Summary of GR18 Numerical Relativity parallel sessions (B1/B2 and B2), Sydney, 8-13 July 2007

Carsten Gundlach

School of Mathematics, University of Southampton, Southampton, SO17 1BJ, UK

The numerical relativity session at GR18 was dominated by physics results on binary black hole mergers. Several groups can now simulate these from a time when the post-Newtonian equations of motion are still applicable, through several orbits and the merger to the ringdown phase, obtaining plausible gravitational waves at infinity, and showing some evidence of convergence with resolution. The results of different groups roughly agree. This new-won confidence has been used by these groups to begin mapping out the (finite-dimensional) initial data space of the problem, with a particular focus on the effect of black hole spins, and the acceleration by gravitational wave recoil to hundreds of km/s of the final merged black hole. Other work was presented on a variety of topics, such as evolutions with matter, extreme mass ratio inspirals, and technical issues such as gauge choices.

I. INTRODUCTION

In the numerical simulation of comparable-mass black hole binary mergers, after a decade of struggle with unstable codes, several groups are now obtaining reliable gravitational wave signals, and are competing closely to investigate the parameter space.

This topic dominated the plenary talk by B. Brügmann, as well as about half of the talks submitted to the parallel session B2 Numerical Methods. To celebrate the fact that binary black hole simulations are at last obtaining astrophysics results, a session was held jointly with the B1 Relativistic Astrophysics parallel session. 7 contributed talks were given in B1/B2 and 24 more in B2. There were 6 posters in B2.

Much work is also being done on numerical simulations of astrophysical scenarios involving matter in general relativity, in particular core collapse and neutron star binary mergers, but at GR18 this interesting and active area was represented by few contributed talks. There were no talks on computer algebra.

Given the remarkable sudden progress in binary black hole evolutions since 2005, and the remarkable similarity of the results from different groups, I begin with an overview of this field. I have aimed to make the astrophysical and historical overviews (Secs. II A and II B) accessible to researchers outside numerical relativity, and a description of the state of the art (Sec. II C) useful to beginning graduate students in the field. After that, the summaries of individual talks are likely to be too technical to be accessible to any but the specialists, and too short to tell the specialists anything new. Therefore, where I could identify a paper or e-print related to a specific talk, I give that reference.

II. BLACK HOLE BINARY MERGERS

A. Astrophysical background

Many stars are in binary systems, and many of these are expected to consist, at the end of their life, of two black holes, two neutron stars, or a neutron star and a black hole. All binary systems lose orbital energy through emitting gravitational radiation, and merge eventually, although the time scale on which this happens depends on their initial separation and masses. Compact objects such as neutron stars and black holes approach each other very closely before merger, which means that they can emit significant amounts of gravitational radiation just before merger, releasing an energy up to a few percent of the total mass of the system.

These gravitational waves are very much harder to detect than the same energy in light, basically because the frequencies are very much lower. This has the twin consequences that the instrument has to be very much larger, and that, unlike a CCD camera for light, it cannot detect individual quanta. For these reasons, no gravitational waves have yet been measured directly. (However, the time series of several pulsars in close binaries agrees very accurately with the energy loss through gravitational waves predicted by general relativity.)

Mergers of compact object binaries are expected to be the principal source of gravitational waves with frequencies of tens to thousands of Hertz. The large interferometric detectors LIGO, GEO, VIRGO and TAMA are now observing in the hundreds of Hertz frequency; they were reviewed in the plenary talk of S. Whitcomb. LIGO is funded for an upgrade that should allow it to see binary mergers in the local group; this should give it enough sources to see something. (See also the plenary talks by D. Shadock on space-based detectors, and by M. A. Papa on gravitational wave astronomy.)

In the mid 1990s, the binary black hole problem seemed the natural problem for numerical relativity to tackle as soon as computers were big enough to allow simulations in 3D: it is entirely described by smooth solutions of the

Einstein equations, without the need to model matter. While the two black holes are still far apart each is approximately a Kerr black hole, and is parameterised by only its mass and spin, and their orbit is parameterised by only its ellipticity and size. The gravitational field itself has an infinite number of degrees of freedom, but after a few orbits the gravitational wave field is effectively determined by the history of the orbital motion. This means that the parameter space of binary mergers which start sufficiently far apart and where accretion into the black holes can be neglected is effectively finite-dimensional.

B. History of numerical simulations

Ten years and perhaps 500 person-years later, the problem had not been solved because the numerical simulations were stopped by numerical instabilities that could not be overcome by just using more resources. In hindsight, there were overlapping problems concerning the mathematical formulation which interacted with, and were sometimes confused with, problems of numerical discretisation. These were mainly of two types: the use of ill-posed formulations of the initial-boundary value problem for general relativity, and gauge choices not appropriate to the symmetries of the problem. The evolution time before the code broke down was gradually extended, with the first full orbit achieved by Brügmann in 1997 [1], but only a very few orbits became possible until 2005.

A first stable simulation with an essentially unlimited number of orbits up to and through the merger was announced by F. Pretorius in March 2005 [2]. He solved the Einstein equations in modified harmonic coordinates. Their principal part is then a set of wave equations with characteristics on the physical lightcones. However, some lower-order terms were modified in order to modify the harmonic time slicing. Other key elements of his success include, but were not limited to, imposing outer boundary conditions by compactifying the numerical domain at spatial infinity, the excision of a spacetime region inside each black hole (singularity excision), the use of adaptive mesh refinement, and damping the constraints by friction-like lower-order terms.

In November 2005, an unlimited number of orbits plus merger was achieved again with very different methods, at the same time but independently by groups at the University of Texas/Brownsville [3] and at NASA/Goddard [4]. These groups used the "BSSN" formulation of the Einstein equations [5], which had already been successful in neutron star and core collapse simulations (see for example [6]). The key additional ingredient for binary black hole simulations was a new version of an old method for dealing with the singular interior of the black holes called the "puncture method" [7], where inside each black hole is a wormhole, rather than a collapsed star.

C. State of the art

These two formulations of the problem are still the dominant ones today with only minor changes. The BSSN formulation is now always used together with a "Bona-Massó 1+log" type slicing condition and a "hyperbolic Γ-driver" type shift condition. With this gauge, BSSN is hyperbolic, although some of its characteristics are spacelike and some timelike with respect to the physical light cones. There seems to be a trend to gauge drivers with time derivatives in the direction normal to the slice, rather than along the lines of constant spatial coordinates. For example, $(\partial_t + \beta^i \partial_i)\alpha = -2\alpha K$, rather than $\partial_t \alpha = -2\alpha K$, and similarly for the evolution of the shift. This avoids a breakdown of hyperbolicity when gauge speeds coincide, and allows the final black hole to settle down to a Killing coordinate system.

Encouraged by Pretorius' success, other groups are now experimenting with modified harmonic coordinates both for vacuum and matter evolutions, and his work seems to have influenced even the evolution of the linearised Einstein equations for modelling extreme mass ratio inspirals, such as stellar mass black holes falling into supermassive black holes.

The most reliable initial data now seem to be produced by converting a snapshot of a post-Newtonian (PN) orbit into "puncture" type initial data for full general relativity by solving the Hamiltonian and momentum constraint. This PN orbit is typically constructed by calculating the adiabatic shrinking of a conservative PN orbit. To reduce eccentricity, the post-Newtonian equations of motion can be evolved over hundreds of orbits until the eccentricity has been radiated away. It is puzzling that the conversion works so well given that this also involves a translation of gauge from the post-Newtonian to the full general relativity framework.

Outer boundaries remain primitive - some in common use are not known to lead to well-posed initial-boundary value problems, and are known not to be compatible with the constraint equations. Nevertheless, numerical boundary conditions exist that are empirically stable, and the strategy of moving them as far out as possible, typically by using nested boxes of Cartesian coordinate grids, seems to work. The location of the outer boundary does not seem to seriously affect the gravitational wave signals.

Waves are extracted on an approximately spherical surface either by finding the Zerilli gauge-invariant linear perturbation with respect to a Schwarzschild background, or by constructing an appropriate tetrad and calculating the Newman-Penrose scalar Ψ_4. Here it does seem to be important to push this sphere as far out as possible.

Black hole masses and spins are calculated by finding a dynamical horizon, and constructing approximate Killing vectors for use in Komar-type integrals, by energy and angular momentum balance arguments, or by fitting quasinormal ringdowns to the known ringdown of

Kerr. Approximate ADM quantities integrated over a large sphere also seem to agree with these diagnostics.

D. Contributed talks

The contributed talks were dominated by physics results which appear to agree between research groups. Most talks focused on the effect of the spin of the two black holes on gravitational recoil ("kicks"). It appears that velocity of several hundred km/s are generic, and several thousand km/s are possible. This is astrophysically relevant as it may completely eject the merged black hole from its galaxy. Note that as velocity is naturally measured in units of the speed of light, and vacuum gravity is scale-invariant, these velocities are invariant under an overall scaling of the initial data. Therefore the results reported here would apply equally to stellar mass and supermassive black hole binaries, as long as the two masses are of the same order of magnitude.

P. Diener [9] reported on joint work by the AEI and LSU groups. Initial data were quasi-circular, determined by an effective potential method. Kicks up to $175km/s$ were achieved for non-spinning unequal mass binaries (mass ratio 0.36), while kicks up to $440km/s$ were obtained with equal mass black holes with spins anti-aligned parallel to the orbital angular momentum. The post-Newtonian analysis of gravitational recoil by Kidder [8] was found to be qualitatively valid beyond its domain of applicability.

M. Hannam reviewed the work of the Jena group. They had concentrated on a configuration with equal masses, and spins anti-aligned in the orbital plane ("superkicks") [10]. The magnitude depended sinusoidally on the angle of the spins in the orbital plane. Because of the high symmetry, most of the energy was radiated as $l = 2, m = \pm 2$ waves, and the kick could be estimated through the energy difference between these. The velocities obtained were of the order of $2500km/s$ (for $J = 0.723M^2$), in agreement with what had been found by the Brownsville group [3] (which was not represented at GR18).

Work was also under way on a bank of numerical template wave forms for non-spinning mergers [11]. Full numerical evolutions based on post-Newtonian initial data could be compared to a continued post-Newtonian evolution for up to 9 more orbits before the merger, and agreed very accurately. Going to sixth order accuracy in the spatial finite differencing had been crucial to obtain a small enough phase error (2 degrees over 9 orbits) for this. Moreover, very accurate "hybrid" wave forms could be constructed by matching a post-Newtonian inspiral to black hole ringing, using information from numerical simulations to match them. This might be used to fill the parameter space with templates.

P. Laguna [12] reported work from the PSU group on spins parallel to the orbital angular momentum (up to $400km/s$) and superkick configurations, and stressed that the Kidder post-Newtonian formula for spin-orbit interactions, although out of the domain of its validity, could be used as a heuristic formula with a small number of free parameters to be determined by full numerical simulations. Quasi-circular orbits were compared to post-Newtonian ones over 9 orbits, and agreed, although to lower precision than that obtained by the Jena group. The moving punctures method was robust. Waveform comparisons between groups were now needed.

B. Kelly presented the work of the Goddard group. Hybrid waveforms agreed accurately with fully numerical ones. The code was fifth-order accurate. The effects of unequal masses and spins were being analysed. He described an analysis of the gravitational wave recoil in multipoles [13], with the force seen as a sum of mainly three products of pairs of multipoles, and stressed that the integral of momentum over time was not monotonic ("antikicks"). This model could also explain why spins in the orbital plane produced larger kicks. However, accretion would tend to align spins with orbital angular momentum. Finally, the effective one-body approach was a step forward in covering the parameter space with approximate wave forms.

M. Scheel [14] presented work by the Caltech group that uses a modified harmonic formulation. (In contrast to Pretorius, their formulation is reduced to first order in space and time, and consistent boundary conditions are applied at finite radius. In contrast to all other groups except the Meudon group they use spectral methods rather than finite differencing in space.) Currently their code experienced coordinate problems during merger, but spectral methods allowed for the currently most accurate inspiral simulations in full general relativity. Post-Newtonian based initial data (equal mass, no spin, zero eccentricity) evolved for 30 orbits until merger agree accurately for the first 15 orbits, and then significant disagreement could be shown, after taking into account the different gauges of what is being compared. It was found that quasi-circular initial data are actually slightly eccentric.

F. Pretorius [15] was interested in unusual threshold behaviour in binary black hole merger, rather than astrophysical wave forms. For non-spinning black holes of comparable mass, at the threshold of immediate merger, prompt merger was delayed and replaced by a whirling phase in which the number of orbits n depends on the impact parameter b as $\exp n \sim |b - b_*|^\gamma$. Although fully nonlinear, this behaviour was also shown by point particle geodesics in Kerr. During the whirl, 1-1.5% of the energy was radiated per orbit. He also offered an explanation of superkicks in terms of each black hole experiencing frame dragging by the other. He expected no more surprises in generic orbits, except perhaps in the limit of extreme rotation, or infinite boost.

D. Pollney [16] presented work of the AEI and LSU group on spin interactions. Distinctive features in the wave forms could be associated with the spins. The momentum flux could be estimated in particular by the terms $Q_{22}^+ Q_{3-3}^+$ and $Q_{2-2}^+ Q_{21}^\times$.

D. Shoemaker [17] presented a quantitative study by the PSU group of the influence of spurious radiation in the initial data on the merger, by artificially adding $l = m = 2$ Teukolsky waves inside the binary. This changed the ADM mass but not the angular momentum, with the merger and ringdown relatively unaffected.

I. Hinder from the PSU group focused on the comparison with post-Newtonian evolutions over 9 orbits to merger. Quasi-circular puncture initial data actually showed slight eccentricity. Disagreement with PN evolutions became more noticeable in the last 3 orbits. The gravitational waves matched the post-Newtonian ones well, although the eccentricity showed, as well as a dependence on resolution towards the end. The wave form still converged to 3rd to 4th order until the merger.

S. Husa [18] from the Jena group spoke on a variety of topics arising in evolutions. Higher order finite differencing was needed for accuracy and, surprisingly, seemed to work well with punctures, which are not smooth. 4th order Kreiss-Oliger dissipation was important, as well as using an advection stencil for the shift terms. Phase error converged to 6th order, but not amplitude error; this could be fixed by re-parameterising by phase. Very low eccentricity in the initial data (obtained by evolving the post-Newtonian equations over many orbits) was needed to see any disagreement between full numerical and 3.5PN evolutions.

J. Gonzalez, also at Jena, reported on unequal mass mergers, with a mass ratio of up to 10. Gauge problems arose which may be related to the damping term in the shift driver. The kick speeds agreed well with [19]. As the mass ratio increased, less energy was radiated in $l = 2$, and more in $l = 3, 4, 5$.

W. Tichy [20] reported on work at FAU on binaries with $J = 0.8M^2$, resulting in kicks of up to $2500 km/s$. With 10 levels of mesh refinement, evolutions with an outer boundary at $240M$ could be run on a 32Gb workstation. The largest error seemed to come from the extraction radius.

III. CONTRIBUTED TALKS ON OTHER TOPICS

A. Physics results

R. de Pietri [21] described simulations of instabilities in rapidly rotating stars with a Γ-law equation of state (initially polytropic, and no shocks form). The threshold of instability was well approximated by a Newtonian analysis. A bar ($m = 2$ dominated) appeared initially, but later resolved into an $m = 1$ or $m = 3$ dominated structure.

S. Liebling described the application of a parallel adaptive mesh refinement code in harmonic coordinates to different physics problems. Binary boson stars [22] could usefully be compared with binary black holes to see how the gravitational waves emitted depend on the internal structure. Here, the relative phase of the two stars provides an additional arbitrary parameter. Neutron star binaries had also been simulated [23]. The old problem of the critical collapse of Brill waves had been taken up again, although the critical amplitude had only been approached to within 1%. Subcritical scaling gave $\gamma \simeq 0.23$, $\Delta \simeq 0.75$, which is quite different from the original results [24] $\gamma \simeq 0.36$, $\Delta \simeq 0.60$. Magneto-hydrodynamics had been implemented [25].

U. Sperhake [26] reported simulations of head-on collisions of black holes which had different internal structure, namely Brill-Lindquist (BL) initial data (each black hole is a wormhole to a separate internal infinity), Misner data (both black holes connect to the same internal infinity) and approximate initial data obtained by superposing two Kerr-Schild (KS) slices through Schwarzschild (where the slices end at the future spacelike singularity). (His is the only BSSN code that can stably move excised black holes, which is necessary for evolving these, non-puncture initial data. Pretorius also uses moving excision, but in the harmonic formulation). KS had much more spurious initial radiation. BS and Misner agreed closely, but there was a small discrepancy between these and KS in the wave form which might be due to numerical error.

B. Kol reviewed a number of problems in higher dimensions, often string-inspired, which might usefully be investigated by numerical relativity. In particular he speculated on a possible relation between Choptuik's critical solution in the gravitational collapse of a scalar field in 4D, and the pinching off of a black string in $R^{d-1,1} \times S^1$ [27].

B. Gauge and boundaries

The talks by D. Brown [28] and D. Garfinkle [29] were closely related to the talk of S. Husa [30] given in parallel session A3: Why does the "moving punctures" approach to representing black holes work in numerical relativity? The initial data [7] represent a wormhole opening out to another universe in the Kruskal solution. However, evolving with the slicing condition now used by most binary black hole groups ("Bona-Massó 1+log"), numerical error allowed the solution inside the black hole to jump to a slicing of the Kruskal spacetime where the wormhole is asymptotically cylindrical and ends at the internal future null infinity, rather than at the internal spacelike infinity. The slicing could then become Killing both inside and outside the black hole. At the same time a "Γ-driver" type shift condition removed grid points from the interior, producing in effect excision by under-resolution.

There were two other talks on coordinate choices for black hole evolutions. D. Hilditch [31] discussed slicings of asymptotically flat spacetimes that are asymptotically null combined with radial compactification, such that the speed of outgoing waves remains constant. Numerical tests of the spherical wave equation on Schwarzschild re-

solved outgoing waves to arbitrarily large radii on a small number of grid points. L. Lindblom discussed generalisations of the "harmonic coordinate driver" coordinate conditions used by Pretorius.

J. Seiler presented an implementation by the AEI group of boundary conditions for the generalised harmonic formulation of the Einstein equations which are at once compatible with the constraint and result in a well-posed initial-boundary value problem [32]. These had been implemented using summation by parts techniques for finite differencing and the code was now being tested.

L. Brewin [33] reviewed his work on smooth lattice numerical relativity: in contrast to Regge calculus, spacetime is not locally flat, and the legs of the simplices are geodesics.

C.   Evolution codes

J. Thornburg review ongoing work on an adaptive mesh refinement code in double null coordinates for the linearised vacuum Einstein equations in harmonic gauge. The code has been tested with the scalar self-force on circular orbits in Schwarzschild [34]. It is to be applied to the calculation of the self-force on generic particle orbits in the Schwarzschild, and later Kerr spacetime.

B. Zink reviewed progress on the application of high order, multipatch, summation-by-parts finite differencing techniques [35] to the simulation of spacetimes with perfect fluid and magnetohydrodynamics matter. Initial applications are to a rotating black hole with an accretion disk. A particular strength of the multipatch approach was that it allowed the combination of high resolution in the radial direction with low resolution in the angular directions.

J. Novak [36] reviewed a fully constrained nonlinear 3D evolution code where only two variables are evolved: in a suitable linearised limit these represent the two polarisations of gravitational waves. The spatial gauge is Dirac gauge, defined with respect to a flat background metric. The code uses spherical coordinates and spectral methods. Boundary conditions are imposed at finite distance which are absorbing for quadrupolar waves on Minkowski. It was planned to evolve a single black hole with horizon boundary conditions, and to include matter.

D.   Initial data

J. Read [37] reported on numerical work in the helical Killing field approximation to the binary inspiral problem, which could be seen as an alternative to the waveless (conformally flat) approximation. A scalar field toy model and a neutron star binary had been implemented. The equations are mixed hyperbolic-elliptic. Convergence at present required the outer boundary to be very close in, but this might be improved by an outer patch where $\partial_t h_{ij} = 0$ is imposed instead.

B. Kelly [38] presented initial data of the puncture type in full general relativity but based on a snapshot of a post-Newtonian binary orbit, and with approximately correct gravitational wave content also based on that orbit. The constraints are currently not solved, but are violated only at $O[(v/c)^5]$.

E.   Analysis

J. L. Jaramillo [39] described the application of a refined Penrose inequality in axisymmetry, namely $A \leq 8\pi(M^2 + \sqrt{M^4 + J^2})$, as a geometric test for a new apparent horizon finder [40], and for the outermost character of marginal trapped surfaces. Numerical evidence of Dain's proposal for characterizing Kerr data by the saturation of such inequality was also presented.

G. Cook [41] presented methods for finding an approximate rotational Killing vector in numerical evolutions, which could then be used in a Komar-type integral to compute the spin of a black hole. These are constructed by minimising an appropriated measure of the residue in the Killing equation, resulting in a system of coupled elliptic equations. This was being tested, and might provide a new tool for analysing the horizon geometry.

N. Bishop presented progress on the extraction of gravitational waves at scri by matching to an outgoing null grid. Problems with numerical noise had been overcome, and the extraction had been tested in the head-on collision of two black holes in a harmonic code with constraint-preserving boundary conditions. Full Cauchy-characteristic matching now seemed feasible.

A. Norton [42] presented animations of a spatial coordinate system that rotates with a rotating star in a central region but is fixed at infinity, without getting increasingly entangled: it is based on the double cover of $SO(3)$ by S^3, and so returns to its original state after two rotations.

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