K fields, compactons, and thick branes

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Abstract

K fields, that is, fields with a non-standard kinetic term, allow for soliton solutions with compact support, i.e., compactons. Compactons in 1+1 dimensions may give rise to topological defects of the domain wall type and with finite thickness in higher dimensions. Here we demonstrate that, for an appropriately chosen kinetic term, propagation of linear perturbations is completely suppressed outside the topological defect, confining the propagation of particles inside the domain wall. On the other hand, inside the topological defect the propagation of linear perturbations is of the standard type, in spite of the non-standard kinetic term. Consequently, this compacton domain wall may act like a brane of finite thickness which is embedded in a higher dimensional space, but to which matter fields are constrained. In addition, we find strong indications that, when gravity is taken into account, location of gravity in the sense of Randall–Sundrum works for these compacton domain walls. When seen from the bulk, these finite thickness branes, in fact, cannot be distinguished from infinitely thin branes.

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1 Introduction

In the last two decades the notion of a universe with more than three space dimensions has excited some interest both in cosmology and in theoretical high energy physics. One way to reconcile this idea with the observational fact that only three space dimensions are perceived is the assumption that the additional dimensions are too small to be detected. Another possibility is to assume that matter and (at least, non-gravitational) interactions are restricted to a lower dimensional (concretely, three dimensional) subspace. This subspace may be either strictly lower dimensional, in which case the name "three-brane" has become customary. Some recent reviews about these cosmological three-branes can be found, e.g., in [1] - [3], which also serve as sources for further references on the subject (which are too numerous to be quoted here). Or the subspace may have a finite, although probably very small, extension in the additional dimensions, that is, it is of the topological defect type [4], [11]. In the latter case, the name "thick brane" has been coined recently for this subspace on which the matter propagation takes place (see, e.g., [6] - [10] for some recent work). At least for a "thick brane", a dynamical mechanism certainly has to be identified which provides the confinement of all matter fields to the subspace. It is the purpose of this letter to provide an alternative mechanism. Our proposal bears some similarity to the work of others (e.g. [11]), in the sense that the "thick brane" is formed by a topological defect, but the mechanism of confinement of the matter fields is different. The basic idea is, in fact, very easy to understand. Usually, the restriction of the propagation to a topological defect is achieved by a potential which becomes very strong outside this topological defect. But the propagation of a field is generally determined by a balance between potential and kinetic terms. Another possibility for restricting the propagation to the topological defect is, therefore, a kinetic term which becomes very small away from the defect, thereby enforcing that the field takes its vacuum value in that region. This is precisely what may happen in the case of K fields, that is, field theories with a non-standard kinetic term.

K fields already play a rather prominent role in cosmology, where they offer a possible mechanism for early time inflation (under the name of K inflation) [12] - [14], as well as a possible explanation both for the value of the cosmological constant (problem of smallness, coincidence), and for the late-time acceleration ([15] - [18]), which has been found to be a property of our universe in the last decade. In the latter case, these models are nowadays known as K essence theories.

2 The model

To illustrate our proposal, we shall consider the simplest possible setting where space time is 4+1 dimensional Minkowski space, and the topological defect will be a simple domain wall which is effectively three dimensional. The field theory is given by the Lagrangian density

$$\mathcal{L} = 4|X|X - V(\xi) \tag{1}$$

where

$$X \equiv \frac{1}{2}\partial_M \xi \partial^M \xi, \tag{2}$$

$$V(\xi) \equiv 3\lambda^4(\xi^2 - a^2)^2, \tag{3}$$

ξ is a scalar field, λ and a are positive constants, and $M = 0, \ldots, 4$. Further, we use the Minkowski metric $g_{MN} = \mathrm{diag}(1, -1, -1, -1, -1)$. The choice of the non-standard kinetic term equal to $4|X|X$ instead of $4X^2$ is not important for the purpose of this letter (i.e., for the existence of the compacton solution and for the linear perturbation analysis), but it *is* important for the global stability of the field theory (1). For the kinetic term $4X^2$, the energy is not bound from below, see [19] for a detailed discussion.
A first fact about the theory (1) which we need is that when restricted to 1+1 dimensions, it has soliton solutions with compact support. Indeed, choosing e.g. $x^4 \equiv y$, the theory has the solution

$$\xi(y) = \begin{cases} -a & y \leq -\frac{\pi}{2\lambda} \\ a \sin \lambda y & -\frac{\pi}{2\lambda} \leq y \leq \frac{\pi}{2\lambda} \\ a & y \geq \frac{\pi}{2\lambda}, \end{cases} \tag{4}$$

which interpolates between the two distinct vacuum values $-a$ and a, see [19] for details (for a more general discussion of compactons, we refer to [20] - [22]). This compacton solution is continuous and has continuous first derivative. It is a domain wall solution in the 4+1 dimensional Minkowski space.
Remark: The compacton configuration has non-continuous second derivative at the compacton boundary. In the field equations, however, this discontinuity is multiplied by zero whenever it shows up (i.e., multiplied by some power of the first derivative), so that the expression for the field equation is continuous everywhere.
Remark: The Cauchy problem at the compacton boundary (i.e., for the initial conditions $\xi(y_0) = a$, $\xi_y(y_0) = 0$) is not well-defined. The determining equation for $\xi_{yy}(y_0)$ is, in fact, a cubic equation with the three roots $\xi_{yy}(y_0) = 0, \pm a\lambda^2$, corresponding to the vacuum, compacton and anticompacton, respectively. Once this ambiguity is resolved, the solution is unique

in a finite neighborhood of y_0 (e.g., up to the other boundary of the compacton). Observe that y_0 is arbitrary due to translation invariance.
In a next step we want to study the behavior of linear perturbations about the domain wall (i.e., the compacton). Here one simply inserts the field

$$\xi(x^M) = \xi_0(y) + \eta(x^M) \tag{5}$$

(where ξ_0 is the compacton and η is the fluctuation) into the action of the theory (1). The resulting linear equation for the fluctuation field for a general Lagrangian density is (see [19], [23])

$$\partial_M(\mathcal{L}_X\delta^M_N + \mathcal{L}_{XX}\xi_0^M\xi_0^N)\eta_N - \mathcal{L}_{\xi\xi}\eta = 0 \tag{6}$$

where $\mathcal{L}_X \equiv \partial_X\mathcal{L}$, $\xi_0^M \equiv g^{MN}\partial_N\xi_0$, etc. Further, we already took into account that there are no mixed terms in the Lagrangian, that is, $\mathcal{L}_{X\xi} = 0$. In the above expression the derivatives of the Lagrangian have to be evaluated for the compacton field. For our model we have concretely

$$\mathcal{L}_X = 8|X|\,, \quad \mathcal{L}_{XX} = 8\,\mathrm{sign}\,(X)\,, \quad \mathcal{L}_{\xi\xi} = -12\lambda^4(3\xi^2 - a^2). \tag{7}$$

Taking into account that for the compacton $\xi_M = \delta^4_M\xi_{x^4}$, etc., we find the equation (where again $x^4 \equiv y$)

$$-24\,\mathrm{sign}\,(X)X_y\eta_y - 24|X|\eta_{yy} + 8|X|\partial^\mu\partial_\mu\eta + 12\lambda^4(3\xi_0^2 - a^2)\eta = 0 \tag{8}$$

where X, etc. have to be evaluated for the compacton field, and $\mu = 0, \ldots, 3$.
In the region outside the compacton, where ξ_0 takes its vacuum values $\pm a$, all terms involving derivatives of the linear perturbation η are multiplied by zero, because $X = 0$ and $X_y = 0$ in that region. There we are left with

$$12\lambda^4(3\xi_0^2 - a^2)\eta = 24\lambda^4a^2\eta = 0 \tag{9}$$

which has the only solution $\eta = 0$. There are, therefore, no linear perturbations in that region, that is, all particle propagation is completely suppressed.
Inside the compacton we need the expressions

$$\partial_y\xi_0 = a\lambda\cos\lambda y\,, \quad X = -\frac{1}{2}a^2\lambda^2\cos^2\lambda y \tag{10}$$

and

$$X_y = a^2\lambda^3\sin\lambda y\cos\lambda y \tag{11}$$

to arrive at the equation (after a division by $12a^2$)

$$-\cos^2\lambda y\,\eta_{yy} + 2\lambda\sin\lambda y\cos\lambda y\,\eta_y + \lambda^2(3\sin^2\lambda y - 1)\,\eta + \frac{1}{3}\cos^2\lambda y\,\partial^\mu\partial_\mu\eta = 0 \tag{12}$$

or, after the variable change $z = \lambda y$,

$$H\eta = -\frac{1}{3\lambda^2}\cos^2 z\, \partial^\mu \partial_\mu \eta \tag{13}$$

where the differential operator H is

$$H \equiv -\cos^2 z\, \partial_z^2 + 2\sin z \cos z\, \partial_z + 3\sin^2 z - 1. \tag{14}$$

For a further evaluation, we use the separation of variables ansatz $\eta = \bar{\eta}(z)\Phi(x^\mu)$, which leads to the equations

$$H\bar{\eta} = \omega^2 \cos^2 z\, \bar{\eta} \tag{15}$$

and

$$\partial^\mu \partial_\mu \Phi + 3\omega^2 \lambda^2 \Phi = 0. \tag{16}$$

Before further discussing these equations, we have to determine the space of functions on which the operator H is supposed to act. We want the perturbation η to be continuous at the boundary of the compacton, i.e., at $z = \pm\pi/2$, therefore the space of functions is

$$\bar{\eta}(z) = \begin{cases} 0 & z \leq -\frac{\pi}{2} \\ \sum_{n=1}^{\infty} b_n \cos nz & -\frac{\pi}{2} \leq z \leq \frac{\pi}{2} \\ 0 & z \geq \frac{\pi}{2}. \end{cases} \tag{17}$$

Observe that the operator H maps the space of functions (17) into itself, so its action is well-defined on this space. Observe also that the first derivative is not continuous at the compacton boundary, which is consistent with the fact that the compacton itself is continuous together with its first derivative. Observe, finally, that there is no discontinuous term in Eq. (15), because whenever a discontinuous factor appears in that equation, it is multiplied by zero.

It is easy to prove that the operator H is positive semi-definite on the space of functions (17), see Section 4.4 of Ref. [19]. It has, in fact, one zero mode $\bar{\eta}_0(z) = \cos z$, i.e., $H\cos z = 0$, and is positive definite on the subspace $\eta = \sum_{n=2}^{\infty} b_n \cos nz$. Therefore, Eq. (15) has the general solutions

$$H\bar{\eta}_n = \omega_n^2 \cos^2 z\, \bar{\eta}_n\, , \quad \omega_0^2 = 0\, , \quad \omega_n^2 > 0 \text{ for } n \geq 1. \tag{18}$$

As a consequence, the field equations (16) for the fields $\Phi(x^\mu)$ on the domain wall (the "thick brane") are just a collection of ordinary Klein–Gordon equations. There exists one massless field due to the zero mode $\omega_0 = 0$ in Eq. (18), which is just the Goldstone field for the spontaneously broken translational invariance in the x^4 direction. The other Klein–Gordon equations are for positive square masses $m_n^2 = 3\omega_n^2\lambda^2$, so there are no tachyons on the brane. The propagation of perturbations on the brane is, therefore, completely standard in spite of the non-standard kinetic term of the model. The only way in which the original, nonstandard theory enters at this stage is in the determination of the values for the masses m_n^2, which depend on the parameters of the original theory. The whole setting is, in fact, quite similar to the reduction in the familiar Kaluza–Klein case, in spite of the noncompact fifth dimension in our case. Finally, the modes for nonzero m_n^2 can always be removed from the physically accessible spectrum by choosing λ sufficiently large.

3 Backreaction and localization of gravity

In the sequel, we shall investigate the K-field equations in the presence of gravity, and the possibility of localizing gravity in compacton solutions like the one described above. We will couple the scalar field to gravity minimally, and include the dynamics of the gravitational sector in the form of a canonical 5D Einstein term. The action is

$$S = \int d^5x\sqrt{-g}\left(\kappa^{-2}(R-\Lambda) + 4|X|X - V(\xi)\right) \tag{19}$$

where Λ is a cosmological constant and X now includes the metric

$$X = \frac{1}{2}g^{MN}\partial_M\xi\partial_N\xi \tag{20}$$

The resulting Einstein equations read

$$\kappa^{-2}G_{MN} = 4|X|\partial_M\xi\partial_N\xi - \frac{1}{2}g_{MN}\left(-\kappa^{-2}\Lambda + 4|X|X - V(\xi)\right) \tag{21}$$

We will choose a 5D metric ansatz with a Minkowskian 4D slice, written in the form

$$ds^2 = e^{-A(y)}\left(dt^2 - d\vec{x}^{\,2}\right) - dy^2 \tag{22}$$

while for the scalar field we assume $\xi = \xi(y)$ as before. The independent components of the Einstein equations now read

$$\begin{aligned}\frac{3}{4}A_{yy} - A_y^2 &= \frac{1}{3}\left(\Lambda + \kappa^2 V(\xi)\right)\\ \frac{3}{4}A_{yy} &= \kappa^2\xi_y^4.\end{aligned} \tag{23}$$

The next step would be to solve the boundary value problem for the above coupled system (23) to prove that the compacton solution is not spoiled by the gravitational backreaction. Since the solution of the full system will be more complicated, we will just assume in a first step that the solution exists and explore the effects of the compacton on the gravitational degrees of freedom.
By definition, outside the support of the compacton the K field ξ is in its vacuum $\xi = \pm a$. There the equations for A take the form

$$\begin{aligned} A_y^2 &= -\frac{1}{3}\Lambda \\ A_{yy} &= 0 \end{aligned} \tag{24}$$

that immediately imply the vacuum AdS solution $A = \sqrt{-\Lambda/3}\,|y| + constant \equiv \bar{A}$. This represents exactly the same bulk solution as in the case on an infinitely thin brane [24].
The metric perturbation analysis for the case of a thick brane has been performed in [6]. The main result is that the 4D graviton decouple from scalar field perturbations and that, to see wether there is localization of gravity, one should prove that the graviton wave function e^{-A} is normalizable. In other words we should verify that

$$\int dy\, e^{-A} < \infty \tag{25}$$

This is obviously satisfied for the above solution, provided it is completed inside the compacton with an integrable function. The corresponding 4D Plank mass reads

$$M_4^2 = \kappa^{-2} \int dy\, e^{-A} \tag{26}$$

Separating the above integral into its contributions inside the compacton and outside it, using the fact that the exterior solution for A is that of an infinitely thin brane $\bar{A}$, and adding and subtracting the interior contributions for $\bar{A}$, we get

$$M_4^2 = \bar{M}_4^2 + \kappa^{-2} \int_{int} dy \left(e^{-A} - e^{-\bar{A}} \right) \tag{27}$$

where $\bar{M}_4$ is the 4D Plank mass for an infinitely thin brane with the same bulk solution. The function in parenthesis vanishes at the boundary of the compacton, and its first derivative also vanishes. Its second derivative is negative for positive y and positive for negative y. The only possible conclusion is that the function itself is negative inside the support, giving a negative contribution to the effective four dimensional Plank mass.

The conclusion is that, if the compacton solution still exists in the backreacting system, it represents a finite thickness brane-world that is indistinguishable from an infinitely thin brane when it is seen from the bulk. The observer on the brane, on the other hand, measures localized 4D gravity with a Plank mass suppressed with respect to that of the infinitely thin case.
We still should comment on the solution of the full system and on the related stability of the compacton under gravitational backreaction. First of all, it still remains true that for vacuum boundary conditions (i.e., for $\xi(y_0) = a$, $\xi_y(y_0) = 0$), the second derivative is not uniquely defined but, instead, obeys a cubic equation with the three roots $\xi_{yy}(y_0) = 0, \pm a\lambda^2$. This indicates that it should still be possible to join the vacuum with the compacton or anti-compacton boundary, respectively. Secondly, both a power series expansion about the compacton boundary and a numerical integration from a point very near the compacton boundary (i.e., with the inclusion of the second derivative $\xi_{yy}(y_0)$ in order to have a well-defined solution) up to the center of the compacton (a point y_1 such that $\xi(y_1) = 0$) lead to very reasonable results. The radius of the compacton $|y_1 - y_0|$ is very similar to the case without gravitation for sufficiently small values of the cosmological constant Λ and gravitational coupling κ. Further, Λ tends to increase the compacton radius, whereas κ tend to shrink it, in complete agreement with general expectations. A detailed analytical and numerical discussion of the full K field and gravitation system is beyond the scope of this letter and shall be presented elsewhere.

4 Discussion

We have proposed a simple and efficient mechanism for the production of thick branes, that is, topological defects within a higher dimensional space, to which the propagation of linear perturbations is confined. These thick branes have the interesting property that they are of strictly finite extension in the additional dimension. The main ingredients of the proposal are the use of a model with a non-standard kinetic term and the observation that topological defects with a compact support (compactons) exist in such models. The total suppression of the propagation of fields outside the support of the compacton is an automatic result of the model. Furthermore, propagation inside the topological defect (i.e., inside the brane) is standard in spite of the non-standard kinetic term. Specifically, there are no tachyons on the brane, and the evolution of linear perturbations is both unitary and causal. Inside the brane, the only remaining effect of the original K field theory resides in the values of the masses of the (Klein–Gordon type) linear fluctuation field.

A study of the dynamical evolution of the full system, i.e., the inclusion of time dependence would therefore be very interesting, in order to discriminate the resulting physics on the brane from other scenarios.
Let us emphasize that in this letter our main purpose was to present the generic mechanism of thick brane generation via K fields and compactons. For a possible use of this idea in cosmological or particle physics considerations, additional structures have to be added. First of all, the existence of compacton solutions and the suppression of propagation in a vacuum background is a rather generic feature of K field theories. All that is needed is that the kinetic term remains non-standard in a specific way for fields near their vacuum value [19]. There exists, therefore, a large class of K field theories which show essentially the same features. Secondly, it will be of interest to add fermions to the theory, which probably give rise to the presence of fermionic zero modes, as in the case of a standard background topological defect [11]. This may also open the way for introducing supersymmetry for K field theories. Thirdly, the system with gravity included should be further analysed. We already found that, provided the compacton domain wall is not destabilized by the addition of gravity, bulk gravity solutions of the Randall–Sundrum type (that is, localization of gravity on the brane) do exist. Further, we found strong indications for the stability of the full system with gravity included. Still, this latter issue should be investigated in more detail. A fourth issue not touched in this letter is the question whether theories with a non-standard kinetic term of the type required for the existence of compactons may be induced as effective low energy theories from some more fundamental theories at higher energies. These and many more problems are subject to further investigations.

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