

“Mass Inflation” With Lightlike Branes

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Abstract

We discuss properties of a new class of p -brane models, describing intrinsically *lightlike* branes for any world-volume dimension, in various gravitational backgrounds of interest in the context of black hole physics. One of the characteristic features of these lightlike p -branes is that the brane tension appears as an additional nontrivial dynamical world-volume degree of freedom. Codimension one lightlike brane dynamics requires that bulk space with a bulk metric of spherically symmetric type must possess an event horizon which is automatically occupied by the lightlike brane while its tension evolves exponentially with time. The latter phenomenon is an analog of the well known “mass inflation” effect in black holes.

Keywords: *non-Nambu-Goto lightlike p -branes; dynamical brane tension; black hole’s horizon “straddling”; exponential inflation/deflation*

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I. INTRODUCTION

The behavior of matter near horizons of black holes has been the subject of several interesting investigations [1]-[5]. One particularly intriguing effect was the “mass inflation” [4]-[5] which takes place, for example, for matter accumulating (blue shifting) near the inner Reissner-Nordström horizon.

In the context of the problem where we consider matter living close to, or in fact on, the horizons of black holes, the notion of lightlike branes becomes particularly relevant. Let us recall that lightlike branes (*LL-branes*, for short) are of particular interest in general relativity primarily due to their role: (i) in describing impulsive lightlike signals arising in cataclysmic astrophysical events [6]; (ii) as basic ingredients in the so called “membrane paradigm” theory [7] of black hole physics; (iii) in the context of the thin-wall description of domain walls coupled to gravity [8]-[11].

More recently, *LL-branes* became significant also in the context of modern non-perturbative string theory, in particular, as the so called *H-branes* describing quantum horizons (black hole and cosmological) [12], as well appearing as Penrose limits of baryonic *D*(=Dirichlet) branes [13].

In the original papers [8]-[11] *LL-branes* in the context of gravity and cosmology have been extensively studied from a phenomenological point of view, *i.e.*, by introducing them without specifying the Lagrangian dynamics from which they may originate. In a recent paper [14] brane actions in terms of their pertinent extrinsic geometry have been proposed which generically describe non-lightlike branes, whereas the lightlike branes are treated as a limiting case. On the other hand, we have proposed in a series of recent papers [15]-[21] a new class of concise Lagrangian actions, among them – *Weyl-conformally invariant* ones, providing a derivation from first principles of the *LL-brane* dynamics.

In Section 2 of the present paper we extend our previous construction (which was restricted to odd world-volume dimensions) to the case of *LL-brane* actions for *arbitrary* world-volume dimensions.

In Section 3 we discuss the properties of *LL-brane* dynamics moving as test brane in generic gravitational backgrounds. The case with two extra dimensions (codimension two *LL-branes*) was studied in a recent paper [22] from the point of view of “braneworld” scenarios [23]-[28] (for a review, see [29]-[30]). Unlike conventional braneworlds, where the

underlying branes are of Nambu-Goto type (*i.e.*, describing massive brane modes) and in their ground state they position themselves at some fixed point in the extra dimensions of the bulk space-time, our lightlike braneworlds perform in the ground state non-trivial motions in the extra dimensions – planar circular, spiral winding *etc* depending on the topology of the extra dimensions. In the present paper we concentrate on the special case of codimension one *LL-branes* which is qualitatively different and needs separate study. Here the consistency of *LL-brane* dynamics as test brane moving in external gravitational fields dictates that the bulk space-time with a bulk metric of spherically symmetric type (see Eq.(33) below) must possess an event horizon which is automatically occupied by the *LL-brane* (“horizon straddling” according to the terminology of ref.[10]). This is a generalization for any $(p + 1)$ world-volume dimensions of the results previously obtained in refs.[15]-[21] for lightlike membranes ($p = 2$) in $D = 4$ bulk space-time.

In Section 4 we study several cases of “horizon straddling” solutions obtained from our *LL-brane* world-volume action (1). For the inner Reissner-Nordström horizon we find a time symmetric “mass inflation” scenario, which also holds for de Sitter horizon. In this case the dynamical tension of the *LL-brane* blows up as time approaches $\pm\infty$ due to its exponential quadratic time dependence. For the Schwarzschild and the outer Reissner-Nordström horizons, on the other hand, we obtain “mass deflationary” scenarios where the dynamical *LL-brane* tension vanishes at large positive or large negative times. Another set of solutions with asymmetric (w.r.t. $t \rightarrow -t$) exponential linear time dependence of the *LL-brane* tension also exists. In the latter case, by fine tuning one can obtain a constant time-independent brane tension as a special case.

II. WORLD-VOLUME ACTIONS OF LIGHTLIKE BRANES

We propose the following reparametrization invariant action describing intrinsically light-like p -branes for any world-volume dimension $(p + 1)$ (for previous versions, cf.[15]-[21]):

$$S = - \int d^{p+1} \sigma \Phi(\varphi) \left[\frac{1}{2} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) - L(F^2) \right] \quad (1)$$

using notions and notations as follows:

- Alternative non-Riemannian integration measure density $\Phi(\varphi)$ (volume form) on the

p -brane world-volume manifold:

$$\Phi(\varphi) \equiv \frac{1}{(p+1)!} \varepsilon_{I_1 \dots I_{p+1}} \varepsilon^{a_1 \dots a_{p+1}} \partial_{a_1} \varphi^{I_1} \dots \partial_{a_{p+1}} \varphi^{I_{p+1}} \quad (2)$$

instead of the usual $\sqrt{-\gamma}$. Here $\{\varphi^I\}_{I=1}^{p+1}$ are auxiliary world-volume scalar fields; γ_{ab} ($a, b = 0, 1, \dots, p$) denotes the intrinsic Riemannian metric on the world-volume, and $\gamma = \det \|\gamma_{ab}\|$. Note that γ_{ab} is *independent* of φ^I .

- $X^\mu(\sigma)$ are the p -brane embedding coordinates in the bulk D -dimensional space time with bulk Riemannian metric $G_{\mu\nu}(X)$; $\mu, \nu = 0, 1, \dots, D-1$, ($\sigma \equiv (\sigma^0 \equiv \tau, \sigma^1, \dots, \sigma^p)$), $\partial_a \equiv \frac{\partial}{\partial \sigma^a}$.
- Auxiliary $(p-1)$ -rank antisymmetric tensor gauge field $A_{a_1 \dots a_{p-1}}$ on the world-volume with p -rank field-strength and its dual:

$$F_{a_1 \dots a_p} = p \partial_{[a_1} A_{a_2 \dots a_p]} \quad , \quad F^{*a} = \frac{1}{p!} \frac{\varepsilon^{aa_1 \dots a_p}}{\sqrt{-\gamma}} F_{a_1 \dots a_p} \quad . \quad (3)$$

- $L(F^2)$ is *arbitrary* function of F^2 with the short-hand notation:

$$F^2 \equiv F_{a_1 \dots a_p} F_{b_1 \dots b_p} \gamma^{a_1 b_1} \dots \gamma^{a_p b_p} \quad . \quad (4)$$

Let us note the simple identity:

$$F_{a_1 \dots a_{p-1} b} F^{*b} = 0 \quad , \quad (5)$$

which will play a crucial role in the sequel.

Remark 1. For the special choice $L(F^2) = (F^2)^{1/p}$ the action (1) becomes manifestly invariant under *Weyl (conformal) symmetry*: $\gamma_{ab} \longrightarrow \gamma'_{ab} = \rho \gamma_{ab}$, $\varphi^I \longrightarrow \varphi'^I = \varphi'^I(\varphi)$ with Jacobian $\det \left\| \frac{\partial \varphi'^I}{\partial \varphi^J} \right\| = \rho$. In what follows we will consider the generic Weyl *non*-invariant case.

Remark 2. In our previous papers [15]-[21] we have used a different form for the Lagrangian of the auxiliary world-volume gauge field in the brane action (1):

$$L(F^2) = \sqrt{F_{ab} F_{cd} \gamma^{ac} \gamma^{bd}} \quad \text{with} \quad F_{ab} = \partial_a A_b - \partial_b A_a \quad , \quad (6)$$

i.e., with ordinary vector gauge field for any p . However, it has been shown in ref.[21] that for the choice (6) the action (1) describes consistent brane dynamics only for *odd* $(p+1)$

world-volume dimensions. This is due to the following relation (Eq.(13) in ref.[21], which is a consequences from the equation of motion w.r.t. γ_{ab} – the counterpart of Eq.(11) below):

$$\det \| (\partial_a X \partial_b X) \| = (-4L'(F^2))^{p+1} (-\det \| \gamma_{ab} \|) (\det \| iF_{ab} \|)^2 . \quad (7)$$

The latter relation implies that for $(p+1)$ =even world-volume dimensions the r.h.s. of (7) is strictly positive (because of the Lorentzian signature of the intrinsic metric γ_{ab}) contradicting the requirement that the determinant of the induced metric in the l.h.s of (7) should be negative conforming with the Lorentzian signatures of both world-volume and embedding space-time metrics. Henceforth, we will employ our new action (1) with the $(p-1)$ -rank auxiliary world-volume antisymmetric tensor gauge fields (3).

Rewriting the action (1) in the following equivalent form:

$$S = - \int d^{p+1}\sigma \chi \sqrt{-\gamma} \left[\frac{1}{2} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) - L(F^2) \right] , \quad \chi \equiv \frac{\Phi(\varphi)}{\sqrt{-\gamma}} \quad (8)$$

with $\Phi(\varphi)$ the same as in (2), we find that the composite field χ plays the role of a *dynamical (variable) brane tension*. Let us note that the notion of dynamical brane tension has previously appeared in different contexts in refs.[31]-[33].

Before proceeding, let us mention that both the auxiliary world-volume scalars φ^I entering the non-Riemannian integration measure density (2), as well as the intrinsic world-volume metric γ_{ab} are *non-dynamical* degrees of freedom in the action (1), or equivalently, in (8). Indeed, there are no (time-)derivates w.r.t. γ_{ab} , whereas the action (1) (or (8)) is *linear* w.r.t. the velocities $\dot{\varphi}^I$. Thus, (1) (or (8)) is a constrained dynamical system, *i.e.*, a system with gauge symmetries including the gauge symmetry under world-volume reparametrizations (about the Hamiltonian treatment of (1), see the remarks after Eq.(11) below). On the other hand, the dynamical brane tension χ (8), being a ratio of two world-volume scalar densities, is itself a well-defined reparametrization-covariant *world-volume scalar field*.

Introducing a short-hand notation for the induced metric:

$$(\partial_a X \partial_b X) \equiv \partial_a X^\mu \partial_b X^\nu G_{\mu\nu} . \quad (9)$$

we can write the equations of motion obtained from (1) w.r.t. measure-building auxiliary scalars φ^I and γ^{ab} as:

$$\frac{1}{2} \gamma^{cd} (\partial_c X \partial_d X) - L(F^2) = M , \quad (10)$$

where M is an integration constant;

$$\frac{1}{2} (\partial_a X \partial_b X) - p L'(F^2) F_{aa_1 \dots a_{p-1}} \gamma^{a_1 b_1} \dots \gamma^{a_{p-1} b_{p-1}} F_{bb_1 \dots b_{p-1}} = 0 . \quad (11)$$

Since, as mentioned above, both φ^I and γ^{ab} are non-dynamical degrees of freedom, both Eqs.(10)–(11) are in fact *non-dynamical constraint* equations (no second-order time derivatives present). Their meaning as constraint equations is best understood within the framework of the Hamiltonian formalism for the action (1) (or (8)). The latter can be developed in strict analogy with the Hamiltonian formalism for a simpler class of modified p -brane models based on the alternative non-Riemannian integration measure density (2), which was previously proposed in [34] (for details, we refer to Sections 2 and 3 of [34]). In particular, Eqs.(11) can be viewed as p -brane analogues of the string Virasoro constraints.

Thus, Eqs.(10)–(11) are particular manifestation in the case of (1) of the general property in any dynamical system with gauge symmetries, *i.e.*, a system with constraints à la Dirac [35]–[37] – variation of the action w.r.t. non-dynamical degrees of freedom (Lagrange multipliers) yields non-dynamical constraint equations.

Taking the trace in (11) and comparing with (10) implies the following crucial relation for the Lagrangian function $L(F^2)$:

$$L(F^2) - p F^2 L'(F^2) + M = 0 , \quad (12)$$

which determines F^2 (4) on-shell as certain function of the integration constant M (10), *i.e.*

$$F^2 = F^2(M) = \text{const} . \quad (13)$$

The second and most profound consequence of Eqs.(11) is due to the identity (5) which implies that the induced metric (9) on the world-volume of the p -brane model (1) is *singular* (as opposed to the induced metric in the case of ordinary Nambu-Goto branes):

$$(\partial_a X \partial_b X) F^{*b} = 0 \quad , \quad \text{i.e.} \quad (\partial_F X \partial_F X) = 0 \quad , \quad (\partial_\perp X \partial_F X) = 0 \quad (14)$$

where $\partial_F \equiv F^{*a} \partial_a$ and ∂_\perp are derivatives along the tangent vectors in the complement of F^{*a} .

Thus, we arrive at the following important conclusion: every point on the surface of the p -brane (1) moves with the speed of light in a time-evolution along the vector-field F^{*a} which justifies the name *LL-brane* (Lightlike-brane) model for (1).

Before proceeding let us point out that we can add [22] to the *LL-brane* action (1) natural couplings to bulk Maxwell and Kalb-Ramond gauge fields. The latter do not affect Eqs.(10) and (11), so that the conclusions about on-shell constancy of F^2 (13) and the lightlike nature (14) of the p -branes under consideration remain unchanged.

The remaining equations of motion w.r.t. auxiliary world-volume gauge field $A_{a_1 \dots a_{p-1}}$ and X^μ produced by the action (1) read:

$$\partial_{[a} (F^{*c} \gamma_{b]c} \chi L'(F^2)) = 0 ; \quad (15)$$

$$\partial_a (\chi \sqrt{-\gamma} \gamma^{ab} \partial_b X^\mu) + \chi \sqrt{-\gamma} \gamma^{ab} \partial_a X^\nu \partial_b X^\lambda \Gamma_{\nu\lambda}^\mu(X) = 0 \quad (16)$$

Here χ is the dynamical brane tension as in (8),

$$\Gamma_{\nu\lambda}^\mu = \frac{1}{2} G^{\mu\kappa} (\partial_\nu G_{\kappa\lambda} + \partial_\lambda G_{\kappa\nu} - \partial_\kappa G_{\nu\lambda}) \quad (17)$$

is the Christoffel connection for the external metric, and $L'(F^2)$ denotes derivative of $L(F^2)$ w.r.t. the argument F^2 .

Invariance under world-volume reparametrizations allows to introduce the standard synchronous gauge-fixing conditions:

$$\gamma^{0i} = 0 \quad (i = 1, \dots, p) \quad , \quad \gamma^{00} = -1 . \quad (18)$$

Also, in what follows we will use a natural ansatz for the auxiliary world-volume gauge field-strength:

$$F^{*i} = 0 \quad (i = 1, \dots, p) \quad , \quad \text{i.e.} \quad F_{0i_1 \dots i_{p-1}} = 0 , \quad (19)$$

the only non-zero component of the dual field-strength being:

$$F^{*0} = \frac{1}{p!} \frac{\varepsilon^{i_1 \dots i_p}}{\sqrt{\gamma^{(p)}}} F_{i_1 \dots i_p} , \quad (20)$$

$$\gamma^{(p)} \equiv \det \|\gamma_{ij}\| \quad (i, j = 1, \dots, p) \quad , \quad F^2 = p! (F^{*0})^2 = \text{const} .$$

According to (14) the meaning of the ansatz (19) is that the lightlike direction $F^{*a} \partial_a \simeq \partial_0 \equiv \partial_\tau$, i.e., it coincides with the brane proper-time direction. Bianchi identity $\nabla_a F^{*a} = 0$ together with (19)–(20) implies:

$$\partial_0 F_{i_1 \dots i_p} = 0 \quad \longrightarrow \quad \partial_0 \sqrt{\gamma^{(p)}} = 0 . \quad (21)$$

Using (18) and (19) the equations of motion (11), (15) and (16) acquire the form, respectively:

$$(\partial_0 X \partial_0 X) = 0 \quad , \quad (\partial_0 X \partial_i X) = 0 \quad , \quad (\partial_i X \partial_j X) - 2a_0 \gamma_{ij} = 0 \quad (22)$$

(Virasoro-like constraints), where the M -dependent constant a_0 :

$$a_0 \equiv F^2 L'(F^2) \Big|_{F^2=F^2(M)} \quad (23)$$

must be strictly positive;

$$\partial_i \chi = 0 ; \quad (24)$$

$$\begin{aligned} & -\sqrt{\gamma^{(p)}} \partial_0 (\chi \partial_0 X^\mu) + \partial_i \left(\chi \sqrt{\gamma^{(p)}} \gamma^{ij} \partial_j X^\mu \right) \\ & + \chi \sqrt{\gamma^{(p)}} (-\partial_0 X^\nu \partial_0 X^\lambda + \gamma^{kl} \partial_k X^\nu \partial_l X^\lambda) \Gamma_{\nu\lambda}^\mu = 0 . \end{aligned} \quad (25)$$

III. LIGHTLIKE BRANE DYNAMICS IN GRAVITATIONAL BACKGROUNDS

Let us split the bulk space-time coordinates as:

$$(X^\mu) = (x^a, y^\alpha) \equiv (x^0 \equiv t, x^i, y^\alpha) \quad (26)$$

$$a = 0, 1, \dots, p \quad , \quad i = 1, \dots, p \quad , \quad \alpha = 1, \dots, D - (p + 1)$$

and consider background metrics $G_{\mu\nu}$ of the form:

$$ds^2 = -A(t, y)(dt)^2 + C(t, y)g_{ij}(\vec{x})dx^i dx^j + B_{\alpha\beta}(t, y)dy^\alpha dy^\beta . \quad (27)$$

Here we will discuss the simplest non-trivial ansatz for the *LL-brane* embedding coordinates:

$$X^a \equiv x^a = \sigma^a \quad , \quad X^{p+\alpha} \equiv y^\alpha = y^\alpha(\tau) \quad , \quad \tau \equiv \sigma^0 . \quad (28)$$

With (27) and (28), the constraint Eqs.(22) yield:

$$-A + B_{\alpha\beta} \dot{y}^\alpha \dot{y}^\beta = 0 \quad , \quad Cg_{ij} - 2a_0 \gamma_{ij} = 0 , \quad (29)$$

where $\dot{y}^\alpha \equiv \frac{d}{d\tau} y^\alpha$. Second Eq.(29) together with the last relation in (21) implies:

$$\frac{d}{d\tau} C(y(\tau)) = \left(\partial_t C + \dot{y}^\alpha \frac{\partial C}{\partial y^\alpha} \right) \Big|_{t=\tau, y=y(\tau)} = 0 . \quad (30)$$

The second-order Eqs.(25) for $X^0 \equiv t$ and $X^{p+\alpha} \equiv y^\alpha$ yield accordingly:

$$\partial_\tau \chi + \frac{\chi}{A} \left[\frac{1}{2} \partial_t A + \dot{y}^\alpha \frac{\partial A}{\partial y^\alpha} + \frac{1}{2} \dot{y}^\alpha \dot{y}^\beta \partial_t B_{\alpha\beta} + \frac{p a_0}{C} \dot{y}^\alpha \frac{\partial C}{\partial y^\alpha} \right] \Big|_{t=\tau, y=y(\tau)} = 0, \quad (31)$$

$$\partial_\tau \left(\chi \dot{y}^\alpha \right) + \chi \left[B^{\alpha\beta} \left(\frac{1}{2} \frac{\partial A}{\partial y^\alpha} + \dot{y}^\gamma \partial_t B_{\beta\gamma} + \frac{p a_0}{C} \frac{\partial C}{\partial y^\beta} \right) + \dot{y}^\beta \dot{y}^\gamma \Gamma_{\beta\gamma}^\alpha \right] \Big|_{t=\tau, y=y(\tau)} = 0, \quad (32)$$

where $\Gamma_{\beta\gamma}^\alpha$ is the Christoffel connection for the metric $B_{\alpha\beta}$ in the extra dimensions (cf. (27)).

LL-brane equations (29)–(32) for codimension two (i.e., for $D - (p + 1) = 2$) have been studied in ref.[22] from the braneworld point of view. The case of codimension one *LL-branes* moving in gravitational backgrounds (i.e., for $D = p + 2$) is qualitatively different and is the subject of the discussion in what follows.

In the latter case the metric (27) acquires the form of a general spherically symmetric metric:

$$ds^2 = -A(t, y)(dt)^2 + C(t, y)g_{ij}(\vec{\theta})d\theta^i d\theta^j + B(t, y)(dy)^2, \quad (33)$$

where $\vec{x} \equiv \vec{\theta}$ are the angular coordinates parametrizing the sphere S^p .

Eqs.(29)–(31) now take the form:

$$-A + B \dot{y}^2 = 0, \text{ i.e. } \dot{y} = \pm \sqrt{\frac{A}{B}}, \quad \partial_t C + \dot{y} \partial_y C = 0 \quad (34)$$

$$\partial_\tau \chi + \chi \left[\partial_t \ln \sqrt{AB} \pm \frac{1}{\sqrt{AB}} \left(\partial_y A + p a_0 \partial_y \ln C \right) \right] = 0, \quad (35)$$

whereas Eq.(32) becomes a consequence of the above ones.

In what follows we will consider the following subclasses of background metrics (33):

(i) Static spherically symmetric metrics in standard coordinates:

$$A = A(y), \quad B(y) = A^{-1}(y), \quad C(y) = y^2, \quad (36)$$

where $y \equiv r$ is the radial-like coordinate. In the case of (36), Eqs.(34) imply:

$$\dot{y} = 0, \text{ i.e. } y(\tau) = y_0 = \text{const}, \quad A(y_0) = 0. \quad (37)$$

In other words, the equations of motion of the *LL-brane* require that the latter positions itself on a spherical-like hypersurface (second Eq.(37)) in the bulk space-time which in addition must be a horizon of the background metric (last Eq.(37), cf. (33)).

The next Eq.(35) reduces in the case of (36) to:

$$\partial_\tau \chi \pm \chi \left(\partial_y A \Big|_{y=y_0} + \frac{2p a_0}{y_0} \right) = 0 \quad (38)$$

with the obvious solution:

$$\chi(\tau) = \chi_0 \exp\left\{\mp\tau\left(\partial_y A \Big|_{y=y_0} + \frac{2p a_0}{y_0}\right)\right\} , \quad \chi_0 = \text{const} . \quad (39)$$

Thus, we find a time-asymmetric solution for the dynamical brane tension which (upon appropriate choice of the signs (\mp) depending on the sign of the constant factor in the exponent on the r.h.s. of (39)) exponentially “inflates” for large times. In the particular case of fine tuning of parameters:

$$\partial_y A \Big|_{y=y_0} + \frac{2p a_0}{y_0} = 0 \quad (40)$$

we obtain a constant solution $\chi = \chi_0$.

(ii) Spherically symmetric metrics in Kruskal-like coordinates:

$$A = B , \quad A = A(y^2 - t^2) , \quad C = C(y^2 - t^2) , \quad (41)$$

where (t, y) play the role of Kruskal’s (v, u) coordinates for Schwarzschild metrics [38]-[39]. In the case of (41), Eqs.(34) yield:

$$\dot{y} = \pm 1 , \quad \text{i.e. } y(\tau) = \pm\tau , \quad (y^2 - t^2) \Big|_{t=\tau, y=y(\tau)} = 0 , \quad (42)$$

i.e., again the *LL-brane* locates itself *automatically* on the horizon. Eq.(35) reduces accordingly to:

$$\partial_\tau \chi + \tau \frac{2p a_0 C'(0)}{A(0)C(0)} \chi = 0 , \quad (43)$$

$$\text{i.e. } \chi(\tau) = \chi_0 \exp\left\{-\tau^2 \frac{p a_0 C'(0)}{A(0)C(0)}\right\} \quad (44)$$

Thus, we find a time-symmetric “inflationary” or “deflationary” solution for the dynamical brane tension depending on the sign of the constant factor in the exponent on the r.h.s. of (44).

(iii) “Cosmological”-type metrics:

$$A = 1 , \quad B = S^2(t) , \quad C = S^2(t) f^2(y) , \quad (45)$$

i.e.:

$$ds^2 = -(dt)^2 + S^2(t) \left[(dy)^2 + f^2(y) g_{ij}(\vec{\theta}) d\theta^i d\theta^j \right] \quad (46)$$

with θ^i parametrizing the p -dimensional sphere S^p . In this case Eqs.(34) give:

$$\dot{y} = \pm \frac{1}{S(\tau)} \quad , \quad C \Big|_{t=\tau, y=y(\tau)} \equiv S^2(\tau) f^2(y(\tau)) = \frac{1}{c_0^2} \quad , \quad c_0 = \text{const} \quad , \quad (47)$$

implying:

$$\dot{y} = c_0 f(y(\tau)) \quad . \quad (48)$$

Eq.(35) reduces in the case of (45) to:

$$\partial_\tau \chi + \chi \frac{\partial_\tau S}{S} (1 - 2p a_0) = 0 \quad \rightarrow \quad \chi(\tau) = \chi_0 (S(\tau))^{2p a_0 - 1} \quad . \quad (49)$$

Here again, for the special choice of the integration constant M (10) such that the constant a_0 (23) is fine-tuned as $a_0 = \frac{1}{2p}$, we obtain a constant solution $\chi = \chi_0$.

IV. EXAMPLES

As a first example of lightlike brane tension's ‘‘inflation’’/‘‘deflation’’ (44) let us consider de Sitter embedding space metric in Kruskal-like (Gibbons-Hawking) coordinates [40]:

$$ds^2 = A(y^2 - t^2) [-(dt)^2 + (dy)^2] + R^2(y^2 - t^2) g_{ij}(\vec{\theta}) d\theta^i d\theta^j \quad , \quad (50)$$

$$A(y^2 - t^2) = \frac{4}{K(1 + y^2 - t^2)^2} \quad , \quad R(y^2 - t^2) = \frac{1}{\sqrt{K}} \frac{1 - (y^2 - t^2)}{1 + y^2 - t^2} \quad . \quad (51)$$

Substituting:

$$A(0) = \frac{4}{K} \quad , \quad C(0) \equiv R^2(0) = \frac{1}{K} \quad , \quad C'(0) \equiv 2R(0)R'(0) = -\frac{4}{K} \quad (52)$$

into expression (44) we get for the dynamical brane tension (recall that the cosmological constant K from (51) and the constant a_0 (23) are strictly positive):

$$\chi(\tau) = \chi_0 \exp\{\tau^2 p a_0 K\} \quad , \quad (53)$$

i.e., exponential ‘‘inflation’’ at $\tau \rightarrow \pm\infty$ for the brane tension of lightlike branes occupying de Sitter horizon.

The second example is Schwarzschild background metric in Kruskal coordinates [38]-[39],[41] (here we take $D = p + 2 = 4$, *i.e.*, $i, j = 1, 2$):

$$ds^2 = A(y^2 - t^2) [-(dt)^2 + (dy)^2] + R^2(y^2 - t^2) g_{ij}(\vec{\theta}) d\theta^i d\theta^j \quad , \quad (54)$$

$$A = \frac{4R_0^3}{R} \exp\left\{-\frac{R}{R_0}\right\} \quad , \quad \left(\frac{R}{R_0} - 1\right) \exp\left\{\frac{R}{R_0}\right\} = y^2 - t^2 \quad , \quad R_0 \equiv 2G_N m \quad (55)$$

Calculating $A(0)$, $C(0) \equiv R^2(0)$ and $C'(0) \equiv 2R(0)R'(0)$ from (55) we obtain for (44):

$$\chi(\tau) = \chi_0 \exp\left\{-\tau^2 \frac{a_0}{R_0^2}\right\}, \quad (56)$$

i.e., exponential “deflation” at $\tau \rightarrow \pm\infty$ for the brane tension of lightlike branes sitting on the Schwarzschild horizon.

Next, we consider Reissner-Nordström background metric in two different Kruskal-like coordinate systems of the general form (here again we take $D = p + 2 = 4$, *i.e.*, $i, j = 1, 2$):

$$ds^2 = A(y^2 - t^2) [-(dt)^2 + (dy)^2] + R^2(y^2 - t^2) g_{ij}(\vec{\theta}) d\theta^i d\theta^j. \quad (57)$$

The first one is appropriate for the region around the outer Reissner-Nordström horizon $R = R_{(+)}$, *i.e.*, for $R > R_{(-)}$, the latter being the inner $R = R_{(-)}$ Reissner-Nordström horizon:

$$y^2 - t^2 = \frac{R - R_{(+)}}{(R - R_{(-)})^{R_{(-)}^2/R_{(+)}^2}} \exp\left\{R \frac{R_{(+)} - R_{(-)}}{R_{(+)}^2}\right\}, \quad (58)$$

$$A(y^2 - t^2) = \frac{4R_{(+)}^4 (R - R_{(-)})^{1+R_{(-)}^2/R_{(+)}^2}}{(R_{(+)} - R_{(-)})^2 R^2} \exp\left\{-R \frac{R_{(+)} - R_{(-)}}{R_{(+)}^2}\right\}. \quad (59)$$

Accordingly, the second Kruskal-like coordinate system is appropriate for the region around the inner Reissner-Nordström horizon $R = R_{(-)}$, *i.e.*, for $R < R_{(+)}$:

$$y^2 - t^2 = \frac{R - R_{(-)}}{(R - R_{(+)})^{R_{(+)}^2/R_{(-)}^2}} \exp\left\{R \frac{R_{(-)} - R_{(+)}}{R_{(-)}^2}\right\}, \quad (60)$$

$$A(y^2 - t^2) = \frac{4R_{(-)}^4 (R_{(+)} - R)^{1+R_{(+)}^2/R_{(-)}^2}}{(R_{(-)} - R_{(+)})^2 R^2} \exp\left\{-R \frac{R_{(-)} - R_{(+)}}{R_{(-)}^2}\right\}. \quad (61)$$

Formula (44) for the brane tension in the case of (58)–(59) specializes to:

$$\chi(\tau) = \chi_0 \exp\left\{-\tau^2 \frac{a_0}{R_{(+)}^2} \left(1 - \frac{R_{(-)}}{R_{(+)}}\right)\right\}, \quad (62)$$

i.e., we find exponentially “deflating” tension for a lightlike brane sitting on the outer Reissner-Nordström horizon – a phenomenon similar to the case of lightlike brane sitting on Schwarzschild horizon (56). In the case of (60)–(61) formula (44) becomes:

$$\chi(\tau) = \chi_0 \exp\left\{\tau^2 \frac{a_0}{R_{(-)}^2} \left(\frac{R_{(+)}}{R_{(-)}} - 1\right)\right\}, \quad (63)$$

i.e., we obtain exponentially “inflating” tension for a lightlike brane occupying the inner Reissner-Nordström horizon – an effect similar to the exponential brane tension “inflation”

on de Sitter horizon (53). In the case of extremal Reissner-Nordström horizon, *i.e.* when $R_{(+)} = R_{(-)}$, where both “deflating” (62) and “inflating” (63) solutions should match, the only solution for the brane tension is the constant one $\chi = \chi_0$.

Finally, as an example for “inflation”/“deflation” behavior of the dynamical lightlike brane tension χ in cosmological-type embedding space-time (46) let us consider Friedman-Robertson-Walker metrics, *i.e.*, background metrics of the form (46), where (see *e.g.* [42]):

$$f(y) = y \quad , \quad f(y) = \sin y \quad , \quad f(y) = \sinh y \quad . \quad (64)$$

Solving Eqs.(47)–(48) yields for each choice (64) of $f(y)$ correspondingly:

$$f(y) = y \quad \rightarrow \quad y(\tau) = y_0 e^{c_0 \tau} \quad , \quad S(t) = \pm \frac{1}{c_0 y_0} e^{-c_0 t} \quad , \quad (65)$$

$$f(y) = \sin y \quad \rightarrow \quad y(\tau) = 2 \arctan \left(e^{c_0(\tau+\tau_0)} \right) \quad , \quad S(t) = \pm \frac{1}{c_0} \cosh (c_0(t + \tau_0)) \quad , \quad (66)$$

$$f(y) = \sinh y \quad \rightarrow \quad y(\tau) = \ln \frac{1 + e^{-c_0(\tau+\tau_0)}}{1 - e^{-c_0(\tau+\tau_0)}} \quad , \quad c_0 > 0 \quad , \quad S(t) = \mp \frac{1}{c_0} \sinh (c_0(t + \tau_0)) \quad , \quad (67)$$

where $y_0, \tau_0 = \text{const}$. Inserting the expressions (65)–(67) for $S(t)$ into Eq.(49) yields a time-asymmetric “inflation”/“deflation” of the brane tension χ at $\tau \rightarrow \pm\infty$, except for the “fine tuned” case $a_0 = \frac{1}{2p}$ where we get a constant $\chi = \chi_0$.

Let us recall that the metrics (46) with any of the three choices (64) for $f(y)$ and the corresponding expressions for $S(t)$ given by (65)–(67) represents de Sitter space-time in various coordinatizations different from the Gibbons-Hawking one (50)–(51) (here $|c_0| = K$ with c_0 and K from (65)–(67) and (51), respectively). Let us also stress the qualitative difference between the solutions for the brane tension of lightlike branes occupying de Sitter horizons: time-asymmetric “inflation”/“deflation” behavior (49) with exponential linear time dependence in Friedman-Robertson-Walker coordinates versus strictly “inflationary” behavior (53) with exponential quadratic time dependence in Gibbons-Hawking (Kruskal-like) coordinates.

V. DISCUSSION AND CONCLUSIONS

In the present paper we presented a systematic Lagrangian formulation of lightlike p -branes in arbitrary $(p + 1)$ world-volume dimensions, whose brane tension becomes an additional nontrivial dynamical degree of freedom. Further, we have shown that codimension

one lightlike branes can move in gravitational backgrounds of spherically symmetric type provided the latter possess event horizons and, moreover, these horizons are automatically occupied (“straddled”) by the lightlike branes.

For more conventional type of matter, a process known as “mass inflation” [4]-[5] leads to matter accumulation on certain horizons (like the inner Reissner-Nordström horizon) and, therefore, is somewhat similar to the phenomenon of automatic positioning of lightlike branes on black hole or cosmological horizons. For the standard “mass inflation” one defines a mass function (not related to the external mass of the black hole) which grows without bound as the matter focuses on the horizon. The natural analog of the mass function in the case of lightlike branes appears to be the dynamical brane surface tension. We study the time dependence of the dynamical brane tension of lightlike branes occupying diverse horizons.

Employing appropriate ansätze for various sets of Kruskal-like coordinates (Gibbons-Hawking coordinates [40] in the case of de Sitter space) we find solutions for the lightlike branes of (1) located at the inner Reissner-Nordström horizon or at the de Sitter cosmological horizon, respectively, such that the dynamical brane tension undergoes time-reflection symmetric “mass inflation”, *i.e.*, it approaches exponentially arbitrary large values for $\tau \rightarrow \pm\infty$. Although the present result for dynamical brane tension “inflation” at the inner Reissner-Nordström horizon parallels (except for the time-reflection symmetry here obtained) the known “mass inflation” phenomenon for standard matter, the accompanying result about brane tension “inflation” at de Sitter space horizon represents something totally new with no analog within the standard matter “mass inflation” and, therefore, it is a unique feature of lightlike branes.

In contrast, using the same ansätze with Kruskal-like coordinates, we find that lightlike branes undergo “mass deflation”, *i.e.*, their dynamical brane tension going to zero for $\tau \rightarrow \pm\infty$ when they are located at the outer Reissner-Nordström or the Schwarzschild horizon.

Other types of ansätze natural for standard coordinates show that for all kinds of horizons there are time-asymmetric “mass inflation” or “mass deflationary” solutions for the dynamical lightlike brane tension and, for a fine tuning – also solutions with constant brane tension do exist. In particular, for de Sitter horizon in cosmological (Friedman-Robertson-Walker) coordinates we obtain time-asymmetric “inflation”/“deflation” with exponential linear time dependence in contrast to the strict “mass inflation” at Sitter horizon in Gibbons-Hawking

(Kruskal-like) coordinates with exponential quadratic time dependence.

Let us stress that in the present paper we have discussed the properties of *LL-brane* dynamics as *test branes* moving in various gravitational backgrounds, *i.e.*, we have not taken into account the back-reaction of *LL-branes* on the geometry and the physical properties of the embedding space-time. In a forthcoming paper we are studying the important issue of self-consistent solutions for bulk gravity-matter systems (*e.g.*, Einstein-Maxwell-type) coupled to lightlike branes, *i.e.*, accounting for its back-reaction, where the latter: (i) serve as a source for gravity and electromagnetism, (ii) dynamically produce space-varying cosmological constant, and (iii) trigger non-trivial matching of two different geometries of de-Sitter/black-hole type across common horizon spanned by the lightlike brane itself. In fact, we have already started the above study in our previous papers [15],[19]-[21] in the simplest case of horizon matching of two different spherically-symmetric space-times where the pertinent lightlike brane occupying the common horizon has *constant* dynamical tension (“static soldering” in the terminology of ref.[10]). One of the physically interesting cases is a solution with de Sitter interior region with dynamically generated cosmological constant through the coupling to the *LL-brane*, and an outer region with Schwarzschild or Reissner-Nordström geometry, *i.e.*, a *non-singular* black hole.

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