

**EINSTEIN-ÆTHER GRAVITY:
THEORY AND OBSERVATIONAL CONSTRAINTS**

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Einstein-æther theory is general relativity coupled to a dynamical unit timelike vector field. A brief review of current theoretical understanding and observational constraints on the four coupling parameters of the theory is given.

1. Introduction

In general relativity (GR), spacetime structure is determined by a dynamical metric tensor field g_{ab} and nothing else, and the theory is both diffeomorphism invariant and locally Lorentz invariant. Einstein-æther theory is the extension of GR that incorporates a dynamical unit timelike vector field u^a —the “æther”—which breaks the local Lorentz symmetry down to a 3d rotation subgroup. Direct coupling of matter to the æther would violate local Lorentz symmetry yet preserve diffeomorphism invariance. This paper presents a brief overview of the current theoretical and observational status of this theory, assuming that matter does not couple directly to the æther.

The action involving metric and æther is highly constrained. Besides the cosmological constant term, the only independent diffeomorphism invariant local terms containing no more than two derivatives are

$$S = -\frac{1}{16\pi G} \int \sqrt{-g} (R + K_{mn}^{ab} \nabla_a u^m \nabla_b u^n) d^4x, \quad (1)$$

where R is the Ricci scalar, K_{mn}^{ab} is defined as

$$K_{mn}^{ab} = c_1 g^{ab} g_{mn} + c_2 \delta_m^a \delta_n^b + c_3 \delta_n^a \delta_m^b + c_4 u^a u^b g_{mn} \quad (2)$$

with dimensionless coupling constants c_i , and the unit timelike constraint on the æther is implicit. (The metric signature is $(+---)$ and the speed of

light defined by the metric g_{ab} is unity.) Higher derivatives would be suppressed by powers of a (presumably) small length, e.g. the Planck length. It is assumed here that the æther is aligned at large scales with the rest frame of the microwave background radiation.

Einstein-æther theory—“æ-theory” for short—is similar to the vector-tensor gravity theories studied by Will and Nordvedt,¹ but with the crucial difference that the vector field is constrained to have unit norm. This constraint eliminates a wrong-sign kinetic term for the length-stretching mode,² hence gives the theory a chance to be viable. An equivalent theory using the tetrad formalism was first studied by Gasperini,³ and in the above form it was introduced by Jacobson and Mattingly.⁴

2. Newtonian and post-Newtonian limits

In the weak-field, slow-motion limit æ-theory reduces to Newtonian gravity,⁵ with a value of Newton’s constant G_N related to the parameter G in the action (1) by

$$G_N = \frac{G}{1 - c_{14}/2}, \quad (3)$$

where $c_{14} \equiv c_1 + c_4$. (Similar notation is used below for other additive combinations of the c_i .) For any choice of the c_i , all parameterized post-Newtonian (PPN) parameters⁶ of æ-theory agree with those of GR^{7,8} except the preferred frame parameters $\alpha_{1,2}$ which are given by⁸

$$\alpha_1 = \frac{-8(c_3^2 + c_1 c_4)}{2c_1 - c_1^2 + c_3^2} \quad (4)$$

$$\alpha_2 = \frac{\alpha_1}{2} - \frac{(c_1 + 2c_3 - c_4)(2c_1 + 3c_2 + c_3 + c_4)}{c_{123}(2 - c_{14})} \quad (5)$$

(This particular way of expressing α_2 was given in Ref. 9. The small c_i form of α_2 was first computed in Ref. 10.)

Observations currently impose the strong constraints $\alpha_1 \lesssim 10^{-4}$ and $\alpha_2 \lesssim 4 \times 10^{-7}$.⁶ Since æ-theory has four free parameters c_i , we may set $\alpha_{1,2}$ exactly zero by imposing the conditions⁸

$$c_2 = (-2c_1^2 - c_1 c_3 + c_3^2)/3c_1 \quad (6)$$

$$c_4 = -c_3^2/c_1. \quad (7)$$

With (6,7) satisfied, *all* the PPN parameters of æ-theory are equivalent to those of GR. (The parameters $\alpha_{1,2}$ can also be set to zero by imposing $c_{13} = c_{14} = 0$, but this case is pathological, as discussed in section 8.)

3. Homogeneous isotropic cosmology

Assuming spatial homogeneity and isotropy, u^a necessarily coincides with the 4-velocity of the isotropic observers, and the æther stress tensor is just a certain combination of the Einstein tensor and the stress tensor of a perfect fluid with energy density proportional to the inverse square of the scale factor, like the curvature term in the Friedman equation.^{11,5} The latter contribution plays no important cosmological role since the spatial curvature is small, while the former renormalizes the gravitational constant appearing in the Friedman equation, yielding⁵

$$G_{\text{cosmo}} = \frac{G}{1 + (c_{13} + 3c_2)/2}. \quad (8)$$

Since G_{cosmo} is not the same as G_{N} the expansion rate of the universe differs from what would have been expected in GR with the same matter content. The ratio is constrained by the observed primordial ⁴He abundance to satisfy $|G_{\text{cosmo}}/G_{\text{N}} - 1| < 1/8$.⁵ When the PPN parameters $\alpha_{1,2}$ are set to zero by (6,7), it turns out that $G_{\text{cosmo}} = G_{\text{N}}$, so this nucleosynthesis constraint is automatically satisfied.⁸

4. Linearized wave modes

When linearized about a flat metric and constant æther, æ-theory possesses five massless modes for each wave vector: two spin-2, two spin-1, and one spin-0 mode. The squared speeds of these modes relative to the æther rest frame are given by¹²

$$\text{spin-2} \quad 1/(1 - c_{13}) \quad (9)$$

$$\text{spin-1} \quad (c_1 - \frac{1}{2}c_1^2 + \frac{1}{2}c_3^2)/c_{14}(1 - c_{13}) \quad (10)$$

$$\text{spin-0} \quad c_{123}(2 - c_{14})/c_{14}(1 - c_{13})(2 + c_{13} + 3c_2) \quad (11)$$

The corresponding polarization tensors were found in one gauge in Ref. 12 and in another gauge in Ref. 9. The energy density of the spin-2 modes is always positive, while for the spin-1 modes it has the sign of $(2c_1 - c_1^2 + c_3^2)/(1 - c_{13})$, and for the spin-0 modes it has the sign of $c_{14}(2 - c_{14})$.^{13,9} (These reduce to the results of Ref. 14 in the decoupling limit where gravity is turned off.)

These squared speeds correspond to $(\text{frequency}/\text{wavenumber})^2$, so must be non-negative to avoid imaginary frequency instabilities. They must moreover be greater than unity (super-luminal), to avoid the existence of vacuum Čerenkov radiation by matter.² (The strongest constraints arise

from the existence of ultra high energy cosmic rays.) And the mode energy densities should be positive, to avoid dynamical instabilities. With the $\alpha_{1,2} = 0$ conditions (6,7) imposed, all of these conditions are met for all of the modes if and only if $c_{\pm} = c_1 \pm c_3$ are restricted by the inequalities⁸

$$0 \leq c_+ \leq 1 \quad (12)$$

$$0 \leq c_- \leq c_+/3(1 - c_+). \quad (13)$$

Interestingly, if the mode speeds are instead required to be *less* than unity (sub-luminal), then the spin-1 and spin-0 energy densities are negative. Hence not only the Čerenkov constraint, but also energy positivity (together with $\alpha_{1,2} = 0$) requires mode speeds greater than unity.

Note that when (7) holds, we have $c_{14} = 2c_+c_-/(c_+ + c_-)$, which satisfies $0 \leq c_{14} < 2$ when the constraints (12,13) hold. Thus in particular the condition for attractive gravity mentioned in section 2 need not be separately imposed, and c_{14} is non-negative.

5. Primordial perturbations

Given the same G_N , and assuming the PPN parameters $\alpha_{1,2}$ vanish, the primordial power in cosmological spin-0 and spin-1 perturbations is unchanged relative to GR, while the power in spin-2 perturbations differs from that in GR by the factor $(1 - c_{14}/2)(1 - c_{13})^{1/2}$.^{14,15} When the constraints (12,13) are satisfied this factor is smaller than unity, hence these spin-2 perturbations are even more difficult to detect than in GR. As for the late time evolution of these perturbations, neutrino stresses in the radiation dominated epoch source the spin-1 mode, which leads to modified matter and CMB spectra. The effect is rather small however, and is degenerate with matter-galaxy bias and with neutrino masses.¹⁵

6. Radiation damping and strong self-field effects

If the fields are weak everywhere (including inside the radiating bodies), and the PPN parameters $\alpha_{1,2}$ vanish, radiation is sourced only by the quadrupole. Waves of spins 0, 1 and 2 are radiated, and the net power is given by $(G_N \mathcal{A}/5)\ddot{Q}_{ij}^2$, where Q_{ij} is the quadrupole moment and $\mathcal{A} = \mathcal{A}[c_i]$ is a function of the coupling parameters c_i that reduces to unity in the case of GR.⁹ Agreement with the damping rate of GR (confirmed to $\sim 0.1\%$ in binary pulsar systems⁶) can be achieved by imposing the condition $\mathcal{A}[c_i] = 1$, which is consistent with the constraints (12,13).

Compact sources with strong internal fields such as neutron stars or black holes can be handled¹⁶ using an “effective source” dynamics specified by a worldline action integral

$$S = -m_0 \int d\tau [1 + \sigma(v^a u_a - 1) + \sigma'(v^a u_a - 1)^2 + \dots], \quad (14)$$

where v^a is the 4-velocity of the body, u_a is the local background value of the æther, and σ and σ' are constants characterizing the body, called a “sensitivity parameters” or just “sensitivities”. The sensitivities scale as c_i for small c_i .

The effects of nonzero sensitivities on two-body dynamics and radiation rates lead to a number of phenomena that are constrained by observations, including violations of the strong equivalence principle, modifications of the post-Newtonian dynamics, modifications of quadrupole sourced radiation, and both monopole and dipole sourced radiation. When $\alpha_{1,2} = 0$, all of these constraints are met provided the sensitivities are less than ~ 0.001 , which will certainly be the case if $c_i \lesssim 0.01$.^{16a} To be more precise would require knowing the actual dependence of the sensitivities on the c_i , which has so far only been determined for σ and only at leading order (where σ vanishes when $\alpha_{1,2} = 0$). (The speed V of the observed binaries with respect to the background æther frame can be neglected in formulating these constraints provided $V \lesssim 10^{-2}$, which is easily satisfied for any known proper motion relative to the rest frame of the microwave background radiation.¹⁶)

7. Spherically symmetric stars and black holes

Unlike GR, æ-theory has a spherically symmetric mode, corresponding to radial tilting of the æther. For each mass, there is a two parameter family of spherically symmetric static vacuum solutions, rather than a unique solution as in GR.¹⁸ Asymptotic flatness reduces this to a one parameter family.^{7,18} The solution outside a static star is the unique solution for a given mass in which the æther is aligned with the Killing vector.¹⁸ This “static æther” vacuum solution depends on the c_i only through the combination c_{14} , and was found analytically (up to inversion of a transcendental equation).¹⁸ It is stable to linear perturbations under the same conditions as for stability of flat spacetime, with the exception of the case $c_{123} = 0$.¹⁹

^aThis corrects an error in version 1 of Ref. 16, where σ is said to scale as c_i^2 . (Also the a prefactor c_{14} in Eqn. (70) should be deleted.) As a result of this correction, the likely constraints on c_i are an order of magnitude stronger, as stated here.¹⁷

The solution inside a fluid star has been found by numerical integration, both for constant density¹⁸ and for realistic neutron star equations of state.²⁰ The maximum masses for neutron stars range from about 6 to 15% smaller than in GR when $c_{14} = 1$, depending on the equation of state. The corresponding surface redshifts can be as much as 10% larger than in GR for the same mass. Measurements of high gravitational masses or precise surface redshifts thus have the potential to yield strong joint constraints on c_{14} and the equation of state. The radius of the innermost stable circular orbit (ISCO) differs from the GR value $6G_N M$ by a small term of relative order about $0.03c_{14}$.

For black holes, the condition of regularity at the spin-0 horizon selects a unique solution from the one-parameter family for a given mass.²¹ When a black hole forms from collapse of matter, the spin-0 horizon develops in a nonsingular region of spacetime, where the evolution should be regular. This motivated the conjecture that collapse produces a black hole with nonsingular spin-0 horizon, which has been confirmed for some particular examples in numerical simulations of collapse of a scalar field.²²

The black holes with nonsingular spin-0 horizons are rather close to Schwarzschild outside the horizon for a wide range of couplings; for instance, the ISCO radius differs by a factor $(1 + 0.043c_1 + 0.061c_1^2)$, in the case with $c_3 = c_4 = 0$ and c_2 fixed so that the spin-0 speed is unity.²³ (This expansion is accurate at least when $c_1 \leq 0.5$. No solution with regular spin-0 horizon exists in this case when $c_1 \gtrsim 0.8$.) Inside the horizon the solutions differ more, but like Schwarzschild they contain a spacelike singularity. Black hole solutions with singular spin-0 horizons have been studied in Ref. 24. These solutions can differ much more outside the horizon. Quasi-normal modes of black holes in \mathfrak{a} -theory have been investigated in Refs. 25.

8. Special values of c_i ?

The first case to be examined in detail^{26,4} was $c_{13} = c_2 = c_4 = 0$, i.e. the ‘‘Maxwell action’’ together with the unit constraint on the vector. The PPN result for α_2 (5) is infinite in this case, and the spin-0 mode speed is zero. The perturbation series used in the PPN analysis is thus evidently not applicable. Independently of that however, other problems with this case have been identified, such as the formation of shock discontinuities^{4,27} and a possibly related instability.¹⁹

Assuming now that $\alpha_{1,2} = 0$ and the constraints (12,13) are satisfied, and putting aside the case $c_1 = c_3 = 0$ which is not covered by existing

PPN analyses, all but one of the cases in which one of the c_i vanishes, or in which one of c_{13} , c_{14} , or c_{123} vanishes, have the property that the spin-1 mode speed (10) diverges while the energy of that mode is nonzero. It seems very unlikely that such cases are observationally viable, although they have not been examined carefully. The exception is the special case $c_3 = c_4 = 2c_1 + 3c_2 = 0$, with $2/3 < c_1 < 1$. This large value of c_1 is probably inconsistent with the strong field constraints from orbital binaries, but as mentioned above those are not yet precisely known because the sensitivity parameters have not yet been computed for neutron stars, so this case is not yet ruled out.

9. Conclusion

Einstein-æther theory is an intriguing theoretical laboratory in which gravitational effects of possible Lorentz violation can be meaningfully studied. There is a large (order unity) two-parameter space of Einstein-æther theories for which (i) the PPN parameters are identical to those of GR, (ii) the linear perturbations are stable and carry positive energy, (iii) there is no vacuum Čerenkov radiation, (iv) the dynamics of the cosmological scale factor and perturbations differ little from GR, (v) non-rotating neutron star and black hole solutions are close to those of GR, but might be distinguishable with future observations. Radiation damping from binaries, imposes an order 0.001 constraint on one combination of the parameters. Strong self-field effects in neutron stars and black holes produce violations of the strong equivalence principle and higher order post-Newtonian effects which will constrain all the parameters c_i to be less than around 0.01, presuming that the sensitivity parameters for neutron stars (which have not yet been computed with the required precision) turn out to have the expected magnitude.

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