## Fiber-Optical Analogue of the Event Horizon

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## Abstract

The physics at the event horizon resembles the behavior of waves in moving media. Horizons are formed where the local speed of the medium exceeds the wave velocity. We use ultrashort pulses in microstructured optical fibers to demonstrate the formation of an artificial event horizon in optics. We observed a classical optical effect, the blue-shifting of light at a white-hole horizon. We also show by theoretical calculations that such a system is capable of probing the quantum effects of horizons, in particular Hawking radiation. Laboratory analogues of black holes [1, 2, 3] are inspired by a simple and intuitive idea [4]: the space-time geometry of a black hole resembles a river [5, 6], a moving medium flowing towards a waterfall, the singularity. Imagine that the river carries waves propagating against the current with speed c'. The waves play the role of light where c' represents c, the speed of light in vacuum. Suppose that the closer the river gets to the waterfall the faster it flows and that at some point the speed of the river exceeds c'. Clearly, beyond this point waves can no longer propagate upstream. The point of no return corresponds to the horizon of the black hole. Imagine another situation: a fast river flowing out into the sea, getting slower. Waves cannot enter the river beyond the point where the flow speed exceeds the wave velocity; the river resembles an object that nothing can enter, a white hole.

Nothing, not even light, can escape from a gravitational black hole. Yet according to quantum physics, the black hole is not entirely black, but emits waves in thermal equilibrium [7, 8, 9]. The waves consist of correlated pairs of quanta, one originates from inside and the other from outside the horizon. Seen from one side of the horizon, the gravitational black hole acts as a thermal black-body radiator sending out Hawking radiation [7, 8, 9]. The effective temperature depends on the surface gravity [7, 8, 9] that, in the analogue model, corresponds to the flow-velocity gradient at the horizon [1, 2, 3, 4, 5].

The Hawking temperature of typical black holes lies far below the temperature of the cosmic microwave background, so an observation of Hawking radiation in astrophysics seems unlikely. However, laboratory demonstrations of analogues of Hawking radiation could be feasible. One type of recent proposal [10, 11, 12] suggests the use of ultracold quantum gases such as alkali Bose-Einstein condensates or ultracold alkali Fermions [12]. When a condensate in a waveguide is pushed over a potential barrier it may exceed the speed of sound (typically a few mm/s) and is calculated to generate a Hawking temperature of about 10nK [11]. Helium-3 offers a multitude of analogues between quantum fluids and the standard model, including Einsteinian gravity [2]. For example, the analogy between gravity and surface waves in fluids [13] has inspired ideas for artificial event horizons at the interface between two sliding superfluid phases [14], but, so far, none of the quantum features of horizons has been measured in Helium-3. Proposals for optical black holes [15, 16] have relied on slowing down light [17] such that it matches the speed of the medium [15] or on bringing light to a complete standstill [16], but in these cases absorption may pose a severe problem near the horizon where the spectral transparency window [17] vanishes.

On the other hand, is it necessary to physically move a medium for establishing a horizon? What really matters are only the effective properties of the medium. If they change, for example as a propagating front, but the medium itself remains at rest, the situation is essentially indistinguishable from a moving medium. Such ideas have been discussed for moving solitons and domain walls [18] in superfluid Helium-3 [2] and more recently for microwave transmission lines with variable capacity [19], but they have remained impractical so far.

Here we report the first experimental observation of the classical optical effects of horizons, the blue-shifting of light at a white-hole horizon, and show theoretically that our scheme combines several promising features for demonstrating quantum Hawking radiation in the optical domain. Our idea, illustrated in Fig. 1, is based on the nonlinear optics of ultrashort light pulses

in optical fibers [20] where we exploit the remarkable control of the nonlinearity, birefringence and dispersion in microstructured fibers [22, 21].

Using a Titanium-Sapphire laser we create 70fs-long non-dispersive pulses (solitons) at 803nm carrier wavelength and 80MHz repetition rate inside 1.5m of microstructured fiber (NL-PM-750B from Crystal Fibre A/S). Each pulse modifies the optical properties of the fiber due to the Kerr effect [20]: the effective refractive index of the fiber,  $n_0$ , gains an additional contribution  $\delta n$  that is proportional to the instantaneous pulse intensity I at position z and time t,

$$n = n_0 + \delta n, \quad \delta n \propto I(z, t).$$
 (1)

This contribution to the refractive index n moves with the pulse. The pulse thus establishes a moving medium, although nothing material is moving. This effective medium naturally moves at the speed of light in the fiber, because it is made by light itself.

We also launch a continuous wave of light, a probe, that follows the pulse with slightly higher group velocity, attempting to overtake it. In order to distinguish the probe from the pulse, it oscillates at a significantly different frequency  $\omega$ . Our probe-light laser is tunable over wavelengths  $2\pi c/\omega$  from 1460nm to 1540nm. While approaching the pulse, the Kerr contribution  $\delta n$  slows down the probe until the probe's group velocity reaches the speed of the pulse. The trailing end of the pulse establishes a white-hole horizon, an object that light cannot enter, unless it tunnels through the pulse. Conversely, the front end creates a black-hole horizon for probe light that is slower than the pulse. As  $\delta n$  is small, the initial group velocity of the probe should be sufficiently close to the speed of the pulse. In microstructured fibers [22] the group-velocity dispersion of light is engineered by arrangements of air holes (sub- $\mu$ m wide hollow cylinders along the fiber). We selected a fiber where the group velocity of pulses near the 800nm carrier wavelength of mode-locked Ti:Sapphire lasers matches the group velocities of probes in the infrared telecommunication band around 1500nm, whereas standard optical fibers [20] do not have this property.

At the horizon of an astrophysical black hole light freezes, reaching wavelengths shorter than the Planck scale where the physics is unknown. (The Planck length is given by  $\sqrt{2\pi\hbar G/c^3}$ where G is the gravitational constant.) Some elusive Trans-Planckian mechanism must regularize this behavior [23, 24]. In our case, the fiber-optical analogue of Trans-Planckian physics is known and simple — it is contained in the frequency-dependance of the refractive index n, the dispersion of the fiber: at the trailing end of the pulse the incoming probe modes are compressed, oscillating with increasing frequency; they are blue-shifted. In turn, the dispersion limits the frequency shifting by tuning the probe out of the horizon. In the case of normal group-velocity dispersion the blue-shifted light falls behind. At the black-hole horizon the reverse occurs: a probe slower than the pulse is red-shifted and then moves ahead of the pulse.

Figure 2 shows the difference in the spectrum of the probe light — incident with  $\omega_1$  — with and without the pulses, clearly displaying a blue-shifted peak at  $\omega_2$ . To quantitatively describe this effect, we consider the frequency  $\omega'$  in a co-moving frame that is connected to the

laboratory-frame frequency  $\omega$  by the Doppler formula

$$\omega' = \left(1 - \frac{nu}{c}\right)\omega. \tag{2}$$

For a stable pulse,  $\omega'$  is a conserved quantity, whereas  $\omega$  follows the contours of fixed  $\omega'$  when  $\delta n$  varies with the intensity profile of the pulse (Fig. 3). For sufficiently large  $\delta n$ , the frequency  $\omega$  completes an arch from the initial  $\omega_1$  to the final  $\omega_2$ ; it is blue-shifted by the white-hole horizon. At a black-hole horizon, the arch is traced the other way round from  $\omega_2$  to  $\omega_1$ . For the frequency at the center of the arches an infinitesimal  $\delta n$  is sufficient to cause a frequency shift; at this frequency the group velocity of the probe matches the group-velocity of the pulse. Figure 2 shows that both the blue-shifted and probe light are spectrally broadened. These features are easily explained: the horizon acts only during the time while probe and pulse propagate in the fiber, where only a finite fraction of the probe is frequency-shifted, forming a blue-shifted pulse and also a gap in the probe light, a negative pulse; these pulses have a characteristic spectrum with a width that is inversely proportional to the fiber length. We compared the measured spectra with the theory of light propagation in the presence of horizons, and found very good agreement [25].

Imagine instead of a single probe a set of probe modes. The modes should be sufficiently weakly excited such that they do not interact with each other via the Kerr effect, but they experience the cross Kerr effect of the pulse, the presence of the medium (1) moving with the velocity u. The modes constitute a quantum field of light in a moving medium [25, 26]. Classical light is a real electromagnetic wave. So, according to Fourier analysis, any amplitude oscillating as  $exp(-i\omega\tau)$  at a positive angular frequency  $\omega$  must be accompanied by the complex conjugate amplitude at  $-\omega$ . In quantum field theory [8, 9, 25], the positive-frequency modes correspond to the annihilation and the negative-frequency modes to the creation operators [26]. Processes that mix positive and negative frequencies in the laboratory frame (in the glass of the fiber) thus create observable light quanta.

In the near ultraviolet around 300nm, the dispersion of microstructured fibers [22] is dominated by the bare dispersion of glass where  $n_0$  rapidly grows with frequency [20], exceeding c/u. For such ultraviolet modes, the medium moves at superluminal speed. We see from the Doppler formula (2) that these superluminal modes oscillate with negative frequencies  $\omega'$  in the co-moving frame for positive frequencies  $\omega$  in the laboratory frame, and vice versa. Moreover, each subluminal mode with positive  $\omega$  has a superluminal partner oscillating at the same comoving frequency  $\omega'$ , but with negative laboratory frequency. The pulse does not change  $\omega'$ , but it may partially convert sub- and superluminal partner modes into each other, thus creating photons [8, 9]. Even if all the modes are initially in their vacuum states, the horizon spontaneously creates photon pairs. This process represents the optical analogue of Hawking radiation [7] and it occurs at both the black-hole and white-hole horizon [25]. Photons with positive  $\omega'$ correspond to the particles created at the outside of the black hole [8, 9], while the negativefrequency photons represent their partners beyond the horizon. In our case, the photon pairs are distinguishable from the intense pulse, because their polarization can be orthogonal to the pulse and their frequencies differ from the carrier frequency by an octave. Furthermore, one can discriminate the Hawking effect from other nonlinear optical processes, such as Four Wave Mixing, because it is not subject to their phase-matching conditions [20]. Moreover, in addition to observing Hawking radiation per se, one could detect the correlations of the Hawking partners — a feat that is impossible in astrophysics, because there the partner particles are lost beyond the horizon of the black hole.

In order to give a quantitative argument for the Hawking effect in optical fibers, we describe the propagation distance z in terms of the time  $\zeta$  it takes for the pulse to traverse it,  $\zeta = z/u$ , and introduce the retarded time  $\tau = t - z/u$ . The phase  $\varphi$  of each mode evolves as

$$\varphi = -\int \left(\omega \mathrm{d}\tau + \omega' \mathrm{d}\zeta\right). \tag{3}$$

We assume that the mode conversion occurs in a narrow interval of retarded time  $\tau$  near a horizon around  $\tau = 0$ , where we linearize  $\delta n$  in  $\tau$  such that

$$1 - \frac{nu}{c} = \alpha' \tau \,. \tag{4}$$

We obtain from the phase integral (3) and the Doppler formula (2) the characteristic logarithmic phase at a horizon [8, 9]. We use the standard result [8, 9, 25]: Hawking radiation is Planck-distributed with the temperature

$$k_B T' = \frac{\hbar \alpha'}{2\pi} \,, \tag{5}$$

where  $k_B$  denotes Boltzmann's constant. For evaluating  $\alpha'$  we consider  $\delta n$  at  $\tau = 0$ , where

$$\alpha' = -\frac{u}{c} \left. \frac{\partial n}{\partial \tau} \right|_0 = -\frac{u}{c} \left. \frac{\partial \delta n}{\partial \tau} \right|_0 \,. \tag{6}$$

Note that T' denotes the Hawking temperature in the co-moving frame, defined with respect to the Doppler-shifted frequencies  $\omega'$ , a temperature that is characterized by the Doppler-shifted Hawking frequency  $\alpha'$  in regions away from the pulse. We use the Doppler formula (2) with the refractive index (1) and the linearization (4) taken at  $\tau = 0$ , and obtain

$$\alpha' = \left(1 - n_0 \frac{u}{c}\right) \alpha = \frac{u}{c} \,\delta n \Big|_0 \,\alpha \,. \tag{7}$$

Consequently, the Hawking temperature T in the laboratory frame is

$$k_B T = \frac{\hbar \alpha}{2\pi}, \quad \alpha = -\frac{1}{\delta n} \frac{\partial \delta n}{\partial \tau} \bigg|_0.$$
 (8)

As T does not depend on the magnitude of  $\delta n$ , even the typically very small refractive-index variations of nonlinear fiber optics [20] may lead to a substantial Hawking temperature when  $\delta n$  varies on the scale of an optical wavelength. This is achievable with few-cycle optical pulses [27, 28].

Note that the Kerr nonlinearity [20] not only influences the probe modes, but the pulse as well [20]. This self Kerr effect shapes the pulse while it propagates in the fiber. Regions of high intensity lag behind, because for them the effective refractive index is increased. The blackhole horizon at the front is stretched, but the trailing edge becomes extremely steep, infinitely steep in theory [20]: the pulse develops an optical shock [20]. The steep white-hole horizon will dominate the Hawking effect of the pulse. In practice, dispersion combined with other nonlinear optical processes in the fiber, in particular Stimulated Raman Scattering [20], limit the optical shock. Assuming that the steepness at the shock front is comparable to twice the frequency of the pulse carrier,  $8 \times 10^{14}$ Hz, the Hawking temperature (8) reaches  $10^3$ K, many orders of magnitude higher than condensed-matter analogues of the event horizon [10, 11, 12, 18].

Our scheme thus solves two problems at once in a natural way: how to let an effective medium move at superluminal speed and how to generate a steep velocity profile at the horizon; the various aspects of the physics of artificial black holes conspire together, in contrast to most other proposals [1, 2, 3, 4, 10, 11, 12, 13, 14, 15, 16].

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Figure 1: Fiber-optical horizons. A a light pulse in a fiber slows down infrared probe light attempting to overtake it. The diagrams below are in the co-moving frame of the pulse. B Classical horizons. The probe is slowed down by the pulse until its group velocity matches the pulse speed at the points indicated in the figure, establishing a white-hole horizon at the back and a black-hole horizon at the front of the pulse. The probe light is blue-shifted at the white hole until the optical dispersion releases it from the horizon. C Quantum pairs. Even if no probe light is incident, the horizon emits photon pairs corresponding to waves of positive frequencies from the outside of the horizon paired with waves at negative frequencies from beyond the horizon. An optical shock has steepened the pulse edge, increasing the luminosity of the white hole.



Figure 2: Measurement of blue shifting at a white-hole horizon. The black curve shows the power spectrum of probe light that has not interacted with the pulses, while the green curve displays the result of the interaction; both curves are represented on a logarithmic scale. The difference between the spectra on a linear scale, shown in red, exhibits a characteristic peak around the blue-shifted wavelength ( $\omega_2$ ) and another peak around the spectral line of the probe laser ( $\omega_1$ ) due to a gap in the probe light; both features indicate the presence of a horizon.



Figure 3: Doppler contours. The Doppler-shifted frequency  $\omega'$  of the probe is a conserved quantity. The pulse shifts the laboratory frequency  $\omega$  along the contour lines of  $\omega'$  as a function of the instantaneous  $\delta n$ ; the same applies to the wavelength  $\lambda = 2\pi c/\omega$ . If the pulse is sufficiently intense such that  $\delta n$  reaches the top of a contour, the probe light completes an arch on the diagram while leaving the pulse; it is red- or blue-shifted, depending on its initial frequency.