Erratum: "Tidal Love numbers of neutron stars" (2008, ApJ, 677, 1216)

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There are typographical errors in Eqs. (20) and (23), and some incorrect entries in Table (1). I thank Ryan Lang for pointing these out.

Equation (20) should read as follows:

$$H = c_1 \left(\frac{r}{M}\right)^2 \left(1 - \frac{2M}{r}\right) \left[-\frac{M(M-r)(2M^2 + 6Mr - 3r^2)}{r^2(2M-r)^2} + \frac{3}{2} \log\left(\frac{r}{r - 2M}\right) \right] + 3c_2 \left(\frac{r}{M}\right)^2 \left(1 - \frac{2M}{r}\right).$$

Equation (23) should be replaced by the following:

$$k_2 = \frac{8C^5}{5} (1 - 2C)^2 [2 + 2C (y - 1) - y] \times \left\{ 2C (6 - 3y + 3C(5y - 8)) + 4C^3 [13 - 11y + C(3y - 2) + 2C^2(1 + y)] + 3(1 - 2C)^2 [2 - y + 2C(y - 1)] \log (1 - 2C) \right\}^{-1},$$

The corrected values for the Love numbers in Table (1) are given in the table below.

Table 1. Relativistic Love numbers k_2

n	M/R	k_2	
0.3	10^{-5}	0.5511	
0.3	0.1	0.294	
0.3	0.15	0.221	
0.3	0.2	0.119	
0.5	10^{-5}	0.4491	
0.5	0.1	0.251	
0.5	0.15	0.173	
0.5	0.2	0.095	
0.5	0.25	0.0569	
0.7	10^{-5}	0.3626	
0.7	0.1	0.1779	
0.7	0.15	0.1171	
0.7	0.2	0.0721	
0.7	0.25	0.042	
1.0	10^{-5}	0.2599	
1.0	0.1	0.122	
1.0	0.15	0.0776	
1.0	0.2	0.0459	
1.0	0.2	0.0253	
1.2	10^{-5}	0.2062	
1.2	0.1	0.0931	
1.2	0.15	0.0577	
1.2	0.2	0.0327	

1. Tidal Love numbers of neutron stars

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For a variety of fully relativistic polytropic neutron star models we calculate the star's tidal Love number k_2 . Most realistic equations of state for neutron stars can be approximated as a polytrope with an effective index $n \approx 0.5-1.0$. The equilibrium stellar model is obtained by numerical integration of the Tolman-Oppenheimer-Volkhov equations. We calculate the linear l=2 static perturbations to the Schwarzschild spacetime following the method of Thorne and Campolattaro. Combining the perturbed Einstein equations into a single second order differential equation for the perturbation to the metric coefficient g_{tt} , and matching the exterior solution to the asymptotic expansion of the metric in the star's local asymptotic rest frame gives the Love number. Our results agree well with the Newtonian results in the weak field limit. The fully relativistic values differ from the Newtonian values by up to $\sim 24\%$. The Love number is potentially measurable in gravitational wave signals from inspiralling binary neutron stars.

2. Introduction and Motivation

A key challenge of current astrophysical research is to obtain information about the equation of state (EoS) of the ultra-dense nuclear matter making up neutron stars (NSs). The observational constraints on the internal structure of NSs are weak: the observed range of NS masses is $M \sim 1.1-2.2 M_{\odot}$ (Lattimer & Prakash 2007), and there is no current method to directly measure the radius. Some estimates using data from X-ray spectroscopy exist, but those are highly model-dependent (e. g. Webb & Barret (2007)). Different theoretical models for the NS internal structure predict, for a neutron star of mass $M \sim 1.4 M_{\odot}$, a central density in the range of $\rho_c \sim 2-8 \times 10^{14} {\rm g cm^{-3}}$ and a radius in the range of $R \sim 7-16 {\rm km}$ (Lattimer & Prakash 2007). Potential observations of pulsars rotating at frequencies above 1400Hz could be used to constrain the EoS if the pulsar's mass could also be measured (e. g. Zdunik et al. (2007)).

Direct and model-independent constraints on the EoS of NSs could be obtained from gravitational wave observations. Coalescing binary neutron stars are one of the most important sources for ground-based gravitational wave detectors (Cutler & Thorne 1993). LIGO observations have established upper limits on the coalescence rate per comoving volume (Abbott et al. 2007), and at design sensitivity LIGO II is expected to detect inspirals at a

rate of $\sim 2/\text{day}$ (Kalogera et al. 2004).

In the early, low frequency part of the inspiral ($f \leq 100$ Hz, where f is the gravitational wave frequency), the waveform's phase evolution is dominated by the point-mass dynamics and finite-size effects are only a small correction. Toward the end of the inspiral the internal degrees of freedom of the bodies start to appreciably influence the signal, and there have been many investigations of how well the EoS can be constrained using the last several orbits and merger, including constraints from the gravitational wave energy spectrum (Faber et al. 2002) and from the NS tidal disruption signal for NS-black hole binaries (Vallisneri 2002). Several numerical simulations of the hydrodynamics of NS-NS mergers have studied the dependence of the gravitational wave spectrum on the radius and EoS (see, e.g. Baumgarte & Shapiro (2003) and references therein). However, trying to extract EoS information from this late time regime presents several difficulties: (i) the highly complex behavior requires solving the full nonlinear equations of general relativity together with relativistic hydrodynamics; (ii) the signal depends on unknown quantities such as the spins and angular momentum distribution inside the stars, and (iii) the signals from the hydrodynamic merger are outside of LIGO's most sensitive band.

During the early regime of the inspiral the signal is very clean and the influence of tidal effects is only a small correction to the waveform's phase. However, signal detection is based on matched filtering, i. e. integrating the measured waveform against theoretical templates, where the requirement on the templates is that the phasing remain accurate to ~ 1 cycle over the inspiral. If the accumulated phase shift due to the tidal corrections becomes of order unity or larger, it could corrupt the detection of NS-NS signals or alternatively, detecting a phase perturbation could give information about the NS structure. This has motivated several analytical and numerical investigations of tidal effects in NS binaries (Bildsten & Cutler 1992; Kokkotas & Schafer 1995; Kochanek 1992; Taniguchi & Shibata 1998; Mora & Will 2004; Shibata 1994; Gualteri et al. 2001; Pons et al. 2002; Berti et al. 2002). The influence of the internal structure on the gravitational wave phase in this early regime of the inspiral is characterized by a single parameter, namely the ratio λ of the induced quadrupole to the perturbing tidal field. This ratio λ is related to the star's tidal Love number k_2 by $k_2 = 3G\lambda R^{-5}/2$, where R is the star's radius. Flanagan & Hinderer (2007) have shown that for an inspiral of two non-spinning $1.4M_{\odot}$ NSs at a distance of 50 Mpc, LIGO II detectors will be able to constrain λ to $\lambda \leq 2.01 \times 10^{37} \mathrm{g \, cm^2 s^2}$ with 90% confidence. This number is an upper limit on λ in the case that no tidal phase shift is observed. The corresponding constraint on radius would be $R \leq 13.6 \,\mathrm{km}$ (15.3 km) for a n = 0.5 (n = 1.0) fully relativistic polytrope, for $1.4M_{\odot}$ NSs (Flanagan & Hinderer 2007).

Because neutron stars are compact objects with strong internal gravity, their Love num-

bers could be very different from those for Newtonian stars that have been computed previously, e. g. by Brooker & Olle (1955).

Knowledge of Love number values could also be useful for comparing different numerical simulations of NS binary inspiral by focusing on models with the same masses and values of λ .

In Flanagan & Hinderer (2007), the l=2 tidal Love numbers for fully relativistic neutron star models with polytropic pressure-density relation $P=K\rho^{1+1/n}$, where K and n are constants, were computed for the first time. The present paper will give details of this computation. Using polytropes allows us to explore a wide range of stellar models, since most realistic models can be reasonably approximated as a polytrope with an effective index in the range $n \sim 0.5-1.0$ Lattimer & Prakash (2007). Our prescription for computing λ is valid for an arbitrary pressure-density relation and not restricted to polytropes. In Sec. 3, we start by defining λ in the fully relativistic context in terms of coefficients in an asymptotic expansion of the metric in the star's local asymptotic rest frame and discuss the extent to which it is uniquely defined. In Sec. 4, we discuss our method of calculating λ , which is based on static linearized perturbations of the equilibrium configuration in the Regge-Wheeler gauge as in Thorne & Campolattaro (1967). Section 5 contains the results of the numerical computations together with a discussion. Unless otherwise specified, we use units in which c=G=1.

3. Definition of the Love number

Consider a static, spherically symmetric star of mass M placed in a static external quadrupolar tidal field \mathcal{E}_{ij} . The star will develop in response a quadrupole moment Q_{ij}^{-1} . In the star's local asymptotic rest frame (asymptotically mass centered Cartesian coordinates) at large r the metric coefficient g_{tt} is given by (Thorne 1998):

$$\frac{(1 - g_{tt})}{2} = -\frac{M}{r} - \frac{3Q_{ij}}{2r^3} \left(n^i n^j - \frac{1}{3} \delta^{ij} \right) + O\left(\frac{1}{r^3}\right) + \frac{1}{2} \mathcal{E}_{ij} x^i x^j + O\left(r^3\right), \tag{1}$$

¹ The induced quadrupolar deformation of the star can be described in terms of the star's l=2 mode eigenfunctions of oscillation.

where $n^i = x^i/r$; this expansion defines \mathcal{E}_{ij} and Q_{ij} . In the Newtonian limit, Q_{ij} is related to the density perturbation $\delta \rho$ by

$$Q_{ij} = \int d^3x \delta \rho(\mathbf{x}) \left(x_i x_j - \frac{1}{3} r^2 \delta_{ij} \right), \tag{2}$$

and \mathcal{E}_{ij} is given in terms of the external gravitational potential $\Phi_{\rm ext}$ as

$$\mathcal{E}_{ij} = \frac{\partial^2 \Phi_{\text{ext}}}{\partial x^i \partial x^j}.$$
 (3)

We are interested in applications to fully relativistic stars, which requires going beyond Newtonian physics. In the strong field case, Eqs. (2) and (3) are no longer valid but the expansion of the metric (1) still holds in the asymptotically flat region and serves to define the moments Q_{ij} and \mathcal{E}_{ij} .

We briefly review here the extent to which these moments are uniquely defined since there are considerable coordinate ambiguities in performing asymptotic expansions of the metric. For an isolated body in a static situation these moments are uniquely defined: \mathcal{E}_{ij} and Q_{ij} are the coordinate independent moments defined by Geroch (1970) and Hansen (1974) for stationary, asymptotically flat spacetimes in terms of certain combinations of the derivatives of the norm and twist of the timelike Killing vector at spatial infinity. In the case of an isolated object in a dynamical situation, there are ambiguities related to gravitational radiation, for example angular momentum is not uniquely defined (Wald 1984). For the application to the adiabatic part of a NS binary inspiral, we are interested in the case of a non-isolated object in a quasi-static situation. In this case there are still ambiguities (related to the choice of coordinates) but their magnitudes can be estimated (Thorne & Hartle 1985) and are at a high post-Newtonian order and therefore can be neglected. We are also interested in (i) working to linear order in \mathcal{E}_{ij} and (ii) in the limit where the source of \mathcal{E}_{ij} is very far away. In this limit the ambiguities disappear.

To linear order in \mathcal{E}_{ij} , the induced quadrupole will be of the form

$$Q_{ij} = -\lambda \mathcal{E}_{ij}. (4)$$

Here λ is a constant which is related to the l=2 tidal Love number (apsidal constant) k_2 by (Flanagan & Hinderer 2007)

$$k_2 = \frac{3}{2}G\lambda R^{-5}. (5)$$

²The l=2 tidal moment can be related to a component of the Riemann tensor $R_{\alpha\beta\gamma\delta}$ of the external pieces of the metric in Fermi normal coordinates at r=0 as $\mathcal{E}_{ij}=R_{0i0j}$ (see Misner et al. (1973)).

Note the difference in terminology: in Flanagan & Hinderer (2007), λ was called the Love number, whereas in this paper, we reserve that name for the dimensionless quantity k_2 .

The tensor multipole moments Q_{ij} and \mathcal{E}_{ij} can be decomposed as

$$\mathcal{E}_{ij} = \sum_{m=-2}^{2} \mathcal{E}_m \mathcal{Y}_{ij}^{2m},\tag{6}$$

and

$$Q_{ij} = \sum_{m=-2}^{2} Q_m \mathcal{Y}_{ij}^{2m},\tag{7}$$

where the symmetric traceless tensors \mathcal{Y}_{ij}^{2m} are defined by (Thorne 1980)

$$Y_{2m}(\theta,\varphi) = \mathcal{Y}_{ij}^{2m} n^i n^j \tag{8}$$

with $\mathbf{n} = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta)$. Thus, the relation (4) can be written as

$$Q_m = -\lambda \mathcal{E}_m. \tag{9}$$

Without loss of generality, we can assume that only one \mathcal{E}_m is nonvanishing, this is sufficient to compute λ .

4. Calculation of the Love number

4.1. Equilibrium configuration

The geometry of spacetime of a spherical, static star can be described by the line element (Misner et al. 1973)

$$ds_0^2 = g_{\alpha\beta}^{(0)} dx^{\alpha} dx^{\beta} = -e^{\nu(r)} dt^2 + e^{\lambda(r)} dr^2 + r^2 \left(d\theta^2 + \sin^2 \theta d\phi^2 \right). \tag{10}$$

The star's stress-energy tensor is given by

$$T_{\alpha\beta} = (\rho + p) u_{\alpha} u_{\beta} + p g_{\alpha\beta}^{(0)}, \tag{11}$$

where $\vec{u} = e^{-\nu/2}\partial_t$ is the fluid's four-velocity and ρ and p are the density and pressure. Numerical integration of the Tolman-Oppenheimer-Volkhov equations (see e.g. Misner et al. (1973)) for neutron star models with a polytropic pressure-density relation

$$P = K\rho^{1+1/n},\tag{12}$$

where K is a constant and n is the polytropic index, gives the equilibrium stellar model with radius R and total mass M = m(R).

4.2. Static linearized perturbations due to an external tidal field

We examine the behavior of the equilibrium configuration under linearized perturbations due to an external quadrupolar tidal field following the method of Thorne & Campolattaro (1967). The full metric of the spacetime is given by

$$g_{\alpha\beta} = g_{\alpha\beta}^{(0)} + h_{\alpha\beta},\tag{13}$$

where $h_{\alpha\beta}$ is a linearized metric perturbation. We analyze the angular dependence of the components of $h_{\alpha\beta}$ into spherical harmonics as in Regge & Wheeler (1957). We restrict our analysis to the l=2, static, even-parity perturbations in the Regge-Wheeler gauge (Regge & Wheeler 1957). With these specializations, $h_{\alpha\beta}$ can be written as (Regge & Wheeler 1957; Thorne & Campolattaro 1967):

$$h_{\alpha\beta} = \text{diag} \left[e^{-\nu(r)} H_0(r), \ e^{\lambda(r)} H_2(r), \ r^2 K(r), \ r^2 \sin^2 \theta K(r) \right] Y_{2m}(\theta, \varphi).$$
 (14)

The nonvanishing components of the perturbations of the stress-energy tensor (11) are $\delta T_0^0 = -\delta \rho = -(dp/d\rho)^{-1}\delta p$ and $\delta T_i^i = \delta p$. We insert this and the metric metric perturbation (14) into the linearized Einstein equation $\delta G_{\alpha}^{\beta} = 8\pi\delta T_{\alpha}^{\beta}$ and combine various components. From $\delta G_{\theta}^{\theta} - \delta G_{\phi}^{\phi} = 0$ it follows that that $H_2 = H_0 \equiv H$, then $\delta G_{\theta}^r = 0$ relates K' to H, and after using $\delta G_{\theta}^{\theta} + \delta G_{\phi}^{\phi} = 16\pi\delta p$ to eliminate δp , we finally subtract the r-r component of the Einstein equation from the t-t component to obtain the following differential equation for $H_0 \equiv H$ (for l=2):

$$H'' + H' \left[\frac{2}{r} + e^{\lambda} \left(\frac{2m(r)}{r^2} + 4\pi r \left(p - \rho \right) \right) \right]$$
$$+ H \left[-\frac{6e^{\lambda}}{r^2} + 4\pi e^{\lambda} \left(5\rho + 9p + \frac{\rho + p}{(dp/d\rho)} \right) - \nu'^2 \right] = 0, \tag{15}$$

where the prime denotes d/dr. The boundary conditions for Eq. (15) can be obtained as follows. Requiring regularity of H at r=0 and solving for H near r=0 yields

$$H(r) = a_0 r^2 \left[1 - \frac{2\pi}{7} \left(5\rho(0) + 9p(0) + \frac{\rho(0) + p(0)}{(dp/d\rho)(0)} \right) r^2 + O(r^3) \right], \tag{16}$$

where a_0 is a constant. To single out a unique solution from this one-parameter family of solutions parameterized by a_0 , we use the continuity of H(r) and its derivative across r = R. Outside the star, Eq. (15) reduces to

$$H'' + \left(\frac{2}{r} - \lambda'\right)H' - \left(\frac{6e^{\lambda}}{r^2} + {\lambda'}^2\right)H = 0, \tag{17}$$

and changing variables to x = (r/M - 1) as in Thorne & Campolattaro (1967) transforms Eq. (17) to a form of the associated Legendre equation with l = m = 2:

$$(x^{2} - 1)H'' + 2xH' - \left(6 + \frac{4}{x^{2} - 1}\right)H = 0.$$
 (18)

The general solution to Eq. (18) in terms of the associated Legendre functions $Q_2^2(x)$ and $P_2^2(x)$ is given by

$$H = c_1 Q_2^2 \left(\frac{r}{M} - 1\right) + c_2 P_2^2 \left(\frac{r}{M} - 1\right), \tag{19}$$

where c_1 and c_2 are coefficients to be determined. Substituting the expressions for $Q_2^2(x)$ and $P_2^2(x)$ from Abramowitz & Stegun (1964) yields for the exterior solution

$$H = c_1 \left(\frac{r}{M}\right)^2 \left(1 - \frac{2M}{r}\right) \left[-\frac{M(M-r)(2M^2 + 6Mr - 3r^2)}{r^2(2M-r)^2} + \frac{3}{2} \log\left(\frac{r}{r - 2M}\right) \right] + 3c_2 \left(\frac{r}{M}\right)^2 \left(1 - \frac{2M}{r}\right).$$
(20)

The asymptotic behavior of the solution (20) at large r is

$$H = \frac{8}{5} \left(\frac{M}{r}\right)^3 c_1 + O\left(\left(\frac{M}{r}\right)^4\right) + 3\left(\frac{r}{M}\right)^2 c_2 + O\left(\left(\frac{r}{M}\right)\right), \tag{21}$$

where the coefficients c_1 and c_2 are determined by matching the asymptotic solution (21) to the expansion (1) and using Eq. (9):

$$c_1 = \frac{15}{8} \frac{1}{M^3} \lambda \mathcal{E}, \quad c_2 = \frac{1}{3} M^2 \mathcal{E}.$$
 (22)

We now solve for λ in terms of H and its derivative at the star's surface r = R using Eqs. (22) and (20), and use the relation (5) to obtain the expression:

$$k_{2} = \frac{8C^{5}}{5} (1 - 2C)^{2} [2 + 2C (y - 1) - y] \times$$

$$\left\{ 2C (6 - 3y + 3C(5y - 8)) + 4C^{3} [13 - 11y + C(3y - 2) + 2C^{2}(1 + y)] + 3(1 - 2C)^{2} [2 - y + 2C(y - 1)] \log (1 - 2C) \right\}^{-1},$$
(23)

where we have defined the star's compactness parameter $C \equiv M/R$ and the quantity $y \equiv RH'(R)/H(R)$, which is obtained by integrating Eq. (15) outwards in the region 0 < r < R.

4.3. Newtonian limit

The first term in the expansion of the expression (23) in M/R reproduces the Newtonian result:

$$k_2^N = \frac{1}{2} \left(\frac{2-y}{y+3} \right), \tag{24}$$

where the superscript N denotes "Newtonian". In the Newtonian limit, the differential equation (15) inside the star becomes

$$H'' + \frac{2}{r}H' + \left(\frac{4\pi\rho}{dp/d\rho} - \frac{6}{r^2}\right)H = 0.$$
 (25)

For a polytropic index of n = 1, Eq. (25) can be transformed to a Bessel equation with the solution that is regular at r = 0 given by $H = A\sqrt{r/R} J_{5/2}(\pi r/R)$, where A is a constant. At r = R, we thus have $y = RH'/H = (\pi^2 - 9)/3$, and from Eq. (23) it follows that

$$k_2^N(n=1) = \left(-\frac{1}{2} + \frac{15}{2\pi^2}\right) \approx 0.25991,$$
 (26)

which agrees with the known result of Brooker & Olle (1955).

5. Results and Discussion

The range of dimensionless Love numbers k_2 obtained by numerical integration of Eq. (23) is shown in Fig. 1 as a function of M/R and n for a variety of different neutron star models, and representative values are given in Table 2. These values can be approximated to an accuracy of $\sim 6\%$ in the range $0.5 \le n \le 1.0$ and $0.1 \le (M/R) \le 0.24$ by the fitting formula

$$k_2 \approx \frac{3}{2} \left(-0.41 + \frac{0.56}{n^{0.33}} \right) \left(\frac{M}{R} \right)^{-0.003}$$
 (27)

Both Fig. 1 and Table 2 illustrate that the dimensionless Love numbers k_2 depend more strongly on the polytropic index n than on the compactness C = M/R. This is expected since the weak field, Newtonian values k_2^N given by Eq. (24) just depend on n (through the dependence on y). The additional dependence on the compactness for the Love numbers k_2 in Eq. (23) is a relativistic correction to this. For $M/R \sim 10^{-5}$ our results for k_2 agree well with the Newtonian results of Brooker & Olle (1955). Figure 2 shows the percent difference

³Note, however, that LIGO measurements will yield the combination k_2R^5 and therefore will be more sensitive to the compactness than the polytropic index.

 $(k_2^N - k_2)/k_2$ between the relativistic and Newtonian dimensionless Love numbers. As can be seen from the figure, the relativistic values are lower than the Newtonian ones for higher values of n. This can be explained by the fact that the Love number encodes information about the degree of central condensation of the star. Stars with a higher the polytropic index n are more centrally condensed and therefore have a smaller response to a tidal field, resulting in a smaller Love number.

Some estimates of the masses and radii of neutron stars, given in Table 3, have been inferred from X-ray observations (Ozel 2006; Webb & Barret 2007) using the information from three measured quantities: the Eddington luminosity, the surface redshift of spectral lines, and the quiescent X-ray flux. The range of the numbers λ for these stars is shown in Fig. 3. LIGO II detectors will be able to establish a 90% confidence upper limit of $\lambda \leq 2.01 \times 10^{37} \mathrm{g \, cm^2 s^2}$ for an inspiral of two nonspinning $1.4 M_{\odot}$ NSs at a distance of 50 Mpc in the case that no tidal phase shift is observed (Flanagan & Hinderer 2007).

The author thanks Éanna Flanagan for valuable discussions and comments.

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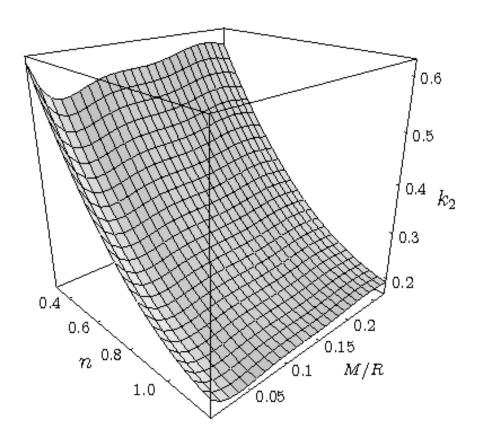


Fig. 1.— The relativistic Love numbers k_2 .

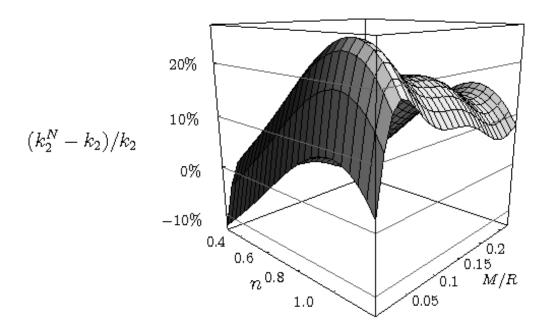


Fig. 2.— The difference in percent between the relativistic dimensionless Love numbers k_2 and the Newtonian values k_2^N .

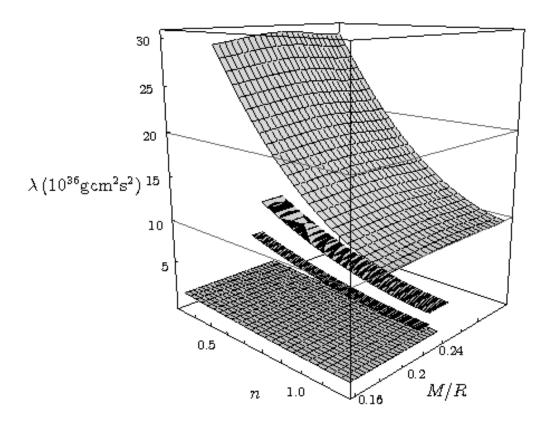


Fig. 3.— The range of Love numbers for the estimated NS parameters from X-ray observations. Top to bottom sheets: EXO0748-676, ω Cen, M 13, NGC 2808. For an inspiral of two $1.4M_{\odot}$ NSs at a distance of 50 Mpc, LIGO II detectors will be able to constrain λ to $\lambda \leq 20.1 \times 10^{36} \mathrm{g \ cm^2 s^2}$ with 90% confidence (Flanagan & Hinderer 2007).

Table 2. Relativistic Love numbers k_2

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1.0	10^{-5}	0.2599	
1.0	0.1	0.122	
1.0	0.15	0.0776	
1.0	0.2	0.0459	
1.0	0.2	0.0253	
1.2	10^{-5}	0.2062	
1.2	0.1	0.0931	
1.2	0.15	0.0577	
1.2	0.2	0.0327	

Table 3: Estimated neutron star parameters from X-ray observations

	1	v	
Cluster / object	$M(M_{\odot})$	$R(\mathrm{km})$	M/R
ω Cen ^a	1.61 ± 0.15	10.99 ± 0.71	0.18 ± 0.04
M 13 $^{\rm a}$	1.36 ± 0.04	9.89 ± 0.08	0.2
NGC 2808 $^{\rm a}$	0.84 ± 0.12	7.34 ± 0.96	0.22 ± 0.01
EXO 0748-676 $^{\rm b}$	$\geq 2.1 \pm 0.28$	$\geq 13.8 \pm 1.8$	0.2256

Note. — Parameters used to generate Fig. (3).

^aThe parameters for these stars are the averages from the best fit values of the data in Webb & Barret (2007) for their three different spectral fits. The errors given here reflect only the deviations among the best fit values for the fits.

 $^{{}^{}b}$ The values are taken from Ozel (2006).