Possibly Large Corrections to the Inflationary Observables

N. Bartolo[1,2] and A. Riotto[2,3]

[1] *Dipartimento di Fisica, Universitá di Padova,*
Via Marzolo, 8 - I-35131 Padua – Italy

[2] *INFN, Sezione di Padova,*
Via Marzolo, 8 - I-35131 Padua – Italy
and
[3] *CERN, Theory Division,*
Genéve 23, CH-1211 Switzerland

We point out that the theoretical predictions for the inflationary observables may be generically altered by the presence of fields which are heavier than the Hubble rate during inflation and whose dynamics is usually neglected. They introduce corrections which may be easily larger than both the second-order contributions in the slow-roll parameters and the accuracy expected in the forthcoming experiments.

Cosmological inflation [1] has become the dominant paradigm within which one can attempt to understand the initial conditions for Cosmic Microwave Background (CMB) anisotropies and structure formation. In the inflationary picture, the primordial cosmological perturbations are created from quantum fluctuations which are "redshifted" out of the horizon during an early period of accelerated expansion. Once outside the horizon, they remain "frozen" until the horizon grows during a later radiation- or matter-dominated era. After falling back inside the horizon they are communicated to the primordial plasma and hence are directly observable as temperature anisotropies in the CMB. These anisotropies have been mapped with spectacular accuracy by the Wilkinson Microwave Anisotropy Probe (WMAP) [2] and even a better accuracy will be reached by the Planck satellite [3] and its successors. This has allowed to put more and more stringent constraints on inflationary models [2, 4, 5, 6].

Given the present and future level of accuracy of the cosmological observations in the CMB anisotropies and in the large scale structure of the universe, the theoretical predictions for the inflationary observables need to be as precise as possible. Within single field models of inflation this is not necessarily a hard task since the power spectrum $\mathcal{P}_\zeta$ of the curvature perturbation ζ, the spectral index n_ζ, the amount of tensor modes and the way these observables run with the scale may be computed with the required level of accuracy in terms of series of powers of the slow-roll parameters [7]. Comparing cosmological data with the predictions of a given model of inflation to decide whether they are compatible seems therefore quite straightforward. In this short note, we would like to point out that this may not be the case: there are generic corrections to the theoretically predicted inflationary observables which have been neglected so far and which may, in principle, be calculated within a given inflationary model once the latter is rooted in a well-defined particle physics model.

Let us briefly explain the nature of such corrections. In any inflationary model it is a common lore to neglect the dynamics of those fields which are heavier than the Hubble rate H during inflation. This is because the heavy fields are stabilized at the minimum of their potential and are thought to play no active role in the inflationary dynamics. This is certainly correct, they do not play any major role. However, the correct question is if they influence the inflationary predictions at the level of accuracy needed for a fair comparison with the observations. We believe the answer may be positive. This may be either fortunate or unfortunate according to the various cases. The key point is that during inflation the vacuum expectation value (VEV) of these heavy fields is not likely to be constant; on the contrary it is expected to adiabatically and slowly changing with time to follow the change of the Hubble rate. This simple effect changes the inflationary predictions.

Let us describe the effect in some detail. Consider the inflationary dynamics driven by a scalar field ϕ with potential $V(\phi)$. We now assume the presence of a heavy scalar field Φ with mass m and VEV Φ_0 in the present-day vacuum. We will also assume that the mass of the heavy field is larger than the Hubble rate during inflation, $H \lesssim m$. The important observation is that, in general, the inflaton field and the heavy field are not totally decoupled from each other. On the contrary, one expects that during inflation the total potential assumes the form

$$U(\phi, \Phi) = V(\phi)f(\Phi/M) + \frac{1}{2}m^2(\Phi - \Phi_0)^2 , \qquad (1)$$

where we have expanded the potential of the heavy field around its present VEV Φ_0 (assuming consistently that $V(\phi)$ is much larger than the heavy field potential at Φ_0) and the function $f(\Phi/M)$ parametrizes the interaction

between the inflaton and the heavy fields. The scale M may be the reduced Planck scale M_p if the interaction is of gravitational nature, but may be smaller if $f(\Phi/M)$ arises from the exchange of some heavy fields. The kind of interaction parametrized by the function f may arise, for instance, in any model of inflation incorporated within supergravity. Indeed, radiative corrections to the Kähler potential lead to terms in the effective Lagrangian of the form

$$\delta \mathcal{L} = \pm \frac{C^2}{M_p^2} \int d^4\theta \, \phi^\dagger \phi \, \Phi^\dagger \Phi = \mp C^2 \frac{V(\phi)}{M_p^2} \Phi^\dagger \Phi, \tag{2}$$

where C^2 is a coefficient which can be larger than unity. We can now Taylor-expand the function $f(\Phi/M)$ around Φ_0 to find how much the heavy field VEV is displaced from its present-day value. The full potential becomes

$$U(\phi, \Phi) \simeq V(\phi)f_0 + V(\phi)\frac{f_0'}{M}(\Phi - \Phi_0) + V(\phi)\frac{1}{2}\frac{f_0''}{M^2}(\Phi - \Phi_0)^2 + \frac{1}{2}m^2(\Phi - \Phi_0)^2, \tag{3}$$

where $f_0 = f(\Phi_0/M)$, f_0' denotes the derivative of f with respect to its argument evaluated at $\Phi = \Phi_0$ and so on. One can easily show that the VEV of the heavy field is shifted from Φ_0 to

$$\Phi \simeq \Phi_0 - \frac{f_0' V(\phi)/M}{f_0'' V(\phi)/M^2 + m^2}. \tag{4}$$

Plugging this new VEV back into Eq. (3) one finds the potential

$$U(\phi) \simeq V(\phi)f_0 - \frac{1}{2}\frac{(f_0')^2}{M^2 m^2}V^2(\phi), \tag{5}$$

where we have assumed $|f_0''|V(\phi)/M^2 \lesssim m^2$ for the sake of simplicity. We obtained what advertised earlier: because of the small change during inflation of the VEV of the heavy field with respect to its value in the present vacuum, the starting inflaton potential receives a generic correction

$$\delta V(\phi) \sim (V^2(\phi)/M^2 m^2) \sim (H(\phi)/m)^2 (M_p/M)^2 V(\phi). \tag{6}$$

Let us now compute the corresponding changes in the inflationary observables. First of all, the number of e-folds to go till the end of inflation becomes

$$N = \int dt' \, H(t') \simeq \frac{1}{M_p^2} \int^{\phi_N} d\phi \, \frac{V}{V'}\left(1 + \frac{1}{2}\frac{(f_0')^2}{f_0 M^2 m^2}V\right), \tag{7}$$

where $V' = dV(\phi)/d\phi$ and so on. The slow-roll parameters become

$$\epsilon \simeq \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2 \left(1 - \frac{(f_0')^2 V}{f_0 M^2 m^2}\right),$$

$$\eta \simeq M_p^2\left(\frac{V''}{V}\right)\left(1 - \frac{1}{2}\frac{(f_0')^2 V}{f_0 M^2 m^2}\right) - M_p^2\left(\frac{V'}{V}\right)^2 \frac{(f_0')^2 V}{f_0 M^2 m^2}, \tag{8}$$

and are to be computed at $\phi = \phi_N$ when there are N e-folds to go till the end of inflation. From these expressions we conclude that the spectral index $n_\zeta = 1 + 2\eta - 6\epsilon$, the tensor-to scalar ratio r and their running with the scales (or e-folds) dn_ζ/dN and dr/dN get corrections of the form

$$\delta n_\zeta = \mathcal{O}(\epsilon,\eta)\frac{(f_0')^2}{f_0}\frac{M_p^2}{M^2}\frac{H^2}{m^2},$$

$$\delta r = \mathcal{O}(\epsilon)\frac{(f_0')^2}{f_0}\frac{M_p^2}{M^2}\frac{H^2}{m^2},$$

$$\delta\left(\frac{dn_\zeta}{dN}\right) = \mathcal{O}(\epsilon^2,\eta^2,\epsilon\eta)\frac{(f_0')^2}{f_0}\frac{M_p^2}{M^2}\frac{H^2}{m^2},$$

$$\delta\left(\frac{dr}{dN}\right) = \mathcal{O}(\epsilon^2,\eta^2,\epsilon\eta)\frac{(f_0')^2}{f_0}\frac{M_p^2}{M^2}\frac{H^2}{m^2}. \tag{9}$$

These corrections may be large as H is not necessarily much smaller than the mass m of the heavy field, M can be smaller than the Planck scale and there may be many heavy fields. The corrections (9) have to be compared to the corrections which are second-order in the slow-roll parameters, e.g. $\delta n_\zeta = \mathcal{O}(\epsilon^2,\eta^2,\epsilon\eta)$. The corrections due to the presence of the heavy field can be clearly larger.

Consider for example the large-field models of inflation characterized by a potential $V(\phi) = (\mu^{4-p}/p)\phi^p$ where μ is a mass scale and p is a positive integer. One can easily check that the spectral index n_ζ is not modified as a function of the number of e-folds N given in Eq. (7) and we find $n_\zeta - 1 = -(2+p)/(2N)$ [8]. However, the tensor-to-scalar perturbation ratio r is modified. Since in single-field inflation $r = 16\epsilon$, we can easily calculate

$$r = \frac{8p}{p+2}(1-n_\zeta)\left(1 - \frac{3(p+1)}{p+2}\frac{(f_0')^2}{f_0}\frac{M_p^2}{M^2}\frac{H^2(\phi_N)}{m^2}\right). \tag{10}$$

Notice that the correction is always negative (this in fact holds for any inflaton potential). Thus, for example, for a quartic potential, $V(\phi) \propto \phi^4$, which has been put under strong pressure from the recent data analyses, this means that a slight red tilt could still be accommodated with an amplitude of the gravity waves that is lower than the usual standard predictions. Also, we conclude that the corrections from the presence of the heavy fields may be even larger than the expected accuracy $r \sim 10^{-4}$ of forthcoming experiments aimed to measure the presence of tensor modes through the B-type polarization of the CMB [9].

The running of r will also change (while that of n_ζ is not affected by the presence of the heavy field) into

$$\frac{dr}{dN} = -\frac{16p}{(p+2)^2}(1-n_\zeta)^2 - \frac{3}{8}\frac{(p-2)(p+1)}{p(p+2)}r^2\frac{(f_0')^2}{f_0}\frac{M_p^2}{M^2}\frac{H^2(\phi_N)}{m^2}. \tag{11}$$

Notice that for $p \gg 1$, the running of r is due only to the heavy field correction. As another example, let us consider an inflaton potential of the form

$$V(\phi) = V_0\left(1 + \alpha\ln\frac{\phi}{\Lambda}\right), \tag{12}$$

which is typical in supersymmetric hybrid inflation [1] (here α is usually a loop-factor and Λ is a mass scale). After some simple algebra, one finds that

$$n_\zeta - 1 = -\frac{1}{N}\left(1 + \frac{3}{2}\alpha\right) - \frac{3}{4}\frac{\alpha}{N}\frac{(f_0')^2}{f_0}\left(\frac{M_p}{M}\right)^2\left(\frac{H}{m}\right)^2. \tag{13}$$

The almost unavoidable presence of heavy fields during inflation leads therefore to corrections to the inflationary observables which may be larger than both the second-order contributions in the slow-roll parameters and of the accuracy of future experiments. Indeed, while $H \lesssim m$ for the field Φ to be considered heavy during inflation, one might have easily $M \lesssim M_p$ and a large contribution from the $(f_0')^2/f_0$. Moreover, while here we have just considered the case of a single heavy scalar field, the size of these effects could be increased by the presence of many of such heavy fields (for instance, in inflationary string-motivated scenarios one has plenty of moduli fields, such as the complex structure moduli, which are usually disregarded). In a well-defined model of inflation, well-rooted in a given particle theory model, these corrections can (and should) be computed. On the other hand, if one wishes to see if a generic (toy) inflaton potential is in agreement with present and future observations, these corrections should be regarded as an unavoidable source of uncertainty in the theoretical predictions. This is rather unfortunate. However, one could also try to make use of these corrections to gain something. For instance, since the corrections to r, ϵ and η are generically negative, one could try to relax the so-called Lyth bound on the total variation of the inflaton field [10] and to obtain inflation from potentials which are too steep.

———————————————————

[1] For a review, see D. H. Lyth and A. Riotto, Phys. Rept. **314**, 1 (1999).

[2] D. N. Spergel *et al.* [WMAP Collaboration], Astrophys. J. Suppl. **170**, 377 (2007) [arXiv:astro-ph/0603449].

[3] See http://planck.esa.int/.

[4] W. H. Kinney, E. W. Kolb, A. Melchiorri and A. Riotto, Phys. Rev. D **74**, 023502 (2006) [arXiv:astro-ph/0605338].

[5] J. Martin and C. Ringeval, JCAP **0608**, 009 (2006) [arXiv:astro-ph/0605367].

[6] F. Finelli, M. Rianna and N. Mandolesi, JCAP **0612**, 006 (2006) [arXiv:astro-ph/0608277].

[7] See, for example, J. E. Lidsey, A. R. Liddle, E. W. Kolb, E. J. Copeland, T. Barreiro and M. Abney, Rev. Mod. Phys. **69**, 373 (1997) [arXiv:astro-ph/9508078].

[8] It is worth mentioning that there might be specific cases when the corrections to the spectral index cancel each other thus leaving the expression of the spectral index unchanged as a function of the number of e-folds N. Taking into account the corrections appearing in Eqs. (7)-(8), the slow-roll parameters can be expressed a $\epsilon = \epsilon(N)(1 + \delta\epsilon(N))$ (and the same for η), where $\epsilon(N)$ and $\eta(N)$ are parametrically the same functions of the number of e-folds as the ones predicted withouth any correction to the potential $V(\phi)$. In other words, $\epsilon(N) = \epsilon_0(N_0 \rightarrow N)$, where the subscript 0 stands for the quantities computed withouth invoking any corrections. It is thus easy to see that the form of the spectral index $n_\zeta = 1 + 2\eta - 6\epsilon$ is left unchanged as a function of N if the following condition is satisfied $\delta\eta(N)/\delta\epsilon(N) = 3\epsilon_0(N_0)/\eta_0(N_0)$.

[9] M. Amarie, C. Hirata and U. Seljak, Phys. Rev. D **72**, 123006 (2005) [arXiv:astro-ph/0508293]; U. Seljak and C. M. Hirata, Phys. Rev. D **69**, 043005 (2004) [arXiv:astro-ph/0310163] ; F. Stivoli, C. Baccigalupi, D. Maino and R. Stompor, Mon. Not. R. astr. Soc. **372**, 615 (2006), [arXiv:astro-ph/0505381].

[10] D. H. Lyth, Phys. Rev. Lett. **78**, 1861 (1997) [arXiv:hep-ph/9606387].