A Conformally Invariant Approach to Estimation of Relations Between Physical Quantities

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Abstract

A.V.Pushkin's approach based on conformal geometrodynamics (CGD) to calculation of quantitative relations between physical quantities is presented and analyzed. In the simplest cases of the stationary solutions to the CGD equations the approach implies separation of internal and external parts (relative to a certain boundary) from the solutions and using inverse transformations transforming the parts into each other. For the quasi-stationary (metastable) states, the possibility of the nonperturbative calculation of their lifetimes is shown. The approach is illustrated by several examples. In particular, it is shown that the Dirac "large number hypothesis" is a consequence of the approach. Also, the evaluated radiation lifetime of the first excited level of 2p hydrogen atom and neutron lifetime are presented.

1. Introduction

This paper addresses a conformally invariant approach to the estimation of relations between fundamental physical quantities. As far as we know, A.V.Pushkin was the first to employ the approach (see, e.g., [1], [2]), that is why we will name the approach after him. Unfortunately, the approach itself is presented fragmentarily, without appropriate elucidation in the literature. The objective of this paper is to bridge the gap.

Eventually, Puskin's approach is based on the analysis of symmetry of the conformal geometrodynamics equations with energy-momentum tensor of purely geometric nature and symmetry of the conformal quantum field theory with the same tensor as a vacuum polarization tensor. According to Pushkin, in the quantum field theory a symmetry group termed the "Monster" group acts. Reasoning from the above considerations, ref. [1] estimates fine structure constant α and proton to electron mass ratio m_P/m_e and ref. [2] evaluates the background radiation temperature.

For convenience of the reader, to whom papers [1], [2] may be unavailable, present the net result of these works. Thus, according to Pushkin, the fine structure constant is

$$\alpha^{-1}_{theory} = \frac{\bar{N} + \Delta}{dim\Omega_2} = 137.03599079... \quad ,$$

where $\bar{N} = N_{tot} - N_1 = 274$, $N_{tot} = \sum\limits_{i=1}^{6} N_i$=286, N_i is the total number of Killing vectors for all i-th order subgroups of the 15-parameter conformal group, $\Delta = (_M - 1)^{-\frac{1}{2}}$ is the quantum anomalous dimension, $C_M = \bar{N} - N_2 =$ 194 is the amount of the Monster conjugacy classes, $dim\Omega_2$=2 is merely the dimension of 2D surfaces.

The following relation is valid for the proton to electron mass ratio:

$\frac{m_p}{m_e} = \frac{d_B}{120 - N_1} + \frac{\frac{\bar{N}}{2}\left[N_{tot} - \frac{\bar{N}}{2}\right]}{(N_1 - 4)C_M} = 1836.1527..,$

where d_B=196884 is the dimension of Griess algebra. The reciprocal of this dimension controls the accuracy of the calculation of Δ and $(N_1 - 4)\, C_M$ in the first and second formulas, respectively, and, as a consequence, of the ultimate results.

Ref. [2] calculates the ratio of temperature of cosmological (relict) radiation to electron rest energy, so that the temperature is

$T = k^{-1} \frac{m_e c^2}{\rho S_{Mac}} = 2.736\ K,$

where ρ=696729600 is the number of Weyl group elements, viz. the symmetry group of lattice E_8, $S_{Mac} = 7 \cdot \frac{1}{3} \cdot 2 \cdot \frac{2}{3} = \frac{28}{9}$ is a characteristic of the first excited state of the internal space characterized by the lattice E_8. In view of the aforesaid, no relation presented includes any phenomenological (adjustable) parameters; only algebraic characteristics of the conformal group and Monster group enter into the relations, with the characteristics of these groups relating with each other.

Discussions of the papers by Pushkin suggest that it is the estimations of the physical quantities and relations between them that provoke, on the one hand, the greatest interest and, on the other hand, most emotional sentences and questions. Therefore it seems reasonable to try to present in a form as much systematized as possible at least some arguments and considerations needed to understand Pushkin's approach to the estimation of relations between physical quantities. The arguments and considerations make up a certain "construct". According to Pushkin, it is the presence of this construct that differentiates his method from a formal numerology, which is a manipulation of numbers in order to obtain needed relations. Besides, it seems reasonable to illustrate the application of Pushkin's method with several specific examples.

That intention proved hard to implement. Pushkin's method includes a wide range of techniques, among which by no means all can be explained by us. Our way to overcome this obstacle is to restrict ourselves only to the part of the computational technique which is clear to us. Actually, this means that we restrict our consideration to that analysis part, which accounts only for the CGD equation symmetry properties corresponding to the conformal transformations and differentiable changes of coordinates. In so doing the properties governed by the Monster group are not taken into consideration explicitly. Besides, according to the data available to us, after the untimely death of Pushkin in 2004 no systematized statement of his method was left, therefore at some points we will have to conjecture for the author. The non-author version of presentation will inevitably include subjective points stemming from the differences in understanding of the approach under discussion. So Pushkin's method version suggested in this paper should not be viewed as the only possible. Despite these reservations, the paper may prove helpful in the attempt to understand the outputs placed by Pushkin in his papers as well as unpublished ones which Pushkin discussed with his colleagues.

2. Conformally inverse transformations of the CGD equations

2.1. Conformal geometrodynamics equations

In this paper by the conformal geometrodynamics (CGD) is meant the theory based on equation [3]

$$\left.\begin{array}{l} R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} = T_{\alpha\beta} \\ T_{\alpha\beta} \equiv -2A_\alpha A_\beta - g_{\alpha\beta} A^2 - 2 g_{\alpha\beta} A^{\varepsilon}_{;\varepsilon} + A_{\alpha;\beta} + A_{\beta;\alpha} \end{array} \right\}. \tag{1}$$

where $R_{\alpha\beta}$ is the Ricci tensor in four-dimensional Riemannian space; the semicolon means the covariant differentiation performed using Christoffel symbols. Tensor $T_{\alpha\beta}$ that is typically associated with the matter energy-momentum tensor is determined in this case by Weyl vector field A_α.

Equations (1) are self-similar, which reflects the absence of any absolutely dimensional scale in the broad variety of effects described by them, with the gauge invariance being local: equations (1) are invariant under coordinate-dependent conformal transformations

$$g_{\alpha\beta}(x) \to g'_{\alpha\beta}(x) = g_{\alpha\beta}(x) \cdot \phi(x)\,, \quad A_\alpha \to A'_\alpha = A_\alpha - \frac{\partial_\alpha ln\phi(x)}{2\partial x^\alpha}. \tag{2}$$

Weyl himself originally interpreted vector A_α as an electromagnetic field potential, which led to the well-known criticism on the part of Einstein. There is, however, another approach to the understanding of the physical meaning of gauge vector A_α, i.e. the one discussed in refs. [4]-[9], [10], [11] and elsewhere. In this approach the meaning of vector A_α is determined through analysis of the solutions to equations (1) with using no a priori assumptions. It is just this approach that we will adhere to in this paper.

2.2. Explanations to the conformal inverse transformations

The conformal inverse transformations refer to the category of transformations (2) and can be applied to stationary solutions of the CGD equations. Illustrate the action of the conformal inverse transformations by the example of the conformally flat solutions.

We start from the Minkowski space with zero Weyl vector. Given Cartesian coordinates in this space, the metric is

$$\eta_{\alpha\beta} = diag\,(-1, 1, 1, 1)\,. \tag{3}$$

Consider the simplest type of the inverse conformal transformations - the ones that include a combination of the following two transformations:

(1) Inverse coordinate transformation $\{x^\alpha\} \to \{x'^\alpha\}$:

$$x^\alpha \to x'^\alpha = \frac{a^2}{r^2} x^\alpha, \tag{4}$$

where a is a parameter having dimension of length, $r^2 \equiv \eta_{\alpha\beta} x^\alpha x^\beta$. From (4) it follows that

$$x^\alpha = \frac{a^2}{r'^2} x'^\alpha, \tag{5}$$

where $r'^2 \equiv \eta_{\alpha\beta} x'^\alpha x'^\beta = a^4/r^2$. If originally the Riemann tensor and vector A_α were zero, then upon transformations (4), (5) the Riemann tensor and vector A_α remain zero as well. In these transformations the metric tensor, of course, changes. The new metric tensor $g'^{\alpha\beta}$ depends on coordinates x'^α and is determined with formula

$$g'^{\alpha\beta} = \frac{\partial x'^\alpha}{\partial x^\mu} \frac{\partial x'^\beta}{\partial x^\nu} \eta^{\mu\nu}. \tag{6}$$

From (4) it follows that

$$\frac{\partial x'^\alpha}{\partial x^\beta} = \frac{a^2}{r^2} \delta^\alpha_\beta - 2 \frac{a^2}{r^4} x^\alpha \eta_{\beta\nu} x^\nu. \tag{7}$$

Substitution of (7) into (6) results in

$$g'^{\alpha\beta} = \frac{a^4}{r^4} \eta^{\alpha\beta}, \quad g'_{\alpha\beta} = \frac{r^4}{a^4} \eta_{\alpha\beta}. \tag{8}$$

In terms of primed coordinates we have:

$$g'_{\alpha\beta} = \frac{a^4}{r'^4} \eta_{\alpha\beta}. \tag{9}$$

(2) The dilaton conformal transformation of the following form:

$$g'_{\alpha\beta} \to \hat{g}'_{\alpha\beta} = g'_{\alpha\beta} \cdot \frac{r'^4}{a^4} = \eta_{\alpha\beta}. \tag{10}$$

The conformal factor in the transition from $g'_{\alpha\beta}$ to $\hat{g}'_{\alpha\beta}$ is

$$\phi\left(x'\right) = \left(\sigma\left(x'\right)\right)^2 = \frac{r'^4}{a^4}. \tag{11}$$

As a result of transformations (4), (9) we arrive at a space with metric (3). The point with coordinates $\{x^\alpha\}$ has transferred to the point with coordinates $\{x'^\alpha\}$ which is calculated by formula (4).

Despite the fact that upon the transformations the form of the space metric remained unchanged, nevertheless, one significant change occurred: a nonzero field of vector A_α appeared. The field of vector A_α is given by

$$A_\alpha (x') = -\frac{(\sigma (x'))_{,\alpha}}{\sigma (x')} = -2\eta_{\alpha\mu}\frac{x'^\mu}{r'^2}. \tag{12}$$

It can be shown that on the substitution of the expression for vector A_α in form (12) the energy-momentum tensor for the CGD equation vanishes. Thus, as a result of the inverse conformal transformation of the simplest type we arrive at a plane space, in which vector A_α appears as a gradient and the energy-momentum tensor remains zero.

Emphasize once again that the conformally plane spaces are exact solutions to the CGD equations that involve no approximations.

The inverse conformal transformations are not exhausted with the ones of the form considered in this item. A wider class of the transformations is discussed in ref. [2].

Note on terminology. In many papers by the conformally plane space is meant the space produced from Minkowski space merely using the conformal transformation. Upon this transformation the metric differs from that of the Minkowski space, therefore thus produced space is not a conformally plane space in our treatment.

Also note that the procedure of introduction of the nonzero Weyl field and inverse conformal transformations admits a generalization consisting in replacement of the Minkowski space with an arbitrary Riemannian (pseudo-Riemannian) space.

3. Stationary solutions and Dirac "large number hypothesis"

Consider the issue of properties of the transformation described by the static spherically symmetric solution of CGD equations. We will use the term of particle with meaning by it a region localized in space, inside and outside of which fields $g_{\alpha\beta}(x)$, $A_\alpha (x)$ are described by the branches of the general static spherically symmetric solution to the CGD equations. We proceed from the assumption that the solution is regular over the entire space, excluding the appearing discontinuity surface. This solution type can exist because, first, there are no connections to initial data in setting up the Cauchy problem for the CGD equations and, second, the velocity of perturbation propagation in continuum described by geometrodynamic energy-momentum tensor (1) can be as fast as light velocity. For a more detailed discussion of this issue see ref. [13].

According to ref. [13], there are three types of the general solution to the static spherically symmetric problem, each of which is described by five integration constants. Some of the constants can be assumed zero. In the simplest case the particle is characterized only by two constants: gravitational radius (Gm/c^2) and radius of the Universe. Whatever the constant set, however, we obtain the solution having singularity for some value of radial variable z.

In the range of small values of the radial variable the discontinuous solution is close to the de Sitter solution known in the general relativity, while in the range of large ones to the Schwarzschild solution in the space coinciding asymptotically with the de Sitter space. At a certain value of the radial variable these two solution branches get sewn. Denote the sewing surface radius by a. A specific value of the a is determined from additional physical considerations (see [13]), which we will not analyze here. To us only the fact itself of the discontinuity surface existence will be important.

Constants (Gm/c^2), (c/H) can be interpreted as follows. For the value of the radial variable z equal to (Gm/c^2), the external part of the solution would become singular, if it were continued to the range of small z. For the value of the radial variable z equal to (c/H), the internal part of the solution would become singular, if it were continued to the range of large z.

If the described situation takes place, then a conformal transformation exists that swaps the internal and external solution parts with the discontinuity surface remaining unchanged. This transformation refers to the category of inverse invariant transformations. Length a should therewith satisfy the "golden" section rule

$$a = Const \cdot \sqrt{\left(Gm\big/c^2\right) \cdot (c/H)} = Const \cdot \sqrt{\frac{Gm}{cH}}. \qquad (13)$$

Constant $Const$ that has appeared in the expression for a is a number close to unity. Here we will not take up the calculation of the $Const$. We only note that its value is related to the amount of the discontinuity surface implementation methods and, evidently, differs for different particles. It will be assumed equal to unity except as otherwise noted.

The obtained value of length a is the discontinuity surface radius for the simplest spherical symmetric static particle. However, no particles offering the above properties exist in the Nature. All nonzero mass particles possess spin, electric charge and other quantum numbers and, of course, cannot be described by the considered spherically symmetric static solution.

Example 1

In the zeroth approximation we can neglect the presence of spin in electron and identify the sewing surface radius a with the classic electron radius (e^2/mc^2), i.e. assume that

$$\sqrt{\frac{Gm}{cH}} \approx \frac{e^2}{mc^2}. \tag{14}$$

It turns out that equality (14) obtained from the identification is neither more nor less than the relation of the Dirac "large number hypothesis". Indeed, the relation is written, as a rule, in the form of equality of two large numbers $N_1 \approx 10^{40}$ and $N_2 \approx 10^{40}$, where

$$N_1 = \frac{e^2}{Gm^2}; \quad N_2 = \frac{(c/H)}{(e^2/mc^2)}. \tag{15}$$

The first number is the ratio of Coulomb force acting between two electrons to Newtonian force of their attraction. The second number is the radius of the Universe expressed in the units of the classic electron radius. Using expressions (15) for N_1 and N_2 and equating N_1 and N_2, we arrive immediately at approximate relation (14).

Thus, the Dirac large number hypothesis can be viewed as a consequence of the CGD equations that owes its existence to the conformal symmetry of the Universe.

Example 2

More realistic is the class of axially symmetric (AXS) solutions of the CGD equations. Although no general solutions to the AXS problem for the CGD equations have been found, it can be stated almost definitely that in the simplest case the AXS solution is determined by two constants: gravitational radius (Gm/c^2) and Compton length ($\hbar/mc$). Reasoning similar to that for the spherically symmetric case shows that discontinuity surface radius a is

$$a = Const \cdot \sqrt{\left(Gm\big/c^2\right) \cdot (\hbar/mc)} = Const \cdot \sqrt{\frac{G\hbar}{c^3}}. \tag{16}$$

Radius a agrees with the Plank length with an accuracy to a constant. It is noticeable that the a is independent of particle mass. In other words, the radius of the surface of sewing of two branches of the ASS solution is the same in all particles with spin $\hbar/2$.

Thus, a consequence of the conformal geometrodynamics is the relation among gravitational radius, Compton length and Plank length which is well known in physics.

4. A nonperturbative method for estimation of relations between physical quantities

Let us next assume that, besides geometric quantities, a physical situation is described by some physical field φ, which depends on the metric tensor and Weyl field in some, maybe complex manner. Let field φ be correspondent with some physical quantity $\Phi(\varphi)$.

Denote the physical quantities corresponding to solutions *Large* and *meso*by Φ_{Large} and Φ_{meso}, respectively. As in the CGD equations there is no dimensional constant, all the physical quantities can be expressed in terms of length, that is the dimension of is a certain degree of length. Therefore preserved quantity Φ_{little} should be related with Φ_{Large}, Φ_{meso} by the same relation, which holds for lengths (radii):

$$(S_{little} \cdot \Phi_{little}) \cdot (S_{Large} \cdot \Phi_{Large}) \sim (S_{meso} \cdot \Phi_{meso})^2 . \tag{17}$$

Quantities S_{little}, S_{meso}, S_{Large} have a meaning close to that of statistical weights (combinatorial multipliers), i.e. methods for realization of states *little*, *meso*, *Large*, respectively, while their reciprocals have the meaning of probabilities of the relevant states.

The form of the relation between physical quantities is easy to use, where all weights (or probabilities) are combined into a single reduced "normalization" factor. Then the relation can be given by

$$\Phi_{little} = \xi \frac{\Phi_{meso}^2}{\Phi_{Large}}. \tag{18}$$

Formula (18), which we will term as Pushkin's relation, provides a basis for what follows. In specific cases the exact determination of $\xi_{little},\ \xi_{Large},\ \xi_{meso}$ can be complex enough. As for the methods for determination of multipliers $\xi_{little},\ \xi_{Large},\ \xi_{meso}$, in his book [9] Pushkin writes that they appeared for three principal reasons:

" a) unit vector enumeration combinatoric analysis;
b) presence of various types of reflective symmetries, including mere inversion of one or more spatial unit vectors;
c) possibility of exchange of a temporal unit vector for a spatial in one signature sector (for example, $t \to ir$and simultaneously $r \to it$).

Inclusion of these symmetries is a simplest method for bundle averaging of trajectories (solutions), where the comparison of two solution sets proceeds over "rough" invariants, which do not distinguish them by these signs. Of course, the primary source of these symmetries are topological and differential-topological properties of the manifolds under consideration. Arithmetically, the summation or averaging over bundle trajectories manifests itself, primarily, in simple combinatoric factors like $\frac{1}{4!} = \frac{1}{24}$; $\frac{2}{3}$; etc. in algebraic formulas relating solution invariants."

To be more specific regarding the above quotation note that in some cases weight factor ξ in (18) can take on both very large and vary small values. This can happen, when the group of symmetries of states *little*, *meso*, *Large* has a large order. Such was the case, for example, in the consideration in [14] of the vacuum solutions corresponding to the lower orbit of lattice E_8; in that case the weight factor was close to the order of Weyl group of algebra E_8, i.e. to $\sim 0.7 \cdot 10^9$.

Clear that all of the aforesaid holds not only in regard to the scalar physical field φ with a certain Weyl weight k, but also the physical field of any nature, for example, bispinor field or gauge field (like Yang-Mills field). The only constraint is that the physical fields should be geometric objects of the Weyl space, in particular, have a certain Weyl weight.

Thus understood physical fields should not, generally speaking, be identified with metric $g_{\alpha\beta}$ and Weyl vector A_α. The physical fields (bispinor, gauge, etc. fields) are prescribed in the Weyl space, in which there are tensor $g_{\alpha\beta}$ and vector A_α, whose explicit form is found as a result of solving the CGD equations. The physical fields obey their dynamics which can relate in a very complex fashion with dynamics of $g_{\alpha\beta}$and A_α fields. Practically, the phenomenological techniques of field description that have been elaborated by theoretical physics can be made use of to determine it. It should be kept in mind that whatever solution in terms of physical fields we consider, a certain solution to the CGD equations exists simultaneously with it as well in each space-time domain.

The parallel existence of a solution in terms of physical fields and a solution to the CGD equations in each spatial domain can cause appearance of relations between physical quantities, if the assumptions specified in subsection 2.2 are fulfilled.

From the above procedure of Pushkin's formula derivation it follows that Pushkin's method under discussion is applicable only to states related by inverse conformal transformations. The search for the states of this kind is an independent problem.

We have conducted a search (far from complete) for inverse conformal states. The search led us to conclude that Pushkin's method under discussion can be attempted to apply to the following processes:

(1) Decaying of the first excited level of hydrogen atom, i.e. the process of transition of the hydrogen atom from state $2p$ to state $1s$.

(2) Emitting of 21-cm radio line by hydrogen atom.

(3) Decaying of neutron in free state.

All the three processes belong to a single class characterized with the following.

A) The states under consideration are causally connected. That is, they refer to processes occurring with one and the same physical object, with at least one of the states being quasi-stationary (quasi-stable). In his book [9] Pushkin correlates these bound or quasi-bound states with solutions in the Euclidean sector of signature $(++++)$. The process of the transition from the metastable state to the stable is described by a Lorentz metric solution. In the geometrodynamics, the initial and final states are related by general (not spherically symmetric) solution in signature sector $(-+++)$. The solution in signature sector $(-+++)$ is an "instanton" with respect to the one in Euclidean sector $(++++)$.

B) The processes under consideration are of dissipative nature, i.e. the nature, in which the processes under consideration cannot be explained in a natural fashion within the framework of the standard versions of effective theories. Note that the CGD equations include the dissipative processes without violation of the causality principle. Responsible for the dissipative process resulting in the transition from one state to another are the conformally invariant interactions.

) The states under consideration are salient physically. The salience (fundamentality) of the states means that the states refer to the primary elementary particles and nuclei and their lowest energy levels.

In what follows we consider each of the above processes and show how relations between characteristics of the processes can be determined.

5. Nontrivial examples of Pushkin's method application

5.1. Width of the first excited level of hydrogen atom

Let us consider the hydrogen atom and ask ourselves if relation (18) can be used as a basis to obtain an estimator of radiation lifetime τ of the first excited level 2p of the hydrogen atom. In the quantum theory, the ground state of atom, its excited state 2p and the process of the transition from the excited state to the ground one are considered to obtain the estimator. In the energy representation, the excited state is characterized with energy E_1 counted from the ground state and equal energy of photons emitted in transition $2p \to 1s$. The lifetime $\tau$is correspondent with level width $\Delta E_1 = \frac{\hbar}{\tau_1}$. The ground state is characterized with energy of the atom as a whole, in the atom rest frame the energy is $E_0 = M_H c^2$.

As we said earlier, Pushkin correlates the excited and ground states with solutions in Euclidean sector of signature $(+ + ++)$. The process of the transition from the metastable state to the stable is described by the solution with Lorentz metric. In the geometrodynamics, the initial and final states are related by general (not spherically symmetric) solution in signature sector $(- + ++)$. For the hydrogen atom this is the geometrodynamic representation of photon emission. The solution in signature sector $(- + ++)$ is an “instanton” with respect to the one in Euclidean sector $(+ + ++)$.

Each of the above characteristics is correspondent with triply connected CGD solutions according to (4.1). The characteristics of this connection are determined by inequality

$$E_0 >> E_1 >> \Delta E_1. \tag{19}$$

In this case formula (18) will be written as

$$\Delta E_1 = \frac{E_1^2}{E_0}\xi_s, \tag{20}$$

and the lifetime of level $2p$ is given by

$$\tau = \frac{\hbar E_0}{\xi_s E_1^2}. \tag{21}$$

In the substitution of the known values of $\hbar$, E_0, E_1 into the right-hand side we obtain$\tau = \frac{5.93}{\xi_s} \cdot 10^{-9}s$, while the experimental value is $\tau_e = 1.6 \cdot 10^{-9}s$[15]. For $\xi_s = 4$ the theoretical value is $\tau = 1.48 \cdot 10^{-9}s$. In our opinion based on quantum mechanics intuition, this value of ξ_s is correspondent with $S_0 = 1, S_1 = 1, S_\Delta = 2$, however the current determination of ξ_s requires plunging into the deep methods of analysis of the geometrodynamics equations which were being developed by Pushkin.

Among the three states appearing in the consideration performed for the hydrogen atom, one refers to the atom as a whole. The question is appropriate: Why on earth does the proton-related term $M_H c^2$ appear in the electron level related estimators? When answering a question of this kind, one should keep in mind that the typically used factorization of the wave function in the form of product of the wave function of nucleus by the wave function of electrons is nothing more than a supposition. In the framework of CGD the atom is a connected system, for which there is no wave function factorization in the entire space-time domain; the wave function can be represented approximately in one spatial domain and cannot in another. Therefore, when considering properties of electrons entering into the composition of the atom, the use of $M_H c^2$ can prove quite appropriate.

Additional "food for thought" is provided by the analysis of the triplet: hydrogen, deuterium and tritium. Formula (28) includes atomic mass. On this evidence it may seem that for deuterium the lifetime of the first excited state is 2 times as long as that in hydrogen, and in tritium this is 3 times as long. It is well known, however, that the lifetime of the first excited level of hydrogen, deuterium and tritium atoms is the same. The seeming contradiction is removed by the fact that for deuterium the additional multiplier 2!=2 due to the proton and neutron transposition should be included in the left-hand side. For tritium the multiplier is equal (with taking into account the identity of two neutrons) to 3=3!/2!.

5.2. Estimation of hydrogen radio line emission time

Imagine that the transition of hydrogen atom from state $\uparrow\uparrow$ to state $\uparrow\downarrow$ proceeds under the action of microwave background radiation, which is a heat reservoir ("bath") for the electron.

We will estimate background radiation energy density ε_γ assuming that:

1) 10^9 background radiation photons are contained in 1 m^3.

2) Order-of-magnitude energy of each of the photons is $\frac{1}{2}kT$.

Assuming $T = 2.73$ K, we obtain that energy density ε_γ is

$$\varepsilon_\gamma = 1.88 \cdot 10^{-13} erg/cm^3. \tag{22}$$

We will determine the width of level $\uparrow\uparrow$, which will be denoted by $\Delta E_{\uparrow\uparrow}$, with Pushkin's formula (18); in this case it will take the form

$$\Delta E_{\uparrow\uparrow} = \left((\Delta E_{atom})^2 \Big/ E_{electron} \right). \tag{23}$$

Here:

ΔE_{atom} is the difference of background radiation energies contained in the initial and final states,

$E_{electron}$ is the background radiation energy contained in the electron volume.

$E_{electron}$ can be estimated as follows:

$$E_{electron} = (\hbar/mc)^3 \cdot \varepsilon_\gamma. \tag{24}$$

As for electron Compton length is $(\hbar/mc) = 3.8 \cdot 10^{-11} cm$, then for $E_{electron}$ we obtain:

$$E_{electron} = 1.03 \cdot 10^{-44}\, erg. \tag{25}$$

ΔE_{atom} will be estimated as

$$\Delta E_{atom} = E'_{atom} - E''_{atom}, \tag{26}$$

where E'_{atom}, E''_{atom} are the background radiation energies contained in the initial and final states. E'_{atom}, E''_{atom} are:

$$E'_{atom} = \frac{4}{3}\pi r_0^3 \cdot \varepsilon_\gamma; \quad E''_{atom} = \frac{4}{3}\pi \left(r_0 - \Delta r_0\right)^3 \cdot \varepsilon_\gamma \tag{27}$$

Here $r_0 = \left(\hbar^2 \big/ me^2\right) = 5.29 \cdot 10^{-9}\ cm$ is the radius of the first Bohr orbit. As for Δr_0, it has the meaning of decrease in the radius of the first Borh orbit in the transition corresponding to photon emission of wavelength $\lambda = 21.1\ cm$ (i.e. hydrogen radio line). The emitted photon energy $\hbar\omega$ is determined by formula

$$\hbar\omega = (2\pi\hbar c/\lambda) = 0.929 \cdot 10^{-17} Erg.$$

Then our reasoning is as follows: if electron binding energy $E_1 = 13.6\ eV = 2.18 \cdot 10^{-11}\ erg$ on the first Bohr orbit is correspondent with radius r_0, then binding energy $E_1 + \hbar\omega$ should be correspondent with radius $r_0 - \Delta r_0$. Whence

$$r_0 - \Delta r_0 = r_0 \frac{E_1}{E_1 + \hbar\omega}.$$

Since $\hbar\omega << E_1$,

$$\Delta r_0 = r_0 \frac{\hbar\omega}{E_1} = 2.25 \cdot 10^{-15}\ cm. \tag{28}$$

For ΔE_{atom} we find:

$$\Delta E_{atom} = 4\pi r_0^2 \left(\Delta r_0\right) \cdot \varepsilon_\gamma = 1.49 \cdot 10^{-43}\ erg. \tag{29}$$

The substitution of the determined values of ΔE_{atom} and $E_{electron}$ into the formula for $\Delta E_{\uparrow\uparrow}$ yields the following:

$$\Delta E_{\uparrow\uparrow} = \left(\left(\Delta E_{atom}\right)^2 \big/ E_{electron}\right) = 2.16 \cdot 10^{-42}\ erg$$

Knowing line width $\Delta E_{\uparrow\uparrow}$, determine average lifetime τ.

$$\tau = (\hbar/\Delta E_{\uparrow\uparrow}) = 4.8 \cdot 10^{14}\ sec = 1.5 \cdot 10^7\ years. \tag{30}$$

The evaluated average lifetime τ is close to the $\tau \approx 10^7\ years$determined from astrophysical considerations (see [16]). This result points to both the validity of the physical notions of the background radiation role and the validity of Pushkin's formula for this case.

5.3. Estimation of neutron lifetime

The process of free neutron decay by scheme

$$n \to p + e + \tilde{\nu}$$

is governed by electroweak interactions. The reaction goes through generation of intermediate charged boson, photons cannot be directly involved in this decay.

However, in the primeval form (before spontaneous break of symmetry) electroweak interactions can be formulated in the conformally invariant form. Therefore it is of interest to treat this process by Pushkin's method.

Two quantities with energy dimension are associated with the decay process. First, quantity

$$(\Delta M)\, c^2 = (M_n - M_p)\, c^2 = 1.29\ MeV = 2.06 \cdot 10^{-6}\ Erg.$$

Second, background radiation energy E_{b-g}contained in neutron volume

$$E_{b-g} = \left(\frac{\hbar}{M_n c}\right)^3 \cdot \varepsilon_\gamma = \left(2.08 \cdot 10^{-14}\ cm\right)^3 \left(1.88 \cdot 10^{-13}\ \frac{Erg}{cm^3}\right) =$$

$$= 1.69 \cdot 10^{-54}\ Erg.$$

Determine level width ΔE by Pushkin's formula, which in this case is given by:

$$\Delta E = \sqrt{(\Delta M)\, c^2 \cdot E_{b-g}} = \sqrt{(2.06 \cdot 10^{-6}\ Erg) \cdot (1.69 \cdot 10^{-54}\ Erg)} =$$

$$= 1.87 \cdot 10^{-30}\ Erg.$$

This level width is correspondent with level lifetime T of

$$T = \frac{\hbar}{\Delta E} = \frac{(1.04 \cdot 10^{-27}\ Erg \cdot sec)}{(1.87 \cdot 10^{-30}\ Erg)} = 560\ sec.$$

The experimental neutron lifetime is $(888 \pm 10)\ \ sec.$ The comparison of T with the experimental value shows that Pushkin's method does work in this case as well, but its accuracy, as expected, is not high. This may be because in the neutron decay the background radiation photons play an auxiliary role - as a mechanism of neutron perturbation in free state resulting in the neutron decay.

5.4 Biological addendum

Pushkin [9] presents an illustration of a chain of processes, for which the conformal geometrodynamics provides tools for construction of solutions. The chain begins with elementary particles and ends with global cosmological structures, that is begins with the smallest scales and ends with the largest. The conformal geometrodynamics symmetry allows the solution invariants not only of the nearest, but also of not nearest neighbors in the chain to be connected. Therefore, even having no explicit solutions, we can ask ourselves about connection of quantities that are far from each other in scales. Actually, we have already considered problems of this kind in Section 3.

In this statement a natural question arises: What will the average scale be, if for the large scale we take the size of the Universe

$L_H = cH^{-1} = 4300Mpc = 1.33 \cdot 10^{28}$ cm,

and for the small scale the Plank length

$l_{Pl} = \left(\frac{\hbar G}{c^3}\right)^{\frac{1}{2}} = 1.61 \cdot 10^{-33}$ cm.

Then by formula (22) with ξ=1 we obtain the following for the medium scale:

$l_c = \sqrt{L_H l_{Pl}} = 4.6 \cdot 10^{-3}$ cm. (31)

Surprising as it may seem, the human cell sizes lie precisely in the 5-100m range. Moreover, the spermatozoid and ovicell nucleus sizes range within 50-60 m. It turns out that the objects of paramount importance to the existence of the mankind as well as all living things are equidistant from the "dangerous" largest and smallest sizes.

The journey to biology can be continued, but we will restrict our consideration only to one more example. The man consists mainly of water molecules. Take for the small size the water molecule diameter

$l_{H_2O} = 3 \cdot 10^{-8}$ cm,

for the large size the man's medium height

$L_{hb} = 160$ cm ,

then

$l_c^1 = \sqrt{l_{H_2O} L_{hb}} = 22$ m.

The small cells are more in number in a body than the large. This estimator objectively characterizes the harmonicity (optimality) of the human body construction. In this consideration the elephant is definitely overheavy, while the mouse is too light.

The examples given in this section are in essence well known. Their interpretation becomes more visual, if we write the golden section formula in the logarithmical form. Then, for example, relation (31) will become

$$lnl_c = \frac{1}{2}\left(lnl_{Pl} + lnL_H\right),$$

that is on the scale axis the cells of living organisms are located exactly in the middle between the Universe and Plank length. Many details of the analysis of life and human in the scale of the Universe in the logarithmical form are described in ref. [22]. Of course, the subject of the scale harmony is quite ancient, a great many studies are devoted to it, and many books are written about it. All art and architecture are inconceivable without the harmony. There are different approaches to its description. The approach discussed by us seminally links the scale harmony of the World with Weyl geometry and its conformal symmetry.

6. Conclusion

In this paper we made an attempt to represent separate aspects of Pushkin's approach to the estimation of fundamental physical constants and relations between physical quantities. Pushkin's approach is based on the analysis (group, geometrical, functional) of conformal geometrodynamics. We restricted our consideration to those estimators which follow immediately from CGD equation solution symmetry about the conformal transformations and differentiable changes of coordinates. Finest features of the method relating to the analysis of symmetry of the conformal quantum field theory with the vacuum polarization tensor coinciding with the energy-momentum tensor of the CGD equations are therewith left aside. According to Pushkin, a symmetry group termed the "Monster" group acts in this quantum field theory. The Monster group has sufficient "building material" ($\sim 8.08 \cdot 10^{53}$ elements) for the calculations with an accuracy competitive with that of most precision experiments of the modern physics. Yet, we restricted ourselves to the order-of-magnitude estimations.

This paper is based on several nontrivial rigorously proven facts. The proven facts include the presence of a specific symmetry in the stationary solutions of the CGD equations which is due to the conformal inverse transformations. The symmetry leads to appearance of a connection between lengths characterizing the position of the singularities of the internal and external solution parts. The connection, in its turn, leads to appearance of the relation between physical quantities characterizing either of the above solution parts. Besides, in CGD there is a possibility of self-consistent description of decay processes, that is the possibility to describe the transition between two (quasi-) bound states using the general solution to the CGD equations. This solution also leads to appearance of the relation between the physical characteristics of the process and states which has the form of the generalized golden ratio.

This paper uses the above facts and some assumptions to consider two commonly known (”large number hypothesis” and relation among gravitational radius, Plank radius, and Compton length) and three nontrivial examples of using Pushkin’s relation (18) between physical quantities. These examples, of course, do not exhaust all possible “golden section” relations. In this paper it was important to us to demonstrate that certain physical CGD models stand behind Pushkin’s relation (18), that the relations are not merely a result of fitting or guessing. The examples given in the paper, as we believe, have accomplished this task.

Speaking about the significance of the relations between physical quantities that follow from the conformally inverse symmetry, two aspects may be mentioned.

- In some cases these relations can allow us to obtain a quantitative estimation of a physical quantity, like this was done for the lifetime of the first excited state of hydrogen atom: if we had not known that time, we could have estimated it.

- Dimensionless relations between physical quantities can serve prompts to reveal the implication of interrelations between effects and processes. Attention is drawn to this aspect in many papers (see, e.g., [17]-[21]). For example, the standard model of electroweak interactions cannot be viewed as a fundamental theory of the interactions, as the model includes phenomenological parameters that are not deduced from it itself. The dimensionless relations between physical quantities, as it seems to us, can be demanded both in improvement of this model and in development of a higher-level theory.

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