

# Spherical symmetric charged solution with cosmological constant

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## Abstract

A spherically symmetric charged ideal fluid solution of Einstein field equation is given in the presence of the cosmological constant and two well known example of this type of solution is presented. If the matter is confined in a region, the exterior spacetime is considered as RN-de Sitter (Reissner-Nordström de Sitter) and to complete solution matching conditions are examined. We show that the function which is related to the dynamics of the system will determine the fate of the system: expansion, contraction or bouncing situations may occur for different configurations. The initial conditions of the matter determine the final form of the system and therefore the nature of the singularities in the presence of the electric charge and the cosmological constant is examined to reveal their effects on the singularity formation during collapse.

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# 1 Introduction

Cosmological observations indicate that the expansion of the universe is accelerating. A simple phenomenological interpretation of the data in terms of dark energy has been successful thus far [1]; dark energy is assumed to have negative pressure, unlike ordinary mass-energy, and thus leads to a negative force that may account for the acceleration of the universe. Recent observations indicate that universe is dominated by dark energy ( $\sim 70\%$ ), which can be thought of as a perfect fluid with an energy-momentum tensor given by

$$T_D^{\mu\nu} = (\mu_D + p_D) u^\mu u^\nu + p_D g^{\mu\nu}, \quad (1)$$

where  $\mu_D + p_D = 0$  and  $p_D < 0$ . The inclusion of source term (1) in Einstein's field equations amounts simply to a cosmological constant  $\Lambda$  given by  $\Lambda = -p_D > 0$ . In this way, dark energy may be represented by a positive cosmological constant  $\Lambda$  in the Einstein field equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = T_{\mu\nu}. \quad (2)$$

The cosmological constant has a long history in Newtonian and relativistic gravity theories. It was introduced into general relativity by Einstein as a means of balancing the gravitational attraction of the matter on cosmological scales, leading to the Einstein static universe model. Alternatively, it can be thought of as a measure of the energy density of the vacuum.

In this paper the gravitational motion of charged matter is considered in the presence of a cosmological constant. To simplify matters, a *perfect fluid* distribution is considered that is electrically charged and undergoes spherically symmetric collapse or expansion in the absence of external fields.

The exterior field is given by the Reissner-Nordström-de Sitter metric

$$ds^2 = -\left(1 - \frac{2M}{\hat{r}} + \frac{Q^2}{\hat{r}^2} - \frac{\Lambda}{3}\hat{r}^2\right)d\hat{t}^2 + \frac{d\hat{r}^2}{\left(1 - \frac{2M}{\hat{r}} + \frac{Q^2}{\hat{r}^2} - \frac{\Lambda}{3}\hat{r}^2\right)} + \hat{r}^2(d\hat{\theta}^2 + \sin^2\hat{\theta}d\phi^2) \quad (3)$$

where  $M$  is the net mass and  $Q$  is the net charge of the system and  $\Lambda$  is the cosmological constant of the RN-de Sitter spacetime.

The gravitational collapse phenomena is still open problem in the general relativity and it has not been taken its final form yet. Throughout the studies it is shown that the initial conditions are determinative of the final fate of the collapse [2]. If the collapse can not be stopped in an equilibrium state and allows to formation of the singularities as a result, the end state of the collapse can either be black hole or naked singularity depending on the character of the singularities. If all singularities are hidden behind an event horizon singularities can not be seen by distant observer and the end state becomes "a black hole", or if the singularities are bare and can be visible by distant observer, the end state becomes "naked singularity" [2].

One of the physical features affected on the gravitational collapse is shear. In [3], shear effects on the gravitational collapse of the spherical massive cloud with non-radial pressure are studied and shown that sufficiently strong shear effects near singularity delay the formation of the apparent horizon and allow the formation of the naked singularity.

In [4] effects of the cosmological constant on the gravitational collapse of the pressureless matter is studied and shown that positive gravitational constant plays repulsive role and slows down the collapse process.

As a property of matter how does electric charge affect the collapse phenomenon? This question finds some answers through the following papers and needs to be completed. A solution for the spherical symmetric charged

stars are considered in point of view formation of the black holes and voids in [5]. In [6], the gravitational collapse of the spherical symmetric charged radiating Vaidya-RN type spacetimes are studied. To avoid singularities due to charge of the matter in spherical symmetric collapse is examined in [7]. As a stellar model, relativistic structure, stability and gravitational collapse of the charged fluid is studied in [8], the specific values of the electric charge of the fluid allows formation of naked singularity besides black hole formation. A spherically symmetric charged ideal fluid is examined in [9] due course of gravitational collapse. Similar to solution presented in [9], in this work, we give a solution of Einstein field equations in de Sitter spacetime which is in isotropic form. Furthermore, we examine the effects of the electric charge on the collapse phenomenon in the presence of the cosmological constant.

In the following section, a spherically symmetric solution of the Einstein-Maxwell equations is given in the presence of the cosmological constant. The exterior spacetime of the charged fluid sphere is considered the RN-de Sitter spacetime and matching conditions about two distinct (interior and exterior) regions are examined in section 3. Two well known examples of this type of solutions, RN-de Sitter and Mc Vittie-de Sitter are given. Section 4 is devoted to the gravitational collapse of the charged fluid and formation of the singularities. Since the co-moving character of the spacetime may give coordinate dependent results, it is necessary to use coordinate free "null geodesics method" where the nature of the singularities does not change its character. Therefore, the nature of the singularities are investigated null geodesic method. Throughout the paper we use units such that  $c = 1$  and  $(8\pi G = 1)$ .

## 2 Interior solution

Imagine a co-moving system of coordinate for the interior  $(t, \rho, \theta, \phi)$  that remains at rest with the moving charged matter and is given by

$$ds^2 = -a^2 dt^2 + b^2 d\rho^2 + R^2 \rho^2 d\theta^2 + R^2 \rho^2 \sin^2 \theta d\phi^2 \quad (4)$$

where  $a = a(t, \rho)$ ,  $b = b(t, \rho)$  and  $R = R(t, \rho)$  are arbitrary positive functions of time coordinate  $t$  and radial coordinate  $\rho$ . To consider shear free motion of the matter that is the ideal fluid, we take  $R = \rho b(t, \rho)$ . (4) is assumed to be the solution of the field equations (2) with a cosmological constant  $\Lambda_0$  and a source  $T_{\mu\nu} = T_{\mu\nu}^{\text{m}} + T_{\mu\nu}^{\text{em}}$ , where

$$T_{\mu\nu}^{\text{m}} = (\mu + p) u_\mu u_\nu + p g_{\mu\nu} \quad (5)$$

and the electromagnetic energy-momentum tensor is defined by

$$T_{\mu\nu}^{\text{em}} = 2(g^{\alpha\beta} F_{\alpha\mu} F_{\beta\nu} - \frac{1}{4} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}). \quad (6)$$

Here electromagnetic field tensor is  $F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu$  and  $A_\mu$  is the vector potential. The spherical symmetry of the spacetime ensures existence of a radial electric field in general.

After a suitable choice of the gauge is  $A_i = 0$ ,  $A_t = \Phi(t, \rho)$  chosen, according to the metric (4) the only non-zero component of  $F_{\mu\nu}$  becomes  $F_{t\rho} = -\partial\Phi/\partial\rho$ . Then, non-vanishing components of the electromagnetic energy momentum tensor for the spacetime with the gauge chosen above are given by

$$\begin{aligned} T_t^t &= -\frac{q^2}{\rho^4 b^4}, \\ T_t^t &= T_\rho^\rho = -T_\theta^\theta = -T_\phi^\phi \end{aligned} \quad (7)$$

where  $q$  is the total charge of the fluid. It is easily seen that  $T = \text{tr } T_\mu^\nu = 0$  as expected. The Maxwell equations

$$\frac{\partial}{\partial x^\mu} [\sqrt{-g} F^{\nu\mu}] = 4\pi \sqrt{-g} J^\nu \quad (8)$$

with  $\sqrt{-g} = ab^3 \rho^2 \sin \theta$  for the spacetime (4) become

$$\frac{\partial}{\partial t} \left( \frac{b}{a} \frac{\partial \Phi}{\partial \rho} \right) = 0. \quad (9)$$

It is seen that total charge  $q$  defined by

$$\frac{b}{a} \frac{\partial \Phi}{\partial \rho} = \frac{q}{\rho^2} \quad (10)$$

is independent of time and related to the charge density  $\zeta$  by the equation

$$\frac{dq}{d\rho} = 4\pi \rho^2 b^3 \zeta. \quad (11)$$

The electric current is given by  $J^t = \zeta u^t$  in terms of the electric charge density and 4-velocity  $u^t = a^{-1}$ . Since the total energy momentum tensor is the sum of the electromagnetic  $T_{\mu\nu}^{\text{em}}$  and matter part  $T_{\mu\nu}^{\text{m}}$ , the total energy-momentum components will be obtained as

$$\begin{aligned} T_t^t &= -\mu - \frac{q^2}{\rho^4 b^4}, & T_\rho^\rho &= p - \frac{q^2}{\rho^4 b^4}, \\ T_\theta^\theta &= p + \frac{q^2}{\rho^4 b^4}, & T_\phi^\phi &= p + \frac{q^2}{\rho^4 b^4} \end{aligned} \quad (12)$$

where  $\mu$  are  $p$  are the matter-energy density and pressure respectively. It is supposed that the matter field satisfies the weak energy condition. For any timelike vector  $v^\mu$

$$T_{\mu\nu}^{\text{m}} v^\mu v^\nu \geq 0 \quad (13)$$

which gives

$$\mu \geq 0, \quad \mu + p \geq 0. \quad (14)$$

Furthermore, the conservation of the energy  $u_\mu T^{\mu\nu}{}_{;\nu} = 0$  gives the relation

$$\frac{1}{(\mu + p)} \frac{\partial \mu}{\partial t} = - \frac{3}{b} \frac{\partial b}{\partial t}. \quad (15)$$

Now we are going to examine the geometry part of the problem given by the metric (4) and consider the Einstein tensor  $G_{\mu\nu} = R_{\mu\nu} - R g_{\mu\nu}/2$  as follows

$$\begin{aligned} G_{tt} &= 3 \frac{\dot{b}^2}{b^2} - \frac{a^2}{b^2} \left[ 2 \frac{b''}{b} - \left( \frac{b'}{b} \right)^2 + \frac{4}{\rho} \frac{b'}{b} \right] \\ G_{t\rho} &= -2a \left( \frac{\dot{b}}{ab} \right)' \\ G_{\rho\rho} &= \left( \frac{b'}{b} \right)^2 + 2 \frac{a'}{a} \frac{b'}{b} + \frac{2}{\rho} \left( \frac{b'}{b} + \frac{a'}{a} \right) - \frac{b^2}{a^2} \left( 2 \frac{\ddot{b}}{b} + \frac{\dot{b}^2}{b^2} - 2 \frac{\dot{a}}{a} \frac{\dot{b}}{b} \right) \\ G_{\theta\theta} &= \rho^2 \left[ \frac{1}{\rho} \left( \frac{a'}{a} + \frac{b'}{b} \right) + \frac{a''}{a} + \frac{b''}{b} - \frac{b'^2}{b^2} + \frac{b^2}{a^2} \left( 2 \frac{\dot{a}}{a} \frac{\dot{b}}{b} - \frac{\dot{b}^2}{b^2} - 2 \frac{\ddot{b}}{b} \right) \right] \\ G_{\phi\phi} &= \sin^2 \theta G_{\theta\theta}. \end{aligned} \quad (16)$$

where “dot” and “prime” represent derivatives with respect to  $t$  and  $\rho$ , respectively.

By using the energy-momentum tensor of the fluid obtained in (12), the field equations (2) take the form

$$G_{t\rho} = 0, \quad (17)$$

$$\frac{1}{a^2} G_{tt} - \Lambda_0 = \mu + \frac{q^2}{\rho^4 b^4}, \quad (18)$$

$$\frac{1}{b^2} G_{\rho\rho} + \Lambda_0 = p - \frac{q^2}{\rho^4 b^4}, \quad (19)$$

$$G_{\rho\rho} - \frac{1}{\rho^2} G_{\theta\theta} = - \frac{2 q^2}{\rho^4 b^2} \quad (20)$$

where  $\Lambda_0$  is the cosmological constant for the interior region. For the sake of the generality, the cosmological constant of the interior region  $\Lambda_0$  is taken to be different from the cosmological constant of the exterior region  $\Lambda$  in the beginning.

Then, the trace of field equations becomes

$$(3p - \mu) - \frac{2}{b^2} \left( \frac{a''}{a} + \frac{a' b'}{a b} - \frac{b'^2}{b^2} + \frac{2b''}{b} + \frac{2a'}{a \rho} + \frac{4b'}{b \rho} \right) - \frac{6}{a^2} \left( \frac{\dot{a} \dot{b}}{a b} - \frac{\dot{b}^2}{b^2} - \frac{\ddot{b}}{b} \right) = 4\Lambda_0. \quad (21)$$

Equation (17) has the solution

$$\dot{b} = a b k(t), \quad (22)$$

where  $k(t)$  is an arbitrary function of time, then equation (20) becomes

$$\left( \frac{a''}{a} + \frac{b''}{b} \right) - \left( \frac{1}{\rho} + 2 \frac{b'}{b} \right) \left( \frac{a'}{a} + \frac{b'}{b} \right) = \frac{2 q^2}{b^2 \rho^4}. \quad (23)$$

A solution of the full Einstein field equations can be found by following the method given in the reference [9] as follows:

$$a = \frac{1 - \frac{\nu \lambda^2}{r^2}}{1 + \frac{\lambda}{r} + \frac{\nu \lambda^2}{r^2}}, \quad b = \frac{1}{W^{1/2}} \frac{\lambda_0}{\lambda} r \left( 1 + \frac{\lambda}{r} + \frac{\nu \lambda^2}{r^2} \right) \quad (24)$$

where  $\nu = \frac{1}{4} \left( 1 - \frac{\eta_0^2}{\lambda_0^2} \right)$ ,  $\lambda(t) = \frac{\lambda_0}{f(t)}$ ,  $W = (\alpha - \gamma r^2)(\delta r^2 - \beta)$

and  $(\alpha\delta - \beta\gamma) > 0$ .  $\alpha, \beta, \delta, \gamma, \eta_0$  are all real and  $\lambda_0 > 0$ ,  $\nu \geq 0$  are positive constants. Moreover,  $f(t)$  is positive arbitrary function of time and cosmological constant  $\Lambda_0$ .



Since the radial coordinate transformation does not change the co-moving character of the metric, for the sake of the brevity, we used the transformation

$$r = \left( \frac{\alpha \rho^2 + \beta}{\gamma \rho^2 + \delta} \right)^{1/2}. \quad (25)$$

Then, the physical quantities mass-energy density and pressure satisfying relations (18), (19), can be written in following form

$$\begin{aligned} \mu &= -\Lambda_0 + 3 \left( \frac{\dot{f}}{f} \right)^2 + \frac{192 f^2 \left( \alpha \beta (-\eta_0^2 + \lambda_0^2) + 2r(\alpha \beta + \delta \gamma r^4) \lambda_0 f + 4\delta \gamma r^6 f^2 \right)}{N_0^4}, \\ p &= \Lambda_0 - \frac{r^2(5(\eta_0^2 - \lambda_0^2) - 8r\lambda_0 f + 4r^2 f^2) \dot{f}^2}{f^2 N_1} - \frac{2 N_0 \ddot{f}}{f N_1} \\ &\quad + \frac{64 f^2 (\alpha r^2 - \gamma^2)^3 (\beta r^2 - \delta)^2 N_2}{N_0^5 N_1 (\alpha \delta - \beta \gamma)^5 r^{10}} \end{aligned} \quad (26)$$

with

$$N_0 = -\eta_0^2 + \lambda_0^2 + 4r\lambda_0 f + 4r^2 f^2, \quad N_1 = \eta_0^2 - \lambda_0^2 + 4r^2 f^2,$$

$$\begin{aligned} N_2 &= \frac{1}{r^{14} (\beta \gamma - \alpha \delta)^4} \left( 4 f^2 \eta_0^4 (\alpha - r^2 \gamma)^6 (\beta - r^2 \delta)^2 (4 f^2 r^2 + \eta_0^2 - \lambda_0^2) \right. \\ &\quad + (\beta \gamma - \alpha \delta)^4 r^4 \left( -\alpha \beta \eta_0^6 - (2 f r + \lambda_0)^4 (4 f^2 r^6 \gamma \delta - \alpha \beta \lambda_0^2) \right. \\ &\quad + \eta_0^4 (-4 f^2 r^2 (-3 \alpha \beta + r^2 \beta \gamma + r^2 \alpha \delta) + 8 f r \alpha \beta \lambda_0 + 3 \alpha \beta \lambda_0^2) \\ &\quad + \eta_0^2 (2 f r + \lambda_0) \left( 8 f^3 r^5 (-\beta \gamma - \alpha \delta + 3 r^2 \gamma \delta) \right. \\ &\quad \left. \left. + 4 f^2 r^2 (-4 \alpha \beta + r^2 \beta \gamma + r^2 \alpha \delta + r^4 \gamma \delta) \lambda_0 - 10 f r \alpha \beta \lambda_0^2 - 3 \alpha \beta \lambda_0^3 \right) \right) \end{aligned} \quad (27)$$

and the total charge of the fluid is given by

$$q = \frac{\eta_0}{(\alpha \delta - \beta \gamma)^2 r^3} (\alpha - \gamma r^2)^{3/2} (\delta r^2 - \beta)^{3/2}. \quad (28)$$

Consider the spacetime given by the line element (24) describes whole space-time then, by setting of arbitrary constants  $(\alpha, \beta, \delta, \gamma, \eta_0, \lambda_0, \nu)$  and function of time  $f$  in special forms, well known solutions of Einstein field equations such as RN-de Sitter and charged McVittie solutions can be obtained.

### 1) RN-de sitter solution.

The line element of the isotropic RN-de Sitter spacetime in [10] is given by

$$ds^2 = -\frac{\left[1 - \frac{m^2}{w^2 r^2} + \frac{\tilde{q}^2}{w^2 r^2}\right]^2}{\left[\left(1 + \frac{m}{wr}\right)^2 - \frac{\tilde{q}^2}{w^2 r^2}\right]^2} dt^2 + w^2 \left[\left(1 + \frac{m}{wr}\right)^2 - \frac{\tilde{q}^2}{w^2 r^2}\right]^2 (dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta^2 d\phi^2) \quad (29)$$

where  $m$  is the mass and  $\tilde{q}$  is the electric charge of the black hole and  $w = e^H$  is function of the Hubble parameter which is in general a function of time. If we take

$$f = w = e^{\sqrt{\Lambda_0/3} t}, \quad \lambda_0 = 2m, \quad \eta_0 = 2\tilde{q}, \quad \alpha = \delta = 1, \quad \beta = \gamma = 0 \quad (30)$$

the line element (24) reduces to isotropic RN-de Sitter line element (29).

In this configuration  $\eta_0/2$  and  $\lambda_0/2$  are considered as total charge and mass of the black hole, respectively. Since  $\dot{f}/f = \sqrt{\Lambda_0/3} > 0$ , therefore  $\dot{b}/b > 0$ , then the spacetime expands in time and  $H = \text{const.} = \sqrt{\Lambda_0/3}$  corresponds to the cosmological constant of RN-de Sitter spacetime. Furthermore, since  $m$  (or  $\lambda_0$ ) and  $\tilde{q}$  (or  $\eta_0$ ) are constants,  $\mu = p = 0$ , are all zero as expected.

The spacetime (24) contracts only if  $\dot{f}/f$  is negative. If we take  $f = e^{-\sqrt{\Lambda_0/3} t}$ , it will correspond to isotropic RN black hole in anti-de Sitter spacetime. Here

the physical quantities mass-energy density and the pressure are also zero as the previous case.

The cosmological constant is the reason of the time evolution of the problem. Therefore, if we take  $\Lambda_0 = 0$ , the contraction (expansion) of the space-time disappears, it becomes static.

## 2) Charged McVittie-de Sitter solution.

A perfect fluid solution of Einsteins equations corresponding to the Schwarzschild field embedded in a RobertsonWalker background is given by McVittie [11]. In expanding universe the McVittie solution represents a white hole, whereas it represents a black hole in contracting universe [12]. In addition to the repulsive effect of the expansion of the spacetime, the repulsive character of the electric charges (Coulomb force) of the fluid will be greater in small regions and will support formation of white hole in the beginning in the McVittie-de Sitter spacetime.

If we consider ( $\eta_0 \neq 0$   $\beta = 0$ ,  $\alpha$  and  $\delta$  positive) in (24), the charged Mc Vittie-de Sitter solution is obtained and the isotropic metric components become

$$\begin{aligned} a &= \frac{4\alpha r^2 f^2 + (\delta + \gamma r^2)(\eta_0^2 - \lambda_0^2)}{4\alpha r^2 f^2 + 4r\sqrt{\alpha(\delta + \gamma r^2)}f\lambda_0 - (\delta + \gamma r^2)(\eta_0^2 - \lambda_0^2)}, \\ b &= \frac{4\alpha r^2 f^2 + 4r\sqrt{\alpha(\delta + \gamma r^2)}f\lambda_0 - (\delta + \gamma r^2)(\eta_0^2 - \lambda_0^2)}{4\alpha r^2(\delta + \gamma r^2)f}. \end{aligned} \quad (31)$$

The matter density and the pressure can be written as

$$\begin{aligned} \mu &= -\Lambda_0 + 3\left(\frac{\dot{f}}{f}\right)^2 \\ &- \frac{128f^3\left(2f(\alpha - r^2\gamma)^6\eta_0^4 + 3r^5\alpha^4\gamma\delta^3(2fr + \lambda_0)^3 + r^4\alpha^4\delta^3\eta_0^2(2f(\alpha - 4r^2\gamma) - 3r\gamma\lambda_0)\right)}{\alpha^4\delta^2(\eta_0^2 - (2fr + \lambda_0)^2)^4} \end{aligned}$$

$$p = \Lambda_0 - \frac{256 f^4 \delta \gamma r^6}{N_1 N_0^2} + \left( \frac{\dot{f}}{f^2} \right) \frac{5(\lambda_0^2 - \eta_0^2) + 8r\lambda_0 f - 4r^2 f^2}{N_1} - 2 \left( \frac{\ddot{f}}{f} \right) \frac{N_0^3}{N_1} \quad (32)$$

and the charge becomes

$$q = \frac{\eta_0}{\alpha^2 \delta^{1/2}} (\alpha - \gamma r^2)^{3/2}. \quad (33)$$

By taking cosmological constant  $\Lambda_0 = 0$ , and electric charge  $\eta_0 = 0$ , uncharged ordinary McVittie solution can be recovered

$$a = \frac{-2\sqrt{\alpha} r f + \sqrt{\delta + \gamma r^2} \lambda_0}{2\sqrt{\alpha} r f + \sqrt{\delta + \gamma r^2} \lambda_0}, \quad b = \frac{2\sqrt{\alpha} r f + \sqrt{\delta + \gamma r^2} \lambda_0}{4\alpha r^2 (\delta + \gamma r^2) f}. \quad (34)$$

Mass-energy density and the pressure of the fluid become

$$\mu = 3 \left( \frac{\dot{f}}{f} \right)^2 + \frac{384\lambda_0 \delta \gamma r^5 f^3}{(2rf - \lambda_0)^3 (2rf + \lambda_0)^3},$$

$$p = \frac{256 \delta \gamma r^6 f^4}{(2rf - \lambda_0)(2rf + \lambda_0)^5} + \left( \frac{\dot{f}}{f^2} \right) \frac{5\lambda_0 + 8\lambda_0 r f - 4r^2 f^2}{(2rf - \lambda_0)(2rf + \lambda_0)} - 2 \left( \frac{\ddot{f}}{f} \right) \frac{(2rf + \lambda_0)^5}{(2rf - \lambda_0)}. \quad (35)$$

Now let us consider the charged fluid is confined in a region in which dynamical evolution of the system is described by the time dependent function  $f(t)$  and the exterior spacetime to the confined matter is RN-de Sitter. Exterior and interior regions separate the spacetime into two distinct parts such that they meet on the boundary surface. To complete the solution, it is necessary to show that the distinct solutions must satisfy boundary conditions on the boundary surface. In the following section we will investigate matching conditions.

### 3 Matching conditions

Let us consider a spherical boundary surface which divides spacetime into two distinct four-dimensional manifolds which admit  $\Sigma$  as their boundaries at  $r_\Sigma = b\rho_\Sigma = \hat{r}_\Sigma = \text{const.}$ .

Let  $\hat{t}$ ,  $\hat{r}$ ,  $\hat{\theta}$ ,  $\hat{\phi}$  be the Reissner-Nordström de Sitter coordinates for the matter-free region. Then, the metric is

$$ds_+^2 = -A d\hat{t}^2 + A^{-1} d\hat{r}^2 + \hat{r}^2 (d\hat{\theta}^2 + \sin^2 \hat{\theta} d\hat{\phi}^2) \quad (36)$$

where

$$A = 1 - \frac{2M}{\hat{r}} + \frac{Q^2}{\hat{r}^2} - \frac{\Lambda}{3} \hat{r}^2$$

with the interior metric is in original form and given by

$$ds_-^2 = -a^2 dt^2 + b^2 d\rho^2 + b^2 \rho^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (37)$$

where  $a$  and  $b$  are functions of coordinates  $\rho$  and  $t$ . Under the coordinate transformations

$$\hat{\theta} = \theta, \quad \hat{\phi} = \phi \quad (38)$$

motion of the boundary surface can be given by the following equations

$$f^+ : \hat{r} - \hat{r}_\Sigma(\hat{t}) = 0, \quad f^- : \rho - \rho_\Sigma = 0. \quad (39)$$

To match the exterior spacetime with the interior spacetime we use the Israel junction conditions [13]. These conditions require the interior and exterior solutions of the gravitational field equations to be joined smoothly up to a coordinate transformation but the partial derivatives may change discontinuously across the boundary surface of the matter.

Let  $ds_\Sigma^2$  be the line element of the boundary surface  $\Sigma$ ,  $ds_+^2$  represent the exterior and  $ds_-^2$  represent the interior spacetime line elements. The junction conditions which state the equality of the first fundamental forms and the discontinuity of the second fundamental forms can be given as

$$ds_\Sigma^2 = ds_-^2|_\Sigma = ds_+^2|_\Sigma, \quad (40)$$

$$K_{ij}^+ - K_{ij}^- - g_{ij} K = \tau_{ij}$$

where  $K = g^{ij} (K_{ij}^+ - K_{ij}^-)$  and  $\tau_{ij}$  is the surface energy momentum tensor. In case of vanishing surface energy momentum tensor i.e. for  $\tau_{ij} = 0$ , the discontinuity condition reduces to the equality of forms i.e., equality of the extrinsic curvatures

$$K_{ij}^+ = K_{ij}^- . \quad (41)$$

The continuity of the metric components in (40) at  $\rho = \rho_\Sigma$  on the boundary surface gives the following relations

$$\hat{r}_\Sigma = \rho_\Sigma b(t, \rho_\Sigma), \quad d\tau = a(t, \rho_\Sigma) dt, \quad (42)$$

$$d\tau = \sqrt{A(\hat{r}_\Sigma) - \frac{1}{A(\hat{r}_\Sigma)} \left( \frac{d\hat{r}_\Sigma}{d\hat{t}} \right)^2} d\hat{t}, \quad (43)$$

and

$$\frac{d\hat{r}}{d\tau} = \rho \frac{db}{d\tau} = \frac{\rho}{a} \frac{db}{dt} |_\Sigma, \quad (44)$$

$$\left( \frac{d\tau}{d\hat{t}} \right)^2 = \frac{A^2}{A + (d\hat{r}/d\tau)^2} |_\Sigma . \quad (45)$$

The non-zero second fundamental forms for the interior and the exterior regions are given by

$$K_{\theta\theta}^- = \sin^2 \theta K_{\phi\phi}^- = \rho (b\rho)' \quad (46)$$

$$K_{\theta\theta}^+ = \sin^2 \theta K_{\phi\phi}^+ = \frac{\hat{r} A}{\sqrt{A - \frac{1}{A} \left( \frac{d\hat{r}}{d\hat{t}} \right)^2}} \quad (47)$$

and

$$K_{\tau\tau}^- = -\frac{a'}{a b} \quad (48)$$

$$K_{\tau\tau}^+ = \frac{d\hat{r}}{d\tau} \frac{d^2\hat{t}}{d\tau^2} - \frac{d\hat{t}}{d\tau} \frac{d^2\hat{r}}{d\tau^2} + \frac{3}{2A} \frac{\partial A}{\partial \hat{r}} \left( \frac{d\hat{r}}{d\tau} \right)^2 \frac{d\hat{t}}{d\tau} - \frac{A}{2} \frac{\partial A}{\partial \hat{r}} \left( \frac{d\hat{t}}{d\tau} \right)^3. \quad (49)$$

By using the equality of the angular components of the second fundamental forms (46, 47) we obtain the following relations

$$A \hat{r} \frac{d\hat{t}}{d\tau} = \rho (b \rho)', \quad (50)$$

$$A = \left( \frac{\rho}{b} \frac{\partial b}{\partial \rho} + 1 \right)^2 - \frac{\rho^2}{a^2} \left( \frac{\partial b}{\partial t} \right)^2. \quad (51)$$

Since  $b$  is function of time dependent function  $f$ , (51) gives a condition that  $f$  should satisfy at any time  $t$ . Furthermore, the equality of the timelike components of the extrinsic curvatures, (48) with (49), gives

$$\frac{(A d\hat{t}/d\tau)_{,\tau}}{d\hat{r}/d\tau} = \frac{1}{ab} \frac{\partial a}{\partial \rho}, \quad (52)$$

in other words it corresponds to  $G_{t\rho} = 0$  that is, no new information is obtained from timelike components of  $K_{\mu\nu}^\pm$ . Since the radial pressure of the fluid is zero on the boundary surface, this condition reduces to the continuity of the energy momentum tensors in radial direction  $T_\rho^\rho = T_{\hat{r}}^{\hat{r}}|_\Sigma$  which gives

$$p(t, \rho) - \Lambda_0 = \frac{q^2(\rho) - Q^2}{b^4 \rho^4} - \Lambda. \quad (53)$$

For the sake of the generality we started with different cosmological constants for interior and exterior regions, but the continuity of the energy momentum

tensor in the radial direction compels their equality “ $\Lambda_0 = \Lambda$ ”. Then, the electric charge distribution can be written as

$$q = \begin{cases} \eta_0 W_\Sigma^{3/2} / \Delta^2 \rho_\Sigma^3 = Q & \rho \geq \rho_\Sigma \\ \eta_0 W^{3/2} / \Delta^2 \rho^3 & \rho < \rho_\Sigma \end{cases}$$

where  $Q$  is the total electric charge of the fluid confined in the region  $\rho \leq \rho_\Sigma$ , and  $W_\Sigma$  is the value of  $W$  at  $\rho = \rho_\Sigma$  given by (24).

## 4 Gravitational collapse

If the collapse phenomena allows their formation two types of singularities may form during collapse: physical and spacetime singularities [2]. Physical singularities make physical quantities (such as mass-energy density, pressure) singular and the space-time singularities make the metric components and the curvature indefinite. In the gravitational collapse manner, among the spacetime singularities the shell focusing and shell crossing singularities are being considered. The shell crossing singularity occurs at distances where change of the radius of the fluid sphere in radial direction is zero  $R' = 0$  (with  $R > 0$ ), and the shell focusing singularity forms at distances which make radius of the fluid sphere zero ( $R \rightarrow 0$ ). The shell crossing singularities can be considered weak with respect to the shell focusing singularity in the gravitational collapse treatment [14]. Therefore, we are only interested in the formation of the shell focusing singularity, i.e.,  $R \rightarrow 0$  as  $\rho \rightarrow 0$  with  $R' > 0$ .

In the literature many factors effective on the formation of the naked singularities are examined [2], [3], [15] and it is pointed out that, one way of a singularity to be naked is to disturb the apparent horizon surface and delay its formation [3]. According to the idea, if the trapped surface forms



before the singularity surface then, the singularity becomes hidden inside a black hole. Otherwise, the trapped surface forms after the singularity surface and the singularity becomes naked. In another words, if the time period for the formation of the event horizon is longer than the time period for the formation of the singularities, singularities become bare and they can be seen by distant observer. This criterion is probably easy and efficient, but it is not clear if this is always equivalent to naked singularity formation and in a way it is coordinate dependent statement. In one coordinate system these two timings may be related in a certain way, but may not be related in another coordinate system. Therefore, the coordinate independent and a full proof condition "the families of null geodesics come out of the singularity" should be examined [2]. We will give this analysis in the next section.

Let us write metric components of (24) in the radial coordinates  $\rho$  explicitly to examine under which circumstances the situation corresponds to collapse, expansion or bounce

$$\begin{aligned} a &= \frac{4f^2(\alpha\rho^2 + \beta) - (\lambda_0^2 - \eta_0^2) (\gamma\rho^2 + \delta)}{4f^2(\alpha\rho^2 + \beta) + 4f\lambda_0\sqrt{(\alpha\rho^2 + \beta)(\gamma\rho^2 + \delta)} + (\lambda_0^2 - \eta_0^2) (\gamma\rho^2 + \delta)}, \\ b &= \frac{4f^2(\alpha\rho^2 + \beta) + 4f\lambda_0\sqrt{(\alpha\rho^2 + \beta)(\gamma\rho^2 + \delta)} + (\lambda_0^2 - \eta_0^2) (\gamma\rho^2 + \delta)}{4f(\alpha\rho^2 + \beta)(\gamma\rho^2 + \delta)} \end{aligned} \quad (54)$$

and define the physical radius  $R$ , the radius of 2-sphere

$$R = b\rho = \rho \frac{4f^2(\alpha\rho^2 + \beta) + 4f\lambda_0\sqrt{(\alpha\rho^2 + \beta)(\gamma\rho^2 + \delta)} + (\lambda_0^2 - \eta_0^2) (\gamma\rho^2 + \delta)}{4f(\alpha\rho^2 + \beta)(\gamma\rho^2 + \delta)}. \quad (55)$$

As the radial coordinate  $\rho \rightarrow 0$  the physical radius shrinks to zero. We know that if the spacetime is in the isotropic form the shear tensor is zero then it

reduces to

$$\frac{\dot{R}}{R} = \frac{\dot{b}}{b}. \quad (56)$$

By means of shear free property of the spacetime, the change of rate of the physical radius that is, the expansion rate of the spacetime is given by

$$\theta = \frac{3}{a} \frac{\dot{R}}{R} = \frac{3\dot{f}}{f}. \quad (57)$$

This equality states the dynamics of this isotropic collapse problem is related to ratio  $\dot{f}/f$ , and collapse (expansion) situation can only occur for the negative (positive) values of  $\dot{f}/f$ . Since  $f$  is a positive function of time, only if  $\dot{f} < 0$  ( $> 0$ ) the negative (positive) expansion rate corresponds to contracting (expanding) physical radius  $R$ . If  $\dot{f} < 0$  changes its sign after a time period then, it is called bounce.  $\dot{f} = 0$  will be static solution. The time evolution of the physical radius is explicitly written by

$$\frac{\dot{R}}{R} = \frac{a\dot{f}}{f} = \frac{\dot{f}}{f} \frac{4f^2(\alpha\rho^2 + \beta) - (\lambda_0^2 - \eta_0^2)(\gamma\rho^2 + \delta)}{4f^2(\alpha\rho^2 + \beta) + 4f\lambda_0\sqrt{(\alpha\rho^2 + \beta)(\gamma\rho^2 + \delta)} + (\lambda_0^2 - \eta_0^2)(\gamma\rho^2 + \delta)}. \quad (58)$$

from equations (56) and (57). By analyzing the time dependency of the  $R$  we see that four different situation can be obtained: collapse, expansion, stable and bouncing cases.

In the literature it is shown that positive cosmological constant delays the formation of the singularities [16, 4]. By considering slowing down effect of the positive cosmological constant on the collapse process let us take time dependent function as  $f = e^{-(c - \sqrt{\Lambda_0/3})t}$  where  $c$  is a positive constant. Since  $\dot{R}/R = -(c - \sqrt{\Lambda_0/3})a$ , with  $\sqrt{g_{tt}} = a > 0$ , the expansion (contraction) becomes dependent directly to the values of the cosmological constant  $\Lambda_0$ . If  $3c^2 > \Lambda_0$  the collapse will continue to a certain radius and be stopped by the

Coulomb's repulsive force by the charge and it will reach to the central point for uncharged matter. If  $3c^2 < \Lambda_0$  radius will expand until it reaches to the boundary of the exterior region that is to the apparent horizon stated by the matching condition (51). Furthermore  $3c^2 = \Lambda_0$  corresponds to static case. In this example all three possible situations are obtained: expansion, crunch and stable cases.

If the cosmological constant effects are dominant in the dynamics, it will be more convenient to choose sample function as  $f = e^{ct - \sqrt{\Lambda_0/3}t^2}$  ( $t \geq 0$ ) to emphasize the cosmological constant dependency. In this model the change of the radius with time  $\dot{R}/R = (-c + 2\Lambda_0 t)a$ ,  $a > 0$ , is positive in the beginning and negative for late time  $t$ . It means that the radius of the fluid will decrease with time and after a period of time, here for  $t \geq c\sqrt{3/4\Lambda_0}$ , the radius will start to increase so, "bounce" situation is obtained.  $a = 0$  case will be examined in the following part in details.

### Physical singularities.

When pressures are non-zero, dynamical evolutions, as allowed by the Einstein equations, are equally important as the initial data is to determine the final fate of collapse. [17]. Dust solution of the gravitational problem is highly important, but the isotropic form of the spacetime and set up of the problem does not give the dust solution consistent with the conditions.

When their singularities are examined, it is seen that matter and charge densities (26, 28) are regular everywhere but the pressure which is subjected to the weak energy condition (14) and satisfying the energy conservation condition (15)

$$p = -\left(\mu + \frac{1}{3} \frac{\dot{\mu}}{a} \frac{\dot{f}}{f}\right) \quad (59)$$

diverges at the distance  $\rho = \rho_s$  which makes the metric component

$$a = \sqrt{g_{tt}} = 0$$

$$4f^2(\alpha\rho^2 + \beta) - (\lambda_0^2 - \eta_0^2)(\gamma\rho^2 + \delta) = 0, \text{ or } \rho_s = \sqrt{\frac{-4\beta f^2 + \delta(\lambda_0^2 - \eta_0^2)}{4\alpha f^2 - \gamma(\lambda_0^2 - \eta_0^2)}}. \quad (60)$$

At  $\rho_s$  the physical region is split into two parts i.e. matter part is confined in  $\rho \leq \rho_s$  and exterior part starts from  $\rho > \rho_s$  where  $a$  is non-negative.

One may think that the physical singularity starts from the origin which makes  $\rho_s = 0$  (60). But it can not be allowed due to Coulomb's repulsive force of the fluid, or the fluid can only be compressed to a radius at which Coulomb interactions balance the the gravitational collapse effects. This restriction can be seen from conservation relation (15), that is time derivative of  $\mu$  must be positive. By taking the charge parameter limit highly big values ( $\eta_0 \rightarrow \infty$ ) in  $\dot{\mu}$ , it gives us a relation to be satisfied by  $f$

$$\frac{\dot{f}}{f} \left( \frac{\dot{f}}{f} \right) \geq 0. \quad (61)$$

Since  $f > 0$  and  $\dot{f}/f < 0$  for all  $t$ , that is  $f$  is decreasing function with time, then the equation is always negative or zero. Zero case corresponds to static solution but for the collapse situation we get only negative values. Thus, existence of the electric charge violates the conservation of the energy momentum for  $\rho \rightarrow 0$ . In other words, The conservation of the energy-momentum does not allow formation of the shell focusing singularity. For uncharged fluid  $\eta_0 = 0$ , the central singularity can be reached and  $f = \sqrt{\delta\lambda_0^2/4\beta}$  gives us shell focusing singularity formation period  $f$ .

For example, if we take time dependent function for neutral matter as  $f = e^{-(c-\sqrt{\Lambda/3})t}$ ,  $c > 0$  with  $\beta = \gamma = 0$  in (60), the pressure will be singular at the radial distance  $\rho_s \sim f^{-1} = e^{(c-\sqrt{\Lambda_0/3})t}$  which means singularity will form later than  $\Lambda_0$  free case but the radius of the singularity surface will be greater than that of  $\Lambda_0$ .

### Spacetime singularities.

The Kretschmann scalar which is the square of the Riemann tensor and defined by  $K = R_{\mu\nu\lambda\sigma} R^{\mu\nu\lambda\sigma}$  gives the essential, coordinate independent singularities of the spacetime. For the spherically symmetric isotropic spacetime it can be given as

$$\begin{aligned}
K = & \frac{1}{a^6 b^8 \rho^2} \left[ 12a^4 \rho^2 b'^2 (a^2 b'^2 + b^2 a'^2) + 8b^2 a^4 (3a^2 b'^2 + b^2 a'^2) \right. \\
& + 12b^4 \rho^2 \dot{b}^2 (a^2 \dot{b}^2 + b^2 \dot{a}^2) + 8a^2 b^4 \rho^2 \dot{a} \dot{b} (a' b' + b a'') - 8a^2 b^2 \rho^2 \dot{b}^2 (a^2 b'^2 + 2b^2 a'^2) \\
& - 16a^2 b^3 \rho a' \dot{b} (2a \rho \dot{b} b' - b^2 \dot{a}) + 8a^4 b^3 \rho a' b' (2a' - \rho b'') + 32a^4 b^3 \rho b' \dot{b} (\rho \dot{b}' - \dot{b}) \\
& + 12ab^6 \rho^2 \ddot{b} (a \ddot{b} - 2\dot{a} \dot{b}) + 8a^6 b^2 \rho b'' (\rho + 2b') + 4a^3 b^4 \rho^2 a'' (a a'' - 2b \ddot{b}) \\
& \left. - 16a^4 b \rho^2 b'' (a^2 b'^2 + b^2 \dot{b}^2) - 8a^3 b^4 \rho \ddot{b} a' (\rho b' + 2ba') + 16a^3 b^4 b \rho^2 \dot{b}' (2\dot{b} a' - a \dot{b}') \right] .
\end{aligned} \tag{62}$$

$K$  has polynomial singularities in  $a, b, \rho$  and divergent as  $\sqrt{g_{tt}} = a \rightarrow 0$  (physical singularity), or  $\rho \rightarrow 0$ ,  $R = b\rho \rightarrow 0$  (central singularity).

If the future directed non-spacelike (timelike or null) curves terminate in the past at the singularity then the singularity is called naked otherwise it is covered. The procedure is coordinate-free method. To clarify the nature of the singularities the future directed non-spacelike geodesics are examined, specially null geodesics [2, 14]. Outgoing radial null geodesics of the isotropic spacetime (4) are given by

$$\frac{dt}{d\rho} = \frac{b}{a} . \tag{63}$$

If null geodesic equation is written in terms of physical radius  $R$  and  $u = \rho^\alpha$  we get

$$\frac{dR}{du} = \frac{R'}{\alpha \rho^{\alpha-1}} \left( 1 + \frac{b}{a} \frac{\dot{R}}{R'} \right) . \tag{64}$$

The singularity is naked if the null geodesics terminate in the past at the singularity with positive finite value and it is hidden or covered if the limit

$$\lim_{\rho \rightarrow \rho_s} \frac{dR}{du} = \lim_{\rho \rightarrow \rho_s} \frac{R'}{\alpha \rho^{\alpha-1}} \left( 1 + \frac{b}{a} \frac{\dot{R}}{R'} \right) \quad (65)$$

is negative. As stated before, the existence of the electric charge restricts the formation of the central singularity, only uncharged matter collapse ( $\eta_0 = 0$ ) ends with central singularity. If all constants about the fluid defined in (24) are non-zero except  $\eta_0 = 0$  and for  $a \neq 0$  singularity starts from origin. The second term in the parenthesis becomes zero as  $\rho_s \rightarrow 0$  since  $R' > 0$  and  $\dot{R} = \rho \dot{b}$ . Therefore, limit becomes positive and equal to 1, then the central singularity is naked. If we take  $\eta_0 = \beta = \gamma = 0$  in (60) at which  $a = 0$  singularity forms at  $\rho_s = \sqrt{\delta \lambda_0^2 / 4\alpha f^2}$ . In this limit the (65), therefore the nature of the singularity becomes parameter dependent.

In this case the limit becomes related to the sign of the time derivative of  $f$  and the constant  $\alpha$  as  $\text{sgn}(\dot{f})/\text{sgn}(\alpha)$ . For ( $\alpha > 0, \dot{f} > 0$ ) or ( $\alpha < 0, \dot{f} < 0$  with  $\delta < 0$ ), limit becomes positive and therefore the central singularity becomes naked. But, for ( $\alpha > 0, \dot{f} < 0$ ) or ( $\alpha < 0, \dot{f} > 0$  with  $\delta < 0$ ) limit is negative therefore, the central singularity is covered. It is possible to give a lot of example such  $f$  functions so that  $\text{sgn} \dot{f}$  changes with time during process other than  $f = e^{ct - \sqrt{\Lambda_0/3} t^2}$  where "bounce" situation occurs. Otherwise the process will be collapse or expansion.

Furthermore, it should be also emphasize that the physical singularity ( $a \rightarrow 0$  singularity) coincides with the apparent horizon for the extremal case  $\lambda_0 = \eta_0$  and  $\beta = 0$ . Apparent horizon is the boundary surface of the trapped regions and makes (51) zero

$$A = b^{-2} R'^2 - a^{-2} \dot{R}^2 = 0. \quad (66)$$

## 5 Conclusion

In this work spherical symmetric charged solution of Einstein field equations is given in the presence of cosmological constant. The matter is considered as ideal fluid and subjected to the weak energy condition. Two specific examples of this type of solution isotropic RN-(anti)de Sitter and charged Mc Vittie-de Sitter solutions are given. When the matter is confined in a region, the exterior spacetime exterior is taken RN-de Sitter and to complete analysis the matching conditions are examined. In these calculations, in the name of the generality, we started with the different cosmological constant for the interior  $\Lambda_0$  and exterior  $\Lambda$  regions but the junction conditions, continuity of the energy-momentum tensor in the radial direction gives their equality.

In the reference [9], spherical symmetric gravitational collapse of charged fluid is studied in black hole formation point of view, that is, strong cosmic censorship hypothesis is considered. In this work, by considering weak cosmic censorship hypothesis we see that besides formation of the black hole the process allows formation of the naked singularity and the existence of the cosmological constant permits bouncing situations as well. The singularity structure analysis is done by using coordinate free null geodesic method. The initial data about the matter (energy density, pressure and the electric charge) therefore, constants about the solution determine the final fate of the collapse. Existence of the electric charge prevents formation of the central singularity and the cosmological constant causes bouncing situation for both charged and neutral matters. Uncharged matter distribution allows formation of the central singularity and it will be naked.

The results are compatible with the results given in the literature. For further studies, the gravitational collapse phenomena can be studied for the

spacetimes other than isotropic form.

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