

## CONSTRAINTS ON GAUSS-BONNET COSMOLOGIES

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The modified Gauss-Bonnet gravity can be motivated by a number of physical reasons, including: the uniqueness of a gravitational Lagrangian in four and higher dimensions and the leading order  $\alpha'$  corrections in superstring theory. Such an effective theory of scalar-tensor gravity has been modeled in the recent past to explain both the initial cosmological singularity problem and the observationally supported cosmological perturbations. Here I present an overview of the recent developments in the use of modified Gauss-Bonnet gravity to explain current observations, touching on key cosmological and astrophysical constraints applicable to theories of scalar-tensor gravity. The Gauss-Bonnet type modifications of Einstein's theory admits nonsingular solutions for a wide range of scalar-curvature couplings. It also provides plausible explanation to some outstanding cosmological conundrums, including: the transition from matter dominance to dark energy and the late time cosmic acceleration. The focus is placed here to constrain such an effective theory of gravity against the recent cosmological and astrophysical observations.

*Keywords:* String theory and cosmology, Gauss-Bonnet gravity, dark energy

### 1. Introduction

Einstein's general relativity has been very successful as a classical theory of gravitational interactions, especially, in a non-accelerating (or non-expanding) spacetime. In a cosmological background, the theory predicts spacetime singularities, so its modification is inevitable at high energy scales. Further the recently observed accelerating expansion of the universe <sup>1</sup> provides some insight to the possibility that general relativity together with ordinary matter and radiation, described by the standard model of particle physics, cannot fully explain the current observations. The question arises because the current observations <sup>2</sup> require in the fabric of the cosmos the existence of a dark energy component of magnitude about 73%, which does not 'clump' gravitationally. Another 23% of the mass-energy is in the form of mysterious non-baryonic dark matter, which 'clumps' gravitationally.

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The bulk of the universe appears to be dark energy and dark matter. So far there is no fully consistent explanation of these energy components supported by a fundamental theory. The main focus of this meeting is obviously to update our knowledge on DARK matter and DARK energy searches and the physics behind these. The focus of my presentation will be on a possible resolution of dark energy problem within some string-inspired theories of scalar-tensor gravity.

## 2. Accelerating universes and string-inspired models

The discovery that the expansion of the universe is currently accelerating is among the most tantalizing (and perhaps most mysterious) of recent times. Evidence in favour of this accelerated expansion (caused by putative dark energy) has led to a continued interest in scenarios that propose modifications to Einstein's general relativity. The proposals are of differing origins as well as motivations, some are based on theories of higher-dimensional gravity and others on consideration of one or more fundamental scalar fields and their interactions with higher-order curvature terms. Both these ideas are well motivated by supergravity and superstring theories, which incorporate Einstein's theory in a more general framework. There are several theoretical motivations to incorporate string theory into cosmological model building. Notably, gravitational interactions mediated by scalar fields, together with the standard graviton, are the best motivated alternatives<sup>3</sup> to general relativity, as they provide a mathematically consistent framework to test the various observable predictions of higher dimensional theories of gravity, such as, brane inflation.

Typically the low energy limit of string theory or supergravity features scalar fields and their couplings to a unique combination of the three quadratic scalars  $R^2$ ,  $R_{\mu\nu}R^{\mu\nu}$  and  $R_{\mu\nu\rho\lambda}R^{\mu\nu\rho\lambda}$ , composed of the scalar curvature, the Ricci and Riemann tensors:

$$\mathcal{R}^2 \equiv R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\lambda}R^{\mu\nu\rho\lambda},$$

known as the Gauss-Bonnet term. This term arises, almost universally, in all versions of string theory as the leading order  $\alpha'$  correction. An illustrative example is the following four-dimensional heterotic superstring model which describes the dynamics of graviton, dilaton  $S$  and the common (volume) modulus field  $T$ , arising from a compactification of 10D heterotic superstring theory on a symmetric 6D orbifold<sup>4</sup>:

$$\mathcal{L}_{\text{grav}} = \mathcal{L}_0 + \mathcal{L}_1, \quad (1)$$

where the string tree-level Lagrangian  $\mathcal{L}_0$  is

$$\mathcal{L}_0 = \frac{R}{2\kappa^2} - \zeta \frac{2\Delta S \Delta \bar{S}}{(S + \bar{S})^2} - \gamma \frac{2\Delta T \Delta \bar{T}}{(T + \bar{T})^2} + \frac{1}{8}(\text{Re}S)\mathcal{R}^2 + \frac{1}{8}(\text{Im}S) R\tilde{R}, \quad (2)$$

while the modulus  $T$  dependent Lagrangian at the one loop level is

$$\mathcal{L}_1 = \Delta(T, \bar{T}) \mathcal{R}^2 - i\Delta(T, \bar{T}) R\tilde{R}, \quad (3)$$

where  $\kappa$  is the inverse Planck mass  $M_P^{-1} = (8\pi G_N)^{1/2}$ ,  $G_N$  is Newton's constant,  $\zeta$  and  $\gamma$  are numerical constants,  $R\tilde{R} \equiv g^{-1/2} \epsilon^{\mu\nu\rho\lambda} R_{\mu\nu}{}^{\sigma\tau} R_{\rho\lambda\sigma\tau}$  (where  $\epsilon^{\mu\nu\rho\lambda}$  is a totally anti-symmetric tensor) and  $\Delta(T, \bar{T}) \propto \ln [(T + \bar{T})|\eta(iT)|^4]$ . The Dedekind  $\eta$ -function is given by  $\eta(iT) \equiv e^{-\pi T/12} \prod_{n \geq 1} (1 - e^{-2n\pi T})$ . There can be additional terms in the four-dimensional effective Lagrangian, such as,

$$\mathcal{L}_{\text{add}} = -V(S, T) - \dots \quad (4)$$

which includes, within the context of string theory, some supersymmetry breaking non perturbative potentials coming from the dynamics of branes, fluxes and orientifold planes, as well as the back reaction effects from the localized sources. The potential usually consists of sum of exponential terms determined by the fluxes and the curvature terms<sup>5,6</sup>; this is related to the fact that upon dimensional reduction of a gravity theory, the potential is exponential in terms of canonically normalized scalar fields descending from the internal space metric and other modes.

To evaluate field equations obtained by varying a gravitational action, we consider approximately homogeneous and isotropic solutions given by the Friedmann-Robertson-Walker metric:  $ds^2 = -dt^2 + a^2(t) d\mathbf{x}^2$ , where  $a(t)$  is the scale factor of the universe.  $H \equiv \dot{a}/a$  defines the Hubble parameter and the dot denotes a derivative with respect to cosmic time  $t$ .

In a flat FRW background, the terms proportional to  $R\tilde{R}$  give a trivial contribution. Defining  $\text{Re}S = e^\varphi/g_s^2$ ,  $\text{Re}T = e^{2\sigma}$ ,  $\text{Im}S \equiv \tau = \text{const}$  and  $\text{Im}T = 0$ , the four-dimensional effective Lagrangian may be given by<sup>4,7</sup>

$$\mathcal{L}_{\text{eff}} = \frac{R}{2\kappa^2} - \frac{\zeta}{2}(\nabla\varphi)^2 - \frac{\gamma}{2}(\nabla\sigma)^2 + \frac{1}{8}[\lambda f(\varphi) - \delta\xi(\sigma)] \mathcal{R}^2 - V(\varphi, \sigma), \quad (5)$$

where  $\lambda \propto 1/g_s^2$ ,  $g_s$  is four-dimensional string coupling,  $\tau$  is pseudoscalar axion and  $\delta$  is a numerical constant. To leading order in string loop expansion,  $f(\varphi) \propto e^\varphi$  and  $\xi(\sigma) = \ln 2 - \frac{\pi}{3}e^\sigma + \sigma + 4 \sum_{n=1}^{\infty} \ln(1 - e^{-2n\pi e^\sigma})$ . The latter implies that  $d\xi/d\sigma \simeq -\text{sgn}(\sigma) \frac{2\pi}{3} \sinh(\sigma) < 0$ . Several authors have explored special features of the string-derived Lagrangian that might provide some characteristic features of the above model (see for example<sup>7,8,9,10,11,12,13,14,15,16</sup>).

In the discussion below we consider the simplest case of a single modulus, under the assumption that the compactification modulus  $\sigma$  would rapidly evolve along an instantaneous minimum determined by the condition  $dV/d\sigma = 0$ , such that  $V(\varphi, \sigma) \approx \text{const} \times V(\varphi)$ , while  $\varphi$  attains a constant value only at late times<sup>a</sup>.

<sup>a</sup>This assumption may just be reversed and assume that the dilaton  $\varphi$  would evolve more rapidly as compared to  $\sigma$ ; these all depend on an underlying model. Of course, the single field description in terms of  $\varphi$  (or  $\sigma$ ) could underestimate the actual evolution of the universe at early epochs, like

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### 3. Modified Gauss-Bonnet theory

As should be clear from the above discussion, the simplest version of scalar-tensor theories, which is perhaps sufficiently general for explaining the present evolution of our universe<sup>17</sup>, may be given by<sup>18</sup>

$$\mathcal{S}_{\text{eff}} = \int d^4x \sqrt{-g} \left[ \frac{R}{2\kappa^2} - \frac{\zeta}{2} (\nabla\varphi)^2 - V(\varphi) + \frac{\lambda}{8} f(\varphi) \mathcal{R}^2 \right]. \quad (6)$$

Here  $f(\varphi)$  is a function that, although computable in concrete string models, may be taken to be general for the present purpose. The most desirable property of the above type modification of Einstein's theory is that only the terms which are the second derivatives of the metric (or their product) appear in field equations – a feature perhaps most important in order to make a gravitational theory absence of (spin-2) ghosts<sup>19</sup> – thereby ensuring the uniqueness of their solutions<sup>20</sup>. Of course, one can supplement the above action with other higher derivative terms, such as those proportional to  $(\nabla_\mu\varphi\nabla^\mu\varphi)^2$  and higher powers in  $R$ ,  $R_{\mu\nu}$  and  $R_{\mu\nu\rho\lambda}$ <sup>21,22</sup>, but in such cases it would only be possible to get special (asymptotic) solutions, so we limit ourselves to the above action.

Another important direction, which I will not review here, is the quest for a concrete construction of four-dimensional cosmology starting from some five-dimensional Gauss-Bonnet brane world models<sup>23,24,25</sup>. Of course, in spacetime dimensions  $D \geq 5$ , a pure Gauss-Bonnet term can lead to modification of Einstein field equation, even if  $f(\varphi) = \text{const}$ , and hence influence the four-dimensional cosmology defined on the 3-brane. Here we limit ourselves to the four-dimensional action and demand that  $f(\varphi)$  is dynamical. In this case the GB term  $\mathcal{R}^2$  is not topological, rather it can have an interesting dynamics, especially, on largest cosmological scales.

### 4. Cosmological perturbations and stability conditions

To explore the stability of an effective gravitational action, under large cosmological perturbations, one may consider the following perturbed metric about a flat Friedmann-Robertson-Walker (FRW) background:

$$ds^2 = -(1 + 2\varpi)dt^2 + 2a\partial_i\chi dx^i dt + a^2 [(1 + 2\psi)\delta_{ij} + 2\partial_{ij}\eta + 2h_{ij}] dx^i dx^j, \quad (7)$$

where  $\varpi, \chi, \psi, \eta$  denote scalar and  $h_{ij}$  denotes vector components of metric fluctuations, and  $\partial_{ij} \equiv \Delta_i\Delta_j - (1/3)\delta_{ij}\Delta^2$ . A remarkable property of the Gauss-Bonnet

during inflation, because string compactifications invariably involve more than one scalar field, and the four-dimensional potential depends, in general, on all the moduli field of the compactification. Nevertheless, this simple approximation in the string-derived Lagrangian holds some validity as a post-inflation scenario.

gravity is that the linearised action can be expressed (in the absence of matter fields) in the following explicit form <sup>26</sup>:

$$\delta^{(2)}\mathcal{S} \propto \int dt a^3 \left[ -A(t)\mathcal{R}\ddot{\mathcal{R}} + \frac{B(t)}{a^2}\mathcal{R}\Delta^2\mathcal{R} \right], \quad (8)$$

where  $\mathcal{R}$  is a gauge invariant quantity:

$$\mathcal{R} \equiv \psi - \frac{H}{\dot{\varphi}} \delta\varphi, \quad (9)$$

so-called a comoving perturbation. For a linearized theory to be free of ghost and superluminal modes, the following conditions

$$A(t) > 0, \quad B(t) > 0, \quad (10)$$

known as stability conditions, should perhaps be satisfied. For quantum stability of (inflationary) solutions the speeds of propagation of scalar and tensor modes should also remain non-superluminal:

$$0 < c_{\mathcal{R}}^2 = 1 + \frac{\mu^2 \left[ 4\epsilon(1-\mu) - \lambda\kappa^2 (\ddot{f} - \dot{f}H) \right]}{(1-\mu)(2\zeta(1-\mu)\varphi'^2 + 3\mu^2)} \leq 1, \quad (11)$$

$$0 < c_T^2 = \frac{1 + \lambda\kappa^2 \ddot{f}}{1 + \lambda\kappa^2 \dot{f}H} = \frac{1 - \mu' + \epsilon\mu}{1 - \mu} \leq 1, \quad (12)$$

where  $\epsilon \equiv \dot{H}/H^2 = H'/H$ ,  $' \equiv d/d \ln a$  and  $\mu \equiv -\lambda\kappa^2 \dot{f}H = \Omega_f$ . In fact,  $A(t), B(t) < 0$  implies a violation of unitarity, while  $B(t) > A(t)$  implies the existence of a superluminal propagation or an ill-defined Cauchy problem. Moreover, in the case  $A(t)B(t) < 0$ , the system of equations could exhibit an exponential type of instability, leading to an imaginary  $c_T$ . Below we use the above relations for studying the stability of inflationary solutions.

## 5. Inflationary constraints

One may put constraints on the strength of the coupling  $f(\varphi)$  by considering observational limits on the spectral indices of scalar and tensor perturbations. For the present theory, and in the limit that  $c_{\mathcal{R}}^2, c_T^2 \approx \text{const}$ , the spectral indices  $n_{\mathcal{R}}$  and  $n_T$  are approximated by <sup>26,27,28</sup>

$$n_{\mathcal{R}} - 1 = 3 - \left| \frac{3 + \epsilon_1 + 2\epsilon_2 + 2\epsilon_3}{1 - \epsilon_1} \right|, \quad n_T = 3 - \left| \frac{3 - \epsilon_1 + 2\epsilon_4}{1 - \epsilon_1} \right|. \quad (13)$$

where  $\epsilon_1 \equiv -\frac{\dot{H}}{H^2} = -\epsilon$ ,  $\epsilon_2 = \frac{\ddot{\varphi}}{\dot{\varphi}H}$ ,  $\epsilon_3 = \frac{\theta'}{2\theta}$ ,  $\epsilon_4 \equiv -\frac{\mu'}{2(1-\mu)}$ ,  $\theta \equiv \zeta + \frac{3\mu^2}{2(1-\mu)\varphi'^2}$  and  $\mu \equiv -\lambda\kappa^2 \dot{f}H$ . One more quantity of cosmological relevance is the tensor-to-scalar ratio, which, in the limit  $|\epsilon_1| \ll 1$ , is approximated by

$$r \approx 16 \frac{2\zeta x^2(1-\mu) + 3\mu^2}{(2-\mu)^2} \left( \frac{c_{\mathcal{R}}}{c_T} \right)^3. \quad (14)$$

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WMAP data alone puts the constraints  $0.94 < n_{\mathcal{R}} < 0.98$  and  $r < 0.28$  for a single scalar field model.

## 6. Non-singular inflationary solutions

### 6.1. Absence of scalar potential

Let us consider a non-singular inflationary solution obtainable by dropping the scalar potential. To quantify this, one sets  $V(\varphi) = 0$ . One also defines

$$\mathcal{F}(\varphi) \equiv -\lambda f(\varphi)H^2. \quad (15)$$

The magnitude of  $\mathcal{F}$  should decrease with the expansion of the universe, so that all higher-order corrections to Einstein's theory become only sub-leading<sup>b</sup>. With  $\mathcal{F} \equiv \mathcal{F}_0$ , the explicit solution is given by

$$\frac{\dot{H}}{H^2} = -\mathcal{A} + \mathcal{B} \tanh \mathcal{B}(N + C), \quad (16)$$

where  $N \equiv \ln a$ ,  $C$  is an integration constant and

$$\mathcal{A} \equiv \frac{5\mathcal{F}_0 + 1}{2\mathcal{F}_0}, \quad \mathcal{B} \equiv \sqrt{\mathcal{A}^2 - 6\mathcal{A} + 15} \quad (17)$$

The Hubble parameter is  $H \propto e^{-\mathcal{A}N} \cosh \mathcal{B}(N + C)$ . The  $\mathcal{F}_0 > 0$  solution, which allows  $\dot{H} > 0$ , supports a super-luminal expansion, see also<sup>29</sup>. It is possible to get a red-tilted scalar index ( $n_{\mathcal{R}} < 1$ ) for  $\mathcal{F}_0 > -2/3$ .

### 6.2. Inflating with an exponential potential

Consider that  $V(\varphi) \propto e^{-\beta(\varphi/\varphi_0)}$  and  $f_{,\varphi}H^2 \propto \varphi'$ ; the latter choice is motivated by the fact that the coupling takes the form  $f(\varphi) \propto e^{\beta(\varphi/\varphi_0)}$  in the limit  $\varphi' \rightarrow \text{const}$ , or after a few e-folds of inflation. The explicit solution is

$$\varphi = \frac{2}{\beta}\varphi_0 \ln \frac{a/a_i}{\cosh \chi \ln(a/a_i)} + \text{const}, \quad \frac{\dot{H}}{H^2} = \frac{2\zeta\varphi'^2}{6 + \zeta\varphi'^2} - \frac{\beta}{\varphi_0}\varphi'^2, \quad (18)$$

where  $\chi \equiv \sqrt{(2\zeta\varphi_0^2 - 3\beta^2)/2\zeta\varphi_0^2}$ ,  $\varphi' \equiv d\varphi/d(\ln a) = \dot{\varphi}/H$  and  $a_i$  is the initial value of scale factor  $a(t)$ . From this solution we can easily evaluate the indices  $n_{\mathcal{R}}$  and  $n_T$ , using (13). The observation in Fig. 1 that  $n_{\mathcal{R}} \simeq 3$  at some early stage of inflation is not quite correct since in that region  $c_{\mathcal{R}}^2$  and  $c_T^2$  are varying considerably, for which there would be non-trivial corrections to the formulae (13). A result consistent with the WMAP data (e.g.  $n_{\mathcal{R}} \simeq 0.96$  and  $n_T < 0.2$ ) can be obtained for  $|\beta/\varphi_0| < \sqrt{\zeta}/4$ .

<sup>b</sup>Particularly in the case  $\mathcal{F} \simeq \text{const} \equiv \mathcal{F}_0$ , the coupled term  $\lambda f(\varphi)\mathcal{R}^2$  is subleading to the Einstein term  $R/2\kappa^2 = 3M_{\text{P}}^2(2H^2 + \dot{H})$  for  $|\mathcal{F}_0| \ll 1$  or  $\lambda \ll 1$ .

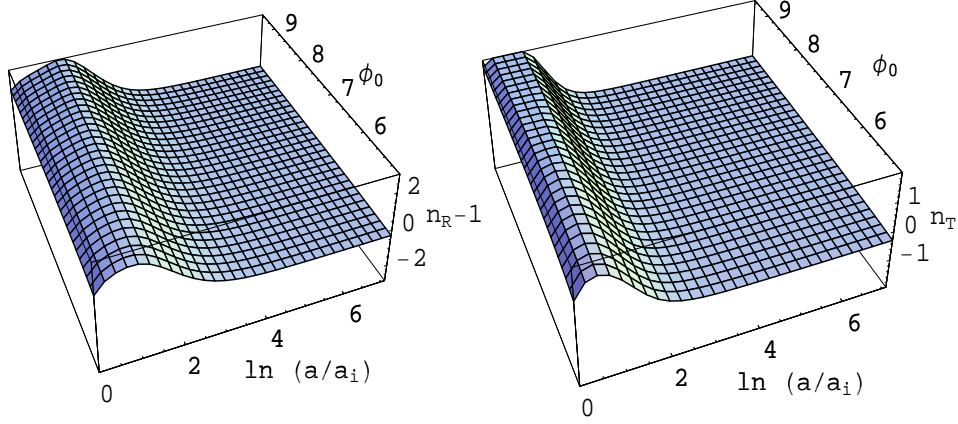


Fig. 1. The spectral indices  $n_R$  and  $n_T$  as the functions of  $\phi_0 = 2\sqrt{\zeta}\varphi_0/\beta$  and  $\ln(a/a_i)$ .

## 7. Matter-scalar couplings

For constructing a realistic late time cosmology, one should consider the ordinary fields (matter and radiation) and also their natural interactions with the scalar field  $\varphi$ .

### 7.1. Minimally coupled scalar field

In a flat FRW spacetime, the Gauss-Bonnet term  $\mathcal{R}^2$  vanishes only at the stage of zero acceleration, and it flips its sign once the universe begins to accelerate. This effect can overturn the slope of the effective potential:

$$\Lambda(\varphi) \equiv V(\varphi) - \frac{\lambda}{8} f(\varphi) \mathcal{R}^2,$$

and push the universe transiently to a phantom era. Such an effect can be seen by considering the effective equation of state:

$$w_{\text{eff}} \equiv -1 - \frac{2\dot{H}}{3H^2} = \frac{p_{\text{tot}}}{\rho_{\text{tot}}} = w_m \Omega_m + w_r \Omega_r + w_\varphi \Omega_\varphi, \quad (19)$$

where  $m = \text{matter}$  and  $r = \text{radiation}$ . With the assumption that the ordinary matter is approximated by a non-relativistic perfect fluid (i.e.  $w_m \simeq 0$  and  $\Omega_r \ll 1$ ), we find  $w_{\text{eff}} \simeq w_\varphi \Omega_\varphi$ . A simple calculation shows  $\rho_\varphi + p_\varphi = \zeta \dot{\varphi}^2 + \lambda H^2 (\ddot{f} - \dot{f}H) + 2\lambda H \dot{H} \dot{f}$ <sup>18</sup>. To this relation, the stability conditions  $1 > \kappa^2 |\lambda \dot{f}|$  and  $|\ddot{f}| \geq |\dot{f}H|$  may be imposed, so as to keep the propagation speed of tensor and scalar modes non-superluminal. Nevertheless, it is possible to get  $p_\varphi + \rho_\varphi < 0$ , or  $w_\varphi < -1$ , without making the cosmic expansion superluminal, or violating the condition  $\dot{H} \leq 0$ . This simple picture has obvious and intuitive appeal.

## 7.2. Non-minimally coupled scalar field

The constraints on the modified Gauss-Bonnet gravity may arise by two different dynamics: one is the standard interaction effect between the scalar field  $\varphi$  and the Gauss-Bonnet term, while the other is the effect of nonminimal coupling between the scalar field  $\varphi$  and matter. The latter effect might perhaps be more significant than the former, especially, while applying the model into high density regions, or solar system experiments. To this reason, let us write the matter Lagrangian in a general form:

$$\mathcal{S}_{\text{matter}} = \mathcal{S}(A^2(\varphi)g_{\mu\nu}, \psi_m), \quad (20)$$

where  $A(\varphi)$  measures the response of the geometry due to a time-variation of the field  $\varphi$ . Ordinary fields (matter and radiation) couple to  $A^2(\varphi)g_{\mu\nu}$  rather than the Einstein metric  $g_{\mu\nu}$  alone. Indeed,  $\varphi$  couples to the trace of the matter stress tensor,  $g_{(i)}^{\mu\mu}T_{\mu\nu}^{(i)}$ , so the radiation term (for which  $w_r = 1/3$ ) does not contribute to the (Klein-Gordon) equation of motion for  $\varphi$ :

$$\dot{\rho}_\varphi + 3H\rho_\varphi(1 + w_\varphi) = -\dot{\varphi}(1 - 3w_i)\alpha_\varphi A(\varphi)\rho_m, \quad (21)$$

where  $\rho_\phi \equiv \frac{\zeta}{2}\dot{\phi}^2 + V(\phi) - 3\lambda H^3 \dot{f}$ ,  $w_\phi \equiv p_\phi/\rho_\phi$  and  $w_i \equiv p_i/\rho_i$ . In order for current experimental limits on verification of the equivalence principle to be satisfied, the quantity

$$\alpha_\varphi \equiv \frac{d \ln A(\varphi)}{d(\kappa\varphi)}, \quad (22)$$

which measures the coupling of  $\varphi$  to background (baryonic and dark) matter, must be much smaller than unity, at least, on cosmological scales. The local GR constraints on  $\alpha_\varphi$  and its derivatives imply that <sup>34</sup>

$$\alpha_\varphi^2 \leq 4 \cdot 10^{-5}, \quad \beta_\varphi = \frac{d\alpha_\varphi}{d\varphi} > -4.5. \quad (23)$$

On large cosmological scales, where  $\rho_m \lesssim \rho_\varphi$  and  $\kappa^2 V(\varphi) \sim 3H_0^2$ ,  $\varphi$  is expected to be sufficiently light, as for *quintessence*,  $m_\varphi \equiv \sqrt{V_{\varphi\varphi}} \sim 10^{-33}$ eV, and the term on r.h.s. of eq. (21) may be safely ignored. In fact, in ref. <sup>35</sup>, smallness of  $\alpha_\varphi$  was found to be linked to the smallness of the horizon-scale cosmological density fluctuation,  $\delta\rho/\rho \sim 5 \times 10^{-5}$  (at the surface of last scattering). However, in high density regions, or within galactic distances,  $\delta\rho/\rho \gg 10^{-5}$  and  $\varphi$  can be massive, like  $m_\varphi \gtrsim 10^{-3}$ eV, in which case the observable deviations from Einstein's gravity are normally quenched on distances larger than a fraction of millimeter.

If  $A(\varphi)$  is sufficiently flat near the current value of  $\varphi = \varphi_0$ , then the matter-scalar coupling can have only modest effects on cosmological scales. Especially, in the case that  $A(\varphi) \propto e^{Q(\varphi/M_P)}$ , the above GR constraints may be satisfied only for a small  $Q$  ( $\ll 1$ ). This restriction on the slope (or strength) of matter-scalar coupling



may not apply to a gravitationally bound system, or in high density regions, where the field  $\varphi$  is not essentially light or weakly coupled to matter degrees of freedom (of the standard model). The latter argument is actually consistent with ideas widely used in recent experiments aimed to detect axion-like particles.

### 8. Late-time cosmology

Making just one simplifying assumption that  $\varphi \equiv \varphi_0 \ln[a(t)] + \text{const}$ , and then inverting the field equations following from (6), we find <sup>7</sup>

$$f(\varphi) = -f_0 e^{\beta(\varphi/\varphi_0)} - f_1, \quad V(\varphi) = \frac{2(\delta - 1)}{3\lambda\kappa^4} \left( \frac{df(\varphi)}{d\varphi} \right)^{-1} \equiv V_0 e^{-\beta(\varphi/\varphi_0)}, \quad (24)$$

where  $\beta = 1 + 3\delta$  and  $\delta = \kappa^2\varphi_0^2/2$ . These simplest choices for the potential and the scalar coupling admit the following simple solution <sup>28</sup>

$$a(t) = a_0 t^{2/\beta}, \quad (25)$$

satisfying the relations:

$$\lambda f_0 = \frac{(\beta - 2\zeta\delta)\beta}{2\kappa^2(\beta + 2)}, \quad V_0 = \frac{24(2 - \beta) + 8\zeta\delta(10 - \beta)}{(\beta + 2)\beta^2\kappa^2}. \quad (26)$$

Acceleration requires  $\beta < 2$  (if it is to be future eternal); thus, for the model to provide a solution to the dark energy problem, the strength of the GB coupling must grow with time,  $f(\varphi) \propto e^{\beta(\varphi/\varphi_0)} \propto a(t)^\beta$ . This is actually consistent with superstring models studied in <sup>4,8,18</sup>. One should, however, note that a growing  $f(\varphi)$  does not necessarily mean that the term  $f\mathcal{R}^2$  will dominate at late times the potential and/or the Einstein-Hilbert term. In the present universe  $H_0 \sim 10^{-60} M_P$ , which leads to  $\kappa^2 V \simeq 10^{-120} M_P^2$ ,  $R/6 \simeq H_0^2 \sim 10^{-120} M_P^2$  and  $\kappa^2 f\mathcal{R}^2 \propto V^{-1} H_0^4 \equiv f_0 e^{-120} M_P^2$ . For  $f_0 \ll 1$ ,  $f\mathcal{R}^2$  is only subleading to  $V$  and  $R/\kappa^2$ . Terms higher powers in Ricci scalar ( $R^n$  with  $n \geq 3$ ) contribute with  $H_0^{2n}$  and are thus subleading to  $f(\varphi)\mathcal{R}^2$ .

Let us consider a specific model for which the dark energy equation of state becomes less than  $-1$ , but only transiently. This example is provided by the choice  $V(\varphi) \propto e^{-\beta(\varphi/\varphi_0)}$  and  $f_{,\varphi} H^2 \propto \varphi'$ . In this case the explicit solution is given by (18). One may take  $a_i = a_0 \equiv 1$ , so that  $a(t) < 1$  in the past. As shown on the left panel of Fig. 2, the equation of state  $w \equiv -1 - \frac{2\dot{H}}{3H^2}$  becomes less than  $-1$ , but only transiently, for  $\phi_0 \equiv 2\sqrt{\zeta}\varphi_0/\beta \gtrsim 5$ . This behaviour may be seen also in the presence of matter field, see ref. <sup>28</sup> for details.

The right panel of Fig. 2 represents a characteristic evolution of the universe for which the Gauss-Bonnet term never becomes dominant, or it contributes only subdominantly. In this plot the coupling  $f(\varphi)$  has been chosen such that  $\varphi' f_{,\varphi} H^2 \simeq \text{const}$  and the Gauss-Bonnet energy density fraction is (almost) constant,  $\Omega_f \sim 10^{-6}$ . With such a small contribution of the coupled GB term  $\lambda f(\varphi)\mathcal{R}^2$ , almost every

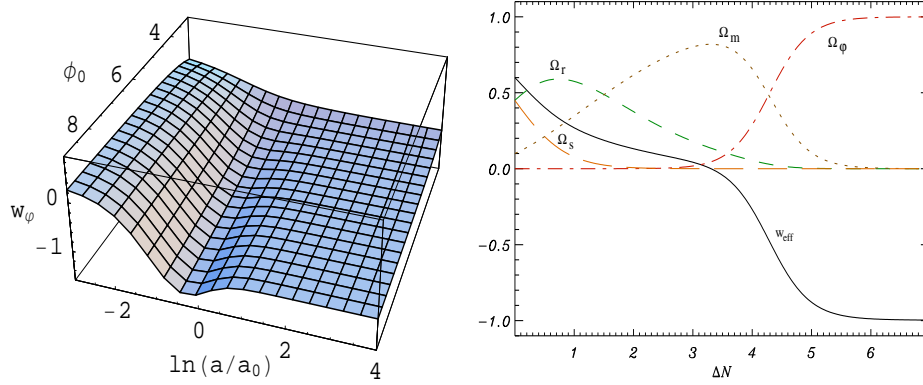


Fig. 2. (Left plot) The dark energy equation of state as a function of  $\phi_0$  and  $\ln(a/a_0)$ . (Right plot) The evolution of the fractional densities:  $\Omega_m$  (dots, brown),  $\Omega_r$  (dashes, green),  $\Omega_s$  (long dashes, orange),  $\Omega_\phi$  (dot-dash, red) and  $\Omega_{\text{GB}} \equiv \Omega_f$  (dot-dot-dot-dash, blue) and the effective equation of state  $w_{\text{eff}} \equiv -1 - 2\dot{H}/3H^2$ , with  $\zeta = 1$ ,  $(\beta/\varphi_0) = 3$ ,  $\alpha_\varphi^2 = 10^{-5}$  (dust) and  $\alpha_\varphi^2 = 10^{-2}$  (stiff matter). The initial values are  $\Omega_f^i = 10^{-6}$ ,  $\varphi_i' = 10^{-7} M_P$ ,  $(V/H^2)_i = 3 \times 10^{-15}$ .  $\Delta N \equiv \ln a + C$ ; here  $C$  may be chosen such that  $\ln a = 0$  corresponds to  $\Omega_m \simeq 0.27$  and  $\Omega_\phi + \Omega_f \simeq 0.73$ .

constraints on the model may be satisfied, including the BBN bound ( $\Omega_\phi(1 \text{ MeV}) < 0.1$ ) and solar system constraints (see below).

We can construct an explicit model by using the parametrization  $f(\varphi) \equiv f_0 e^{\alpha(\varphi/\varphi_0)}$  and  $V(\varphi) \equiv V_0 e^{-\beta(\varphi/\varphi_0)}$ <sup>30</sup> and also replicate many observable properties of the universe from nucleosynthesis to the present epoch<sup>31,28</sup> (see also<sup>32</sup>). A possible drawback of this simple parametrization is, however, that, especially, for large slope parameters, like  $\alpha > \beta \gtrsim \sqrt{3}(\varphi_0/M_P)$ , the model may exhibit some kind of semi-classical instabilities associated with the linearized inhomogeneities or quantum fluctuations that grow explosively as the limit  $c_T^2 < 0$  is approached or the tensor modes start to propagate faster than light's velocity<sup>33</sup>.

This rather undesirable feature of the model is indeed related to the fact that, for  $\alpha \gg \beta$ , the contribution of GB term become appreciable (or non-negligible) at recent times (or even in far future) but only transiently. In the case GB contribution becomes appreciable, even momentarily, one normally observes an oscillatory crossing of  $w_\phi = -1$ . Generally, the amplitude of these oscillations corresponds to the amplitude of the oscillations seen in the Gauss-Bonnet contribution and hence is heavily dependent on the slope of the scalar-GB coupling,  $\alpha$ . For large  $\alpha$  one may observe much larger oscillations, which, however, disappear when the GB contribution becomes negligibly small, and settle to a late time evolution for which  $w_\phi \approx -1$ . In most cases, this limit is approached from above, so the issue inherent with a super-inflation or a violation of unitarity may not be applicable to late time

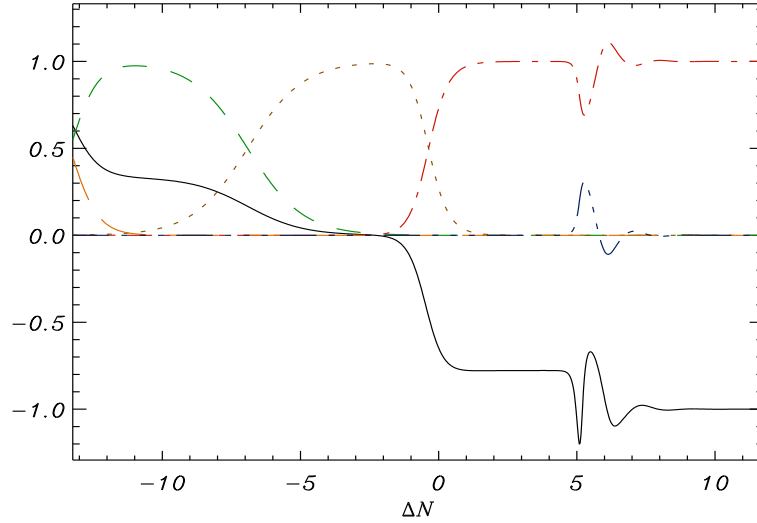


Fig. 3. As in the right panel of Fig. 2, but with the parametrization  $f(\varphi) \propto e^{\alpha(\varphi/\varphi_0)}$ , and the choice  $\alpha = 12\varphi_0 \gg \beta = \sqrt{2/3}\varphi_0$ . For the large  $\alpha$  it is not unnatural that the coupled Gauss-Bonnet term  $f(\varphi)\mathcal{R}^2$  becomes significant (non-negligible) at recent times, or even at distant future. Here  $\Delta N \equiv \ln(a/a_0)$ , we normalize the scale factor such that  $\ln a = 0$  at  $a_0 = 1$ .

cosmologies. At any rate, the appearance of a superluminal mode, though not inevitable, could actually imply that one would have to invoke modifications of the simplest exponential parametrization or should allow only small slope parameters.

### 9. Time-variation of fundamental constants

Scalar-tensor theories of gravity also entertain the result that some of the fundamental constants of nature may vary with time, including the Newton's constant, which are however tightly constrained by observations. On large cosmological scales, it is reasonable to assume that  $A(\varphi) = \text{const}$ . In this case, the growth of matter fluctuations in the Gauss-Bonnet theory can be expressed in the following standard form:

$$\ddot{\delta} + 2\dot{\delta}H = 4\pi G_* \rho_m \delta, \quad (27)$$

where the normalized Newton's constant  $G_*$  may be given by <sup>36</sup>

$$G_* = G \left[ 1 + 3\Omega_f - \frac{\dot{\varphi}}{H} \left( \frac{\ddot{\varphi}}{\dot{\varphi}^2} + \frac{f_{\varphi\varphi}}{f_{\varphi}} \right) \Omega_f \right], \quad (28)$$

where  $\Omega_f \equiv -\lambda\kappa^2\dot{\varphi}Hf_\varphi$ . Unlike the slow roll relations  $\ddot{\varphi}/\dot{\varphi}, \dot{\varphi} \ll 1$ , the ratios like  $\ddot{\varphi}/\dot{\varphi}^2$  and  $\ddot{f}/\dot{f}$ , which appear in the expression

$$\frac{f_{\varphi\varphi}}{f_\varphi} = \frac{d^2f/d\varphi^2}{df/d\varphi} = \frac{H}{\dot{\varphi}} \left( \frac{\ddot{f}}{\dot{\varphi}\dot{f}} - \frac{\ddot{\varphi}}{\dot{\varphi}^2} \right) \quad (29)$$

can be of order unity (in units  $M_P = 1$ ). It is not improbable that  $G_* \approx G$  for present value of the field,  $\varphi_0$ , and the coupling,  $f(\varphi_0)$ . In fact, almost every models of scalar-tensor gravity behave as Einstein's GR supplemented with a cosmological constant term  $\Lambda$ , if  $\varphi' = \frac{\dot{\varphi}}{H} \ll M_P$  holds (at least) after the epoch of big bang nucleosynthesis. In the particular case that  $f(\varphi) \propto e^{\beta(\varphi/\varphi_0)}$ , we obtain

$$G_* = G [1 + \lambda f(\varphi) H^2 (\varphi'' + \epsilon\varphi' - 2\varphi')]. \quad (30)$$

Thus one should satisfy, at least, one of the following conditions: (i)  $|\lambda| \ll 1$ , (ii)  $|f(\varphi)|H^2 \ll 1$ , or (iii)  $|\varphi'| = |\dot{\varphi}/H| \ll M_P$ , in order to get  $G_* \simeq G$  at present. For a specific model studied in <sup>35</sup>, a safe upper bound is found to be  $|\varphi'_0| < 0.84 M_P$ . Nevertheless, within solar system and laboratories distance, there exists a more stronger bound that  $(dG_*/dt)/G_* < 0.01 H_0$  (where  $H_0$  is the Hubble expansion rate at present). This last condition translates to the constraint  $|G_{\text{now}} - G_{\text{nucleo}}|/G_{\text{now}}(t_{\text{now}} - t_{\text{nucleo}}) < 10^{-12}\text{yr}^{-1}$ . The quantity  $dG_*/dt$  is actually suppressed (as compared to  $G_*$ ) by a factor of  $\dot{\varphi}/H$ , so it is necessary to satisfy  $\varphi' \ll M_P$ , at least, on large cosmological scales. Another opportunity for the model to overcome local gravity constraints coming from GR is to have a coupling  $f(\varphi_0)$  which is nearly at its minimum. This is very much the approach one takes in a standard scalar-tensor theory.

## 10. Further constraints

The growth of matter perturbations and the integrated Sachs-Wolfe (ISW) effect are the other effective ways of constraining the model under consideration <sup>36</sup>. In the case  $\lambda f(\varphi)\mathcal{R}^2$  is subdominant to  $V(\varphi)$  (thus  $|\Omega_f| \ll 1$ ), the matter growth factor may be approximated by

$$\left( \frac{\dot{\delta}}{\delta} \right)_{\text{EGB}} \approx \left( \frac{\dot{\delta}}{\delta} \right) \left[ 1 - \left( 1 + \frac{H'}{H} \right) (1 + 0.75 \Omega_m) \Omega_f \right]. \quad (31)$$

In an accelerating spacetime, so  $H'/H \geq -1$ , the Gauss-Bonnet coupling decreases the matter growth factor (as compared to the standard  $\Lambda$ CDM), for  $\Omega_f > 0$ . In view of the observational uncertainty in the growth rate of large scale structures <sup>37</sup>,  $f_{\text{struc}} \equiv \left( \dot{\delta}/\delta \right) = 0.51 \pm 0.1$ , the Gauss-Bonnet energy density fraction  $\Omega_f$  should perhaps not exceed 20% <sup>36</sup>. This last result may apply only to largest cosmological scales, and it is, by no means, applicable to gravitationally bound systems, such as, our solar system.

Under the post-Newtonian approximation:

$$ds^2 = -(1 + 2\Phi)(cdt)^2 + (1 - 2\Psi)\delta_{ij}dx^i dx^j \quad (32)$$

where  $\Phi, \Psi \sim \mathcal{O}(GM/rc^2)$ , the solar system constraints appear to be stronger than astrophysical constraints, mainly, due to a small fractional anisotropic stress<sup>38</sup> (see also<sup>39</sup>):

$$\hat{\gamma} - 1 \equiv \frac{\Psi - \Phi}{\Phi} \approx 2\Omega_f \left( 1 - \frac{\dot{\varphi}}{H} \left( \frac{\ddot{\varphi}}{\dot{\varphi}^2} + \frac{f_{\varphi\varphi}}{f_{\varphi}} \right) \right) < 4 \times 10^{-5}. \quad (33)$$

When applied to solar system distances, the above expression demands that  $\Omega_f \lesssim 10^{-5}$ . There remains the possibility that the classical tests of Newtonian gravity, which typically deal with small perturbations in fixed (or time-independent) backgrounds are almost unaffected by the GB type modification of Einstein's theory.

## 11. Conclusions

The important ingredient of the present approach to dark energy cosmology is the treatment of gravitational coupling between the dynamical scalar field  $\varphi$  and the quadratic Riemann invariant of the Gauss-Bonnet form, which gives rise to nonsingular cosmologies for a wide range of scalar-curvature couplings. The model also provides plausible explanation to some outstanding cosmological conundrums, including: the transition from matter dominance to dark energy and the late time cosmic acceleration. Furthermore, the scalar-curvature coupling can easily trigger onset of a late dark energy domination. Despite these promising signs, it remains to be checked whether the Gauss-Bonnet modification of Einstein's theory will lead to genuine contact between observations and string theory.

String theory is known to be free from ghosts and superluminal modes, at least, in a flat ten-dimensional Minkowski background. This is perhaps not essentially the case in a four-dimensional FRW background. The effective string actions in four dimensions may well exhibit some unwarranted features, such as, short scale instabilities and superluminal propagation of tensor or scalar modes, under inhomogeneous (cosmological) perturbations. The model discussed here is perhaps not an exception; at least, for the potential and the Gauss-Bonnet coupling in simplest exponential forms, one could see that the tensor or scalar modes propagate at a speed faster than light at some stage, especially, for large slope parameters. The appearance of a superluminal mode is generic, and perhaps also acceptable, if such an effect is only transient.

In the present proposal for explaining a crossing of cosmological (dark energy) equation of state,  $w_{DE} = -1$ , and a superluminal propagation of scalar or tensor modes, a number of important, physically falsifiable predictions can be made. These include a transient violation of Lorentz symmetry and the weak equivalence

principle, associated with the microscopic effects of the coupling between  $\varphi$  and background matter in high density regions.

Ninety years after Einstein's proposition of general relativity with a cosmological constant, a modified cosmological scenario with its natural generalization is close to experimental test and possibly an outlet compatible with present experimental data. The coming generation of cosmological experiments, including Dark Energy Survey, will probably rule out the great majority of string-derived models, as well as exclude those class of scalar-tensor theories which give rise to unphysical states. *Time will tell.*

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