

# Thermodynamical description of the interaction between holographic dark energy and dark matter

Bin Wang<sup>1\*</sup>, Chi-Yong Lin<sup>2†</sup>, Diego Pavón<sup>3‡</sup>, Elcio Abdalla<sup>4§</sup>

<sup>1</sup> *Department of Physics, Fudan University, 200433 Shanghai*

<sup>2</sup> *Department of Physics, National Dong Hwa University, Shoufeng, 974 Hualien*

<sup>3</sup> *Department of Physics, Autonomous University  
of Barcelona, 08193 Bellaterra, Barcelona and*

<sup>4</sup> *Instituto de Física, Universidade de São Paulo, CP 66318, 05315-970, São Paulo*

## Abstract

We present a thermodynamical description of the interaction between holographic dark energy and dark matter. If holographic dark energy and dark matter evolve separately, each of them remains in thermodynamic equilibrium. A small interaction between them may be viewed as a stable thermal fluctuation that brings a logarithmic correction to the equilibrium entropy. From this correction we obtain a physical expression for the interaction which is consistent with phenomenological descriptions and passes reasonably well the observational tests.

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\* E-mail address: wangb@fudan.edu.cn

† E-mail address: lcyong@mail.ndhu.edu.tw

‡ E-mail address: diego.pavon@uab.es

§ E-mail address: eabdalla@fma.if.usp.br

A variety of cosmological observations strongly suggest that our Universe is currently undergoing a phase of accelerated expansion [1, 2, 3], likely driven by some exotic component called dark energy (DE) whose main feature is to possess a high negative pressure -see however [4]. Nevertheless, despite the mounting observational evidence, the nature and origin of dark energy remains elusive and it has become a source of vivid debate -see [5] and references therein.

Most discussions on DE rely on the assumption that it evolves independently of other matter fields. One might argue that given the unknown nature of both DE and dark matter (DM), an entirely independent behavior of DE and DM is very special whereby it is not unnatural to suppose that they interact. Studies on the interaction (coupling) between DE and DM have been carried out in [6]-[11] and, in particular, it has been shown that the coupling can alleviate the coincidence problem [6, 11]. Furthermore, it was argued that the appropriate interaction between DE and DM can influence the perturbation dynamics and lowest multi-poles of the cosmic microwave background (CMB) spectrum and account for the observed CMB low  $l$  suppression [9, 12]. The strength of the coupling could be as large as the fine structure constant [9, 13]. Recently it was shown that such an interaction could be inferred from the expansion history of the Universe, as manifested in the supernova data together with CMB and large-scale structure [14].

In contrast to minimally coupled DE models, the coupling between DE and DM not only influences the Universe expansion history but also modifies the structure formation scenario through the coupling to cold DM density fluctuations [7, 15]. Indeed, the growth of DM perturbations can be enhanced due to the coupling [9, 10], which can be used to explain the age ( $\sim 2.1$  Gyr) of the old quasar APM0879+5255 observed at redshift  $z = 3.91$  [9]. Further, lately it was suggested that the dynamical equilibrium of collapsed structures would be affected by the coupling of dark energy to dark matter in a way that could be detected in the galaxy cluster Abell A586 [16]. Through the internal dynamics of galaxy clusters and using reliable x-ray, weak lensing and optical data from 33 galaxy clusters, a much tighter limit on the strength of the coupling between DE and DM has been established [17]. Indeed, it was shown that the coupling is small but positive which indicates that DE may decay into DM. Nevertheless, albeit the interaction hypothesis is gaining ground the observational limits on the strength of the coupling remain weak [18].

The interaction between DE and DM could be a major issue to be confronted in studying

the physics of DE. However, so long as the nature of these two components remain unknown it will not be possible to derive the precise form of the interaction from first principles. Therefore, one has to assume a specific coupling from the outset [10, 11] or determine it from phenomenological requirements [6]. Nevertheless, attempts to provide a Lagrangian description of the interaction have been put forward. These comprise proposals that include the dependence of the matter field on the scalar field [19] or express the cosmological constant as a function of the trace of the energy-momentum tensor [20]; at any rate, the exact form of the dependence stays unspecified.

The main purpose of this Letter is to try to understand such a coupling from thermodynamical considerations. The thermodynamics of black hole physics [21] and de Sitter space [22] are well established. Recently, extensive analysis found that the current data favors DE models with EoS very close to  $w_D = -1$ . This suggests that the present evolution of the universe is practically quasi-de Sitter. Therefore, one may assume that some thermodynamical approach will also apply to eternally accelerating quasi-de Sitter universes [23].

We shall assume that in the absence of a mutual interaction both DE and DM remain in their respective thermodynamic equilibrium states, and that a small coupling between DE and DM may be viewed as small stable fluctuations around equilibrium. (We say “small” because a large, or even moderate, coupling would substantially deviate the model from the  $\Lambda$ CDM concordance model and would be incompatible with observation [7]). Some years ago, Das *et al.* [24] showed that logarithmic corrections to the equilibrium thermodynamic entropy arise in all thermodynamic systems when stable fluctuations around equilibrium are taken into account and that, in particular, it leads to logarithmic corrections to the Bekenstein-Hawking formula for black hole entropy. This idea was later applied to obtain an evolution law for the cosmological constant [25]. We shall present a thermodynamic description of the interaction between DE and DM by building a relation between the logarithmic entropy correction and the interaction. Thus this derivation possesses a solid physical foundation. Next, we will argue that our thermodynamical interpretation of the interaction is consistent with phenomenological approaches and meet observational constraints.

We shall focus on the DE model inspired by the holographic idea that the energy within our horizon cannot exceed the mass of a black hole of the same size [26, 27, 28]. The extension of the holographic principle to a general cosmological setting was first addressed by Fischler and Susskind [29] and subsequently got modified by many authors [30]-[34]. The

idea of holography is viewed as a real conceptual change in our thinking about gravity [35]. There have been a lot of attempts on applying holography in the study of cosmology. It is interesting to note that holography implies a possible value of the cosmological constant in a large class of universes [36]. In an inhomogeneous cosmology holography was also realized as a useful tool to select physically acceptable models [33]. The idea of holography has further been applied to the study of inflation and gives possible upper limits to the number of e-folds [37]. Recently, holography has again been proved as an effective way to investigate dark energy [27, 38]. Thus holography seems a useful tool to investigate cosmology.

Following Li [27], we assume that the holographic dark energy density is given by  $\rho_D = 3c^2/L^2$ , where  $c^2$  is a constant of order unity and  $L$  is an appropriate length scale which we identify with the radius of the future event horizon,

$$R_E = a \int_a^\infty \frac{dx}{H x^2}. \quad (1)$$

Here,  $H \equiv \dot{a}/a$  is the Hubble function and  $a$  the scale factor of the Robertson-Walker metric.

The total energy density is  $\rho = \rho_m + \rho_D$ , where  $\rho_m$  is the matter energy density and  $\rho_D = 3c^2/(8\pi R_E^2)$  is the holographic DE energy density -we neglect radiation and non-dark matter. If holographic DE and DM do not interact, their energy densities satisfy separate conservation laws

$$\dot{\rho}_m + 3H\rho_m = 0, \quad (2)$$

$$\dot{\rho}_D + 3H(1 + w_D^0)\rho_D = 0, \quad (3)$$

where  $w_D^0$  is the EoS of the holographic DE when it evolves independently of DM. Introducing the dimensionless density parameter for DE,  $\Omega_D = 8\pi\rho_D/(3H^2)$ , the event horizon radius can be written as [27]  $R_E = c/(\sqrt{\Omega_D}H)$ . Taking the derivative with respect to  $\ln a$  of last expression and resorting to Eq. (1) we get

$$\frac{H'}{H} = \frac{\sqrt{\Omega_D}}{c} - 1 - \frac{\Omega'_D}{2\Omega_D}. \quad (4)$$

Using Friedmann's equation,  $\Omega_D + \Omega_m = 1$  and (2)-(4), valid for spatially-flat homogeneous isotropic cosmologies, we obtain for the EoS of the holographic dark energy component the expression,  $w_D^0 = -\frac{1}{3} - \frac{2\sqrt{\Omega_D}}{3c}$ . Likewise, it follows that the holographic DE

evolution is governed by [8, 27]

$$\Omega'_D = \Omega_D^2(1 - \Omega_D) \left[ \frac{1}{\Omega_D} + \frac{2}{c\sqrt{\Omega_D}} \right]. \quad (5)$$

Equipped with these relationships, we can determine how much the event horizon changes in one Hubble time,

$$t_H \frac{\dot{R}_E}{R_E} = 1 - \frac{\sqrt{\Omega_D}}{c}, \quad (6)$$

where  $t_H \equiv H^{-1}$ . Provided that  $c = \mathcal{O}(1)$  -as should be expected, see [27]- the event horizon will not change significantly over one Hubble scale whereby the thermodynamical description near equilibrium seems a reasonable approach. It also follows from Eq. (6) that  $w_D^0 > -1 (< -1)$  if  $\dot{R}_E > 0 (< 0)$ , respectively.

The equilibrium entropy of the holographic DE component is related to its energy and pressure by Gibbs' equation [39]

$$TdS_D = dE_D + P_D dV. \quad (7)$$

Considering  $V = 4\pi R_E^3/3$ ,  $E_D = \rho_D V = c^2 R_E/2$  and using the event horizon temperature  $T = 1/(2\pi R_E)$ , we get

$$dS_{D0} = \pi c^2 (1 + 3w_D^0) R_E^0 dR_E^0 = -2\pi c \sqrt{\Omega_D^0} R_E^0 dR_E^0, \quad (8)$$

for the holographic DE entropy when it is not coupled to DM. (A zero superscript or subscript indicates absence of interaction).

However, when holographic DE and DM interact with each other, they cannot remain in their respective equilibrium states. The effect of the interaction may be assimilated to small stable fluctuations around thermal equilibrium. It was shown that due to the fluctuation, there is a leading logarithmic correction,  $S_1 = -\frac{1}{2} \ln(CT^2)$  -with  $C$  the heat capacity-, to the thermodynamic entropy around equilibrium in all thermodynamical systems [24]. In our case, the heat capacity of the DE can be calculated as  $C = T(\partial S_{D0}/\partial T) = -\pi c^2(1 + 3w_D^0)(R_E^0)^2$ , which is positive since for holographic DE one has  $1 + 3w_D^0 < 0$ . Accordingly, the fluctuation is indeed stable and the entropy correction

reads

$$S_{D1} = -\frac{1}{2} \ln \left[ -\frac{c^2}{4\pi} (1 + 3w_D^0) \right] = -\frac{1}{2} \ln \left[ \frac{c}{2\pi} \sqrt{\Omega_D^0} \right]. \quad (9)$$

As mentioned above, we assume that this entropy correction is linked to the DE-DM coupling. Thus, the total entropy of holographic DE enclosed by the event horizon is  $S_D = S_{D0} + S_{D1}$  and from Gibbs' equation we get

$$1 + 3w_D = \frac{1}{c^2 \pi R_E} \frac{dS}{dR_E} = \frac{1}{c^2 \pi R_E} \frac{dS_{D1}}{dR_E} - \frac{2\sqrt{\Omega_D^0}}{c} \frac{R_E^0}{R_E} \frac{dR_E^0}{dR_E}, \quad (10)$$

where  $w_D$  denotes the EoS of holographic DE when it is coupled to DM. If the interaction were turned off, the DE would return to its equilibrium state and we would have that  $w_D \rightarrow w_D^0$  and  $R_E \rightarrow R_E^0$ .

When an interaction between holographic DE and DM exists, their energy densities no longer satisfy independent conservation laws. They obey instead

$$\dot{\rho}_m + 3H\rho_m = Q, \quad (11)$$

$$\dot{\rho}_D + 3H(1 + w_D)\rho_D = -Q, \quad (12)$$

$Q$  denotes the interaction term which is expected to be derived from the entropy correction[44].

We first rewrite last two equations as

$$\Omega'_m + \frac{2H'}{H}\Omega_m + 3\Omega_m = \frac{8\pi Q}{3H^3}, \quad (13)$$

$$\Omega'_D + \frac{2H'}{H}\Omega_D + 3(1 + w_D)\Omega_D = -\frac{8\pi Q}{3H^3}, \quad (14)$$

and insert (4) into (14) to get

$$1 + 3w_D = -\frac{2\sqrt{\Omega_D}}{c} - \frac{8\pi Q}{3H^3\Omega_D}. \quad (15)$$

Then, comparing last expression with Eq. (10), we obtain

$$\frac{8\pi Q}{9H^3} = \frac{\Omega_D}{3} \left[ -\frac{2\sqrt{\Omega_D}}{c} + \frac{2\sqrt{\Omega_D^0}}{c} \frac{R_E^0}{R_E} \frac{dR_E^0}{dR_E} \right] - \frac{1}{\pi c^2 R_E} \frac{\Omega_D}{3} \frac{dS_1}{dR_E} \quad (16)$$

for the interaction term,  $Q$ .

From (9) the evolution of  $S_{D1}$  appearing in the last equation can be written as

$$\frac{dS_1}{dR_E} = -\frac{H}{(c/\sqrt{\Omega_D}) - 1} \frac{(\Omega_D^0)'}{4\Omega_D^0}, \quad (17)$$

where we have made use of  $R_E = c/(H\sqrt{\Omega_D})$ .

We, thus, have built a relation between the DE-DM coupling and the correction,  $S_{D1}$ , to the equilibrium entropy.

To see how the above expression for  $Q$  (equations (16) and (17)) fares when contrasted with observation let us compare it with the interaction term [6]

$$Q = 3b^2 H (\rho_m + \rho_D), \quad (18)$$

where  $b^2$  is a coupling constant, introduced on phenomenological grounds to alleviate the coincidence problem [40]. However, before doing that let us provide a rationale for (18).

The right hand side of (11) and (12), i.e.,  $Q$  and  $-Q$ , must be functions of the energy densities multiplied by a quantity with units of inverse of time. For the latter the obvious choice is the Hubble factor  $H$ , so we have that  $Q = Q(H\rho_m, H\rho_D)$ . By power law expanding  $Q$  and retaining just the first term we get  $Q \simeq \lambda_m H\rho_m + \lambda_D H\rho_D$ . To facilitate comparison of the resulting model with observation it is expedient to eliminate one of the two  $\lambda$  parameters. Thus we set  $\lambda_m = \lambda_D = 3b^2$  and arrive to Eq. (18). The simpler choice,  $\lambda_m = 0$  would not yield a constant dark matter to dark energy ratio at late times. Clearly, the term  $3b^2$  measures to what extent the decay rate of DE into DM differs from the expansion rate of the Universe and also gauges the intensity of the coupling. The lower  $b^2$ , the closer the evolution of the Universe to a non-interacting model is. It should be emphasized that this phenomenological description has proven viable when contrasted with observations, i.e., SNIa, CMB, large scale structure,  $H(z)$ , and age constraints [8, 9, 14, 18], and recently in galaxy clusters [16, 17].

So, to carry out the said comparison we set  $b^2 = \frac{8\pi Q}{9H^3}$ . Accordingly,  $b^2$  is no longer a constant but a variable parameter that evolves according to

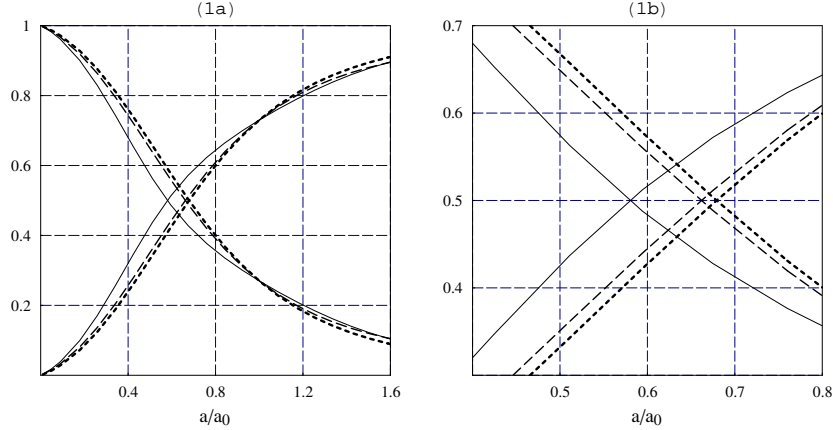


Figure 1: Evolutions of  $\Omega_D$  and  $\Omega_m$  with and without interaction. Lines showing values increasing with  $a$  is  $\Omega_D$ , and the decreasing lines are for  $\Omega_m$ . The solid, dotted, and dashed lines correspond to our scenario, the holographic model without interaction, and the phenomenological interacting model with  $b^2 = 0.06$ , respectively.

$$b^2 = \frac{8\pi Q}{9H^3} = \frac{2\Omega_D^{3/2}}{3c} \left[ -1 + \frac{H^2 \sqrt{\Omega_D}}{(H^0)^2 \sqrt{\Omega_D^0}} \frac{\sqrt{\Omega_D^0/c - 1}}{\sqrt{\Omega_D/c - 1}} \right] + \frac{1}{12\pi c^2} \frac{H^2}{c/\sqrt{\Omega_D}(c/\sqrt{\Omega_D} - 1)} \frac{\Omega_D}{\Omega_D^0} (\Omega_D^0)'. \quad (19)$$

Using Friedmann's equation as well as (4), equation (14) can be recast as

$$\frac{\Omega_D'}{\Omega_D} + (\Omega_D - 1) + \frac{2\sqrt{\Omega_D}}{c} (\Omega_D - 1) = -\frac{8Q}{3H^3} = -3b^2. \quad (20)$$

With the help of Eqs. (19), (20) and (4), we are in position to discuss the dependence of the evolution of holographic DE in terms of the coupling to DM. In the numerical calculations, we set  $c = 1$ . From Fig.1 we learn that because of the interaction between holographic DE and DM,  $\Omega_D$  increases faster, and from Figs.2a and 2b that  $\rho_D$  and  $\rho_m$  follow each other and that the instant at which  $\rho_D = \rho_m$  occurs earlier than in the non-interacting case. The latter feature is more clearly appreciated in Fig. 2c where the dependence of the ratio  $r \equiv \rho_m/\rho_D$  with the scale factor is depicted. The said ratio decreases monotonously with expansion and it varies very slowly at the present era. Compared with the noninteracting case, we find that currently  $r$  decreases slower when there is interaction. This means, on the one hand, that the coincidence problem gets substantially alleviated and, on the other



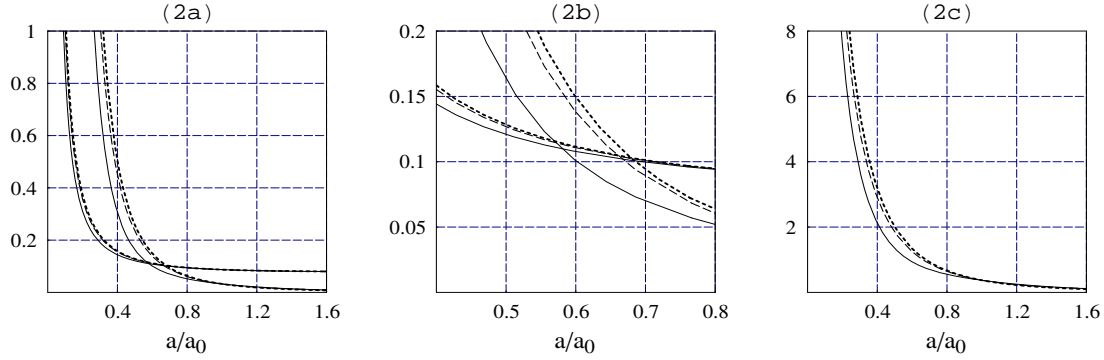


Figure 2: Evolutions of  $\rho_D$  and  $\rho_m$  with and without interaction. Before the crossing point, lines on the left are for  $\rho_D$ , other bunch of lines are for  $\rho_m$ . The solid, dotted, and dashed lines correspond to our scenario, the holographic model without interaction, and the phenomenological interacting model with  $b^2 = 0.06$ , respectively.

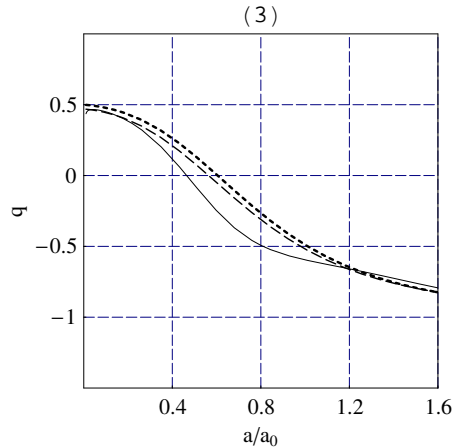


Figure 3: Dependence of the deceleration parameter,  $q = -\ddot{a}/(aH^2)$ , on the interaction. The solid, dotted, and dashed lines correspond to our scenario, the holographic model without interaction, and the phenomenological interacting model with  $b^2 = 0.06$ , respectively.

hand, that in the recent history of the Universe DE is decaying into DM. This is consistent with phenomenological interacting models [6], [9]. The different evolution of the DM due to its interaction with the DE gives rise to a different expansion history of the Universe and a different evolution of the matter density perturbations which alters the standard structure formation scenario as the latter assumes  $\rho_m \propto a^{-3}$ . In [7, 9] the matter density perturbations

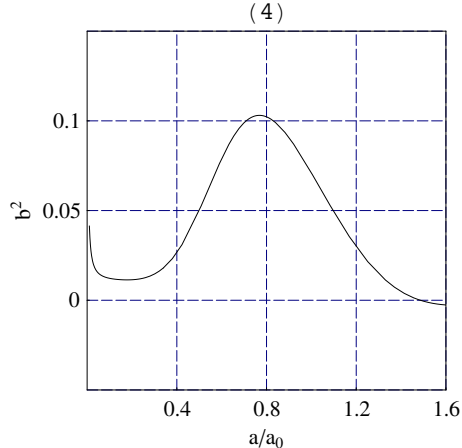


Figure 4: The corresponding coupling  $b^2$  in our scenario by comparing it with the simple phenomenological model.

in interacting models were investigated and in [9] the impact of the interaction on the DM perturbations was used to explain why it is possible, as recently observed [41], for the old quasar APM0879+5255 to exist already at the early stages of the Universe (at  $z = 3.91$ ). As a comparison, in Figs. 1 and 2 we have also included the phenomenological interaction case with constant coupling,  $b^2$  (dashed line). It is seen that the results obtained for the evolution of holographic DE and DM using the phenomenological model and using the interaction derived from the thermodynamical consideration are consistent with each other.

Clearly, the interaction must affect the acceleration history of the Universe. Figure 3 depicts the dependence of the deceleration parameter,  $q = -\ddot{a}/(aH^2)$ , on the coupling. It is seen that the interaction shifts the beginning of the acceleration to earlier times; a result previously obtained by several authors [8], [6], [10], [42].

Now we test this scenario for the interaction between holographic DE and DM by using some observational results. For the comparison with the phenomenological interacting model, in our scenario the coupling between holographic DE and DM can be expressed as a counterpart of  $b$  as in the phenomenological interaction form. Now the coupling is no longer a constant but a time-dependent parameter. Its evolution is depicted in Fig. 4. During an ample period, the effective coupling,  $b$ , remains small and positive, indicating that holographic DE could be decaying into DM. In fact,  $b^2$  lies within the region of the golden supernova data fitting result  $b^2 = 0.00^{+0.11}_{-0.00}$  [8] and the observed CMB low  $l$  data constraint

[9]. In Ref. [9] it was investigated whether this model satisfies the current Universe age constraints and allows a considerably older universe at high redshift to be compatible with the existence of some old objects such as the old quasar APM0879+5255 at redshift  $z = 3.91$  [41]. Its age, at that redshift, was estimated as  $t_g = 2.1 Gyr$ . Using the present WMAP data on the Hubble parameter,  $H_0 = 73.4_{-3.8}^{+2.8}$  [3], the dimensionless age of the quasar  $T_g = H_0 t_g$  is seen to lie in the interval  $0.148 \leq T_g \leq 0.162$ . In our scenario, it is easy to realize that the age of the Universe at  $z = 3.91$  was  $T_z = \int_{3.91}^{\infty} (1+z)^{-1} H^{-1} dz = 0.152$ . This is to say, at that redshift the Universe was old enough to accommodate the existence of this old quasar. These results show that our interacting DE scenario is compatible with observations.

In summary, from thermodynamical considerations we derived an expression for the interaction between holographic DE and DM. We assumed that in the absence of a DE-DM coupling these two components remain in separate thermal equilibrium and that the presence of a small coupling between them can be described as stable fluctuations around equilibrium. Then, resorting to the logarithmic correction to the equilibrium entropy [24] we arrived to an expression for the interaction term, namely, Eq. (16) together with (17). By comparing it with phenomenological proposals, Eq.(18), we concluded that this scenario is compatible with the golden SN Ia data, small  $l$  CMB data and age constraints at different redshifts. The study here is limited to the particular case of the holographic model. Our argument may well not apply to the Chaplygin gas model and its generalizations [43], since the admixture and interaction of the DE and DM in these models does not imply any sort of entropy. However, it would be interesting to generalize our work to models where DE and DM are not intrinsically mixed.

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