GRAVITATIONAL POTENTIAL, INERTIA AND EARTH ROTATION

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INTRODUCTION

Several satellite missions, devoted to the study of the Earth gravity field, have been launched (like CHAMP, recently). This year, GRACE (Gravity Recovery and Climate Experiment) will allow us to obtain a more precise geoid. But the most important is that they will supply the temporal variations of the geopotential coefficients (called Stokes coefficients).
In the poster, we show how the Earth gravitational potential is linked to the Earth rotation parameters. Indeed, through the Earth inertia coefficients, we can connect the variation of LOD and Polar Motion with the temporal variations of the Stokes coefficients. We also consider the nutations, that are related to the gravitational geopotential coefficients.
We discuss the possibility of using the Stokes coefficients in order to improve our knowledge of the Earth rotation.

1. LENGTH-OF-DAY

The excess in the length-of-day can be related to the instantaneous Earth rotation rate $\omega = \Omega\ (1 + m_3)$, where Ω is the mean Earth speed of rotation. Indeed, we have :

$$\Delta(LOD) = LOD - LOD_{mean} = k\ \frac{2\pi}{\omega} - k\ \frac{2\pi}{\Omega} \simeq -k\ \frac{2\pi}{\Omega}\ m_3 \tag{1}$$

where k is the conversion factor from sidereal to mean solar days. That involves :

$$-\ \frac{\Delta(LOD)}{LOD_{mean}} = m_3 \tag{2}$$

By the way of the Liouville's equations, m_3 is linked with L_3, third component of the external torque, with c_{33}, time-dependant difference to the constant part C of the third principal moment of Earth inertia and with h_3, third component of the relative angular momentum of the system. In this case, not considering the external perturbations, we have :

$$\frac{\Delta(LOD)}{LOD_{mean}} = \frac{c_{33}}{C} + \frac{h_3}{C\ \Omega} \tag{3}$$

Furthermore, the variable part c_{33} of the Earth inertia tensor depends on the temporal variation of the Stockes coefficient C_{20} of degree 2 and order 0 (Gross, 2000) :

$$c_{33}(t) = \frac{1}{3}\ \Delta Tr(I) - \frac{2}{3}\ \mathcal{M}\ \mathcal{R}_e^2\ \Delta C_{20}(t) \tag{4}$$

where $\Delta Tr(I)$ is the time-dependent difference to the constant part of the inertia tensor trace. Due to the conditions in which we place (fluid atmospheric and oceanic layers parts of our system), we can consider that there is conservation of the volume of our system under deformations.

For this reason, according to (Rochester and Smylie, 1974), we have : $\Delta Tr(I) = 0$. Finally, according to the equations (3) and (4), but also considering surface loading and rotationnal deformation (Barnes et al., 1983) (factor 0.7), we obtain :

$$\frac{\Delta(LOD)}{LOD_{moyen}} = -0.7 \; \frac{2}{3\;C} \; \mathcal{M} \; \mathcal{R}_e^2 \; \Delta C_{20} + \frac{h_3}{C\;\Omega} \tag{5}$$

2. POLAR MOTION

According to (Gross, 1992), we can connect the polar motion $p = x - iy$ with the motion of the instantaneous rotation axis $m = m_1 + i\; m_2$: $m = p - i/\Omega\; \dot{p}$. Considering (Gross, 2000), this brings to link the polar motion to the temporal variations of some Stockes coefficients :

$$\begin{cases} p + i\; \frac{\dot{p}}{\sigma_r} = \chi \\ \chi = \frac{1}{\Omega\;(C-A)}(\Omega\; c + h) = \frac{1}{\Omega\;(C-A)}\left(-\mathcal{M}\; \mathcal{R}_e^2\; \Omega\; (C_{21} + i\; S_{21}) + 1.43\; h\right) \end{cases}$$

where $C_{21} \equiv \Delta C_{21}$, $S_{21} \equiv \Delta S_{21}$, $c = c_{13} + i\; c_{23}$, and the coefficient 1.43 comes from (Barnes et al., 1983).

3. NUTATIONS

Some authors have derived the nutations of the Earth rotation axis using the harmonic coefficients of the gravity geopotential (Melchior,1973; Bretagnon, 1997; Brezinski and Capitaine, 2001).
The most important Stokes coefficient for such a phonomenon is C_{20}, but we can see that the other zonal coefficients act on the nutations. It would be then interesting to have their temporal variations, even if they are diurnal in the Earth.

DISCUSSION

We have provided the equations linking the temporal variations of the geopotential coefficients with the Earth rotation parameters (variation of LOD, polar motion and nutations). With the new gravity missions, we will be able to determine variations of Stokes coefficients, and to isolate the total variations of the solid Earth moment of inertia, what is totally new.
This is a new method, so comparing our results with those already known will be interesting, but a study of the precision needed for such a work is necesary and will be done.

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