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Evolution of Primordial Black Holes in a radiation and phantom energy environment

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Abstract In this work we extend previous work on the evolution of a Primordial Black Hole (PBH) to address the presence of a dark energy component with a super-negative equation of state as a background, investigating the competition between the radiation accretion, the Hawking evaporation and the phantom accretion, the latter two causing a decrease on black hole mass. It is found that there is an instant during the matter-dominated era after which the radiation accretion becomes negligible compared to the phantom accretion. The Hawking evaporation may become important again depending on a mass threshold. The evaporation of PBHs is quite modified at late times by these effects, but only if the Generalized Second Law of thermodynamics is violated.

Keywords Black holes; Cosmology; Hawking radiation; phantom energy.

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1 Introduction

The now widely accepted accelerated expansion of the Universe in its recent history is yet to be fully explained. Several possibilities to reproduce this effect have been advanced, ranging from “conservative” to very unusual ones requiring new physics. One of the most economical hypotheses that has received a great deal of attention is the late dominance of a fluid with an “anomalous” equation of state, a sort of analogue of the inflationary proposals but at a lower energy scale, the so-called *dark energy*. As is customary to write a

generical equation of state in the form $P = \omega\rho$ and in spite that values of ω larger than -1 are usually considered, some works have raised the possibility that the dark sector may be characterized by a fluid with an equation of state with $\omega < -1$, known throughout the literature as the *phantom energy*.

There are many physical consequences of such phantom component in a variety of physical species present in the Universe, most notably the spacelike singularity known as the *Big Rip* [1,2], or even more fabulous possibilities, like the *Big Trip* [3,4]. Some effort has been made to remove the Big Rip singularity, but it is still premature to rule out or support definitely any scenario.

We work within a general phantom energy scenario in this paper. It has already been acknowledged that, being such an exotic physical species, the phantom energy may also change the accretion regime of black holes [5]. In the present paper we investigate the influence of phantom energy accretion onto primordial black holes (hereafter PBHs) together with the radiation and matter accretion/evaporation formerly addressed.

The PBH interaction with different types of energy in the universe is the continuous subject of several studies, as well as their interaction with cosmological boundary conditions [6]. Several numerical results also work as test fields for alternate gravitational theories, and the questions regarding their very formation at extreme cosmological scenarios are beginning to yield several interesting results [7].

We shall focus on the new features specifically introduced by the phantom era [8], and generally refer to the full evolution of the PBHs across the mass-time plane. Previous attempts to address this problem have been limited to the consideration of the black holes plus phantom fluid only, although there is a more subtle interplay among components when radiation and matter are also included (as will be shown below). It is also of interest to revisit the issue of the black hole behavior in the radiation-dominated and matter-dominated eras (that is, well before the phantom component can be important) for a complete assessment of the fate of PBHs, especially their behavior in the matter-dominated era where dark matter may fuel their growth.

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2 PBHs evolution in the early Universe

2.1 The radiation-only accretion equation

Our starting point will be the evolution equation for PBHs in the radiation-dominated era addressed by several authors (see Custódio and Horvath [9] and references therein), which takes into account the accretion of radiation and the Hawking evaporation at a semiclassical level. Ignoring the (potentially relevant) “grey factors” in the absorption of radiation, the resulting differential equation for the black hole mass M reads quite generally

$$\frac{dM}{dt} = -\frac{A(M)}{M^2} + \frac{27\pi G^2}{c^5} \rho_{\text{rad}}(T) M^2 \quad (1)$$

with t being the cosmological time, $A(M) = \frac{\hbar c^4}{G^2} \alpha(M)$, with $\alpha(M)$ called the *running constant* [10], counting the degrees of freedom of the emitted particles on the Hawking radiation (in CGS units, $A = 7,8 \times 10^{26} \text{ g}^3/\text{s}$ for black holes evaporating today [11]), and $\rho_{\text{rad}}(T)$ the radiation energy density at temperature T at the time t .

In a Universe also filled with phantom energy, the accretion of such exotic component should also be taken into account. Babichev, Dokuchaev and Eroshenko [5] have worked out a differential equation for a black hole accreting phantom energy only, obtaining a counterintuitive result that phantom energy accretion *decreases* the overall black hole mass. The expression is similar to the accretion term in eq. (1), and is given by

$$\frac{dM}{dt} = \frac{16\pi G^2}{c^5} M^2 [\rho_{\text{ph}} + p(\rho_{\text{ph}})] \quad (2)$$

2.2 The complete accretion equation

Considering the radiation accretion and evaporation terms from eq. (1) together with the new phantom energy accretion term in eq. (2), and assuming no interaction between the two different species, the complete equation for the accretion of the different types of energy into the black hole is just

$$\frac{dM}{dt} = -\frac{A(M)}{M^2} + \frac{G^2}{c^5} [27\pi \rho_{\text{rad}}(T) + 16\pi (\rho_{\text{ph}} + p(\rho_{\text{ph}}))] M^2 \quad (3)$$

Using for the phantom energy $p(\rho) = \omega\rho$, $\omega < -1$, the phantom component of the accretion may be written as

$$\rho_{\text{ph}} + p(\rho_{\text{ph}}) = (1 + \omega)\rho_{\text{ph}} \quad (4)$$

and the complete accretion equation becomes

$$\frac{dM}{dt} = -\frac{A(M)}{M^2} + \frac{G^2}{c^5} [27\pi \rho_{\text{rad}}(T) + 16\pi(1 + \omega)\rho_{\text{ph}}] M^2 \quad (5)$$

2.3 Accretion regimes

As is well-known, the Friedmann equation can be solved to follow the cosmological evolution of the phantom energy, as given by Babichev, Dokuchaev and Eroshenko [5].

$$|\rho + p| \propto a^{-3(1+\omega)} \quad (6)$$

neglecting all other contributions. The densities of the radiation, matter and phantom energy terms evolve as represented in the graphic shown in Figure 1.

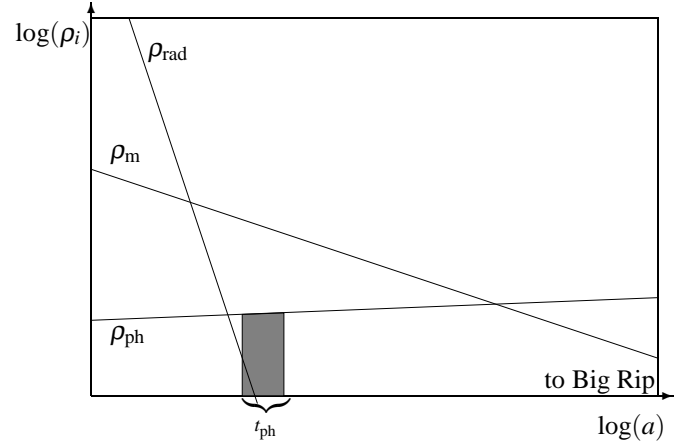


Fig. 1 Evolution of the radiation, matter and phantom energy densities with the scale factor. The exact position of t_{ph} depends on the densities and equation of state of the radiation and phantom energy.

As expected, there is an epoch in which the radiation and phantom energy accretion terms from eq. (5) become comparable. We call such an epoch the *phantom time*, or t_{ph} . It must be noted that this instant is distinct from the one when the lines of Figure 1 cross each other. The phantom time represents the cosmological instant when the phantom energy accretion term dominates the radiation term, changing drastically the black hole evolution dynamics. We can calculate the value of this time as a function of the initial radiation and phantom energy densities.

The radiation density as a function of the scale factor is given by the Friedmann equation, $\rho_{\text{rad}} = \rho_{\text{rad}}^0 \left(\frac{a_0}{a}\right)^4$. During the matter-dominated era, the scale factor as a function of time is given by

$$\frac{a(t)}{a_0} = \left(\frac{3H_0 t}{2}\right)^{2/3} \quad (7)$$

Therefore, the radiation density evolves in the matter-dominated era as

$$\rho_{\text{rad}} = \rho_{\text{rad}}^0 \left(\frac{3H_0 t}{2}\right)^{-8/3} \quad (8)$$

Similarly, with the phantom energy eq. (4) and evolving according to eq. (6), and with the time dependence of the

scale factor evolving as of eq. (7), the phantom energy density as a function of time is

$$\rho_{\text{ph}} = \frac{\rho_{\text{ph}}^0}{|1+\omega|} \left(\frac{3H_0 t}{2} \right)^{-2(1+\omega)} \quad (9)$$

The epoch when the phantom energy accretion is as important as the radiation accretion is the instant when, equating both expressions according to eq. (5)

$$\rho_{\text{rad}} = -\frac{16}{27}(1+\omega)\rho_{\text{ph}} \quad (10)$$

Inserting the time dependences calculated in eq. (8) and eq. (9), this equation yields the *phantom time*.

$$\frac{t_{\text{ph}}}{1 \text{ s}} = \frac{2}{3H_0} \left(\frac{16 \rho_{\text{ph}}^0}{27 \rho_{\text{rad}}^0} \right)^{\frac{8}{3}-2(1+\omega)} \frac{1 \text{ km}}{1 \text{ Mpc} \cdot 1 \text{ s}} \quad (11)$$

with H_0 expressed in $\frac{\text{km}}{\text{s} \cdot \text{Mpc}}$ and, as the initial values ρ_{ph}^0 and ρ_{rad}^0 , calculated at the end of the matter-dominated era.

We can express this transition time in terms of the redshift, using eq. (10), with the initial conditions $\rho_{\text{rad}}^0 = 8.12 \times 10^{-13} \frac{\text{erg}}{\text{cm}^3}$ and $\rho_{\text{ph}}^0 = 1.79 \times 10^{-8} \frac{\text{erg}}{\text{cm}^3}$, appropriate for the obtained conditions, finally coming to $z_{\text{ph}} \simeq 3.1$.

It is reasonable to suppose the transition between radiation and phantom accretion to be instantaneous due to the very steep radiation/phantom density ratio, which can be easily seen by rewriting eq. (11) for an arbitrary epoch.

$$\frac{\rho_{\text{rad}}}{\rho_{\text{ph}}} = \frac{\rho_{\text{rad}}^0}{\rho_{\text{ph}}^0} |1+\omega| \left(\frac{3H_0 t}{2} \right)^{-\frac{8}{3}+2(1+\omega)} \quad (12)$$

The radiation density quickly becomes negligible compared to the phantom energy. The higher the $|\omega|$, the quicker the transition becomes.

3 Effects of dark matter accretion

3.1 General results

Up to this point we have neglected completely the possible effects of (cold) dark matter on the PBHs, which is a popular and reasonable explanation for the structure formation problem. Within the CDM scenario, right after the decoupling of dark matter its accretion onto black holes will depend on the black hole cross-section for point-like particles. Therefore, the time dependence of the mass would be given by

$$\frac{dM}{dt} = \frac{16\pi G^2}{c^2} \frac{\rho_{\text{m}}}{u_{\text{m}}} M^2 \quad (13)$$

where u_{m} is the dark matter particle density, computed after the decoupling

$$u_{\text{m}} \simeq \sqrt{\frac{3k_B T_{\text{dec}}}{m}} \frac{1+z}{(1+z)_{\text{dec}}} \quad (14)$$

Well before the phantom energy becomes important, the PBH mass equation, including now the dark matter contribution, is just

$$\frac{dM}{dt} = -\frac{A}{M^2} + \frac{27\pi G^2}{c^3} \rho_{\text{rad}} M^2 + \frac{16\pi G^2}{c^2} \frac{\rho_{\text{m}}}{u_{\text{m}}} M^2 \quad (15)$$

We must remark that we are always referring to a *diffuse* CDM component, an appropriate assumption prior to any structure formation.

3.2 Numerical predictions

Because we are interested in the fate of a wide range of black hole masses, we should integrate equation (15) numerically for several initial conditions and cosmological parameters.

To solve this equation, we first rewrite it in explicitly time-dependent terms

$$\rho_{\text{m}} = \rho_{\text{m}}^0 (1+z)^3 = \frac{\rho_{\text{dec}}}{(1+z)_{\text{dec}}^3} (1+z)^3 = \rho_{\text{dec}} \left(\frac{t_{\text{dec}}}{t} \right)^{\frac{3}{2}} \quad (16)$$

Letting $m = 100 \text{ GeV}$ and $T_{\text{dec}} \simeq 1 \text{ GeV}$, $(1+z)_{\text{dec}} \simeq 4.26 \times 10^{12}$ yields

$$u_{\text{m}} = \sqrt{\frac{3k_B T_{\text{dec}}}{m}} \frac{1+z}{(1+z)_{\text{dec}}} = 0.173c \left(\frac{t_{\text{dec}}}{t} \right)^{\frac{1}{2}} \quad (17)$$

Setting $\rho_{\text{m}}^0 = \frac{3H_0^2}{8\pi G} \Omega_{\text{m}} = 2.24 \times 10^{-30} \frac{\text{g}}{\text{cm}^3}$ and $\rho_{\text{rad}} = \frac{3}{32\pi G t^2}$, equation (15) reads

$$\frac{dM}{dt} = -\frac{A}{M^2} + \frac{81}{32} \frac{GM^2}{c^3} \frac{1}{t^2} + \frac{16\pi(GM)^2}{c^3} \frac{\rho_{\text{m}}^0 (1+z)_{\text{dec}}^3}{0.173c} \frac{t_{\text{dec}}}{t} \quad (18)$$

which can be solved introducing new variables $y = \frac{M}{M_0}$, $M_0 = \frac{\alpha c^3 t_0}{G}$ and $x = \log\left(\frac{t}{t_0}\right)$, yielding an equation of the form

$$\frac{dy}{dt} = -a_1 y^{-2} e^x + a_2 y^2 e^{-x} + a_3 y^2 \quad (19)$$

with $a_1 = \frac{AG}{\alpha c^3 M_0^2} = \frac{1.30 \times 10^{-13}}{\alpha M_0^2}$, $a_2 = \frac{81}{32} \alpha = 2.53125\alpha$ and $a_3 = 1.38 \times 10^{-42} M_0$.

The dark matter accretion should be taken into account for $x \geq x_{\text{dec}}$. We may also introduce an instant x_* similar to the phantom time, in which the dark matter and radiation accretion have the same value. An estimate for x_{dec} and x_* is given by

$$t_0 = 2.47 \times 10^{-38} M_0 \rightarrow x_{\text{dec}} = \log \left(\frac{6.72 \times 10^{30}}{M_0} \right) \quad (20)$$

$$a_2 y^2 e^{-x} = a_3 y^2 \rightarrow x_* = \log \left(\frac{1.83 \times 10^{41}}{M_0} \right) \quad (21)$$

Table 1 summarizes a few numerical estimates for x_{dec} and x_* , covering most of the important PBH masses.

Table 1 Numerical values for x_{dec} and x_0 for some initial values for the black hole mass, along with calculations for the times of evaporation with ($x_{\text{evap}}^{\text{in}}$) and without (x_{evap}) dark matter.

M_0 (g)	x_{dec}	x_*	x_{evap}	$x_{\text{evap}}^{\text{in}}$
10^8	52.56	76.59	63.99	63.99
10^9	50.26	74.29	68.59	68.59
10^{10}	47.96	71.98	73.20	73.10
10^{11}	45.65	69.68	77.81	77.81
10^{12}	43.35	67.38	82.41	82.41
10^{13}	41.05	65.08	87.01	87.01
10^{14}	38.75	62.77	91.62	91.62
10^{15}	36.44	60.47	96.23	96.22
10^{16}	34.14	58.17	100.83	100.83

An inspection of Table 1 shows that only black holes with masses greater than 10^9 g should be influenced by the dark matter accretion at early times. However, this effect of the dark matter term happens to be small, because it is rapidly overcome by the accretion of radiation. This can be expected on physical grounds because the geometrical dilution of the dark matter component “starves” the PBHs by quickly diminishing the flux of particles coming into them. Note that this particular evolution does *not* refer to much later epochs where dark matter halos had formed, possibly then contributing to the growth of PBHs as seeds for the ultimate supermassive galactic residents.

The numerical results for the evolution through time are depicted in Figure 2 for the highest initial condition, as an example. The resulting bump in the mass (Fig. 2) has been exaggerated for the sake of clarity.

4 Behavior of the critical mass function

With expression eq. (11) for the time, we can calculate the value of the critical mass M_c in the instant t_{ph} . From Custódio and Horvath [9], the expression for the critical mass is

$$M_c(t) \sim 10 M_{\text{Haw}} \left(\frac{t}{1\text{s}} \right)^{\frac{1}{2}} \text{g} \quad (22)$$

During the late phantom energy accretion dominance era, a critical mass function would be meaningless, since there is no longer a relevant mass increase mechanism. Thus, the largest value reachable by the critical mass in a Universe filled only by radiation and phantom energy is

$$M_c^{\text{max}} \sim 10 M_{\text{Haw}} \frac{2}{3 H_0} \left(\frac{16 \rho_{\text{ph}}^0}{27 \rho_{\text{rad}}^0} \right)^{\frac{8}{3} - 2(1+\omega)} \text{g} \quad (23)$$

After this time, the Hawking evaporation is no longer a relevant mechanism for black hole mass decrease, until its mass reaches the transition value discussed in section 5.

It is also convenient to calculate the initial mass of the black hole which disappears at t_{ph} . For that purpose, it is enough to consider only the Hawking term in eq. (1), which yields the well-known solution

$$\tau = \frac{1}{3A(M)} M_i^3 \quad (24)$$

where τ is the evaporation timescale. Restoring the cgs units τ reads

$$\tau \sim 10^{71} \left(\frac{M_i}{M_{\odot}} \right)^3 \quad (25)$$

Combining eq. (22) and eq. (24), we find a third degree equation in M_c , whose solution is the critical mass of the black hole that will evaporate *completely* at $t = t_{\text{ph}}$

$$\frac{M_c^3}{3A(M)} + \frac{M_c^2}{100M_{\text{Haw}}^2} = t_{\text{ph}} \quad (26)$$

We use the numerical values of $A(M) \leq 7,8 \times 10^{26} \frac{\text{g}^3}{\text{s}}$ [9] and $M_{\text{Haw}} \equiv 10^{15}$ g, as well as the numerical values of ρ_{ph} , ρ_{rad} , ω and H_0 necessary to compute t_{ph} . The instant when the critical mass assumes this value is found by inverting eq. (22).

Since the mass gain due to radiation accretion is not substantial [9], all black holes with $M_i \lesssim M_c^{\text{ph}}$, which reach critical mass at $T_{\text{cross}} \lesssim t_c$, will disappear before t_{ph} and will never reach the phantom era.

5 The competition between phantom accretion and Hawking evaporation

We have emphasized before that, since after t_{ph} there is no efficient mechanism that could increase the mass of the black holes, there is no longer a critical mass function. However, due to the presence of a phantom field, there are now *two* distinct regimes of mass decrease, whose relative importance depends on the mass of a given PBH entering the phantom era.

Taking eq. (5) and neglecting the radiation term, we can describe the evolution of black hole masses during the phantom era. Let us define a ratio between the two remaining terms,

$$\xi(M) = \frac{\dot{M}_{\text{ph}}}{\dot{M}_{\text{Haw}}} = \frac{G^2}{c^3} \frac{16\pi(1+\omega)\rho_{\text{ph}}}{A(M)} M^4 \quad (27)$$

or, in terms of a *transition mass*

$$\xi(M) = \left(\frac{M}{M_t}\right)^4 \quad (28)$$

with

$$M_t = \left[\frac{c^3}{16\pi G^2} \frac{A(M)}{(1+\omega)\rho_{\text{ph}}} \right]^{1/4} \quad (29)$$

Substituting numerical values for the constants, we obtain an expression for M_t in terms of the phantom field density

$$M_t \cong 5.5 \times 10^{17} [(1+\omega)\rho_{\text{ph}}]^{-1/4} \text{ g} \quad (30)$$

with ρ_{ph} given in g/cm^3 .

Since both regimes are of mass *decrease*, the black hole mass will diminish mostly due to phantom accretion until it reaches M_t . After this, the predominant effect will be Hawking evaporation, since eq. (28) shows that the change in regimes is sufficiently sudden for us to make this approximation.

To find the time dependence of the transition mass, we must first know the evolution of the phantom density. According to the Friedmann equations for the phantom fluid, we finally obtain [5]

$$(\rho_{\text{ph}})^{-\frac{1}{2}} = (\rho_{\text{ph}}^0)^{-\frac{1}{2}} + \frac{3(1+\omega)}{2} \left(\frac{8\pi G}{3}\right)^{\frac{1}{2}} t \quad (31)$$

with $(\rho_{\text{ph}}^0)^{-\frac{1}{2}}$ being the initial density of the phantom field. Inserting this result on equation eq. (30) the time dependence of M_t is obtained

$$M_t \cong \frac{8.29 \times 10^{21}}{(1+\omega)^{\frac{1}{4}}} \left[(\rho_{\text{ph}}^0)^{-\frac{1}{2}} + \frac{3(1+\omega)}{2} \left(\frac{8\pi G}{3}\right)^{\frac{1}{2}} t \right]^{\frac{1}{2}} \text{ g} \quad (32)$$

The initial value of the transition mass depends on both the initial value of the phantom density and on ω . It is worth remarking that this transition mass is meaningless in the radiation-accretion regime.

The differences between the three regimes is depicted in Figure 2.

It is important to stress that the Hawking evaporation does not become negligible after t_{ph} if taken into account as an independent process. However, the masses for which it becomes important ($M < M_t$) drop by a factor of 10^5 after the transition. This suddenly drives many black holes, but not all, into the new regime. When, however, the black holes reach the Planck mass, a full quantum gravity analysis becomes necessary to properly determine its fate, since it has been shown that the Hawking evaporation no longer behaves as expected on such scales [13, 14].

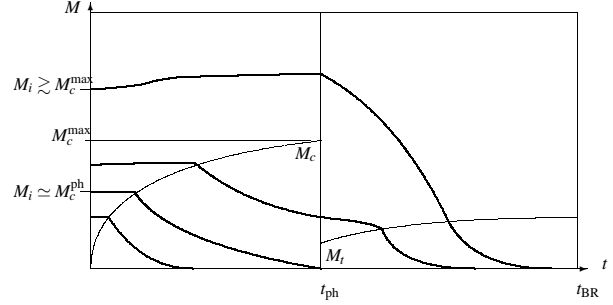


Fig. 2 Primordial black hole evolution in the matter-radiation-phantom energy scenario. The thick lines represent the different trajectories of black holes of different initial masses. The Big Rip singularity occurs at t_{BR} .

6 Conclusions

We have studied in the present work the evolution of PBH for various regimes of accretion/evaporation in the very early and contemporary Universe. In particular, we have extended and clarified the evolution in the radiation-dominated and matter-dominated eras, including the features of diffuse CDM accretion producing only a small bump in the mass of the PBHs at early times. We have generally confirmed previously known features of the semiclassical pictures of PBH evolution from a general point of view. Novel features are introduced in this scenario when a phantom energy component is introduced, as suggested by Babichev, Dokuchaev and Eroshenko [5].

Broadly speaking, a phantom field introduces another evaporation regime that competes with the celebrated Hawking evaporation. We have found that the joint consideration of the relevant terms quenches the asymptotic approach to a common mass resulting from the phantom term only. This conclusion should, however, not be considered as definitive. Its validity rests on the assumption of the entropy for the phantom fluid being negligible, which is not the most general possibility. In fact, the enforcement of the Generalized Second Law (GSL) of thermodynamics would *forbid* the evaporation of the PBHs by phantom accretion [15, 18]. In addition, it is not clear whether the GSL should be valid in presence of the phantom fluid not respecting the dominant energy condition, as pointed out by Izquierdo and Pavón [15], and models may be constructed in which the GSL must be modified. There is a rich variety of behaviors [16, 17] within phantom energy models that remains to be explored in connection with the PBH evolution problem. In particular, late evaporation may conflict with the generalized second law of thermodynamics [18].

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