second method has been employed.

Our failure to find a means of uniquely deriving a very tiny cosmological constant from first principles, combined with the existence of a huge number of string vacua, inclines many to favour an account of the situation analogous to the second method of dartplaying. That is, given a "large" number of possible vacua with suitably spaced vacuum energies, one can hope to show that at least one string vacuum with the observed energy will be internally consistent. (It should be said that actually making this proposal work at a technical level is — contrary to a widespread misapprehension — an extremely non-trivial and delicate matter: see [\[3\]](#page-35-2).)

Now the reader is asked to note the care with which we have chosen our words: we say that the vacuum is *internally consistent*, not that it $exists¹$ $exists¹$ $exists¹$. There is a very clear moral distinction between *conspiring* to cheat as a way of winning a professional darts game, and actually carrying out such a conspiracy. More than this: if one actually tries to score a bullseye by throwing thousands of darts at the board, one needs to explain in detail exactly how one proposes to secure such a large supply of darts.

Similarly, we need a way of actually building the universe whose internal consistency we have asserted. The simplest approach is to find some mechanism that automatically builds essentially every universe in the Landscape. This automatically brings into existence universes with the "right" value of the vacuum energy.

The currently favoured mechanism for building universes is the nucleation of "baby universes," which split off from a given universe under rather general conditions. Baby universes can have properties which differ radically from those of their parents. There are grounds for believing that this variability allows essentially all of the Landscape to be brought into actual existence, with baby universes having suitably spaced vacuum energies, as above [\[3\]](#page-35-2). The idea is that baby universes can themselves eventually have babies; thus, even if a particular universe in the Landscape does not have a particularly small vacuum energy, one or more of its *descendants* may well have a cosmological constant similar in value to the one we observe.

It seems to be generally assumed that, having found a mechanism for producing the right value of the vacuum energy, our task of universe-building is essentially complete; for even after discarding babies with the "wrong" vacuum energy, one still finds an enormous number of babies which do resemble our Universe in that respect. Surely a universe just like ours can be found in this huge ensemble? Let us examine this assumption more closely, however.

Isaac Newton discovered a set of laws which are able to describe, with very great accuracy, the extremely complex motions of all of the objects in our solar system. Yet the laws themselves are extremely simple. How was that possible? It is possible because Newton was able to isolate most of the complexity in the initial conditions of the problem. These initial conditions are apparently not constrained by any law, and hence they can vary in a very complex way. They have to be determined by direct observation. Newton assures us that he can predict the motion of the system provided that we supply him with precise details of the positions and velocities of the objects at some initial time. Conversely, no matter how thoroughly we understand the dynamical laws of a system, we will not be able to describe its motion unless we are provided with the initial conditions.

Similarly here: even if we feel confident that our ensemble of universes contains members with the "right" physical laws and parameters (such as the vacuum energy), we cannot say whether the ensemble is likely to contain even one member resembling our Universe until we can answer this question: how hard is it to produce a baby universe with initial conditions not too dissimilar to those of our Universe?

The answer to this question is (roughly) known: it was given by Roger Penrose [\[7\]](#page-36-1) in 1979. Penrose observed that the second law of thermodynamics, the statement that the entropy of an isolated system can (almost) never decrease, implies that the entropy

¹There is of course a body of philosophical opinion which controverts this distinction. Few physicists sympathise; see however [\[6\]](#page-36-2).

of the Universe at the beginning of time must have been very low, compared with the value it might have had. One can make this more quantitative after the manner familiar in statistical mechanics, as follows.

The basic concept we need is that of the "phase space" of a dynamical system. This is just an abstract mathematical space, introduced (in this context) over a century ago by Josiah Willard Gibbs, in which each point represents one possible state of the system. In practice, many states are indistinguishable, so one speaks of the system occupying a certain volume in phase space. To say that the entropy of a system is low is to say that it occupies a volume in the appropriate phase space which is small compared to the full volume accessible to it. The volume in phase space corresponding to the actual initial conditions of our observable Universe can be roughly estimated, and Penrose computed the full volume, using the then recently developed theory of the thermodynamics of black holes. How much larger is this full phase space volume, the volume that was open to the earliest Universe, of which it somehow failed to take advantage? Penrose computes the ratio to be about

$$
P \approx 10^{10^{123}}.\t(2)
$$

This number represents some fundamental property of our Universe; we propose to name it the "Penrose number". Note that the exponent in the Penrose number is itself exponential. Compared to this number, the Landscape begins to seem positively claustrophobic. Dividing Penrose's number by 10^{500} has, to an excellent approximation, no effect whatever.

The answer to our question — how hard can it be to build a baby universe with initial conditions similar to the initial conditions of our Universe? — has turned out to be: it is almost unimaginably hard. If we should propose to rely on "mere chance" to select for us a universe from the Landscape, then our chances of obtaining the right value for the vacuum energy are excellent, but the probability that this universe will resemble our own is nevertheless essentially zero. For it will almost certainly have the wrong initial conditions.

The conclusion, of course, is that we *cannot* rely on mere chance to build anything that resembles our Universe. The Landscape is far, far too small for anything of that sort to be possible. We commend this fact to the attention of those who maintain, on the basis that 10^{500} is "large", that the Landscape can account for *any* possible observation.

There are many possible responses to this line of argument, and we will discuss some of them below. But the essential point is clear: no "mechanism for building universes" can be considered satisfactory unless it addresses the size of the Penrose number in a convincing way.

As is well known, Penrose's calculation arose from his abiding concern with the problem of the Arrow of Time. The macroscopic features of our world are massively asymmetric in time. Eggs break when dropped; this process has never been seen to occur in reverse. Since the seminal work of Ludwig Boltzmann, we have understood why this is so: in modern terms, the intact egg occupies a vastly smaller domain in phase space than the broken one. That is, being broken is a more generic state than being intact. The breaking of the egg is then the entirely unsurprising process of the egg finding its way from less to more generic states. This evolution from less generic to more generic is precisely what we call the passage of time. (A discussion of this argument, and of the many subtleties we have ignored here, may be found in reference [\[8\]](#page-36-3).)

The amazing aspect of this situation, then, is not the irreversible breaking of the egg — it is the fact that intact eggs ever exist "in the first place". With this basic insight in hand, the existence of the Arrow that we observe now is readily traced back, through a chain of steadily less generic states, to the earliest Universe. (The egg owes its existence to a chicken, which in turn exists only because the Sun supplies us with a reliable source of low-entropy photons, and the Sun exists because of low-entropy conditions billions of years ago, and so on.) The real question is then: why was our Universe born in such an ultra-low entropy state? What is the ultimate origin of the Arrow of Time?

One point we wish to make here is that the string Landscape can only be made to work if we can find a concrete answer to this question. The need to answer it has suddenly become a matter of the greatest urgency. If we cannot understand the Arrow, then we will never know whether the Landscape has any physical relevance at all; for we will not know whether any universe in the Landscape has the right initial conditions.

In this work, we will propose a solution of this problem. It is put forward not as the last word on the subject, but to show that the $-$ undeniably formidable $-$ difficulties which arise can be surmounted. The basic idea is to combine the initial-value theory for gravity with certain deep theorems in global differential geometry to explain precisely how the Arrow came into existence.

Our approach here will be resolutely string-theoretical. The problem of the Arrow of Time can of course be posed in a broader context, and there is a large literature on the subject from various points of view. The classic text, giving a complete treatment of the background material, is that of Heinz-Dieter Zeh [\[9\]](#page-36-4); other indispensable sources are [\[10\]](#page-36-5), [\[11\]](#page-36-6), [\[12\]](#page-36-7), [\[13\]](#page-36-8), and [\[14\]](#page-36-9). The new ideas advanced here are described in technical detail in [\[15\]](#page-36-10)[\[16\]](#page-36-11); some recent alternative approaches are given in [\[17\]](#page-36-12)[\[18\]](#page-36-13).

Let us begin with a survey of the difficulties which must be overcome by any theory of the Arrow.

2. Why Is Understanding The Arrow A Hard Problem?

As befits a question of such fundamental importance, understanding the origin of the Arrow is an extremely difficult problem. Let us try to be as specific as possible as to why it is so hard.

2.1. Inflation Doesn't Make an Arrow.

Inflation is the extremely successful idea [\[19\]](#page-36-14) that our Universe is so large because of a short but fantastically rapid period of expansion very early in the history of the Universe. In this picture, the observed Universe (and, in fact, a vast region beyond it) is obtained, by the passage of time, from a tiny region near the beginning. This theory has its critics, but it is fair to say that it is now the standard theoretical framework for the physics of the very early Universe. Basically, when one has a problem regarding the physics of that era, one looks to Inflation for a solution^{[2](#page-4-0)}. One reason for the difficulty of the Arrow question

²The reader is nevertheless warned that the precise way in which Inflation arises in string theory is not yet fully understood; see [\[20\]](#page-37-0).

is that this is one of the few problems that Inflation does not solve. Let us see why this is so.

Unitarity can be described informally — mathematically precise definitions can be given both in classical and in quantum mechanics — as the principle that information is never completely lost (or created *ex nihilo*) in the evolution of any physical system. All of the ordinary observed^{[3](#page-5-0)} processes in nature obey this principle; which is not to say that it cannot, in principle, be violated by very exotic processes.

Now it does not follow that a description or portrait of an object must have as many possible states as the object itself. An atlas of maps of the Earth certainly has a smaller range of possible states than the Earth itself. This is of course by no means paradoxical, since it is clearly understood that the art of making maps consists in deliberately disregarding irrelevant information in order to present more important data; that is, cartography is a process in which information is discarded, not destroyed. To many physicists, it seems "natural" that the inflationary "initial region" contained very little information: after all, it was extremely small, whereas the current Universe is enormously large; one is tempted to think of the early universe as a sort of plan or map. But — Sean Carroll and Jennifer Chen [\[13\]](#page-36-8) have stressed this simple yet fundamental point — the early Universe was not a map of the present world: it was that world. The latter is obtained from the former simply by means of the passage of time, which is ordinarily taken to be a unitary process. If unitarity holds, then the smallness of the initial region does not explain why its entropy was so low.

Now one can certainly argue that the evolution of the Universe is no ordinary process; perhaps it is precisely an "exotic" process which does violate unitarity, admittedly on a massive scale. Indeed, as is well known, the "exotic" process of the evaporation of black holes was long widely believed to violate unitarity [\[21\]](#page-37-1), and the arguments in favour of that belief were very cogent. This was not considered to be contradictory, because black hole evaporation is certainly outside the range of our ordinary (or other) experience.

Despite this, it is rapidly becoming the consensus view that black hole evaporation *does* not violate unitarity. This unexpected development is due to the discovery of "AdS/CFT" duality" in string theory. Very briefly, this is the idea that quantum-gravitational processes, such as black hole evaporation, can be completely described by a certain ("dual") quantum field theory. Since this latter is fully unitary, so also must be the physics governing the equivalent evaporating black hole [\[22\]](#page-37-2).

This development has convinced many that unitarity is always strictly preserved in string theory. But if one accepts this, it follows, as we have seen, that the mere fact that our Universe has inflated does not explain the Arrow of Time in the context of string theory. Once again, we see that a recent development in string theory has greatly increased the urgency of solving the problem of the Arrow. For if we believe in unitarity, then the initial region must have had the potential to acquire an amount of entropy that is stupefyingly large compared to its size. The situation is analogous to opening an atlas of world maps, putting a certain map under a microscope, and seeing oneself represented accurately on the page. Indeed, it is far more astonishing than that.

 3 Here and henceforth, we use the term "observer" in the metaphorical sense traditional in relativity, that is, to mean a timelike curve, usually a geodesic. Absolutely no "anthropic" connotations are intended; "observers" in our sense exist at all times in every universe.

In short, Inflation means that a recent development in string theory — the confirmation of the universality of unitarity — only really makes sense if we can solve the problem of the Arrow. Conversely, if we wish to solve that problem within string theory, we have to do it without appealing to Inflation.

In fact, the situation is exactly the reverse: Inflation depends on low-entropy initial conditions in order to get started. To see this, recall that Inflation is driven by a scalar field, the "inflaton", denoted φ . Generically, such a field depends strongly on position both in time and in space: that is, generically, its space and time derivatives, $\partial_{\mu} \varphi$ (where μ is an index labelling the three space directions, plus time), will not be close to zero. The distribution of energy and momentum in the inflaton field is given by its stress-energymomentum tensor $T_{\mu\nu}$, which takes the form

$$
T_{\mu\nu} = \partial_{\mu}\varphi \,\partial_{\nu}\varphi - \frac{1}{2}g_{\mu\nu}(g^{\alpha\beta}\,\partial_{\alpha}\varphi \,\partial_{\beta}\varphi - V(\varphi)). \tag{3}
$$

Here $g_{\mu\nu}$ is the metric tensor familiar from General Relativity, there is a summation implied over the indices α and β , and $V(\varphi)$ is a scalar function of φ called the inflaton potential. We need not discuss the details: it is intuitively clear from this expression that the energy stored in φ can be distributed in a vast variety of ways. But for Inflation to begin, the energy must be distributed in an extremely special way; Inflation will only occur if, to an excellent approximation, all of the derivatives of φ are initially zero, and the potential is positive. For then we have

$$
T_{\mu\nu} \approx \frac{1}{2} g_{\mu\nu} V(\varphi), \tag{4}
$$

and it is well known that energies of this kind lead to a rapidly accelerated expansion (since then the potential behaves like a positive cosmological constant). One says, in view of [\(4\)](#page-6-0), that Inflation begins with the inflaton in its "potential-dominated state" [\[10\]](#page-36-5). The point is that the potential-dominated state is clearly an extremely non-generic state for the inflaton: that is, Inflation can only begin if the inflaton has somehow been put into this very low-entropy state. Inflation depends on the existence of an Arrow of Time, which must have been established *before* Inflation began.

This should not be construed as a criticism of Inflation. On the contrary, Inflation gives us an enormous simplification of the problem of explaining the Arrow. As Huw Price has emphasised [\[11\]](#page-36-6), we are by no means entitled a priori to assume that the low initial entropy of our Universe was "stored" in a simple way or confined to a single form of matter. Inflation performs precisely this service: it implies that the entire, potentially vastly complicated problem of accounting for the entropy of the early Universe reduces to the single problem of accounting for the initial state of the inflaton. In the words of Lisa Dyson, Matthew Kleban, and Leonard Susskind [\[12\]](#page-36-7): "Some unknown agent initially started the inflaton high up on its potential, and the rest is history." In fact, at stake here is the reason why there was any history: if we can understand the "unknown agent", then, thanks to Inflation, we will have a theory of the "passing" of time.

In short: the first reason for the extreme difficulty of our problem is that while Inflation is undoubtedly a crucial part of the puzzle — later we shall see that it is even more important than is apparent from this discussion — it assumes a pre-existing Arrow; so we are deprived of a major technical resource.

2.2. Laws of Nature vs Initial Conditions

To say that the very early Universe had "low" entropy is not to say much; this vagueness, indeed, is one of the obstacles to understanding the origin of the Arrow. Fortunately, as we have already discussed, Penrose has solved this problem for us, in terms of the fundamental number P (equation [\(2\)](#page-3-0)). We are searching for an explanation of "low" entropy, where "low" is quantified by the fantastically small number $1/P$.

However, the extreme smallness of this number is itself another factor in the difficulty of explaining the Arrow. We have seen that string theory can naturally give rise to "large" numbers, such as 10^{500} , and the reciprocal of this is, by any normal standard, an extremely small number. But it is still vastly too large compared to $1/P$. What kind of theory could possibly generate numbers of this order?

One possible answer to this question can be stated in simple language as follows: in an eternal universe, *everything* can happen, even something with probability of order $1/P$. Of course, something so improbable will happen rarely, but what of that? Just wait.

This "just wait" argument goes back to Boltzmann himself ([\[23\]](#page-37-3); see [\[11\]](#page-36-6) for a clear discussion of this theory). The idea is simply that the extremely low entropy of the early Universe was due to a random fluctuation, such as must always occur in any system at equilibrium, given enough time. The subsequent history of the Universe is just the normal return to the prior equilibrium state. In essence, Boltzmann and his successors claim to solve the problem of the Arrow by outbidding Penrose: to his number P they respond with a (supposedly) infinite supply of time. (There is also a spatial version of this argument: in a spatially infinite universe, everything, no matter how improbable, will happen somewhere. All of our strictures on the "just wait" argument apply, mutatis mutandis, to this version.)

This idea, with its air of "getting something for nothing", has a strong superficial appeal; but its logical status is, in fact, very problematic. It is certainly far from clear, for example, that *everything* can happen even with unlimited resources of time or space. More seriously: why stop at "explaining" the initial conditions of the universe in this way? The Universe we observe has many other "improbable" features; notably, it has laws of nature; why not "explain" these, too, as apparent regularities such as must arise, by mere chance, if one is prepared to wait sufficiently long? It is possible that Boltzmann's "rare fluctuation" can account for the low entropy at the beginning of time; but it is also possible that the Universe fluctuated into existence last Thursday [\[24\]](#page-37-4), complete with suitable false evidence of being ancient. The mere fact that an explanation "works" does not in any way oblige us to take it seriously. One suspects that the "just wait" technique is applied exclusively to events in the remote past or future for a reason: the phenomena are not present to reproach us with the extreme implausibility of this kind of argument.

Physical systems typically have characteristic time (and length) scales. One does not expect plate tectonics to predict the motions of continents on microsecond time scales or nanometre length scales. Similarly, the "just wait" philosophy can make sense only in the context of a theory which incorporates (possibly vast but) finite characteristic time and space scales. For these scales will impose discipline on the "just wait" philosophy; requests for more time or space than the characteristic scales can be denied (unless one has a mechanism explaining the unusual longevity of a specific system, which is not the case here).

Indeed, string theory does seem to have such scales. The known ways of constructing string vacua resembling our Universe [\[25\]](#page-37-5) (in that they have a positive cosmological constant) certainly do not lead to universes that endure for arbitrarily long periods of time; one says that the vacuum is "metastable". One might wait uncounted aeons for Boltzmann's "rare fluctuation" to occur, only to find that the system being examined has by that time disappeared into baby universes which nucleated in the meantime^{[4](#page-8-0)}. Similarly, the question as to whether the Universe is spatially infinite becomes somewhat ambiguous in string theory: the "holographic principle" (see for example [\[26\]](#page-37-6) and references) appar-ently indicates that the regions beyond a cosmological horizon^{[5](#page-8-1)} are in some sense merely a redundant description of the data inside the horizon. This may well imply that something that is very unlikely in one cosmological domain is in fact unlikely to occur anywhere. In short, it is far from clear that, in reality, we do have the limitless resources of time and space postulated by the "just wait" theories. String theory does allow for periods of time and regions of space which are truly vast by any normal standard. Boltzmann-style fluctuations might be legitimate objects of scientific interest if they occurred within those allowances; this, however, is apparently not the case.

In fact, Boltzmann's argument is usually held to fail [\[11\]](#page-36-6) because of reasons connected with the "anthropic principle", the idea that the existence of intelligent life imposes nontrivial conditions on physical theories. However, this principle has itself been strongly criticised [\[14\]](#page-36-9)[\[27\]](#page-37-7). In our view, it is neither necessary nor desirable to become involved in that highly contentious debate here. In fact, we feel that the entire "just wait" approach is misconceived, both because of the remarks above and because of the following argument.

Normally, when we discover some "highly improbable" pattern in Nature, our reaction is that the seeming improbability is really a sign of the existence of some hitherto unknown law of Nature. When we discover, for example, that planetary orbits have the approximate shape of an ellipse — a shape which has been known to be of mathematical interest for thousands of years — nobody would think it an adequate response to point out that such paths are bound to be traced out by randomly moving particles in the fullness of time. Instead we search for a law, or a collection of laws (in this case, Newton's laws of motion and of gravitation). When these laws are understood, the improbable is transformed into a certainty; elliptical orbits are an unavoidable consequence of the laws. Similarly here: surely the normal scientific response to Penrose's observation is that the initial conditions of our Universe seem improbable only because we do not know the underlying law of Nature.

In our view, Penrose's calculation really means that the initial entropy was as low as it is mathematically possible for it to be: in other words, "no greater than $1/P$ " is really just a way of saying "exactly zero at the classical level". Again, this is the kind of statement which would result from the existence of some law of Nature governing the initial conditions of the Universe.

Another way of stating the case is as follows. One might suppose that the low entropy

⁴One will always be able to find particular "fortunate" systems which evade this fate, since the babies do not devour the entire space [\[13\]](#page-36-8). Sufficiently fortunate systems can endure far beyond the characteristic time scale. Such an argument cannot be disproved; which is precisely why it should be rejected.

⁵A cosmological horizon is a surface surrounding a cosmological observer, such that objects beyond it are forever invisible to that observer. Such horizons arise in cosmological models, such as de Sitter spacetime, in which a positive vacuum energy dominates all other forms of matter.

conditions at the beginning of time may have been selected by some kind of quantummechanical probability distribution (defined on the "set of all initial conditions"). The problem here is, once again, the sheer size of the Penrose number; somehow we have to find a way of concentrating the wave-function over a particular outcome, and we must concentrate it to an unheard-of degree. This suggests that we are dealing with a quantummechanical version of a classical situation in which only a truly unique configuration is possible. (That is, the classical probability distribution is a delta function.) But this is tantamount to saying that we must find a (classical) law of Nature which dictates the nature of this unique set of possible initial conditions.

This point of view has the following major advantage. One of the objections (discussed, for example, in reference [\[28\]](#page-37-8)) to Boltzmann's explanation of the second law of thermodynamics is the following. We said that a system has low entropy if it is confined to a small volume in phase space. This volume is occupied by a set of states which are indistinguishable from each other. But indistinguishability is to some extent a "subjective" matter: it has something to do with the power of the observer to distinguish. (This process of dividing up the phase space into sets of indistinguishable "micro-states" is known as "coarse-graining"; see [\[10\]](#page-36-5).) By requiring the initial state to be absolutely unique at the classical level, the law of Nature we seek helps to resolve this problem.

This imposition of a *law* on initial conditions puts us, however, in a very unfamiliar situation. As we discussed in the Introduction, it was one of Newton's great insights that extremely complex behaviour can be understood in terms of very simple laws precisely by rigorously separating laws from initial conditions. Thus, another reason for the difficulty of our problem is that it requires us to find a way of breaking down this traditional separation.

Of course, one could simply declare that the new law is: "entropy must be as low as possible at the beginning of time". This is not satisfactory because it is not related to, and does not flow from, our other theories of the early Universe: Inflation and string theory. The new law should be connected with Inflation; and it should, ideally, be an inescapable consequence of the mathematical structure of string theory. But these statements do not make the problem any easier.

2.3. Low Entropy is a Geometric Property

Thus far, we have spoken of "low entropy" in a rather vague way. In order to progress, we need to refine this expression. The way to do this was pointed out, once again, by Penrose [\[7\]](#page-36-1).

We begin by noting that, by the time the Universe had evolved to the point where its contents are directly observable by us (that is, the stage called "decoupling"), most of these contents are by no means in a low entropy state. On the contrary, the cosmic microwave background has nearly all of the characteristics of a high entropy state: its spectrum is that of a black body. The only only hint of the extreme "specialness" that defines a low-entropy state is the extreme spatial uniformity of the radiation: to a good approximation, it looks the same in all directions. One interprets this as a sign of low gravitational entropy, as follows.

The precise way in which we should compute the entropy associated with a gravitational field is not yet fully understood, but the existence of black hole entropy (associated with its horizon, see for example [\[29\]](#page-37-9)) assures us that the concept does make sense. Now the natural tendency of gravitational systems (except in circumstances which do not hold here — see section 3.2 below) is to become clumped, indicating that gravitational entropy is somehow related to the "degree of clumping". But the cosmic background radiation, despite its thermal character, is extremely uniformly distributed: it is hardly clumped at all. Thus, in "looking" at the microwave background, we are observing a system in which the gravitational entropy, and only the gravitational entropy, is low. This very strongly suggests that "low entropy" in the very earliest Universe, even when we cannot observe it directly, really means "low gravitational entropy."

Now General Relativity teaches us that gravitation is nothing but a manifestation of the geometry of spacetime. Therefore, low entropy for gravitation must correspond to the spacetime geometry taking some very special form; "specialness" here actually means extreme spatial *uniformity*. This refines our problem very substantially: the problem of explaining the low entropy of the early Universe has been reduced to explaining its spatial uniformity.

This point is of such fundamental importance that we should expand on it. Think of a two-dimensional sphere. It has a characteristic topology, expressed in this simple case by saying that it is compact and has genus zero (it has no "holes"). It also has a distinguished geometry associated with a perfectly round shape. But clearly there are infinitely many other geometries compatible with this topology: one can deform the two-sphere to an arbitrarily complex shape without tearing any holes in it. The perfectly round shape is obviously very "special"; a typical geometry, chosen randomly from the space of all possible shapes with spherical topology, will look nothing like this. Furthermore, two such typical shapes will be hard to distinguish, just as it would be difficult to say whether two clouds resemble each other. In the language we have been using: the perfectly round spherical geometry corresponds to low gravitational entropy. Similar comments apply to other spaces. For example, the familiar flat geometry described by Cartesian coordinates is the geometry with the "lowest possible entropy" on a space with that topology.

The general situation is as follows. For a topological space with a given number of dimensions, it turns out that there is a measure of the amount of symmetry that a given geometry can have in a neighbourhood of any point^{[6](#page-10-0)}. There is a maximum possible "amount of symmetry" in each dimension, and certain topologies allow this maximum to be realised by particular geometries; the topologies of interest to us here are of this kind. Penrose's calculations indicate that the earliest spatial sections of our spacetime were, to a precision given at worst by the reciprocal of Penrose's number, similar to some such maximally symmetric three-dimensional space.

This actually ties in rather beautifully with our earlier discussion of Inflation. We saw that the latter depends on the existence of some "unknown agent" which is capable of putting the inflaton into its potential-dominated state initially. Now the extremely symmetric geometries we are dealing with here have the property of being everywhere locally isotropic: that is, the geometry "looks the same in all directions" to an arbitrar-ily situated observer^{[7](#page-10-1)} (who cannot however see arbitrarily far — hence the "locally").

 6 Technically, this measure is the maximal dimensionality of the isometry groups of the metrics induced on open sets containing the point.

⁷Except in certain very special spacetimes, isotropy can only be seen by a unique family of observers,

Now suppose that we can show that the inflaton field must share the symmetries of the underlying three-dimensional space. Energy cannot flow under such conditions, because the direction of flow would define a "special" local direction, which is precisely what local isotropy forbids. In fact, under these conditions, physical fields are forbidden to define any special direction. But this is a formidable restriction indeed, because most physical fields are defined, at a fundamental level, by vectors, which, by their very nature, do define a direction. In fact, the only way a vector can be isotropic is by being exactly zero. But the spatial derivatives of the inflaton define a vector. Thus, if the field shares the symmetries of the underlying space, a perfectly locally isotropic spatial geometry forces the spatial derivatives of the inflaton to vanish.

This goes very far — though we still have to explain the smallness of the initial time derivative of the inflaton — towards putting the inflaton in its lowest-entropy state, just as we wanted. More than that: this argument strongly suggests that the earliest form of matter probably was a scalar field, precisely as Inflation postulates. For a scalar field can, unlike a fundamental vector field, be non-zero even in conditions of perfect isotropy[8](#page-11-0) . In short, extreme spatial symmetry, interpreted as local isotropy around every point, is an ideal candidate for the "unknown agent" which started Inflation, and the argument we are developing here is, more generally, extremely natural from the inflationary point of view.

In the previous section we asked: what kind of theory can give rise to "low" entropy on the kind of scale measured by $1/P$? Our discussion here reduces this to: what kind of theory can give rise to geometric symmetry (more precisely: local isotropy at each point) with such fantastic accuracy? Notice that, by sharpening the question in this way, we are drastically raising the bar for any proposed explanation. A theory which "merely" predicts ultra-low entropy is not good enough; it has to predict low entropy of a specific, geometric, form.

A good example of what we have in mind here is provided by one of the most carefully thought-out proposals for explaining the Arrow, due to Carroll and Chen [\[13\]](#page-36-8). In this theory, baby universes (of a particular kind, see below) branch off from a mother universe by means of a rare fluctuation in the inflaton field, a fluctuation upwards to a higherenergy equilibrium. The basic idea is that the mother is in a *high* entropy state (which is generic, hence not itself in need of explanation) but the baby is in a low entropy state. As one would expect from this, however, this theory demands Boltzmannian patience^{[9](#page-11-1)} as we await the nucleation of a baby universe with sufficiently low entropy; that is, the Carroll-Chen theory is a "just wait" theory par excellence. In this sense, in fact, the theory is much like Boltzmann's. The difference lies in the claim that Boltzmann's scenario arises naturally as a consequence of the modern understanding of early-universe physics and spacetime dynamics.

One way to explain how this theory works (that is, why the upward fluctuation of the inflaton produces a low-entropy state) is as follows. Let us assume that the spacetimes

who are distinguished by that very fact.

 $8A$ similar argument applies to spinor fields. Note that *tensor* fields can be non-trivial even when they are perfectly isotropic; it is the fact that general relativity represents the gravitational field by a tensor which permits non-trivial isotropic cosmologies.

⁹Carroll and Chen estimate the relevant probability to be $\approx 10^{-10^{10^{56}}}$, a figure which induces awe and presbyopia in equal measure.

of both mother and child can be approximated by de Sitter spacetime, but with a much larger (inflationary) value of the cosmological constant for the baby than for the parent. Now, like the horizon of a black hole, the cosmological horizon of de Sitter spacetime has an entropy associated with it. Large values of the cosmological constant correspond to lower entropy, so this is consistent with the claim that the mother has much higher entropy than the baby. Thus the theory "explains" why the inflaton was in a low-entropy state initially.

This argument is, however, circular. The usual formula [\[30\]](#page-37-10) for de Sitter entropy (in terms of the horizon area) is of course derived using de Sitter spacetime geometry, which is maximally spatially symmetric by assumption. If the baby's spatial sections were, on the contrary, extremely anisotropic, then we would have no grounds for using the de Sitter entropy formula to estimate its entropy (which in that case would in fact not be low, or at least not low enough). Thus, the Carroll-Chen proposal will only work if one can show that the baby is born with very isotropic spatial sections. The above calculation of the baby's entropy in terms of the value of its inflationary potential is certainly consistent with a possible solution of our problem; but in itself it does not provide an explanation, because it does not address the fact that "low entropy" really means "extreme initial spatial isotropy around every point".

In fact, if the mother universe has high entropy, this means that its spatial sections are not highly isotropic around each point; they must be highly irregular (and hence not like those of de Sitter spacetime); and yet the mother must give birth to extremely isotropic babies. Explaining how that can happen is precisely the difficulty.

We will see later that the Carroll-Chen theory can in fact be interpreted so that it deals with this problem. For the moment, the point we are making is that there is little hope of explaining the Arrow unless we can address the problem of spatial regularity directly. The remainder of the explanation will then follow.

Our emphasis on isotropy is motivated by Penrose's "Weyl Curvature Hypothesis" [\[7\]](#page-36-1). Penrose postulates that the initial geometric "specialness" takes the form of a demand that a certain piece of the spacetime curvature tensor, the Weyl tensor, should vanish at initial singularities (and *only* there^{[10](#page-12-0)}). This demand does ensure local isotropy around each point "in" the initial singularity. However, there is a serious problem here: most physicists hope that the presence of singularities in the standard cosmological models is misleading. A full understanding of the initial state should reveal it to be non-singular. How can Penrose's idea be adapted to this case?

In our view, local isotropy around every point is in any case the heart of the matter, and one might as well try to impose it directly. This has the great advantage that it applies also to the case of an entirely non-singular cosmology. But it also forces us to confront a central technical difficulty.

Once we accept that the origin of the Arrow is reducible to explaining the "special" geometry of the earliest spatial sections of the Universe, we can frame the problem in the language of statistical mechanics as follows. Consider the set of all possible threedimensional Riemannian manifolds. Think of this as a "phase space". In this phase space, the subset of manifolds which are everywhere locally isotropic is a fantastically tiny sub-

¹⁰A more precise technical formulation is that spacetime should be "conformally compactifiable" at initial singularities.

space. It consists of familiar geometries like that of the perfectly round three-dimensional sphere, or of perfectly flat spaces, or of geometries like that of hyperbolic space, the space having constant negative curvature. The generic three-dimensional geometry is nothing like any of these, *even approximately*: one should picture it as a wildly distorted "blob", corresponding indeed to just what we mean when we say that something is "shapeless". We stress that it is *not* enough just to study spaces with geometries which are obtained by making small perturbations around the perfectly isotropic spaces. For even these will sample an insignificant region in the full phase space.

To see more concretely what we mean by this, let us consider one of the most sophisticated attempts to derive an Arrow of Time from a more basic formalism, the one due to Stephen Hawking [\[31\]](#page-37-11) and his collaborators [\[32\]](#page-37-12). Here one uses the well-known "no boundary" version of quantum cosmology to study inhomogeneous perturbations around an assumed perfectly isotropic background. A subtle technical argument leads to the conclusion that the perturbations are "born" in their ground state, and thereafter grow, defining an Arrow of the appropriate (geometric) kind. This is undoubtedly an extremely important check of the consistency of the no-boundary approach with the existence of an Arrow of the kind we observe. But in itself it does not solve the main problem, since the perfect isotropy of the background is assumed.

The problem here is that the full phase space is truly vast and very difficult to describe in detail — even in the case of two dimensions, it is hard to describe the "generic" shape into which a sphere can be deformed — and correspondingly "hard to manage". By this we mean something specific: there are very few mathematical theorems which operate at this level of generality. A typical theorem in differential geometry will begin with much more restrictive assumptions: as for example, "consider the set of compact manifolds with curvature constrained in the following manner..." Theorems of the form: "Consider the set of all compact three-dimensional manifolds..." are rare indeed. This set is simply too large and its contents too various and hard to characterize.

Even if we had the relevant theorem, it could only lead us to a conclusion if we were able to apply it to some well-justified physical assumptions. But it is very likely that the conditions we can justify will be extremely weak. They will, for example, pertain only to the spatial surface along which the Universe came into being, not to the entire subsequent history. They will not involve strong assumptions about the behaviour of matter in the beginning, because we have no reason to believe that matter behaved, at that time, in ways familiar to us now.

An example of the kind of argument we have in mind was advanced in an important work of Gary Gibbons and James Hartle [\[33\]](#page-37-13), who attempted to show, using a powerful theorem in global differential geometry, that the initial spatial section of the Universe must have been a perfectly round three-dimensional sphere. The argument does not assume that the geometry must be approximately spherical; it surveys the entire region of the phase space compatible with its physical assumptions. Unfortunately, those assumptions are much too strong, to the extent that they are almost certainly not satisfied by the actual Universe. (See [\[34\]](#page-37-14) for a detailed discussion of these conditions.) Furthermore, in the context of Inflation, the discussion in [\[33\]](#page-37-13) effectively assumes that the inflaton has been put into its minimal-entropy (potential-dominated) state; but we have argued that this state must be a consequence of the existence of an Arrow. So the conditions needed here are far too restrictive.

Thus, we have another reason for the difficulty of our problem: despite our efforts to refine it, it remains at a level of generality such that technical resources are scant. We need a technique which can take the *very mild* physical conditions that we are entitled to impose at the beginning of time, and transform them into an enormously restrictive constraint on the truly vast phase space of initial geometries. In addition, it has to force the inflaton to share the symmetries of the resulting geometry, and it will also have to restrict the way the initial spatial slice is embedded in spacetime (so that it can tell us something about the initial *time* derivative of the inflaton). The reader can be excused for doubting that such a magical technique exists at all.

2.4. In My Beginning Is My End

Thus far, we have spoken as if our sole task is to explain the peculiar conditions which obtained at the creation of a universe. If we can perform this task, however, then we should be able to understand the behaviour of the Arrow everywhere in spacetime, including at the destruction of a universe. Indeed, an inability to answer such questions would be a strong indication that something is wrong with our theory. Note that some universes in the Landscape have negative vacuum energy, which does indeed cause them to destroy themselves; thus we cannot evade this question by noting that our Universe shows no signs of mortality. Furthermore, even in our Universe, spacetime is "destroyed" inside black holes. We need to be able to predict what the Arrow would be like for observers in less salubrious circumstances than our own.

Most of us feel intuitively that initial conditions completely fix the fate of a universe, that its end is implicit in (and different from) its beginning. That is, we feel that there are no "special conditions" when a universe is destroyed; we find it hard to accept that the direction of the Arrow can somehow "flip", as it would have to do if conditions near the destruction of a universe are as special as those near its creation. Can this intuition be justified — or is it no more than a prejudice? Certainly, the arguments that we instinctively raise against a symmetry between creation and destruction seem invariably to convict us of "cosmic hypocrisy": we find ourselves arguing about cosmic destruction in ways which we would not dream of applying to the creation. It is no use saying, for example, that low entropy in (what we call) the future is "highly improbable": that is true, but it is equally true that low entropy in the *past* is "highly improbable"; it is absurdly improbable, yet it was indeed so. Price [\[11\]](#page-36-6) has dissected many fallacious arguments of this kind.

Despite this, we propose to argue that our intuitions are not misleading us in this case: we will claim that there are indeed no "special conditions" at the destruction of a universe. We can begin to clarify these issues by eliminating one major source of confusion: the expansion and (in some cases) contraction of universes.

The observed thermodynamic Arrow runs parallel to the expansion of the Universe. Since the former is generally agreed to have a cosmological origin, one sometimes speaks also of a "cosmological Arrow". It is then natural to ask whether there is some kind of link between the two. In fact, some authors — see [\[28\]](#page-37-8) for a particularly eloquent defence of this idea — have gone to the extreme of arguing that the thermodynamic Arrow is a consequence of the cosmological Arrow.

It is important to understand that this idea cannot be correct if Boltzmann's interpretation of the passage of time, accepted here and by most authors, is valid. For if the thermodynamic Arrow depends on the cosmological Arrow, it follows that the former could not exist if the Universe did not expand. But recall that Boltzmann pointed out that the problem of the Arrow reduces to explaining the special initial conditions of our world; if that can be understood, then the subsequent evolution towards more generic states is only to be expected, and it will occur whether or not the Universe expands. In fact, granted that we can somehow explain the special initial conditions, entropy will increase, at least initially, even if the Universe always contracts.

If we accept Boltzmann's account of the basis of the second law, then, it follows that the expansion and contraction of a universe are irrelevant distractions as far as understanding the Arrow is concerned. The real question is this. Consider a universe which is both created and destroyed: is there any justification for thinking that "special" conditions, such as spatial uniformity, can be imposed at one "end" of the universe and not the other? (While we argue that the expansion of the Universe is not *responsible* for the Arrow, it is of course still possible that both the expansion and the Arrow have a common origin. This is precisely what happens in the theory of the Arrow discussed below and in [\[15\]](#page-36-10).)

If a universe has two "ends", then the only difference between the ends is precisely the difference, if there is one, between creation and destruction. Now it may be that these two words have no meaning apart from the one given to them by the existence of an Arrow: "creation" may be another name for the low-entropy end, "destruction" may be a name for the high-entropy end. But this is not at all obvious. Why should the degree of "specialness" of the state of the universe have anything to do with its coming into existence? How high does the entropy have to be before one is allowed to destroy a universe? One can all too easily imagine a universe that is created in a generic state, and remains in that state until it is destroyed. The alternative claim, that such a universe is a logical impossibility, seems far-fetched. Certainly the onus is on those who assert this to prove it.

We shall assume, therefore, that "creation" and "destruction" have some meanings which are logically prior to the Arrow; that is, they might be meaningful even in universes which do not have an Arrow. Nevertheless, if an Arrow does exist, it has to conform itself to these meanings: it would clearly be unpleasant to find oneself saying eventually that a certain universe "began by being destroyed". It should be possible to show that entropy was low at the creation, and will be high at the destruction. This will have the *consequence* that there are no "special conditions" at the destruction.

All this seems very reasonable. Once again, however, we are asking for a great deal here. First, we must formulate the distinction between creation and destruction in concrete mathematical language. Then we must find a mechanism which imposes special conditions on the "creation end", and which somehow fails to do so at the "destruction end" of a universe. It is one thing to make "reasonable" assertions about these matters, quite another to establish (or even formulate) them mathematically. We have yet another major obstacle to overcome.

2.5. Summary

The difficulties we have discussed give rise to an intimidating list:

- First, we cannot rely on Inflation to help us; Inflation needs an Arrow in order to get started.
- Second, it seems unlikely that we will obtain an Arrow (of the kind we actually observe) simply by waiting, in the manner of a Boltzmann. Instead we need something more akin to a new law of Nature which, unlike any other, constrains the initial conditions of the Universe.
- Third, this law must somehow constrain the *geometry* of the initial spatial section, by means of some technique which has the power to probe the entire vast space of all possible three-dimensional geometries.
- Finally, the theory should determine the behaviour of the Arrow of Time not just in the vicinity of the beginning of time, but throughout spacetime, including near its destruction. If we assert that one end of a universe is dissimilar to the other, we have to prove this.

Having summarized the worst of our difficulties, we can now propose an approach to solving them. We begin by discussing the currently favoured way of making universes, the nucleation of babies.

3. Bringing Up Baby

In the Landscape, a universe begins as the baby of some mother universe. In this section we argue that the baby universe will not spontaneously develop an Arrow; it can only inherit one from its mother. If all mothers were themselves originally babies, it seems that no universe will have an Arrow. Let us explain the argument and see where it directs us.

3.1. The Universe as a Branch System

When we observe a system in an unusually low entropy state, we invariably find that it owes that low entropy to a process involving a larger system. The overall entropy of that larger system need not be small; all that is needed is that (a) the part that goes into making the smaller system should have low entropy, and (b) the process of "splitting" should not itself greatly increase the entropy of that part. For example, if I wish to cool my vodka, I go to the refrigerator (by no means a system with low total entropy, by the standards of the glass of vodka) and split off a low-entropy sub-system, to wit, a block of ice. I convey this sub-system to my vodka, but I must take care to do this without destroying its icy state. In principle one could extract ice from a refrigerator using dynamite, but this is unlikely to produce satisfactory results; the splitting of the system *alone* does not yield a refreshing low-entropy drink: one needs to do it in the right way.

A low-entropy system obtained in this manner is said to be a branch system of the larger system. The existence of branch systems is what allows the enormous complexity of our world, despite the relentless increase of overall entropy demanded by the second law of thermodynamics.

It is natural to guess that our own Universe may be a branch system of some larger system, and that this may account for the low initial entropy of the world we observe. Of course this does not in itself solve the problem of the Arrow, but we may hope to shift that problem to a venue where it may be more easily addressed.

A good example of this way of thinking is provided by the theory of Carroll and Chen [\[13\]](#page-36-8) discussed earlier. Recall that the idea here is that the observed Universe, among countless others, splits off as a baby universe from some much larger and older universe. That older world may have a large overall entropy, but its entropy *density* could be small; if the baby nucleates over a small region, and if it is born in the right way, it will begin with only a small amount of entropy, in the manner typical of a branch system.

Now there are three fundamental questions which have to be answered before this "baby universe as a branch system" idea can be said to work. First, of course, we have to ask why the larger universe should have a low (geometric) entropy density; secondly, we have to determine whether, and in what precise sense, baby universe nucleation does indeed take place in a small region of spacetime; and finally, we must convince ourselves that the splitting off of the baby does not increase its entropy to an unacceptable degree.

3.2. The Importance of Being Baldest

In the case of the Carroll-Chen theory, we can try to answer the first question as follows. We try to use the idea, familiar from discussions of Inflation, that an accelerated expansion of space (due, for example, to the presence of positive vacuum energy) will in some sense make the spatial sections more smooth. The intuitive picture here is that space is like a rubber sheet; as it is stretched, it becomes smoother. Presumably, if one is willing (and able) to wait sufficiently long, this process will produce spatial sections which are as smooth as we know the early spatial sections of our Universe to have been. The end result of this kind of smoothing is usually called *cosmic baldness*.

Now in fact the process of cosmic balding is not as simple as the "rubber sheet" analogy might suggest; see for example [\[35\]](#page-37-15) for a taste of the rather formidable technicalities. It is not true, in particular, that an accelerating spacetime must globally come to resemble de Sitter spacetime. The balding process only takes place within the cosmological event horizon of an individual inertial observer; roughly speaking this is because of the exponential decay of the proportion of a spatial slice accessible to a single observer. (See [\[36\]](#page-37-16) for a clear recent discussion of this crucial point.)

In fact, this more subtle interpretation of cosmic baldness is exactly what is needed for the Carroll-Chen theory to work. For what it implies is that the mother universe, even if it accelerates, never becomes isotropic at the global level: that is, its "total geometric entropy" is large. However, each local inertial observer does see a low geometric entropy in his immediate neighbourhood; the geometric entropy density is small. Thus we see that cosmic baldness, properly interpreted, allows us to answer the objection raised earlier against the Carroll-Chen theory (that it did not appear to give rise to low geometric entropy in the baby universe).

On the other hand, one must be careful regarding the meaning of "observer" here: in cosmology, an "observer" can be an entire galaxy or even a group of galaxies. Even the local smoothing-out effect of the accelerated expansion only operates on larger length scales than this, and, in fact, the observed acceleration in our Universe is not capable of, for example, tearing the Milky Way apart. Actually, even the Local Group of galaxies can resist the repulsion for an indefinite period [\[37\]](#page-38-0). Thus the geometry of space within the Local Group will never be smoothed out by the accelerated expansion taking place on very large scales, as long as the galactic matter remains within the current confines of the Local Group.

Carroll and Chen, however, propose another way to scatter the Local Group to the winds. They ask us to *wait* until essentially all of the matter in these galaxies has fallen into black holes, and then to wait for these black holes to evaporate. Presumably the resulting radiation will cease to be bound. In effect, this process contracts the "observer" from the size of the Local Group to smaller and smaller scales. Eventually, one might hope, the "observer" will become so small that the smoothing effect of cosmic acceleration can operate over the tiny length scales on which baby universes nucleate. After a further period of waiting, this might cause such a patch to become smooth on scales measured by Penrose's number P. A baby which is born at this point has at least some hope of being born with a geometric entropy as low as that of the early stages of our Universe.

We have already expressed our reservations about whether one should really be prepared, particularly if one is a string theorist, to do this truly stupendous amount of waiting, and we need not rehearse that discussion here. The point to be stressed is that this is a very delicate construction, which could easily malfunction even if one is blessed with Boltzmannian patience.

Nevertheless we do seem to be making some progress, so let us proceed to the second question, concerning the extent of the region over which baby universe nucleation takes place. Carroll and Chen propose to rely on the mechanism of baby universe nucleation proposed by Edward Farhi and Alan Guth [\[38\]](#page-38-1) (see also [\[39\]](#page-38-2); note that this mechanism differs radically from the one usually employed in discussions of the Landscape [\[3\]](#page-35-2)). This kind of baby universe is what one needs if, like Carroll and Chen, one wants to fluctuate upwards to a higher inflationary equilibrium. Now the idea that Farhi-Guth baby universes are possible in string theory has in fact been strongly challenged [\[40\]](#page-38-3), and this is a major problem for the Carroll-Chen theory. But let us leave that to one side for the moment, and note that Farhi-Guth baby universes do in fact branch off from a small spatial region in the mother universe. If we can accept the argument that the latter has achieved an ultra-low geometric entropy density by this time, then the baby will indeed begin with the right properties if it can maintain its spatial regularity through the birth process. This brings us to the third question.

3.3. It's Not That Easy Being Born

Farhi and Guth showed that, in the absence of exotic matter (see below), the umbilical cord joining mother and child is swiftly severed by a singularity. What this means is that the spatial sections of the future baby shrink down to literally zero size in a finite time. The presence of a singularity is of course unpleasant, but, for our purposes, the shrinking to zero size itself is the real problem.

To see why, recall that, locally, expanding a universe tends to make its spatial sections more uniform — this was precisely how we are proposing to make the mother universe so smooth, as seen by an individual observer. Equally, however, *shrinking* the spatial sections tends to make them highly anisotropic. Any small blemish becomes increasingly apparent as the sections shrink, and this happens very quickly when they become sufficiently small. It turns out (see for example [\[41\]](#page-38-4)) that the size of a spatial section can be quantified by a certain function of time, $a(t)$, and that, when the sections are small, the anisotropy grows according to $a(t)^{-6}$; this of course diverges rapidly if $a(t)$ is allowed to tend to zero. (It diverges more rapidly than almost anything that might try to defeat it; the exceptions [\[41\]](#page-38-4)[\[42\]](#page-38-5) are not relevant to the Landscape.) If the spatial sections really shrink down to zero size, then even the slightest anisotropy, even one measured on a scale comparable to the reciprocal of Penrose's number, will be magnified to an arbitrary extent. Our careful smoothing of space has been in vain.

This is not very surprising: the birth of a baby universe (of any kind) should be pictured as a highly traumatic event; certainly, if the reader has any reason to suspect that a baby universe has nucleated in his immediate vicinity, then he is urged to keep his distance, if not to take to his heels. But this is just a way of saying that we would not expect the birth of a baby universe to be an event that would give rise to a low-entropy situation, or to preserve any low-entropy systems which pre-date the birth. In other words, one simply expects that the birth of a baby universe respects the second law of thermodynamics, increasing the entropy as it proceeds. But, once again, "entropy" here means *geometric* entropy; so we must expect that the birth tends to make the relevant region less isotropic.

It has been argued [\[43\]](#page-38-6) that quantum-mechanical effects allow the singularity in the Farhi-Guth "wormhole" to be evaded; see [\[44\]](#page-38-7) for a discussion of the merits of this. In effect, the quantum fields behave like a kind of exotic matter, "exotic" in the sense that it violates the *Null Energy Condition*, the requirement that the sum of the energy density and the pressure of a fluid should never be negative; see [\[16\]](#page-36-11) for references.

We can picture this situation by imagining that the exotic matter intervenes to prevent the spatial sections of the baby universe from shrinking to *exactly* zero size. We must still, in accordance with the second law, expect a certain amount of anisotropy to develop; but now at least we have some hope of bounding the increase. A detailed theory would have to show that the increase in the anisotropy is sufficiently small as to allow Inflation to start in the baby. The general point, however, is that we can be sure that there will be an increase. Thus the initial smoothness of the mother universe must have been *even* more extreme than that of our own early Universe.

Many of these comments apply well beyond the particular scenario posited by Carroll and Chen; they apply to any theory in which our Universe appears as a branch system, splitting off as a baby universe in a larger mother universe. The key point is that, in any such theory, we must find some effective way of forcing the mother universe to be or become extremely isotropic. In the case of a baby connected to the mother by a wormhole, it suffices to do this in the immediate vicinity of the place where the baby nucleates; otherwise, the requirement has to be imposed on a much larger region of the mother universe. In either case, the mother needs to be even more isotropic than the baby. (Here and henceforth, "isotropic" means "locally isotropic at the relevant points".)

Since the Carroll-Chen theory is in any case committed to an extreme version of the "just wait" philosophy, the necessary level of maternal isotropy can be achieved by waiting for cosmic baldness to do its duty. (We assume that the entrance to the wormhole remains small, so that its environs can always be described by a single observer.) Thus, putting all of the many pieces together, and assuming that they all work, we do now have a theory of the Arrow. In summary: we wait for a Farhi-Guth baby universe to nucleate in some small region of a mother universe which does not have particularly uniform spatial sections but which is accelerating. While we wait, cosmic baldness ensures that the *local* region becomes smoother, while collapse to black holes and their subsequent evaporation effectively re-defines the meaning of "local", so that, when the baby does nucleate, it does so in a region which is extremely smooth; so smooth that, even when the anisotropy grows as the baby is born, the spatial sections are still as smooth as Penrose demands. The baby universe then has an Arrow arising from this extreme initial smoothness.

This is an ingenious and charming story, but we hope that we can be excused for not believing in it. First, the colossal time scales involved are likely to lead to very serious problems even *without* going into anthropic questions; when we do go into them, extremely long time scales can only make the situation worse (see for example [\[45\]](#page-38-8)). Such long time scales do not seem natural in string theory. Second, the mechanism whereby the definition of "local" is tremendously contracted, from cosmological scales down to the size of a typical region in which a wormhole can nucleate, does not seem to be sufficiently reliable. Finally, it is very doubtful that Farhi-Guth baby universes are compatible with string theory [\[40\]](#page-38-3).

Nevertheless, the Carroll-Chen model does at least address all of the problems we have identified, so it has much to teach us when we consider other proposed ways of building universes. In particular, of course, we are interested in building universes in the string Landscape. Here, too, one uses baby universes, but of a very different kind to the ones discussed above: the ones proposed by Sidney Coleman and Frank De Luccia [\[46\]](#page-38-9). This kind of baby reduces the value of the vacuum energy. It nucleates in a small region but then expands very rapidly, for an indefinite period, into the mother spacetime, instead of disappearing into a wormhole. Its fate therefore cannot be understood in terms of the observations of a single observer^{[11](#page-20-0)}, and so cosmic baldness is of no help to this kind of baby; the baby is no longer protected from outside influences, a fact which has recently attracted some attention [\[47\]](#page-38-10)[\[48\]](#page-38-11). Furthermore, the spatial sections of the baby are *infinitely large*; leaving aside perturbations, each spatial section is a copy of the threedimensional space of constant negative curvature (as we shall explain in more detail later). They therefore present a large "target" for outside influences, and in fact every part of the earliest spatial sections is causally related to events deep in the mother universe.

It is therefore clear that, as in the case of Farhi-Guth baby universes, the second law of thermodynamics will apply here: if the mother universe is not smooth on large scales, nor will the baby be smooth; in fact it will be less $so¹²$ $so¹²$ $so¹²$. The situation here is in fact even worse, because we cannot appeal to cosmic baldness. This means that the *total* entropy

¹¹Technically: because the boundary of the baby expands at a rate asymptotically approaching the speed of light, the baby occupies a region of the relevant conformal diagram which cannot be contained in the cosmological event horizon of a single inertial observer.

 12As in the Farhi-Guth case, the growth of anisotropies has to be controlled by means of violations of the Null Energy Condition; see [\[16\]](#page-36-11).

of the mother universe has to be extremely low if that of the baby is to be low: the second law of thermodynamics can no longer be circumvented, as it was in the Carroll-Chen theory, by contrasting total entropy with entropy density.

To be brief: baby universes are an important feature of the Landscape, but in themselves they do not help us to understand the Arrow of Time. In fact, they merely recapitulate the problem on a higher level.

3.4. Haunted by the Past

A baby in the Landscape, then, will have an Arrow of Time only if its mother does: it has to inherit an Arrow. Of course, it is all too clear that we have not solved the problem of the Arrow in this way. Actually, we may have made it worse, because the mother has to be at least as smooth as the baby. Furthermore, the mother itself may well be the baby of some still larger "grandmother universe", which was as smooth or smoother still; and so on.

In our view, we have arrived at this impasse because we have tried to avoid the conclusion of our discussion in Section 2.2. In fact, we have tried to use the baby universe concept to avoid speaking of the beginning of time in any real sense. This has not worked. Intuitively, anything which has a past will be influenced by that past; it cannot be as smooth as Penrose's calculation demands, unless its past was at least as smooth. The most straightforward conclusion is that Penrose's calculation really means that our Universe ultimately arose from some system which simply had no past. In the context of baby universes, this means that we must trace the origin of the Arrow back through the mother universe, the grandmother, and so on, until we reach the original female ancestor. Let us call this original universe "Eve". By definition, then, Eve had no past whatever. This made it possible, in some way to be explained, for Eve to begin with an Arrow; all subsequent generations have an Arrow *only* because they inherited one from Eve.

Our task, then, is to explain how Eve acquired her Arrow.

4. All About Eve

Eve had no past, but nevertheless was subject to some kind of physical law. How can that be? Can a universe really have no past?

Ideas of this sort first arose when Alexander Vilenkin [\[49\]](#page-38-12) made the daring suggestion that the Universe was created literally from nothing. (The celebrated "no boundary" theory of Hartle and Hawking [\[50\]](#page-38-13) can also be interpreted in a similar way.) It is, to put it mildly, difficult to think about "the physics of nothing", but string theory suggests a way of doing so. It is generally felt [\[51\]](#page-38-14) that, in some sense which today remains rather vague, time itself is emergent in string theory. Of course, much remains to be done to render this idea more precise, but the rough idea is that if one probes sufficiently far back in time, one will reach a region beyond which time simply ceases to be meaningful. Evidently, that region cannot have a past. According to our discussion, the boundary, which we necessarily picture as a spacelike hypersurface, is where we must seek the real origin of the Arrow.

The stringy "pre-emergence" state is not an *alternative* to Vilenkin's idea, so much as a way of giving a deeper analysis of "nothingness". It certainly does seem problematic to speak of the Universe "existing" in the absence of time; see [\[52\]](#page-38-15) for philosophical background on such questions. We shall therefore continue to speak of the Universe being created or destroyed along the surfaces which are boundaries between regions in which time is well-defined and regions in which it is not. Let us try, then, to find a stringy account of creation.

4.1. The Minimal Torus of Creation

What would "spacetime" be like if time did not exist? There is an apparently naive yet ultimately profound answer to this question; it runs as follows. Take Minkowski spacetime, the usual spacetime of Special Relativity. Distances in Minkowski spacetime are measured using the Minkowski metric, given by

$$
g_{-+++}^{M} = -dt^{2} + dx^{2} + dy^{2} + dz^{2}, \qquad (5)
$$

where t denotes time, the other coordinates are the familiar Cartesian ones for the spatial dimensions, and the pattern of signs, $- + + +$, is called the *signature*. Time is the dimension corresponding to the aberrant sign in this formula. "Spacetime without time" would be the " $+ + +$ " version of this formula,

$$
g_{++++}^{M} = +dt^{2} + dx^{2} + dy^{2} + dz^{2}. \qquad (6)
$$

This is described [\[50\]](#page-38-13) as the *Euclidean* version of the original spacetime metric. (The terminology is unfortunate: it means nothing more than that the metric has $+ + + +$ signature; in particular, it certainly does *not* mean that the metric is flat.) Naively, then, we expect time to emerge from a stringy state with Euclidean signature. In terms of this oversimplified example, this means that time emerges at the point at which g_{+++}^M is suddenly^{[13](#page-22-0)} replaced by g_{-+++}^M . Notice that each formula can be obtained from the other by replacing t by it, where i is the usual imaginary unit, $\sqrt{-1}$. One says that this coordinate has been complexified.

Thus we obtain a concrete picture of emergent time. This is a useful idea, however, only if we can say something about the "pre-temporal", Euclidean region.

Vilenkin and Hartle and Hawking made proposals as to the structure of the Euclidean region; more recently, a radically different suggestion, based on string theory, was made by Hirosi Ooguri, Cumrun Vafa, and Erik Verlinde [\[54\]](#page-39-0). They begin far from cosmology, by studying a kind of stringy black hole, leading to a two-dimensional Euclidean metric of the form

$$
g_{++}^{OVV} = K^2 e^{(2\rho/L)} d\tau^2 + d\rho^2.
$$
 (7)

Here one should think of ρ as a coordinate which runs along a line, but of τ as the angular coordinate giving position on a circle, of radius related to the fixed number K. (The constant L measures the curvature of this space.) Now the Euclidean "time" here — that is, the dimension which would normally be complexified — is τ . (It is usual for black hole Euclidean "time" to be compactified in this manner; the periodicity of "time" is ultimately related to the thermodynamics of the black hole.) But Ooguri et al. noticed that this metric looks very much like a *cosmological* metric: if we take ρ to be Euclidean

 13 We would argue that it is analytic that time cannot "emerge gradually". But see [\[53\]](#page-39-1).

"time", then the geometry is like a Euclidean version of an exponentially expanding two-dimensional cosmos with finite (circular) "spatial" sections. The fact that a single Euclidean space can have two spacetime interpretations opens the way to a profound application of stringy black hole theory to cosmology.

The four-dimensional version of this geometry is just

$$
g_{+++}^{OVV} = d\rho^2 + K^2 e^{(2\rho/L)} \left(d\theta_1^2 + d\theta_2^2 + d\theta_3^2 \right).
$$
 (8)

Here $\theta_{1,2,3}$ are angular coordinates on three separate circles. Such a product of circles is called a three-dimensional torus. It may be pictured as the interior of a box with nontrivial topology; as one tries to exit through a side, one instantly re-appears inside the box, entering it from the opposite side; one says that the opposite sides have been *topologically* identified. Thus, the cosmological version of this space will have spatial sections each of which is a copy of a torus.

The torus has a property which turns out to be crucial here: it is finite (compact) yet flat. That is, inside the box, all of the usual laws of flat-space geometry obtain; the space differs from ordinary flat space *only* through its topology. That is why the metric g_{+++}^{OVV} contains two independent length scales: L, which fixes the overall curvature in four dimensions, cannot control the sizes of the "spatial" sections (determined by K), precisely because these sections have no curvature of their own. Since the length scale of the torus is independent of L for this particular (exactly flat) spatial geometry, it must be independent of L even if we begin to distort the torus away from exact flatness. (It should be pointed out here that string theory is defined on higher-dimensional spacetimes, and that the "hidden" dimensions are always taken to be compact, and often to be flat. Thus, the compactness of the "visible" dimensions is particularly natural in string cosmology, a fact strongly emphasised by Ooguri et al. [\[54\]](#page-39-0).)

Now let us follow Ooguri et al. and take this metric as the Euclidean version of a spacetime metric. That is, we truncate the space along the three-dimensional surface $t = 0$, where time emerges; notice that this surface has the topology of a torus. The spacetime on the other side has a metric which is obtained by complexifying both ρ and L (so that $e^{2\rho/L}$ remains real). This works *only* because K is an independent parameter; if the "spatial" sections had themselves been curved, this curvature would have been determined by L , with disastrous consequences when L is complexified. For these simple yet deep reasons, we maintain [\[55\]](#page-39-2) that the Ooguri-Vafa-Verlinde account of the "pretemporal" state requires time to emerge along a torus.

It is surely no coincidence that spaces of toral topology^{[14](#page-23-0)} actually play a very basic role in string theory. String theory has a very unusual symmetry, known as T-duality [\[56\]](#page-39-3). In general terms, this symmetry implies that string theory defined on a torus is insensitive to a replacement $K \to L^2_{string}/K$, where K is the toral radius parameter we have been discussing, and where L_{string} is a fundamental parameter of string theory, the string length scale. "Insensitive" here means that the result of this transformation is the same physics described in a different way. This is often interpreted to mean that, in string theory, it does not make sense for a torus to have a "radius" smaller than about one string

 14 There are actually a few other compact flat three-dimensional manifolds apart from the three-torus, but all of them can be obtained from the torus by means of yet further topological identifications. Henceforth, "torus" means any member of this finite and fully-understood family of spaces.

length. (This slightly vague idea is made explicit in one particularly interesting approach to string cosmology, the "string gas cosmology"; see [\[57\]](#page-39-4).) This distinguished torus of minimal size is the obvious candidate for the surface along which time emerged. (In fact, earlier work on "creation from nothing" [\[49\]](#page-38-12)[\[50\]](#page-38-13) always did assume that the Universe was created along a minimal spacelike surface; here we are saying that this assumption is particularly natural and meaningful in string theory defined on a torus.)

Notice that the existence of a minimal initial spacelike surface automatically solves the problem of having an initial Big Bang-style singularity; that is, the theory implicitly violates one or another assumption of the Singularity Theorems. In fact a toral cosmology can only be non-singular by violating the Null Energy Condition; but we are already committed to this in any case if we intend to make use of baby universes. (Exotic matter which violates the Null Energy Condition will violate any of the conditions assumed by the Singularity Theorems.) The original space considered by Ooguri et al. (with metric given by equation (8) does not contain any minimal surface; the exotic matter removes the singularity and replaces it by a geometry which does have such a surface.

It is implicit in this argument that it is possible for the Universe to come into existence with a size given approximately by the string scale. That seems very reasonable, but we should point out that the string scale is normally thought to be substantially larger than the Planck scale, the scale at which it is usually surmised that quantum gravity effects come to dominate. This suggests that quantum gravity does not in itself fully explain the Arrow. (It does play a role in the theory to be put forward later, but only through small, perturbative effects.) Once again, the fact that the toral scale K is completely independent of the spacetime curvature scale L is of basic importance here.

On the other hand, the string scale may be too short to be the scale at which Inflation starts. Happily, in the cases of interest to us, the toral topology itself allows this problem to be solved, in the following manner [\[15\]](#page-36-10). Essentially what happens is that the toral topology initially prevents cosmological horizons from forming, so that the entire universe can be seen by a single observer. (Recall that a cosmological horizon forms when objects escape beyond the observer's view; but it is difficult to escape from a torus, since the escapee tends to find itself back where it started.) Cosmic baldness then applies to the entire universe, so that, if we can achieve smoothness *initially*, this smoothness will be maintained by that effect. Eventually, however, a cosmic horizon does form, and Inflation in the conventional sense can begin; but, by that time, the universe has expanded to the appropriate size, much larger than the string scale. Thus, toral topology can reconcile the Inflation scale with the string scale. (See [\[58\]](#page-39-5) for the use of tori to "delay" Inflation, though the approach there is different.)

Throughout this discussion, we have been assuming, for simplicity, that the torus is endowed with its exactly flat, and therefore perfectly locally isotropic, geometry. In discussing the Arrow in this context, we cannot of course begin in this way. Fortunately, everything we have said here can be formulated for a completely *arbitrary* geometry on the initial torus, fixing only its topology. With such an arbitrary geometry, it no longer makes sense to speak of the "radius" of the torus, but it still has a well-defined (finite) volume; this fixes a definite length scale, which we continue to denote by K . The overall spacetime in the immediate vicinity of the initial spacelike surface will also have a characteristic length scale, independent of K , which again we continue to denote by L . The concept

of a minimal surface in a general Riemannian manifold is a classical one, defined as follows [\[59\]](#page-39-6). Consider a family of finite (compact) subspaces embedded in a manifold. The subspaces have metrics induced on them, and, as one moves from one subspace to another, this metric will usually change, and so will the volume of the subspace. The rate of change of the metric is measured by an object usually called (in physics) the extrinsic curvature; it can be regarded as a tensor K_b^a defined in each subspace. A subspace is said to be minimal if it minimizes the volume for variations with fixed boundaries, and this requires the trace of this tensor to vanish: we have:

$$
K_a^a = 0,\t\t(9)
$$

with an implied summation on a.

The concept of a minimal surface can be generalized to the situation we are considering here, resulting in the same equation. We claim that time emerges along a spatial slice of minimal volume, a claim which we can now formulate in terms of this equation. (In this discussion we have used *Riemannian* geometry, meaning that we have been examining the boundary surface "from the Euclidean side", but the surface will also be minimal from the spacetime side, in the obvious sense.)

On the Euclidean side, the geometry we are discussing can be described concretely as follows. The vectors perpendicular to the boundary minimal surface allow us to define a coordinate ρ such that the metric, at least locally, takes the form

$$
g(F, h_{ab}, K, L) = d\rho^2 + K^2 h(\rho/L, \theta_c)_{ab} d\theta_a d\theta_b, \qquad (10)
$$

where the $\theta_{1,2,3}$ are the angular coordinates on the torus, and $h(\rho/L, \theta_c)_{ab}$ is the metric on the torus labelled by ρ ; it is completely arbitrary, apart from the condition that its dependence on ρ is subject to the demand that the boundary must be minimal. The corresponding spacetime metric on the other side of the boundary is obtained simply by complexifying ρ and L (but not K).

In this section, we have argued that the Arrow is related to the *emergence* of time itself. The argument is simple: that which has no past cannot be distorted by any "prior" conditions. Guided by the work of Ooguri et al. on the string-theoretic view of the "pretemporal" state, we have been led to a concrete picture of the earliest era of the universe. It would certainly be of great interest to pursue the programme of Ooguri et al., in order to further elucidate the nature of the Euclidean region. (Notice for example that the complexification of L means that, if the spacetime is predominantly positively curved, as it will be if it inflates, then the Euclidean region must be negatively curved; indeed, Ooguri et al. use a space which is locally identical to hyperbolic space.) However, that is not needed for our purposes.

We have seen that string theory may contain a ritual for exorcising the ghosts of the past. The penalty we pay for this exorcism is however a heavy one: whatever the principle responsible for Eve's Arrow may be, it cannot, for obvious reasons, act causally. In the next section we argue that general relativity contains certain features which, in a sense, are acausal.

4.2. The Initial Value Problem for Gravity

We have repeatedly referred to Newton's epochal separation of *dynamical laws* from *initial* conditions. For Newton this was a natural thing to do, because the initial positions and momenta of a collection of classical particles can be prescribed completely arbitrarily. Gravity, however, is different.

At first sight it seems paradoxical to speak of "time evolution" for gravity, since, if one does not already know the spacetime structure, one does not know how time itself behaves. This apparent paradox is resolved in the following extraordinary way. One begins with a three-dimensional manifold Σ , on which is given a Riemannian (that is, signature $+,+,+)$ metric h_{ab} , a symmetric three-dimensional tensor K_{ab} , a function ϱ and a three-dimensional vector field J^a , together with data for the "matter fields". At this point, K_{ab} , ϱ , and J^a "have no meaning"; one simply proceeds to solve the Einstein field equations in a formal way, so that Σ is embedded in the resulting spacetime. One then finds that these fields "turn out to have been", respectively, the extrinsic curvature of the "initial" surface Σ , the value of the energy density of the matter fields on Σ as measured by observers whose worldlines are perpendicular to Σ , and the projection onto Σ of the four-dimensional *energy-momentum flux vector* seen by these observers. Thus these objects acquire their meanings retrospectively.

All of this is strange and suggestive enough. But what is still more interesting is the following. Suppose that we try to prescribe the metric on Σ , the tensor K_{ab} , and so on, completely independently of each other. Then, generically, one will find that the Einstein field equations simply do not have a solution. Solutions exist only if the initial data are related to each other by means of a complicated set of equations, the *initial value* constraints:

$$
D^{a} (K_{ab} - K_{c}^{c} h_{ab}) = -8\pi J_{b}
$$

$$
R(h) + (K_{a}^{a})^{2} - K_{ab} K^{ab} = 16\pi \varrho.
$$
 (11)

Here D^a is the covariant derivative operator in Σ , there is an implied summation on the indices when they are repeated, and $R(h)$ is the scalar curvature defined by h_{ab} . (See [\[60\]](#page-39-7), Chapter 10. We can regard D^a as an operator which measures spatial rates of change within Σ ; we shall discuss the meaning of $R(h)$ in more detail later.) One must not think of these relations as having anything to do with the fields interacting with each other; they are strictly acausal; they are imposed on the initial data by the presumed existence of the "subsequent" spacetime as a structure compatible with the Einstein field equations.

Now the following idea suggests itself. The function ρ has (retrospectively) a direct physical significance, and it is reasonable to suppose that some physical principle will restrict it. Later, for example, we shall suggest that ρ should never be negative along a surface Σ where a universe is being created. The initial value constraints then impose conditions on the geometry of Σ . One might even regard these conditions as *laws of* nature of precisely the kind requested in Section 2.2. In fact, however, this point of view has never been taken. Why?

The reason can be understood by considering a special case, the beginning of time as it appears in the Hartle-Hawking theory [\[50\]](#page-38-13). Here time begins (or emerges) along a spatial section with vanishing extrinsic curvature and with the topology of a three-dimensional sphere. The second equation in [\(11\)](#page-26-0) is then simply

$$
R(h) = 16\pi\varrho. \tag{12}
$$

Thus the scalar curvature and the initial energy density cannot be prescribed independently. But does this really represent much of a restriction? Suppose for example that we are given ρ as an arbitrary non-negative function on the three-dimensional sphere. Can we find a metric such that (12) is satisfied? The answer is that this is, indeed, *always* possible: this is a consequence of a profound theorem due to Jerry Kazdan and Frank Warner [\[61\]](#page-39-8); see [\[15\]](#page-36-10) for a discussion of this theorem. If ρ is an extremely complicated, irregular function, then the geometry of Σ will likewise be extremely irregular. In short, under normal circumstances, the geometry of the topological three-sphere is constrained so weakly by the initial value constraints that the latter cannot be considered worthy of the title of "laws of nature".

So it is, at least, for universes created with spherical topology. One of our main claims in this work is that it is otherwise with the torus. Before explaining this, let us be more precise as to the physical conditions we are going to assume.

4.3. Creation vs Destruction

In order to explain the origin of the Arrow, we must of course make some physical assumptions. These should either follow from string theory, or at least be so plausible (or "mild") that one can hold out hope that they will follow from string theory (when, perhaps, the emergence of time is better understood).

In general relativity, each observer defines at each point a four-dimensional *energy*momentum flux vector T^{μ} . This specifies the flow of energy in various spatial directions, as seen by this observer. Also, the observer's worldline has a tangent vector V^{μ} at each of its points. In general these two vectors need not be parallel, but one can specify whether they point in the same general direction in spacetime by computing the projection of T^{μ} onto V^{μ} ; this projection, if it is not zero, is either parallel to V^{μ} or anti-parallel to it (that is, it points in exactly the opposite direction.)

Now consider the spacelike hypersurface Σ constituting part of the boundary of spacetime, and let N^{μ} be the field of unit vectors, perpendicular to Σ at each point, and pointing inwards from the boundary. (The inwards/outwards distinction can be given a rigorous mathematical definition; see [\[15\]](#page-36-10).) We can think of N^{μ} as defining tangent vectors to the worldlines of a family of observers. Suppose now that we claim that this particular Σ represents the creation of the Universe. Whatever this means precisely, we can formulate it mathematically as follows. If T^{μ} is the energy-momentum flux vector field associated with N^{μ} , then T^{μ} , evaluated on Σ , never points outwards; that is, its projection on N^{μ} is never anti-parallel to N^{μ} . On the other hand, if Σ is a boundary along which the Universe is being destroyed, then we shall take this to mean that T^{μ} , again evaluated on Σ , does point *outwards*. The intuitive picture here — not to be taken too literally — is that while the spacelike components of T^{μ} measure flows of energy within Σ , its timelike component tells us whether energy is "coming into the world" (in a direction perpendicular to Σ) or taking leave of it.

These statements are the *only* input we shall need regarding the energy content of the universe. In order to *prove* them, one would need to have a detailed physical theory of "creation" and "destruction", which in turn would demand an understanding of the "emergence of time" far beyond what we can claim to have. At this point they are just simple postulates about the correct mathematical representation of "creation" and "destruction". They are motivated by the Landscape, in the following sense.

Recall that, in the Landscape, the nucleation of a Coleman-De Luccia baby universe has the effect of *lowering* the value of the vacuum energy. Certain unfortunate babies will "overshoot" the desired value of the cosmological constant, so that they have a negative vacuum energy. These will expand at first, but eventually they will begin to contract; classically, they will terminate in a singularity. (A vacuum which contains nothing but negative vacuum energy is never singular; this is the famous "anti-de Sitter space" beloved of string theorists. If, however, such a spacetime also contains a small amount of "normal" matter, then the collapse will not be so innocuous.) We shall assume that, instead of a singularity, a string-theoretic treatment will lead to a minimal spacelike surface, along which time "submerges", that is, it ceases to be meaningful. Now despite their name, the "initial" value constraints apply in this case; substituting the minimality condition (equation (9)) into the second member of (11) , we obtain

$$
16\pi \varrho = R(h) - K_{ab} K^{ab}.
$$
\n
$$
(13)
$$

Now recall from section 3.3 that the spatial sections of a Coleman-De Luccia baby universe are, if perturbations are ignored, negatively curved. Therefore, we see from this equation that ρ has to be negative in this case, since the term involving the extrinsic curvature can be expressed, at each point, as a sum of squares. But ϱ is just the time-component of T^{μ} as seen by observers with inward-pointing tangent vectors (see [\[60\]](#page-39-7), Chapter 9); its being negative implies that T^{μ} points outward. Hence we see that, according to our postulates, spacetime is indeed being *destroyed* in this case; which is as it should be.

This discussion is for motivation: it does not prove anything. For, as always, we must not assume that the spatial sections of this universe will retain the geometry of an exact space of constant negative curvature. If they did so, then in fact $R(h)$ would become more negative as this universe shrinks. If they fail to do so, then the behaviour of $R(h)$ is less clear; on the other hand, the term involving the extrinsic curvature would then grow, so in either case we can expect ρ to remain negative. Also, independently of this equation, one can argue on general grounds that there must be a strong negative contribution to ρ (in addition to the background vacuum energy, which of course is also negative) from whatever it is that prevents a singularity here. Thus, it does seem that, in the Landscape, the destruction of universes is associated with negative (total) energy densities. Our postulate, based on this evidence, is that this is true in general. Conversely, it is natural to suppose that the total energy density is never negative when a universe is created. (Note that the vacuum energy in the creation case is not negative, and that the topology and geometry of the spatial surfaces are also different in that case.)

We stress the obvious: the conditions we are proposing here are almost *ridiculously* weak, especially when they are compared with the extremely strong restrictions on the geometry of Σ which we hope to extract from them. All they require is that the vector T^{μ} should point in a certain *general* direction; this is no more specific than saying that (either) Cambridge is "northward" of Singapore. Furthermore, they require this only on Σ , not in the bulk of the spacetime itself. This "weakness" is very important, because it means that our arguments are correspondingly robust. If we had demanded physical conditions involving *equations*, then we would have to fear that corrections to those equations might ruin the argument. But all we are requesting here is that the time component of T^{μ} should satisfy a certain *inequality* along Σ . Obviously, inequalities are robust: if a certain number is positive, for example, then so are all nearby numbers. We will shortly see how this works in detail.

4.4. And Then A Miracle Happens: The Arrow Explained

Now let us put all of the pieces together. We assume that the Eve universe was *created* along a *minimal-volume* spacelike surface Σ with the topology of a three-dimensional torus but with otherwise unknown geometry (except that the overall length scale is given by $K \approx L_{string}$. Why should we believe that the geometry of this torus is not generic, that is, extremely irregular?

First, note that we agreed that "being created" means that T^{μ} , the energy-momentum flux vector field defined by the inwards-pointing normal field N^{μ} , should never point outwards. But, because ϱ is the time-component of T^{μ} as seen by these observers, this simply means that, along Σ only, we have

$$
\varrho \geq 0. \tag{14}
$$

Inserting this and the minimality condition $K_a^a = 0$ into the second member of the initial value constraint equations [\(11\)](#page-26-0), we get

$$
R(h) = K_{ab} K^{ab} + 16\pi \varrho \ge 0; \tag{15}
$$

recall that the term involving the extrinsic curvature is essentially a sum of squares.

As expected, our weak physical assumptions have led to an apparently feeble constraint on the metric on Σ . First, as we have stressed, we have a mere inequality. Second, let us recall the definition of the *scalar curvature*. At each point, a three- (or higher-) dimensional manifold can curve in many ways, depending on the direction in which the space is sliced. Thus, even at a fixed point, such a space can be negatively curved in one direction, and positively curved in another. The scalar curvature at a point is simply a constant multiple of the average of all of these various curvatures. Now to say that the average of a collection of numbers is not negative is to say almost nothing: for example, all of them but one could be negative, as long as the one exception is sufficiently large that it outweighs all of the others combined when the total is taken. Thus, we have no right to expect anything interesting from a condition like [\(15\)](#page-29-0); and, as we mentioned earlier, in the case of *spherical* topology one can actually prove that this condition hardly constrains the geometry at all.

But now something extraordinary arises. One has the following theorem, obtained by combining several extremely deep geometric results of Richard Schoen, Shing-Tung Yau, Mikhail Gromov, Blaine Lawson, and Jean-Pierre Bourguignon. (For technical details, and full references, see [\[62\]](#page-39-9) and [\[15\]](#page-36-10).)

THEOREM (Schoen-Yau-Gromov-Lawson-Bourguignon): Consider the set of all possible Riemannian metrics on any torus. In this set, the only metrics with everywhere nonnegative scalar curvature are those which are perfectly flat; that is, their full curvature tensor vanishes exactly.

The proof of this theorem — let us call it the SYGLB theorem for brevity — is very abstruse; appropriately, perhaps, it makes use of techniques originally drawn from physics (particularly the concepts of Dirac operators and spin geometry).

The SYGLB theorem is a genuinely astonishing result. First, the theorem surveys all possible geometries on the torus. Second, and even more startling, it means that the average of the curvatures can only be non-negative if each and every curvature vanishes exactly — the curvatures cannot "average out" to a non-negative value. Third, the result must somehow link the curvature tensor to the specific topology of the torus, since no such result holds in the case of spherical topology.

Suddenly we find ourselves in a different world from the case of the sphere. For now the inequality [\(15\)](#page-29-0) implies the following equations.

First, and of course most importantly, [\(15\)](#page-29-0) now implies that the full curvature tensor of the "initial" three-dimensional surface Σ is exactly equal to zero. The Eve universe had to be created along a spacelike surface which was perfectly flat, which means that it was exactly locally isotropic around each point.

This is of course a major step forward, but we are not yet done. Essentially, we have fixed the initial spatial metric, but we have not yet fixed its "time derivative", as we must do if we wish to have a complete set of initial conditions. Furthermore, while it may be "natural" to assume that the initial state of the inflaton shares the symmetries of the initial spatial section, ideally we should prove this. In principle, the geometry might be perfectly regular while the inflaton field is highly irregular.

Again we refer to [\(15\)](#page-29-0): having deduced that the curvature is zero, the left side must vanish, and so therefore must the right. But since the right side is a sum of terms which cannot be negative, the only way they can add to zero is if all of the terms vanish separately. Thus we find that

$$
K_{ab} = 0; \t\t(16)
$$

that is, the entire extrinsic curvature matrix vanishes initially. This is of course a far stronger condition than the mere vanishing of its trace. The initial "time derivative" of the spatial metric is now fixed: it has to vanish. This implies that the initial moment was a moment of time symmetry $([60], \text{Chapter 10})$ $([60], \text{Chapter 10})$ $([60], \text{Chapter 10})$. Classically, this forces the time derivative of the inflaton field to vanish initially, an important contribution to putting the inflaton into its minimal-entropy state. Of course, the geometry on the other side of Σ is not really the same as that on the spacetime side; but classical local fields would not "know" this. Quantum effects spilling over from the Euclidean domain may not be totally negligible, however, so perhaps it would be more prudent to say that this argument shows that the initial time derivative of the inflaton must be very small. This is acceptable, because the inflaton need not be exactly constant for the approximate relation [\(4\)](#page-6-0) to hold. (In fact we want the inflaton to start (slowly) "rolling".)

Next, we consider the *first* equation in (11) . Now that we know that the extrinsic curvature vanishes everywhere on Σ , so do its derivatives, so this equation simply requires that

$$
J^a = 0. \t\t(17)
$$

Recall that J^a is the projection into Σ of the energy-momentum flux vector. This implies that energy cannot flow in any direction at any point of Σ , which just means that the inflaton field is uniformly distributed over Σ : it does share the uniformity of the underlying space, and so the spatial derivatives of the inflaton field vanish, and the inflaton is indeed initially in its low-entropy, potential-dominated state.

Finally, we now see from [\(15\)](#page-29-0) that the total energy density must vanish exactly everywhere on the initial surface. This is a reminder that some kind of exotic effect, which violates the Null Energy Condition, must act initially in order to invalidate the Singularity Theorems; it must contribute a negative energy density which exactly cancels the energy of the inflaton initially (but not thereafter). A discussion of the nature of this effect would take us too far afield here, but we may mention that the *Casimir effect* arises naturally in spaces with toral topology; see [\[63\]](#page-39-10) for a general discussion of Casimir effects in cosmology, and [\[64\]](#page-39-11)[\[65\]](#page-39-12) for recent discussions from a string-theoretic point of view. The negative energy density need not be precisely a Casimir energy, but it must be something similar: like Casimir energy, it must dilute very rapidly as the Universe expands, so that, after a short period of expansion, its effect on inflationary physics is negligible. (Lest the reader suspect a circular argument here, note that the initial vanishing of the total energy density by no means implies the initial vanishing of the total pressure. Under reasonable physical conditions, the "primordial pressure" will in fact be negative, and this will launch the newborn Eve into an immediate phase of accelerated expansion, even though the inflaton energy density has been (momentarily) canceled. For speculations on the possible identity of the "Casimir-like" effect, see [\[15\]](#page-36-10).)

We can now claim to have fixed the initial conditions of the Eve universe, in a manner which is so restrictive that, as requested in section 2.2, it deserves the name of a "law of nature". For now the initial metric has been fixed (up to relatively inconsequential issues such as the precise global shape of the initial flat torus), and so has its initial "time derivative" (equation [\(16\)](#page-30-0)). The (classical version of the) law simply states that the Eve universe was born along a surface which was perfectly flat and embedded in such a way that its extrinsic curvature was exactly zero. The law has been derived from the string-motivated demands that the topology of this surface should be toral (from the construction of Ooguri et al. [\[54\]](#page-39-0)), that it should be minimal (from T-duality), and that the energy-momentum flux vector on it should never point outwards from spacetime.

This is the origin of the Arrow of Time: the initial geometry has to be smooth, the inflaton has to begin in the potential-dominated state. The Eve universe (eventually) inflates, and gives birth to baby universes which inherit her Arrow. The rest is history.

We conclude this section by asking: is there anything analogous to the SYGLB theorem for other compact three-dimensional manifolds? This problem can be solved with the aid of various theorems due to Gromov, Lawson, Grigori Perelman, and John Milnor; see [\[15\]](#page-36-10) for the details. The answer is "no": all non-toral compact three-dimensional manifolds are unsatisfactory either because they are like the sphere (that is, they support arbitrarily irregular metrics with non-negative scalar curvature) or because, no matter how they are deformed, they have no metric of non-negative scalar curvature (so that they do not permit a universe to be created along them in the first place). Thus, according to the ideas advanced here, we can say that the construction of Ooguri et al. [\[54\]](#page-39-0) leads precisely to the only universes which can have an Arrow.

This, too, is a very remarkable conclusion. For we should not imagine that a torus with a generic geometry will "look very different" from any other space endowed with its generic geometry. (Think of an extremely distorted two-dimensional torus: the "handle"

distinguishing it from a sphere could be very small and inconspicuous, and the fact that it has exactly one handle will be far from obvious.) Thus we see that the Arrow is an intrinsically global phenomenon. That is, in order to determine whether an Arrow will arise in a given set of circumstances, it is not enough to know what is happening in the immediate vicinity of an observer; one has to examine the entire spatial section.

Now in this connection it is important to understand that, while Eve is spatially finite, her babies are not. This strange fact can be roughly explained as follows. We said that a Coleman-De Luccia baby universe expands outward into the mother universe at a rate approaching the speed of light. This means that, in the manner familiar from special relativity theory, there is an increasing disparity between lengths in the spatial sections inside the baby and those of the outside universe. Eventually this effect becomes so extreme that it becomes possible to fit an infinite space inside the mother spacetime — the point being that, as usual in relativity, mother and child have different ways of slicing spacetime into spatial sections. In fact, ignoring perturbations, one finds that the babies have spatial sections which are copies of an infinite space with constant negative curvature; it is well known that this shape can fit inside the forward light cone of any point in Minkowski spacetime, and it fits inside the baby universe in much the same way. (We discussed the consequences of this shape for the baby's spatial slices in sections 3.3 and 4.3.) Even if we do not ignore perturbations, this will only change the geometry, not the topology, of the spatial sections of a baby.

The global structure of the babies is therefore quite different to that of Eve. The topology of the spatial sections near their birth (and thereafter) is that of ordinary infinite space, which is certainly not constrained by any result analogous to the SYGLB theorem. Hence, our argument showing that Eve has an Arrow will not work for the babies. This agrees with our thermodynamic argument to the effect that the babies can only have an Arrow if they can inherit one, ultimately from Eve.

The fact that the Arrow is a global phenomenon also solves one of the most puzzling questions about time: why should the destruction of spacetime not be associated with "special", low-entropy conditions?

4.5. Against "Final" Conditions

We agreed in section 2.4 that a successful theory of the Arrow must explain not just how it arose at the creation, but also what happens to it over the full span of history. Let us see how this works in the theory we have developed here.

A spacetime (like Eve) with a positive vacuum energy cannot be destroyed after it is created, because it never stops expanding. Eventually, however, Eve will have descendants which do destroy themselves. This happens, as we discussed in section 4.3, when a universe in the Landscape "overshoots" as the value of its vacuum energy decreases, so that the value becomes negative. What happens to the Arrow, assuming that this universe has inherited one, near the "destruction end" of such a universe?

The answer is simple: a universe of this kind will have spatial sections which, like those of any other baby universe, do not have toral topology. Thus we cannot apply the SYGLB theorem to the spacelike slice along which this universe ceases to exist. We therefore have no reason to expect that the geometry of this slice will be in any way "special".

The only reason we believe in the existence of "special conditions" at the creation of

our Universe is that Nature gives us no option: the tendency of entropy to increase is a fundamental fact, confirmed "experimentally" at each moment by all of us. By sharp contrast, there is no evidence whatever for special conditions at any time in our future. Since we now have a theory in which an asymmetry between creation and destruction is natural and inevitable, the parsimonious course is to assume that there are no special conditions at any time in our future. The fate of the Universe, from start to finish, is determined by special *initial* conditions alone. In the case of universes in the Landscape which overshoot into negative vacuum energy, we can expect that, as they contract, their anisotropies will grow rapidly, and that their final spatial geometry will be extremely distorted, in agreement with the second law of thermodynamics. In short: the global nature of the Arrow ensures that it points in the same direction throughout the Landscape, even in universes which destroy themselves.

The only other regions where spacetime is "destroyed" is inside black holes [\[11\]](#page-36-6). The local circumstances around black hole singularities ([\[60\]](#page-39-7), Chapter 12) are indeed somewhat like those near to the singularity in a collapsing universe. The resemblance does not stop there, however: the topology of the "final" spatial section inside a black hole is also nontoral, even if the black hole arises from the collapse of a star in a toral cosmology. As before, it can be shown that there is no analogue of the SYGLB theorem for this topology; so there is no reason to think that the Arrow behaves in any unusual way inside black holes^{[15](#page-33-0)}. Again, the global nature of the Arrow is crucial here.

It is clear from this discussion that the only way "special conditions" could ever arise in connection with destruction is if a toral universe is destroyed. We stress that, in our picture of the Arrow, Eve is the only toral universe, and Eve is never destroyed. In that sense, "destruction along a torus" is not an issue. Nevertheless, the logic of our model demands that it should never, even hypothetically, be possible to enforce low-entropy conditions at a surface where a universe is being destroyed; otherwise an observer might find "the destruction of the universe" to be in his past. That would suggest that the theory is internally inconsistent.

The proof that this cannot happen is instructive. Let us suppose that we have a toral universe which is contracting towards what would be, classically, a final singularity. Now in discussing this situation, we are eager to avoid the charge of cosmic hypocrisy, so let us treat it, as far as possible, in exactly the same way as we treated the creation of a universe. We shall assume, then, that when this universe contracts to about the string length scale, the "Casimir-like" negative energy becomes important, violating the Null Energy Condition, and invalidating the Singularity Theorems. (Unlike negative vacuum energy, the "Casimir-like" field has negative pressure, and so its gravitational field is repulsive; thus it has the power to stop the contraction.) This is how ρ can be negative, in accordance with our mathematical formulation of "destruction".

But with *negative* ρ inserted into the second member of [\(11\)](#page-26-0), we find that our earlier argument simply cannot get started: $R(h)$ is equal to an expression which is a sum of a non-negative term with one which is negative. Such a sum could be of either sign, so we cannot apply the SYGLB theorem here, except to put bounds on the squared extrinsic curvature (which cannot, for example, at every point exceed the local value of $16\pi |\varrho|$).

¹⁵This comment applies to universes like our own, with positive vacuum energy. The situation in the negative vacuum energy case is more complex and remains to be fully understood.

Thus we can put very mild constraints on the rate at which the universe rushes to its doom, but we cannot constrain its geometry when it arrives there.

To see this in detail, let us assume for simplicity that the extrinsic curvature of the "final" boundary does vanish. Then the second member of [\(11\)](#page-26-0) requires the scalar curvature to be negative everywhere. We know that demanding *non-negative* scalar curvature imposes strong conditions on the boundary geometry. What are the consequences of having scalar curvature which is negative? Is there a symmetry, in the space of all metrics on the torus, between metrics with scalar curvatures of opposite signs? Here we can use the following theorem, which is an immediate consequence of the Kazdan-Warner theorem mentioned earlier [\[61\]](#page-39-8):

THEOREM: Let M be any compact manifold of dimension at least 3, and let f be any scalar function on M such that f is negative somewhere on M. Then there exists a Riemannian metric on M having f as its scalar curvature.

We see that negative scalar curvature is radically different to the positive case. Nonnegative scalar curvature imposes extremely strong conditions; negative scalar curvature imposes none at all. Fix any negative function on the torus, no matter how convoluted, and there will be some (equally convoluted) geometry such that the scalar curvature is equal to the given function. Thus there is no reason whatever to expect the "destruction" surface to have a "special" geometry or for any physical field to be in a "special" state. Entropy will always be high when a universe is destroyed, even in the case of toral topology. Hence our theory is internally consistent.

What, then, is the ultimate origin of the strange asymmetry between the the size of the entropy at the creation and destruction of a universe, even when the topology is toral? The answer is found partly in the detailed structure of the general-relativistic initial value constraints, but mainly in the extreme asymmetry of the space of all metrics on the threedimensional torus. In this vast space, the set of metrics with non-negative scalar curvature is absurdly small compared to the set of metrics with negative scalar curvature: this is the content of the SYGLB theorem and of the Kazdan-Warner theorem. This geometrically special condition is turned into the physically special state of the early universe by the gravitational initial value constraints and by the way they control the initial state of the inflaton.

5. Conclusion: The Ageing Of The Multiverse

For each one of us, the passing of time is a basic aspect of experience: the transience of life is intimately related to its meaning. In the theory presented here, this phenomenon is a relic of conditions which were established even before our Universe was born, a souvenir of the emergence of time itself.

What we have found here may be described as the final triumph of the second law of thermodynamics. The latter is now seen to rule not just our Universe, but the entire Landscape; in particular, the birth of new universes occurs in a manner that is strictly in accord with the second law; our Universe has low entropy now because it was preceded by a universe which had still lower entropy, and so on back to Eve. In short, the multiverse ages.

What remains is to develop a *quantitative* theory of this ageing process. This will involve (at least) three major steps.

First, we need a precise analysis of the quantum-mechanical process which breaks the perfect symmetry of the spatial sections of the Eve universe. Unfortunately, this process is not yet fully understood at a fundamental level: the perturbations seem to occur on length scales even shorter than the Planck scale. (See [\[12\]](#page-36-7) and [\[57\]](#page-39-4) for discussions of this "transplanckian modes" problem.) More generally, one will want to have a better understanding of the "emergence" of time in terms of the quantum-mechanical version of the main initial-value constraint, namely the Wheeler-De Witt equation, possibly along the lines suggested in [\[66\]](#page-39-13). Perhaps a deeper understanding of the theory of Ooguri et al. [\[54\]](#page-39-0) will help here.

Secondly, we need a much better understanding of the way the geometric entropy grows during the process of the birth of a baby universe, and of the way this fits together with ideas about how baby universes inflate $|67|$.

Finally, we need a clearer picture of the *rate* at which baby universes nucleate, so that we can predict the conditions in the Eve universe at the time when this happens. Note that transitions from larger values of the vacuum energy to smaller values can involve quite large steps, so the number of steps from Eve to our Universe need not be large; this important fact [\[3\]](#page-35-2) needs to be taken into account. In addition, recent developments [\[68\]](#page-39-15)[\[69\]](#page-39-16)[\[70\]](#page-40-0) have suggested that there may exist one or more mechanisms which can greatly shorten the time needed for transitions to occur. Thus it may be possible for babies to be born when Eve is still extremely young. Clearly, much remains to be done.

The long-range hope is that one will eventually be able to *compute*, using greatly refined theories of Inflation and the Landscape, a precise theoretical value for Penrose's number P. That would be clear evidence that we have finally understood the Arrow in all of its aspects.

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