

# Dark Energy in Light of the Cosmic Horizon

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## Abstract

Based on dramatic observations of the cosmic microwave background radiation with WMAP and of Type Ia supernovae with the Hubble Space Telescope and ground-based facilities, it is now generally believed that the Universe's expansion is accelerating. Within the context of standard cosmology, this type of evolution leads to the supposition that the Universe must contain a third 'dark' component of energy, beyond matter and radiation. However, the current data are still deemed insufficient to distinguish between an evolving dark energy component and the simplest model of a time-independent cosmological constant. In this paper, we examine the role played by our cosmic horizon  $R_0$  in our interrogation of the data, and reach the rather firm conclusion that the existence of a cosmological constant is untenable. The observations are telling us that  $R_0 \approx ct_0$ , where  $t_0$  is the perceived current age of the Universe, yet a cosmological constant would drive  $R_0$  towards  $ct$  (where  $t$  is the cosmic time) only once, and that would have to occur right now. In contrast, scaling solutions simultaneously eliminate several conundrums in the standard model, including the 'coincidence' and 'flatness' problems, and account very well for the fact that  $R_0 \approx ct_0$ . We show in this paper that for such dynamical dark energy models, either  $R_0 = ct$  for all time (thus eliminating the apparent coincidence altogether), or that what we believe to be the current age of the universe is actually the horizon time  $t_h \equiv R_0/c$ , which is always shorter than  $t_0$ . Our best fit to the Type Ia supernova data indicates that  $t_0$  would then have to be  $\approx 16.9$  billion years. Though surprising at first, an older universe such as this would actually eliminate several other long-standing problems in cosmology, including the (too) early appearance of supermassive black holes (at a redshift  $> 6$ ) and the glaring deficit of dwarf halos in the local group.

Keywords: cosmic microwave background, cosmological parameters, cosmology: observations, cosmology: theory, distance scale, cosmology: dark energy

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## I. INTRODUCTION

Over the past decade, Type Ia supernovae have been used successfully as standard candles to facilitate the acquisition of several important cosmological parameters. On the basis of this work, it is now widely believed that the Universe’s expansion is accelerating (Riess et al. 1998; Perlmutter et al. 1999). In standard cosmology, built on the assumption of spatial homogeneity and isotropy, such an expansion requires the existence of a third form of energy, beyond the basic admixture of (visible and dark) matter and radiation.

One may see this directly from the (cosmological) Friedman-Robertson-Walker (FRW) differential equations of motion, usually written as

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\rho - \frac{kc^2}{a^2}, \quad (1)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\rho + 3p), \quad (2)$$

$$\dot{\rho} = -3H(\rho + p), \quad (3)$$

in which an overdot denotes a derivative with respect to cosmic time  $t$ , and  $\rho$  and  $p$  represent, respectively, the total energy density and total pressure. In these expressions,  $a(t)$  is the expansion factor, and  $(r, \theta, \phi)$  are the coordinates in the comoving frame. The constant  $k$  is  $+1$  for a closed universe,  $0$  for a flat (or open) universe, and  $-1$  for an open universe.

Following convention, we write the equation of state as  $p = \omega\rho$ . A quick inspection of equation (2) shows that an accelerated expansion ( $\ddot{a} > 0$ ) requires  $\omega < -1/3$ . Thus, neither radiation ( $\rho_r$ , with  $\omega_r = 1/3$ ), nor (visible and dark) matter ( $\rho_m$ , with  $\omega_m \approx 0$ ) can satisfy this condition, leading to the supposition that a third ‘dark’ component  $\rho_d$  (with  $\omega_d < -1/3$ ) of the energy density  $\rho$  must be present. In principle, each of these contributions to  $\rho$  may evolve according to its own dependence on  $a(t)$ .

Over the past few years, complementary measurements (Spergel et al. 2003) of the cosmic microwave background (CMB) radiation have indicated that the Universe is flat (i.e.,  $k = 0$ ), so  $\rho$  is at (or very near) the ‘critical’ density  $\rho_c \equiv 3c^2H^2/8\pi G$ . But among the many peculiarities of the standard model is the inference, based on current observations, that  $\rho_d$  must itself be of order  $\rho_c$ . Dark energy is often thought to be the manifestation of a cosmological constant,  $\Lambda$ , though no reasonable explanation has yet been offered as to why such a fixed, universal density ought to exist at this scale. It is well known that if  $\Lambda$  is associated with the energy of the vacuum in quantum theory, it should have a scale representative of phase transitions in the early Universe—many, many orders of magnitude larger than  $\rho_c$ .

Many authors have attempted to circumvent these difficulties by proposing alternative forms of dark energy, including Quintessence (Wetterich 1988; Ratra and Peebles 1988), which represents an evolving canonical scalar field with an inflation-inducing potential, a Chameleon field (Khoury and Weltman 2004; Brax et al. 2004) in which the scalar field couples to the baryon energy density and varies from solar system to cosmological scales, and modified gravity, arising out of both string motivated (Davli et al. 2000), or General

Relativity modified (Capozziello et al. 2003; Carroll et al. 2004) actions, which introduce large length scale corrections modifying the late time evolution of the Universe. The actual number of suggested remedies is far greater than this small, illustrative sample.

Nonetheless, though many in the cosmology community suspect that some sort of dynamics is responsible for the appearance of dark energy, until now the sensitivity of current observations has been deemed insufficient to distinguish between an evolving dark energy component and the simplest model of a time-independent cosmological constant  $\Lambda$  (see, e.g., Corasaniti et al. 2004). This conclusion, however, appears to be premature, given that the role of our cosmic horizon has not yet been fully folded into the interrogation of current observations.

The purpose of this paper is to demonstrate that a closer scrutiny of the available data, if proven to be reliable, can in fact already delineate between evolving and constant dark energy theories, and that a simple cosmological constant  $\Lambda$ , characterized by a fixed  $\omega_d = \omega_\Lambda = -1$ , is almost certainly ruled out. In § 2 of this paper, we will introduce the cosmic horizon and discuss its evolution in time, demonstrating how measurements of the Hubble constant  $H$  may be used to provide strict constraints on  $\omega$ , independent of Type Ia supernova data. We will compare these results with predictions of the various classes of dark energy models in §§ 3 and 4, and conclude with a discussion of the consequences of these comparisons in § 5.

## II. THE COSMIC HORIZON

The Hubble Space Telescope Key Project on the extragalactic distance scale has measured the Hubble constant  $H$  with unprecedented accuracy (Mould et al. 2000), yielding a current value  $H_0 \equiv H(t_0) = 71 \pm 6 \text{ km s}^{-1} \text{ Mpc}^{-1}$ . (For  $H$  and  $t$ , we will use subscript “0” to denote cosmological values pertaining to the current epoch.) With this  $H_0$ , we infer that  $\rho(t_0) = \rho_c \approx 9 \times 10^{-9} \text{ ergs cm}^{-3}$ .

Given such precision, it is now possible to accurately calculate the radius of our cosmic horizon,  $R_0$ , defined by the condition

$$\frac{2GM(R_0)}{c^2} = R_0, \quad (4)$$

where  $M(R_0) = (4\pi/3)R_0^3\rho/c^2$ . In terms of  $\rho$ ,  $R_0 = (3c^4/8\pi G\rho)^{1/2}$  or, more simply,  $R_0 = c/H_0$  in a flat universe. This is the radius at which a sphere encloses sufficient mass-energy to turn it into a Schwarzschild surface for an observer at the origin of the coordinates  $(cT, R, \theta, \phi)$ , where  $R = a(t)r$ , and  $T$  is the time corresponding to  $R$  (Melia 2007).

When the Robertson-Walker metric is written in terms of these observer-dependent coordinates, the role of  $R_0$  is to alter the intervals of time we measure (using the clocks fixed to our origin), in response to the increasing spacetime curvature induced by the mass-energy enclosed by a sphere with radius  $R$  as  $R \rightarrow R_0$ . The time  $t$  is identical to  $T(R)$  only at the origin ( $R = 0$ ). For all other radii, our measurement of a time interval  $dT$  necessarily comes with a gravitational time dilation which diverges when  $R \rightarrow R_0$ . It is therefore physically impossible for us to see any process occurring beyond  $R_0$ , and this is why the recent observations have a profound impact on our view of the cosmos. For example, from the Hubble measurement of  $\rho(t_0)$ , we infer that  $R_0 \approx 13.5$  billion light-years; this is the

maximum distance out to which measurements of the cosmic parameters may be made at the present time.

Let us now consider how this radius evolves with the universal expansion. Clearly, in a de Sitter universe with a constant  $\rho$ ,  $R_0$  is fixed forever. But for any universe with  $\omega \neq -1$ ,  $R_0$  must be a function of time. From the definition of  $R_0$  and equation (3), we see that

$$\dot{R}_0 = \frac{3}{2}(1 + \omega)c, \quad (5)$$

a remarkably simple expression that nonetheless leads to several important conclusions regarding our cosmological measurements (Melia 2008). We will use it here to distinguish between constant and evolving dark energy theories.

Take  $t$  to be some time in the distant past (so that  $t \ll t_0$ ). Then, integrating equation (5) from  $t$  to  $t_0$ , we find that

$$R_0(t_0) - R_0(t) = \frac{3}{2}(1 + \langle\omega\rangle)ct_0, \quad (6)$$

where

$$\langle\omega\rangle \equiv \frac{1}{t_0} \int_t^{t_0} \omega dt \quad (7)$$

is the time-averaged value of  $\omega$  from  $t$  to the present time.

Now, for any  $\langle\omega\rangle > -1$ ,  $\rho$  drops as the universe expands (i.e., as  $a(t)$  increases with time), and since  $R_0 \sim \rho^{-1/2}$ , clearly  $R_0(t) \ll R_0(t_0)$ . Therefore,

$$R(t_0) \approx \frac{3}{2}(1 + \langle\omega\rangle)ct_0. \quad (8)$$

The reason we can use the behavior of  $R_0$  as the universe expands to probe the nature of dark energy is that the latter directly impacts the value of  $\langle\omega\rangle$ . A consideration of how the cosmic horizon  $R_0$  evolves with time can therefore reveal whether or not dark energy is dynamically generated. Indeed, we shall see in the next section that the current observations, together with equation (8), are already quite sufficient for us to differentiate between the various models.

Before we do that, however, we can already see from this expression why the standard model of cosmology contains a glaring inconsistency (Melia 2008). From WMAP observations (Spergel et al. 2003), we infer that the age  $t_0$  of the universe is  $\approx 13.7$  billion years. Since  $R_0 \approx 13.5$  billion light-years, this can only occur if  $\langle\omega\rangle \leq -1/3$ . Of course, this means that the existence of dark energy (with such an equation of state) is required by the WMAP and Hubble observations alone, independently of the Type Ia supernova data. But an analysis of the latter (see below) reveals that the value  $\langle\omega\rangle = -1/3$  is ruled out, so in fact  $\langle\omega\rangle < -1/3$ . That means that  $R_0 \neq ct_0$ ; in fact,  $R_0$  must be less than  $ct_0$ , which in turns suggests that the universe is older than we think. What we infer to be the time since the Big Bang, is instead the ‘‘horizon’’ time  $t_h \equiv R_0/c$ , which must be shorter than  $t_0$ . As discussed in Melia (2008), this has some important consequences that may resolve several long-standing conflicts in cosmology. Through our analysis in the next section, we will gain a better understanding of this phenomenon, which will permit us to calculate  $t_0$  more precisely.

## FIGURES

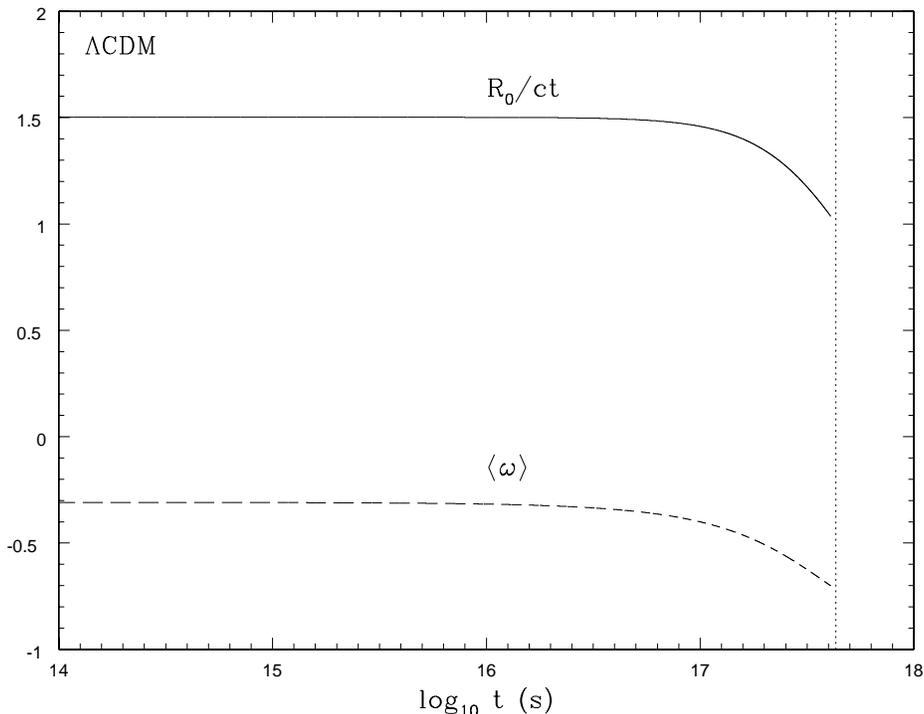


FIG. 1. Plot of the horizon radius  $R_0$  in units of  $ct$ , and  $\langle \omega \rangle$ , the equation of state parameter  $\omega \equiv p/\rho$  averaged over time from  $t$  to  $t_0$ . The asymptotic value of  $\langle \omega \rangle$  for  $t \rightarrow 0$  is approximately  $-0.31$ . These results are from a calculation of the universe's expansion in a  $\Lambda$ CDM cosmology, with matter energy density  $\rho_m(t_0) = 0.3\rho_c(t_0)$  and a cosmological constant  $\rho_d = \rho_\Lambda = 0.7\rho_c(t_0)$ , with  $\omega_d \equiv \omega_\Lambda = -1$ .

### III. THE COSMOLOGICAL CONSTANT

The standard model of cosmology contains a mixture of cold dark matter and a cosmological constant with an energy density fixed at the current value,  $\rho_d(t) \equiv \rho_\Lambda(t) \approx 0.7\rho_c(t_0)$ , and an equation of state with  $\omega_d \equiv \omega_\Lambda = -1$ . Known as  $\Lambda$ CDM, this model has been reasonably successful in accounting for large scale structure, the cosmic microwave background fluctuations, and several other observed cosmological properties (see, e.g., Ostriker and Steinhardt 1995; Spergel et al. 2003; Melchiorri et al. 2003).

But let us now see whether  $\Lambda$ CDM is also consistent with our understanding of  $R_0$ . Putting  $\rho = \rho_m(t) + \rho_\Lambda$ , where  $\rho_m$  is the time-dependent matter energy density, we may integrate equation (5) for a  $\Lambda$ CDM cosmology, starting at the present time  $t_0$ , and going backwards towards the era when radiation dominated  $\rho$  (somewhere around 100,000 years after the Big Bang). Figure 1 shows the run of  $R_0/ct$  as a function of time, along with the time-averaged  $\omega$  given in equation (7). The present epoch is indicated by a vertical dotted line. The calculation begins at the present time  $t_0$ , with the initial value  $R_0 = (3/2)(1 + \langle \omega \rangle_\infty)ct_0$ , where  $\langle \omega \rangle_\infty$  ( $\approx -0.31$ ) is the equation-of-state parameter  $\omega$  averaged over the entire universal expansion, obtained by an iterative convergence of the solution

to equation (5). In  $\Lambda$ CDM, the matter density increases towards the Big Bang, but  $\rho_\Lambda$  is constant, so the impact of  $\omega_d$  on the solution vanishes as  $t \rightarrow 0$  (see the dashed curve in figure 1). Thus, as expected,  $R_0/ct \rightarrow 3/2$  early in the Universe’s development.

What is rather striking about this result is that in  $\Lambda$ CDM,  $R_0(t)$  approaches  $ct$  only once in the entire history of the Universe—and this is only because we have imposed this requirement as an initial condition on our solution. There are many peculiarities in the standard model, some of which we will encounter shortly, but the unrealistic coincidence that  $R_0$  should approach  $ct_0$  only at the present moment must certainly rank at—or near—the top of this list.

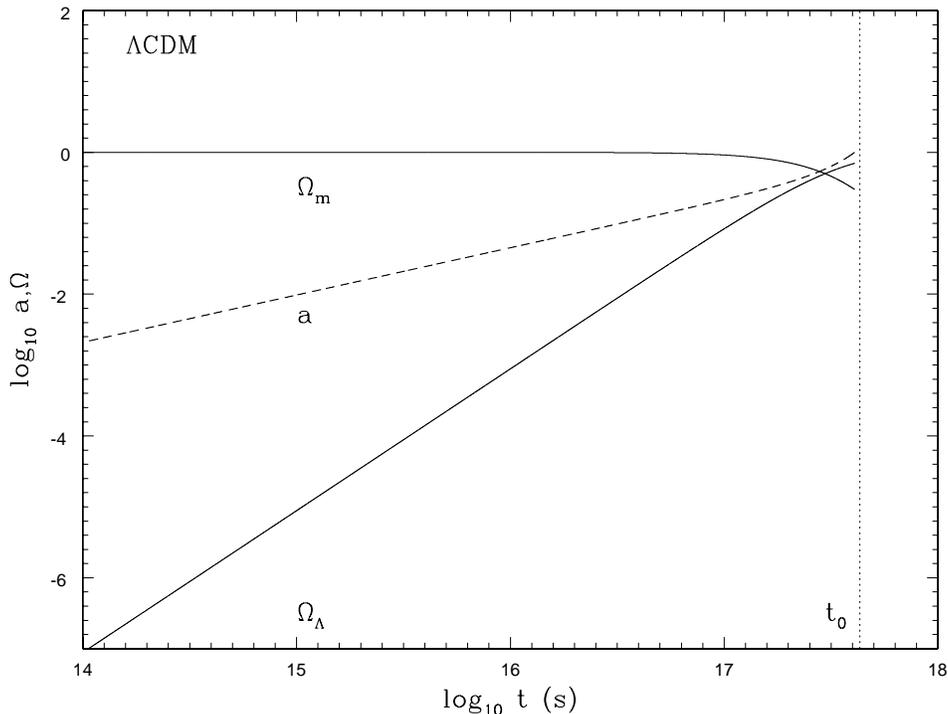


FIG. 2. Plot of the matter energy density parameter  $\Omega_m \equiv \rho_m(t)/\rho_c(t)$ , the cosmological constant energy density parameter  $\Omega_\Lambda \equiv \rho_\Lambda(t)/\rho_c(t)$ , and the expansion factor  $a(t)$ , as functions of cosmic time  $t$  for the same  $\Lambda$ CDM cosmology as that shown in figure 1. Since the energy density associated with  $\Lambda$  is constant,  $\Omega_m \rightarrow 1$  as  $t \rightarrow 0$ . A notable (and well-known) peculiarity of this cosmology is the co-called “coincidence problem,” so dubbed because  $\Omega_\Lambda$  is approximately equal to  $\Omega_m$  only in the current epoch.

Though it may not be immediately obvious, this seemingly contrived scenario is actually related to the so-called “coincidence problem” in  $\Lambda$ CDM cosmology, arising from the peculiar near-simultaneous convergence of  $\rho_m$  and  $\rho_\Lambda$  towards  $\rho_c$  in the present epoch. Just as  $R_0/ct \rightarrow 1$  only at  $t_0$  (see figure 1), so too  $\rho_\Lambda/\rho_m \sim 1$  just now (see figure 2). Even so, it is one thing to suppose that  $\rho_m \sim \rho_\Lambda$  only near the present epoch; it takes quite a leap of faith to go one step further and believe that  $R_0 \rightarrow ct_0$  only now. This odd behavior must certainly cast serious doubt on the viability of  $\Lambda$ CDM cosmology as the correct description of the Universe.

A confirmation that the basic  $\Lambda$ CDM model is simply not viable is provided by attempts (figure 3) to fit the Type Ia supernova data with the evolutionary profiles shown in figures 1 and 2. The comoving coordinate distance from some time  $t$  in the past to the present is  $\Delta r = \int_t^{t_0} c dt/a(t)$ . With  $k = 0$ , the luminosity distance  $d_L$  is  $(1+z)\Delta r$ , where the redshift  $z$  is given by  $(1+z) = 1/a$ , in terms of the expansion factor  $a(t)$  plotted in figure 2.

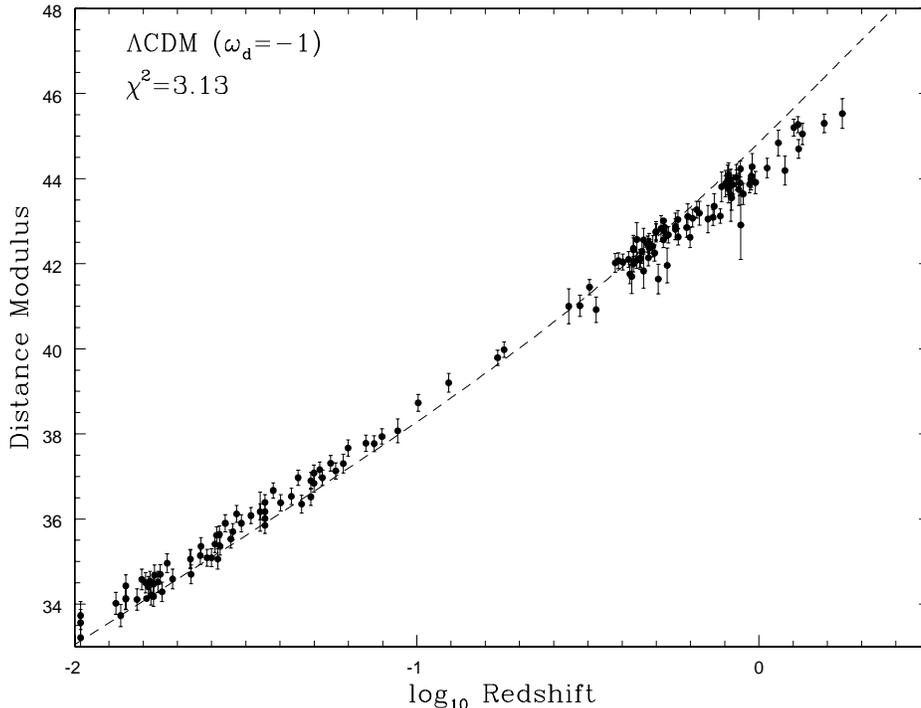


FIG. 3. Plot of the observed distance modulus versus redshift for well-measured distant Type Ia supernovae (Riess et al. 2004). The dashed curve shows the theoretical distribution of magnitude versus redshift for the  $\Lambda$ CDM cosmology described in the text and presented in figures 1 and 2. Within the context of this model, the best fit corresponds to  $\Delta = 26.57$  and a reduced  $\chi^2 \approx 3.13$ , with  $180 - 1 = 179$  d.o.f.

The data in figure 3 are taken from the “gold” sample of Riess et al. (2004), with coverage in redshift from 0 to  $\sim 1.8$ . The distance modulus is  $5 \log d_L(z) + \Delta$ , where  $\Delta \approx 25$ . The dashed curve in this plot represents the fit based on the  $\Lambda$ CDM profile shown in figures 1 and 2, with a Hubble constant  $H_0 = 71 \text{ km s}^{-1} \text{ Mpc}^{-1}$  ( $\Delta$  is used as a free optimization parameter in each of figures 3, 6, and 9). The “best” match corresponds to  $\Delta = 26.57$ , for which the reduced  $\chi^2$  is an unacceptable 3.13 with  $180 - 1 = 179$  d.o.f.

Attempts to fix this failure have generally been based on the idea that there must have been a transition point, at a redshift  $\sim 0.5$ , from past deceleration to present acceleration (Riess et al. 2004). One can easily see from figure 3 that the 16 high-redshift Type Ia supernovae, detected at  $z > 1.25$  with the Hubble Space Telescope, fall well below the distance expected for them in basic  $\Lambda$ CDM. But even if these modifications can somehow be made consistent with the supernova data, in the end they must still comply with the unyielding constraints imposed on us by the measured values of  $R_0$  and  $ct_0$ . No such relief

patch can remove the inexplicable (and simply unacceptable) coincidence implied by the required evolution of  $R_0/ct$  towards 1 only at the present time (figure 1). It is difficult avoiding the conclusion that  $\Lambda$ CDM is simply not consistent with the inferred properties of dark energy in light of the cosmic horizon.

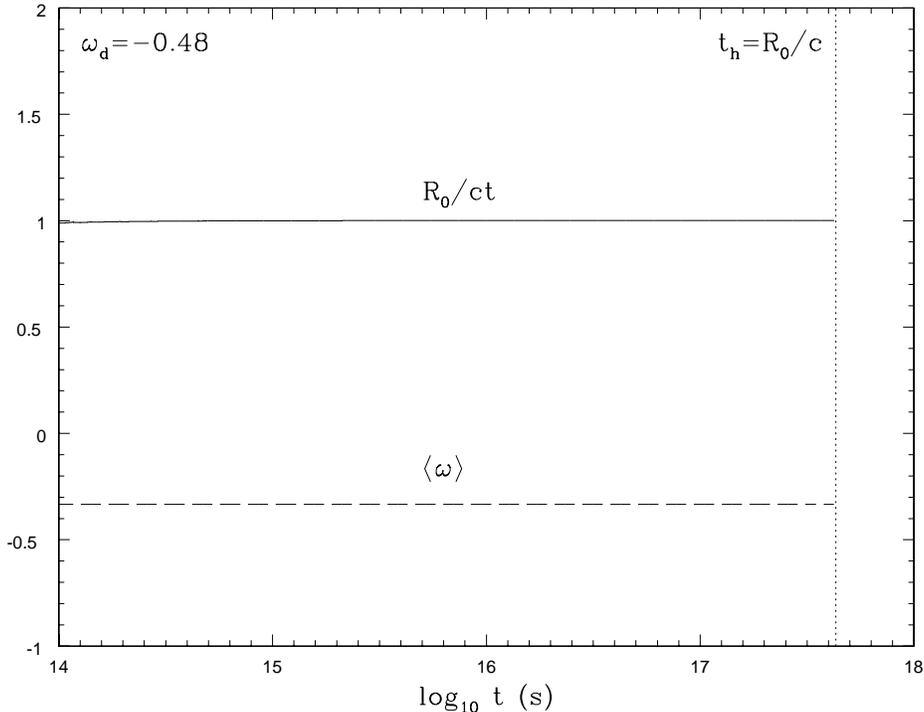


FIG. 4. Same as figure 1, except for a “scaling solution” in which  $\rho_d \propto \rho_m$  and  $\langle \omega \rangle = -1/3$ . The corresponding dark-energy equation-of-state parameter is  $\omega_d \approx -0.48$ . This is sufficiently close to  $-0.5$ , that it may actually correspond to this value within the errors associated with the measurement of  $\rho_m(t_0)/\rho_c(t_0)$  and  $\rho_d(t_0)/\rho_c(t_0)$ . Note that for this cosmology,  $R_0/ct$  is always exactly one. A universe such as this would do away with the otherwise inexplicable coincidence that  $R_0(t_0) = ct_0$  (since it has this value for all  $t$ ), but as we shall see in figure 6, it does not appear to be consistent with Type Ia supernova data.

#### IV. DYNAMICAL DARK ENERGY

Given the broad range of alternative theories of dark energy that are still considered to be viable, it is beyond the scope of this paper to exhaustively study all dynamical scenarios. Instead, we shall focus on a class of solutions with particular importance to cosmology—those in which the energy density of the scalar field mimics the background fluid energy density. Cosmological models in this category are known as “scaling solutions,” characterized by the relation

$$\frac{\rho_d(t)}{\rho_m(t)} = \frac{\rho_d(t_0)}{\rho_m(t_0)} \approx 2.33 \quad (9)$$

(some of the early papers on this topic include the following: Copeland et al. 1998; Liddle and Scherrer 1999; van den Hoogen et al. 1999; Uzan 1999; de la Macorra and Piccinelli 2000; Nunes and Mimoso 2000; Ng et al. 2001; Rubano and Barrow 2001).

By far the simplest cosmology we can imagine in this class is that for which  $\omega = -1/3$ , corresponding to  $\omega_d \approx -1/2$  (within the errors). This model is so attractive that it almost begs to be correct. Unfortunately, it is not fully consistent with Type Ia supernova data, so either our interpretation of current observations is wrong or the Universe just doesn't work this way. But it's worth our while spending some time with it because of the sheer elegance it brings to the table.

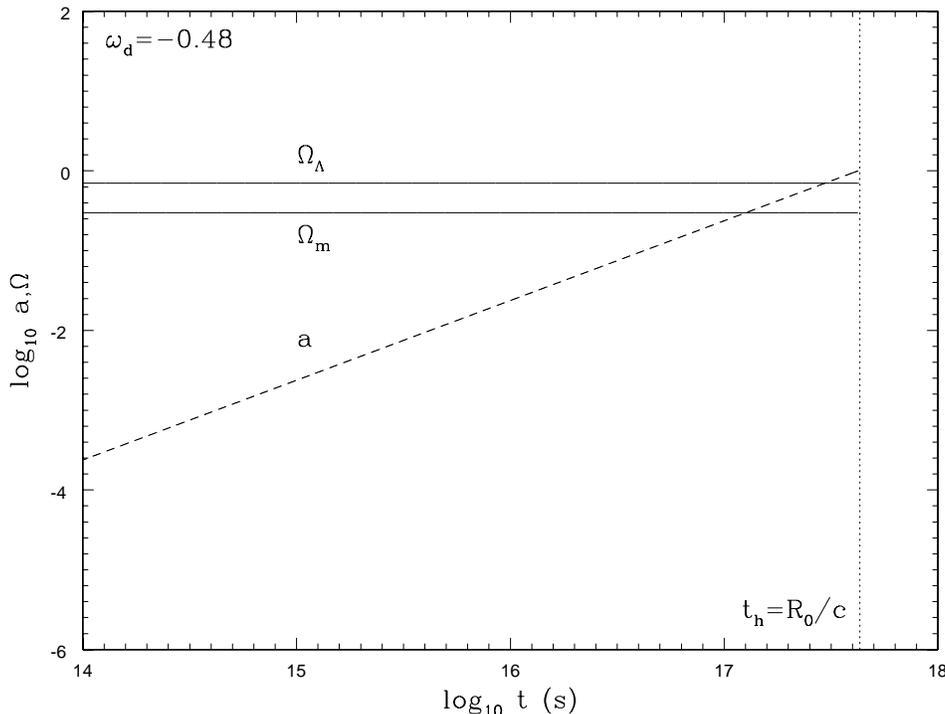


FIG. 5. Same as figure 2, except for a “scaling solution” in which  $\rho_d \propto \rho_m$  and  $\langle \omega \rangle = -1/3$ . The corresponding dark-energy equation-of-state parameter is  $\omega_d \approx -0.48$ .

To begin with, we see immediately from equation (5) (and illustrated in figures 4 and 5) that when  $\omega = -1/3$ , we have  $R_0(t) = ct$ . Instantly, this solves three of the major problems in standard cosmology: first, it explains why  $R_0(t_0)$  should be equal to  $ct_0$  (because these quantities are always equal). Second, it completely removes the inexplicable coincidence that  $\rho_d$  and  $\rho_m$  should be comparable to each other only in the present epoch (since they are always comparable to each other). Third, it does away with the so-called flatness problem (Melia 2008). To see this, let us return momentarily to equation (1) and rewrite it as follows:

$$H^2 = \left(\frac{c}{R_0}\right)^2 \left(1 - \frac{kR_0^2}{a^2}\right). \quad (10)$$

Whether or not the Universe is asymptotically flat hinges on the behavior of  $R_0/a$  as  $t \rightarrow \infty$ . But from the definition of  $R_0$  (equation 4), we infer that

$$\frac{d}{dt} \ln R_0 = \frac{3}{2}(1 + \omega) \frac{d}{dt} \ln a . \quad (11)$$

Thus, if  $\omega = -1/3$ ,

$$\frac{d}{dt} \ln R_0 = \frac{d}{dt} \ln a , \quad (12)$$

and so

$$H = \text{constant} \times \left( \frac{c}{R_0} \right) . \quad (13)$$

We also learn from equation (2) that in this universe,  $\ddot{a} = 0$ . The Universe is coasting, but not because it is empty, as in the Milne cosmology (Milne 1940), but rather because the change in pressure as it expands is just right to balance the change in its energy density.

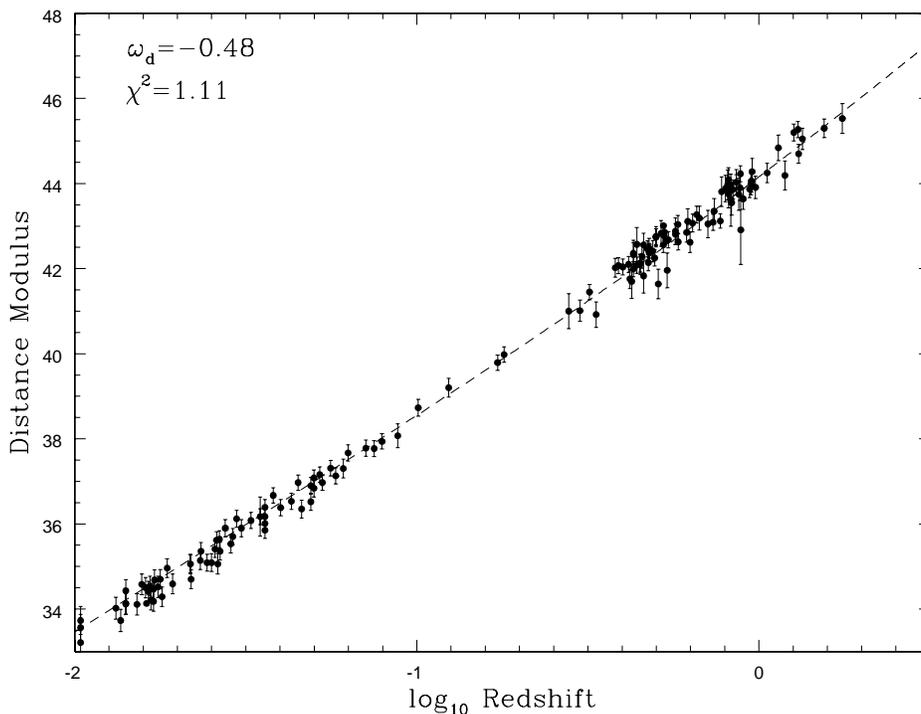


FIG. 6. Same as figure 3, except for a “scaling solution” in which  $\rho_d \propto \rho_m$  and  $\langle \omega \rangle = -1/3$ . The corresponding dark-energy equation-of-state parameter is  $\omega_d \approx -0.48$ . This fit is much better than that corresponding to a  $\Lambda$ CDM cosmology (figure 3) but, with a reduced  $\chi^2 \approx 1.11$  for  $180 - 1 = 179$  d.o.f., is still not an adequate representation of the Type Ia supernova data (Riess et al. 2004).

All told, these are quite impressive accomplishments for such a simple model, and yet, it doesn’t appear to be fully consistent with Type Ia supernova data. Repeating the calculation that produced figure 3, only now for our scaling solution with  $\omega = -1/3$ , we obtain the best fit shown in figure 6. This is a significant improvement over  $\Lambda$ CDM, but the resultant  $\chi^2$  ( $\approx 1.11$ ) is still unacceptable. Interestingly, if we were to find a slight systematic error in

the distance modulus for the events at  $z > 1$ , which for some reason has led to a fractional over-estimation in their distance (or, conversely, a systematic error that has led to an under-estimation of the distance modulus for the nearby explosions), the fit would improve significantly. So our tentative conclusion right now must be that, although an elegant scaling solution with  $\omega = -1/3$  provides a much better description than  $\Lambda$ CDM of the Universe’s expansion, it is nonetheless still not fully consistent with the supernova data.

Fortunately, many of the attractive features of an  $\omega = -1/3$  cosmology are preserved in the case where  $\omega_d = -2/3$ , corresponding to a scaling solution with  $\omega \approx -1/2$ . This model fits the supernova data quite strikingly, but it comes at an additional cost—we would have to accept the fact that the universe is somewhat older (by a few billion years) than we now believe. Actually, this situation is unavoidable for any cosmology with  $\omega < -1/3$  because of the relation between  $R_0$  and  $ct_0$  in equation (8).

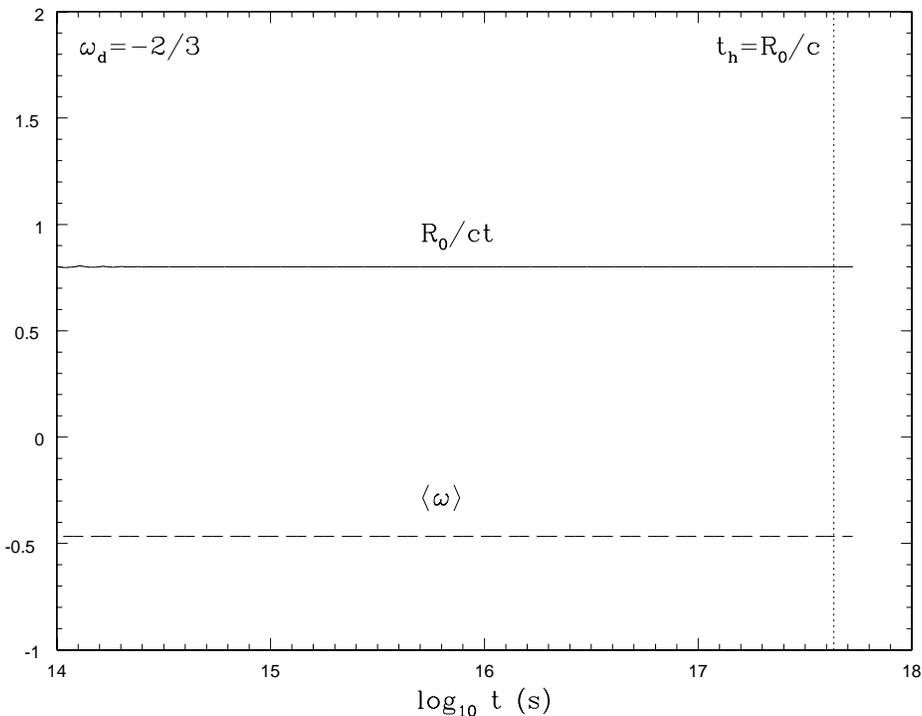


FIG. 7. Same as figure 1, except for a “scaling solution” in which  $\rho_d \propto \rho_m$  and  $\omega_d = -2/3$ . The time-averaged equation-of-state parameter is  $\langle \omega \rangle \approx -0.47$ . Thus,  $R_0 \neq ct_0$ . Instead,  $t_0 \approx 16.9$  billion years, approximately 3.4 billion years longer than the “horizon” time,  $t_h \equiv R_0/c$  ( $\approx 13.5$  billion years). This type of universe is not subject to the “coincidence” problem since  $R_0/ct$  is constant. It provides the best fit to the Type Ia supernova data (see figure 9).

We see in figure 7 that  $R_0/ct$  is constant, but at a value  $(3/2)(1 + \langle \omega \rangle)$ , where the time-averaged  $\omega$  is now  $\approx -0.47$ . Thus, if  $R_0$  is 13.5 billion light-years,  $t_0$  must be approximately 16.9 billion years. The explanation for this (Melia 2008) is that what we believe to be the Universe’s age is actually the horizon time  $t_h \equiv R_0/c$ , which is shorter than its actual age  $t_0$ . In figure 7, the distinction between  $t_h$  and  $t_0$  is manifested primarily through the termination points of the  $R_0$  and  $\langle \omega \rangle$  curves, which extend past the vertical dotted line at  $t = t_h$ .

Ironically, this unexpected result has several important consequences, such as offering an explanation for the early appearance of supermassive black holes (at a redshift  $> 6$ ; see Fan et al. 2003), and the glaring deficit of dwarf halos in the local group (see Klypin et al. 1999). Both of these long-standing problems in cosmology would be resolved if the Universe were older. Supermassive black holes would have had much more time (4 – 5 billion years) to form than current thinking allows (i.e., only  $\sim 800$  million years), and dwarf halos would correspondingly have had more time to merge hierarchically, depleting the lower mass end of the distribution.

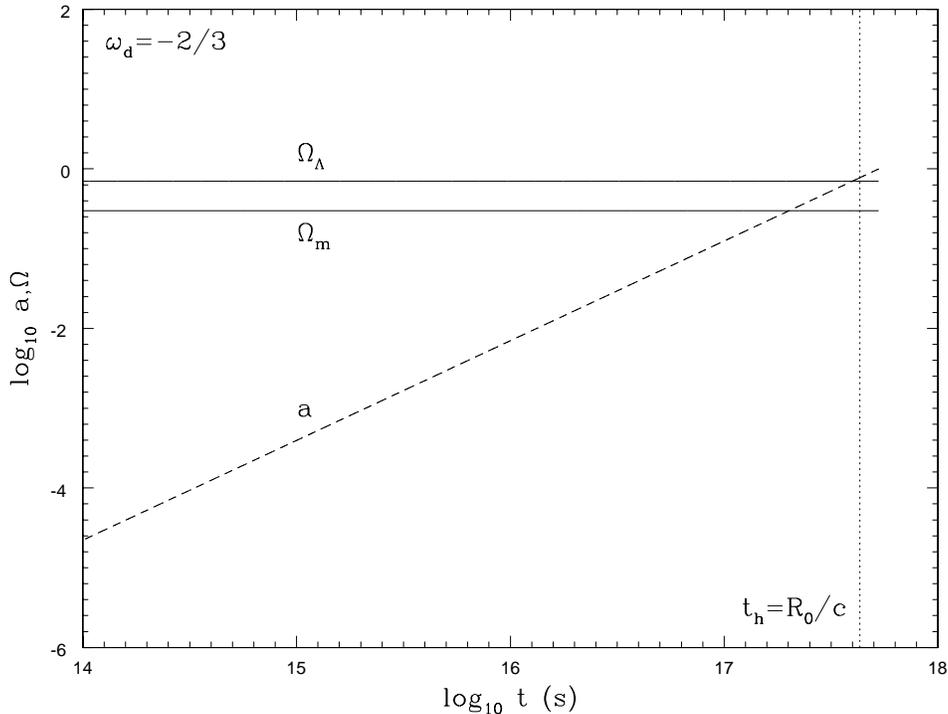


FIG. 8. Same as figure 2, except for a “scaling solution” in which  $\rho_d \propto \rho_m$  and  $\omega_d = -2/3$ .

The matter and dark energy densities corresponding to the  $\omega_d = -2/3$  scaling solution are shown as functions of cosmic time in figure 8, along with the evolution of the scale factor  $a(t)$ . Here too, the “coincidence problem” does not exist, and the flatness problem is resolved since  $kR_0^2/a^2 \rightarrow 0$  as  $t \rightarrow \infty$  in equation (10), so the constant in equation (13) should be  $\approx 1$  at late times, regardless of the value of  $k$ . Very importantly, this model fits the Type Ia supernova data very well, as shown in figure 9. The best fit corresponds to  $\Delta = 25.26$ , with a reduced  $\chi^2 = 1.001$  for  $180 - 1 = 179$  d.o.f.

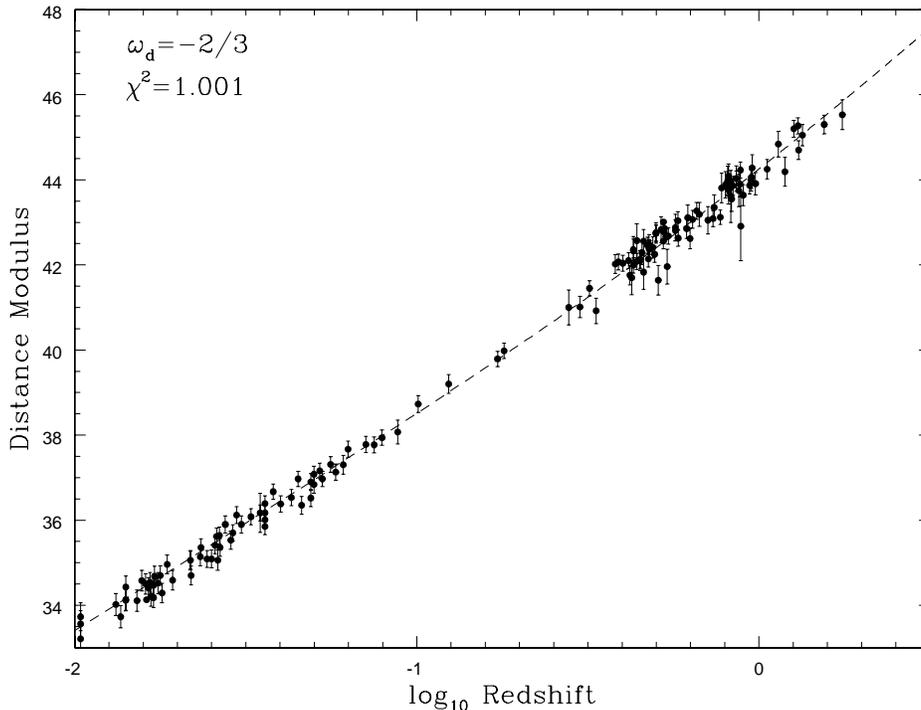


FIG. 9. Same as figure 3, except for a “scaling solution” in which  $\rho_d \propto \rho_m$  and  $\omega_d = -2/3$ . This type of universe provides the best fit to the Type Ia supernova data (Riess et al. 2004), with a reduced  $\chi^2 \approx 1.001$  for  $180 - 1 = 179$  d.o.f. The best fit corresponds to  $\Delta = 25.26$ .

## V. CONCLUDING REMARKS

Our main goal in this study has been to examine what we can learn about the nature of dark energy from a consideration of the cosmic horizon  $R_0$  and its evolution with cosmic time. A principal outcome of this work is the realization that the so-called “coincidence” problem in the standard model is actually more severe than previously thought. We have found that in a  $\Lambda$ CDM universe,  $R_0 \rightarrow ct_0$  only once, and according to the observations, this must be happening right now. Coupled to the fact that the basic  $\Lambda$ CDM model does not adequately account for the Type Ia supernova data without introducing new parameters and patching together phases of deceleration and acceleration that must somehow blend together only in the current epoch, this is a strong indication that dark energy almost certainly is not due to a cosmological constant. Other issues that have already been discussed extensively in the literature, such as the fact that the vacuum energy in quantum theory should greatly exceed the required value of  $\Lambda$ , only make this argument even more compelling. Of course, this rejection of the cosmological-constant hypothesis then intensifies interest into the question of why we don’t see any vacuum energy at all, but this is beyond the scope of the present paper.

Many alternatives to a cosmological constant have been proposed over the past decade but, for the sake of simplicity, we have chosen in this paper to focus our attention on scaling solutions. The existence of such cosmologies has been discussed extensively in the

literature, within the context of standard General Relativity, braneworlds (Randall-Sundrum and Gauss-Bonnett), and Cardassian scenarios, among others (see, e.g., Maeda 2001; Freese and Lewis 2002; and Fujimoto and He 2004).

Our study has shown that scaling solutions not only fit the Type Ia supernova data much better than the basic  $\Lambda$ CDM cosmology, but they apparently simultaneously solve several conundrums with the standard model. As long as the time-averaged value of  $\omega$  is less than  $-1/3$ , they eliminate both the coincidence and flatness problems, possibly even obviating the need for a period of rapid inflation in the early universe (see, e.g., Guth 1981; Linde 1982).

But most importantly, as far as this study is concerned, scaling solutions account very well for the observed fact that  $R_0 \approx ct_0$ . If  $\langle\omega\rangle = -1/3$  exactly, then  $R_0(t) = ct$  for all cosmic time, and therefore the fact that we see this condition in the present Universe is no coincidence at all. On the other hand, if  $\langle\omega\rangle < -1/3$ , scaling solutions fit the Type Ia supernova data even better, but then we have to accept the conclusion that the Universe is older than the horizon time  $t_h \equiv R_0/c$ . According to our calculations, which produce a best fit to the supernova data for  $\langle\omega\rangle = -0.47$  (corresponding to a dark energy equation of state with  $\omega_d = -2/3$ ), the age of the Universe should then be  $t_0 \approx 16.9$  billion years. This may be surprising at first, perhaps even unbelievable to some, but the fact of the matter is that such an age actually solves other major problems in cosmology, including the (too) early appearance of supermassive black holes, and the glaring deficit of dwarf halos in the local group of galaxies.

When thinking about a dynamical dark energy, it is worth recalling that scalar fields arise frequently in particle physics, including string theory, and any of these may be appropriate candidates for this mysterious new component of  $\rho$ . Actually, though we have restricted our discussion to equations of state with  $\omega_d \geq -1$ , it may even turn out that a dark energy with  $\omega_d < -1$  is providing the Universe's acceleration. Such a field is usually referred to as a Phantom or a ghost. The simplest explanation for this form of dark energy would be a scalar field with a negative kinetic energy (Caldwell 2002). However, Phantom fields are plagued by severe quantum instabilities, since their energy density is unbounded from below and the vacuum may acquire normal, positive energy fields (Carroll et al. 2003). We have therefore not included theories with  $\omega_d < -1$  in our analysis here, though a further consideration of their viability may be warranted as the data continue to improve.

On the observational front, the prospects for confirming or rejecting some of the ideas presented in this paper look very promising indeed. An eagerly anticipated mission, SNAP (Rhodes et al. 2004), will constrain the nature of dark energy in two ways. First, it will observe deeper Type Ia supernovae. Second, it will attempt to use weak gravitational lensing to probe foreground mass structures. If selected, SNAP should be launched by 2020. An already funded mission, the Planck CMB satellite, probably won't have the sensitivity to measure any evolution in  $\omega_d$ , but it may be able to tell us whether or not  $\omega_d = -1$ . An ESA mission, Planck is scheduled for launch in mid-2008.

Finally, we may be on the verge of uncovering a class of sources other than Type Ia supernovae to use for dark-energy exploration. Type Ia supernovae have greatly enhanced our ability to study the Universe's expansion out to a redshift  $\sim 2$ . But this new class of sources may possibly extend this range to values as high as 5 – 10. According to Hooper and Dodelson (2007), Gamma Ray Bursts (GRBs) have the potential to detect dark energy

with a reasonable significance, particularly if there was an appreciable amount of it at early times, as suggested by scaling solutions. It is still too early to tell if GRBs are good standard candles, but since differences between  $\Lambda$ CDM and dynamical dark energy scenarios are more pronounced at early times (see figures 3, 6, and 9), GRBs may in the long run turn out to be even more important than Type Ia supernovae in helping us learn about the true nature of this unexpected “third” form of energy.

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