

ASPHERICAL PROPERTIES OF HYDRODYNAMICS AND NUCLEOSYNTHESIS IN JET-INDUCED SUPERNOVAE

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ABSTRACT

Jet-induced supernovae (SNe) have been suggested to occur in gamma-ray bursts (GRBs) and highly-energetic SNe (hypernovae). I investigate hydrodynamical and nucleosynthetic properties of the jet-induced explosion of a population III $40M_{\odot}$ star with a two-dimensional special relativistic hydrodynamical code. The abundance distribution after the explosion and the angular dependence of the yield are obtained for the models with high and low energy deposition rates $\dot{E}_{\text{dep}} = 120 \times 10^{51} \text{ ergs s}^{-1}$ and $1.5 \times 10^{51} \text{ ergs s}^{-1}$. The ejection of Fe-peak products and the fallback of unprocessed materials in the jet-induced SNe account for the abundance patterns of the extremely metal-poor (EMP) stars. It is also found that the peculiar abundance pattern of a Si-deficient metal-poor star HE 1424-0241 is reproduced by the angle-delimited yield for $\theta = 30^{\circ} - 35^{\circ}$ of the model with $\dot{E}_{\text{dep}} = 120 \times 10^{51} \text{ ergs s}^{-1}$. Furthermore, I compare the yield of the jet-induced explosion with that of the spherical explosion and confirm the ejection and fallback in the jet-induced explosion is almost equivalent to the “mixing-fallback” in spherical explosions. In contrast to the spherical models, however, the high-entropy environment is realized in the jet-induced explosion and thus [(Sc, Ti, V, Cr, Co, Zn)/Fe] are enhanced. The enhancements of [Sc/Fe] and [Ti/Fe] improve agreements with the abundance patterns of the EMP stars.

Subject headings: Galaxy: halo — gamma rays: bursts — nuclear reactions, nucleosynthesis, abundances — stars: abundances — stars: Population II — supernovae: general

1. INTRODUCTION

Gamma-ray bursts (GRBs) are phenomena emitting γ -ray for short periods followed by a power-law decaying afterglow. The origin had been covered for a long while, but it has become clear that long GRBs are associated with supernovae (SNe). Three GRB-associated SNe have been observed so far: GRB 980425/SN 1998bw (Galama et al. 1998), GRB 030329/SN 2003dh (Stanek et al. 2003; Hjorth et al. 2003), and GRB 031203/SN 2003lw (Malesani et al. 2004). They are all energetic explosions of massive stars, called hypernovae (e.g., Nomoto et al. 2006, 2007 for a review).

Although the explosion mechanism of GRBs and GRB-associated SNe is still uncovered, the following photometric and spectroscopic observations indicate that they are aspherical explosions with jet(s). (1) The light curve of the GRB afterglow has shown a polychromatic break in the power-law decay. The break is called a “jet break” explained by a deceleration of a relativistic jet and relativistic beaming of light (e.g., Frail et al. 2001; Piran 2005). (2) The nebular spectra of SN 1998bw show narrower [O I] lines than [Fe II] lines (Patat et al. 2001) indicating that O locates in the inner and lower-velocity region than Fe. This is realized in an aspherical explosion but not in a spherical explosion (Maeda et al. 2002, 2006).

The aspherical explosions are indirectly suggested from the abundance patterns of extremely metal-poor

(EMP) stars with $[\text{Fe}/\text{H}] < -3$.³ Such EMP stars are likely to show the nucleosynthesis yields of a single or a few core-collapse SN/SNe (Audouze & Silk 1995; Beers & Christlieb 2005). The abundance patterns are reproduced by the “mixing-fallback” models that assume the extensive mixing of the shocked material before the fallback in a spherical SN model (Umeda & Nomoto 2002, 2003, 2005; Iwamoto et al. 2005; Tominaga et al. 2007b). In particular, the abundance patterns of EMP stars with high [Co/Fe] and [Zn/Fe] (~ 0.5) are explained by energetic explosion models because the high ratios require explosive nucleosynthesis under high entropy (e.g., Umeda & Nomoto 2005; Tominaga et al. 2007b). Since the fallback does not take place in energetic spherical or quasi-spherical explosions (e.g., Woosley & Weaver 1995; Iwamoto et al. 2005), the mixing and fallback are interpreted as a consequence of the aspherical explosion. Indeed, the abundance patterns of the EMP stars are reproduced by the jet-induced explosion model (Tominaga et al. 2007a).

Since the jet-induced explosion contributes to many astronomical phenomena, it is important to make a quantitative prediction on the nucleosynthesis outcome of the jet-induced explosions. In order to follow the jet propagation, fallback, and nucleosynthesis on the site and compare the jet-induced explosion models with the EMP stars, GRBs, and SNe, it is required to calculate multi-dimensional hydrodynamics and nucleosynthesis with the gravity and the special relativity.⁴ Although there are many studies on the GRB jet propagation of the stellar

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³ Here $[A/B] = \log_{10}(N_A/N_B) - \log_{10}(N_A/N_B)_{\odot}$, where the subscript \odot refers to the solar value and N_A and N_B are the abundances of elements A and B, respectively.

⁴ In order to follow the core collapse, jet formation, and nucleosynthesis on the site, it is also required to involve the neutrino, the

mantle using the special relativistic hydrodynamics (e.g., Aloy et al. 2000; Zhang et al. 2004; Mizuta et al. 2006), they did not include the gravity or calculate nucleosynthesis.

To investigate the yields of the jet-induced SNe, it is crucial to include the gravity because the fallback plays an important role on the nucleosynthesis yields. The studies by the use of Newtonian calculations have concluded that the energy deposition rate (\dot{E}_{dep}) sensitively affects SN nucleosynthesis (Maeda & Nomoto 2003; Nagataki et al. 2006). This result has been confirmed for the special relativistic cases (Tominaga et al. 2007a). In particular, Tominaga et al. (2007a) have shown that the jet-induced explosions with various \dot{E}_{dep} can explain both the variations of GRB-associated SNe and the EMP, carbon-enhanced EMP (CEMP, $[\text{C}/\text{Fe}] > 1$), and hyper metal-poor (HMP, $[\text{Fe}/\text{H}] < -5$) stars.

The previous studies have proved mostly the angle-integrated yields and shown that the abundance patterns of the EMP stars are reproduced by the angle-integrated yield. However, the abundance distribution of the jet-induced explosion depends on the direction (e.g., Maeda & Nomoto 2003). Thus, the abundance patterns of the next-generation stars might depend on the direction. I calculate aspherical stellar explosions induced by highly relativistic jets and obtain hydrodynamical and nucleosynthetic structures of such jet-induced explosion models. In particular, I investigate the angular dependence of the yield to compare the yields with the abundance patterns of the metal-poor stars. Furthermore, I compare the jet-induced explosion with the spherical SN model applied the mixing-fallback model and connect properties of the jet-induced explosion to the mixing-fallback model.

In § 2, the applied models are described. In § 3, I present the hydrodynamical and nucleosynthetic structures of the jet-induced explosion model, investigate the angular dependence of the yields, and compared the jet-induced explosion model with the spherical SN model. In § 4, the conclusion and discussion are presented. In Appendixes, the hydrodynamics and nucleosynthesis code is described and tested.

2. MODELS

I investigate a jet-induced SN explosion of a Pop III $40M_{\odot}$ star (Umeda & Nomoto 2005; Tominaga et al. 2007b) by means of a two-dimensional relativistic Eulerian hydrodynamic and nucleosynthesis calculation with the gravity (Appendix A). The nucleosynthesis calculation is performed as a post-processing (Hix & Thielemann 1996, 1999) with the reaction network including 280 isotopes up to ^{79}Br (see Table 1 in Umeda & Nomoto 2005). The thermodynamic histories are traced by marker particles representing Lagrangian mass elements (e.g., Hachisu et al. 1990; Maeda & Nomoto 2003, see also Appendix B). A computational domain initially ranges up to the stellar surface where $R_{\text{star}} = 2 \times 10^7$ km and is captured by 200 logarithmical grids in the r -direction and 100 uniform grids in the θ -direction. A circumstellar matter (CSM) extends from the stellar surface with the slope $\rho \propto r^{-2}$.

general relativity, and possibly the magnetic field (e.g., Pruet et al. 2005; Fröhlich et al. 2006; Janka et al. 2007).

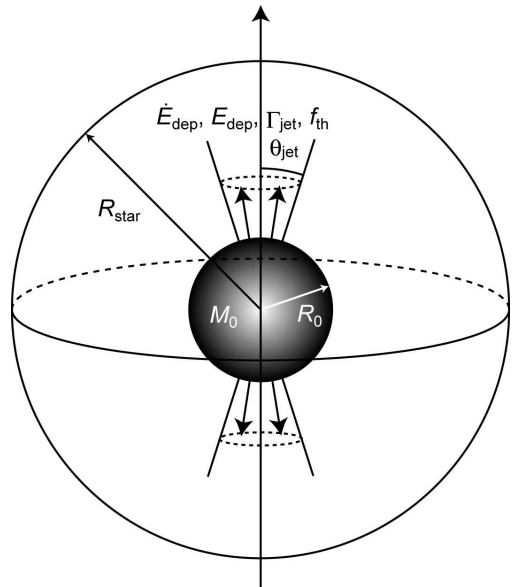


FIG. 1.— Schematic picture of the jet-induced explosion.

The explosion mechanism of GRB-associated SNe is still under debate (e.g., a neutrino annihilation, Woosley 1993; MacFadyen & Woosley 1999; and a magnetorotation, Blandford & Znajek 1977; Brown et al. 2000; Mizuno et al. 2004). Thus, I do not consider how the jet is launched, but the jet is dealt parametrically with the following five parameters (Fig. 1): energy deposition rate (\dot{E}_{dep}), total deposited energy (E_{dep}), initial half angle of the jets (θ_{jet}), initial Lorentz factor (Γ_{jet}), and the ratio of thermal to total deposited energies (f_{th}). The jet is injected from the inner boundary at an enclosed mass M_0 corresponding to a radius R_0 . The density, pressure, and velocities of the jet are described with the five parameters (Appendix C) and put as a boundary condition at the inner boundary. The jet is assumed to consist of the accreted matter and to expand adiabatically below the inner boundary. After the jet is injected into the computational domain, the thermodynamic histories are traced by the marker particles.

In this paper, I show three models; (A) a model with $\dot{E}_{\text{dep},51} = \dot{E}_{\text{dep}}/(10^{51}\text{ergs s}^{-1}) = 120$ and $M_0 = 1.4M_{\odot}$ ($R_0 = 900$ km), (B) a model with $\dot{E}_{\text{dep},51} = 1.5$ and $M_0 = 1.4M_{\odot}$ ($R_0 = 900$ km), and (C) a model with $\dot{E}_{\text{dep},51} = 120$ and $M_0 = 2.3M_{\odot}$ ($R_0 = 2700$ km). The other parameters are same for each model; $E_{\text{dep}} = 1.5 \times 10^{52}$ ergs,⁵ $\theta_{\text{jet}} = 15^\circ$, $\Gamma_{\text{jet}} = 100$ and $f_{\text{th}} = 10^{-3}$. The mass of jets is $M_{\text{jet}} \sim 8 \times 10^{-5}M_{\odot}$.

⁵ Frail et al. (2001) suggested the γ -ray energies (E_{γ}) of GRBs are clustered at $E_{\gamma} \sim 5 \times 10^{50}$ ergs. Although E_{γ} is 30 times smaller than E_{dep} , there are large uncertainties on the relation between E_{γ} and E_{dep} . For example, the energy possessed by the relativistic matter depends on the interactions between the relativistic jet and the stellar mantle and is reduced compared to the injected energy (Zhang et al. 2003, 2004). Moreover, the radiative conversion efficiency of a kinetic energy depends on an unknown γ -ray emission mechanism. The efficiency is estimated to be $\sim 2 - 40\%$ for collisions of internal shocks (Kobayashi et al. 1997) or $> 50\%$ for nonthermalized photospheric emissions (Ioka et al. 2007). E_{dep} adopted in this paper might be appropriate for the former emission mechanism or excessive for the latter emission mechanism. The dependence of the jet-induced explosions on E_{dep} will be studied in future.

TABLE 1
JET-INDUCED EXPLOSION MODELS.

Name	M_0 [M_\odot]	R_0 [km]	\dot{E}_{dep} [10^{51} ergs s^{-1}]	E_{dep} [10^{51} ergs]	θ_{jet} [degrees]	Γ_{jet}	f_{th}	M_{rem} [M_\odot]
A	1.4	900	120	15	15	100	10^{-3}	9.1
B	1.4	900	1.5	15	15	100	10^{-3}	16.9
C	2.3	2700	120	15	15	100	10^{-3}	8.1

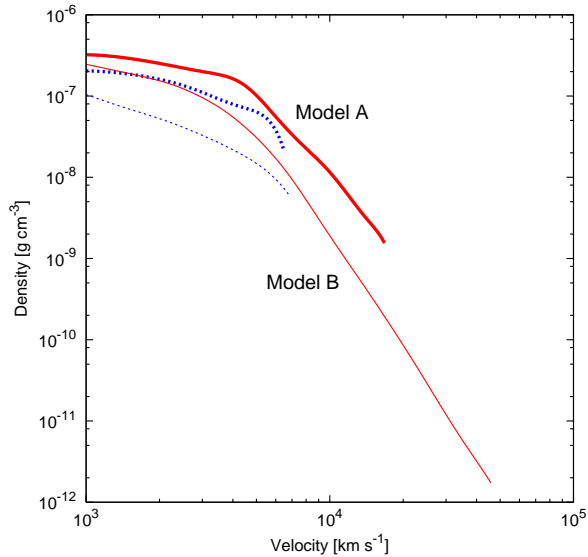


FIG. 2.— Density structures at $t = 10^5$ s along the jet axis (*solid line*) and the equatorial plane (*dashed line*) of model A (*thick line*) and B (*thin line*).

The model parameters and the central remnant mass (M_{rem}) are summarized in Table 1. Models A and B are used in Tominaga et al. (2007a) and they reproduce the abundance patterns of the EMP stars⁶ (Cayrel et al. 2004) and HE 1327–2326 (e.g., Frebel et al. 2005, 2006; Collet et al. 2006), respectively.

3. RESULTS

The hydrodynamical calculations are followed until the homologously expanding structure is reached ($v \propto r$). Then, the ejected mass elements are identified from whether their radial velocities exceed the escape velocities at their positions. The density structures of models A and B at $t = 10^5$ s are shown in Figure 2. The density along the jet axis is higher than the density along the equatorial plane because the matter is more easily ejected along the jet axis than along the equatorial plane. And the SN ejecta of model A is denser and more compact than that of model B because the ejected mass of model A is larger than that of model B.

3.1. Fallback

Figures 3a and 3b show “accreted” regions for models A and B, where the accreted mass elements initially located in the progenitor. The O layer is separated into the two layers: (1) the O+Mg layer with $X(^{24}\text{Mg}) > 0.01$ and (2) the O+C layer with $X(^{12}\text{C}) > 0.1$. The inner matter is ejected along the jet-axis but not along the

⁶ An averaged abundance pattern of four EMP stars, CS 22189–009, CS 22172–002, CS 22885–096, and CD –38 245, is adopted.

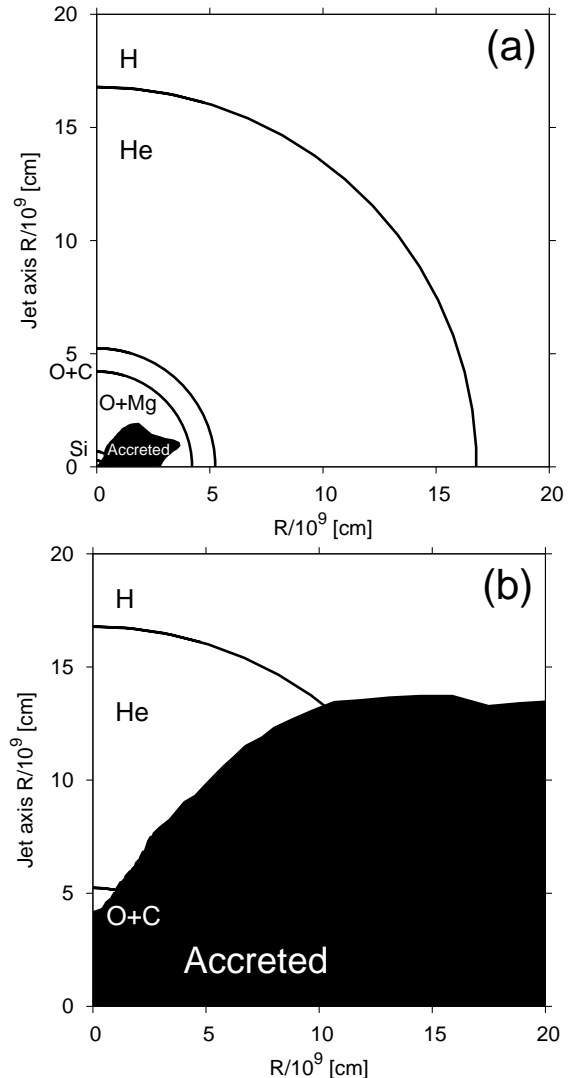


FIG. 3.— Initial locations of the mass elements which are finally accreted (*black*), for (a) model A and (b) model B. The background circles represent the boundaries between the layers in the progenitor star; the H, He, O+C, O+Mg, and Si layers from the outside.

equatorial plane. On the other hand, the outer matter is ejected even along the equatorial plane, since the lateral expansion of the shock terminates the infall as the shock reaches the equatorial plane.

The accreted mass is larger for lower \dot{E}_{dep} . This stems from the balance between the ram pressures of the injecting jet (P_{jet}) and the infalling matter (P_{fall}) (e.g., Fryer 1999; Fryer & Mészáros 2003; Maeda & Tominaga 2008). While P_{jet} depends on \dot{E}_{dep} , P_{fall} depends on the presupernova structure. If the matter falls freely, P_{fall} is proportional to $\rho_r r^{3/2} M_r$ for the matter having located at r

and M_r in the progenitor where the presupernova density is ρ_r . The critical energy deposition rate ($\dot{E}_{\text{dep,cri}}$) giving $P_{\text{jet}}(\dot{E}_{\text{dep,cri}}) = P_{\text{fall}}(M_r)$ is lower for the outer layer (i.e., larger M_r , Fryer & Mészáros 2003; Maeda & Tominaga 2008). Therefore, the jet injection with lower \dot{E}_{dep} is enabled at a later time when the central remnant becomes more massive. Additionally, the lateral expansion of the jet is more efficiently suppressed for lower \dot{E}_{dep} . As a result, the accreted region and M_{rem} are larger for lower \dot{E}_{dep} .

A model with lower \dot{E}_{dep} has larger M_{rem} , higher [C/O], [C/Mg], and [C/Fe], and smaller $M(\text{Fe})$ because of the larger amount of fallback (Tominaga et al. 2007a). The larger amount of fallback decreases the mass of the inner core (e.g., Fe, Mg, and O) relative to the mass of the outer layer (e.g., C, Figs. 3ab). Since O and Mg are synthesized in the inner layers than C, [C/O] and [C/Mg] are larger for the larger infall of the O layer. Also, the fallback of the O layer decreases $M(\text{Fe})$ because Fe is mainly synthesized explosively in the Si and O+Mg layers. Therefore, the variation of \dot{E}_{dep} in the jet-induced explosions predicts that the variations of [C/O], [C/Mg], and [C/Fe] are corresponding to the variation of $M(\text{Fe})$.

3.2. Abundance distribution

Figures 4a and 4b show the abundance distributions and density structures at $t = 10^5$ s for models A and B. I classify the mass elements by their abundances as follows: (1) Fe with $X(^{56}\text{Ni}) > 0.04$, (2) Si with $X(^{28}\text{Si}) > 0.08$, (3) O+Mg with $X(^{16}\text{O}) > 0.6$ and $X(^{24}\text{Mg}) > 0.01$, (4) O+C with $X(^{16}\text{O}) > 0.6$ and $X(^{12}\text{C}) > 0.1$, (5) He with $X(^4\text{He}) > 0.7$, and (6) H with $X(^1\text{H}) > 0.3$. If a mass element satisfies two or more conditions, the mass element is classified into the class with the smallest number.

The abundance distribution and thus the composition of the ejecta depend on the direction. In model A, the O+Mg, O+C, He, and H mass elements locate in the all direction. On the other hand, most of the Fe and Si mass elements locate at $\theta < 10^\circ$ and stratify in this order from the jet axis and a part of them locate at $15^\circ < \theta < 35^\circ$. Interestingly, the Fe mass elements surround the Si mass elements at $15^\circ < \theta < 35^\circ$. In model B, most of the O+C and He mass elements locate at $\theta < 3^\circ$ because they are ejected only along the jet axis, while the Fe mass elements injected as a jet expand laterally up to $\theta \sim 50^\circ$ and the H mass elements are distributed in the all directions. The lateral spread of the Fe mass elements in models A and B are led by the collision with the stellar mantle and the internal pressure of the jet.

3.2.1. Angular dependence of the yields

I investigate the angle-delimited yields integrated over each 10° for models A and B, although the integration range might be too wide to be taken in a single next-generation star. Fe is mainly distributed at $\theta < 40^\circ$ for model A (Fig. 4a) and $\theta < 50^\circ$ for model B (Fig. 4b). Figures 5a and 5b show the abundance patterns of the angle-delimited yields for $\theta < 40^\circ$ of model A and $\theta < 50^\circ$ of model B, respectively. The yields are compared with the abundance patterns of the EMP stars (Cayrel et al. 2004), the CEMP stars (HE 1300+0157,

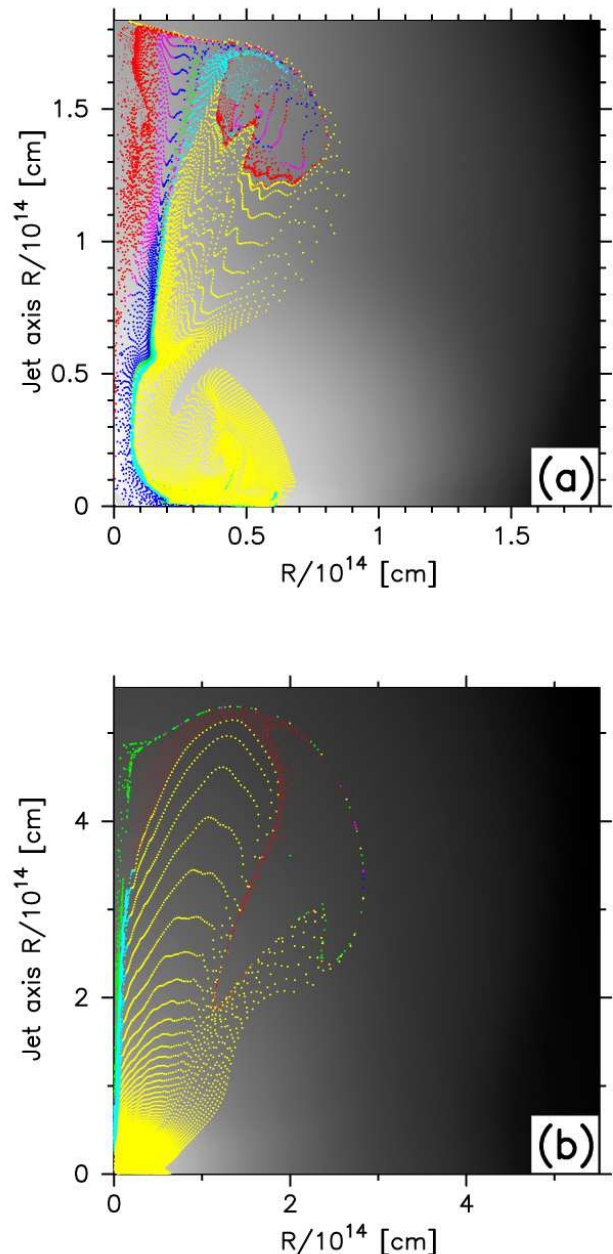


FIG. 4.— Density structure (back ground gray scale) and the positions of the mass elements at $t = 10^5$ s for (a) model A and (b) model B. Color of the marks represents the abundance of the mass element (H: yellow, He: cyan, O+C: green, O+Mg: blue, Si: magenta, and Fe: red). Size of the marks represents the origin of the mass element (the jet: dots, and the shocked stellar mantle: filled circles).

Frebel et al. 2007), and the HMP stars (HE 0107–5240, e.g., Christlieb et al. 2002; Bessell & Christlieb 2005; Collet et al. 2006 and HE 1327–2326, e.g., Frebel et al. 2005, 2006; Collet et al. 2006).

Figure 5a shows the angle-delimited yields of model A. The abundance patterns of the angle-delimited yields are determined by which mass elements are included into the integration. Because of the stratified abundance distribution, the yields for $0^\circ \leq \theta < 10^\circ$ and $10^\circ \leq \theta < 20^\circ$ show low [C/Fe] and [O/Fe] and high [C/Fe] and [O/Fe],

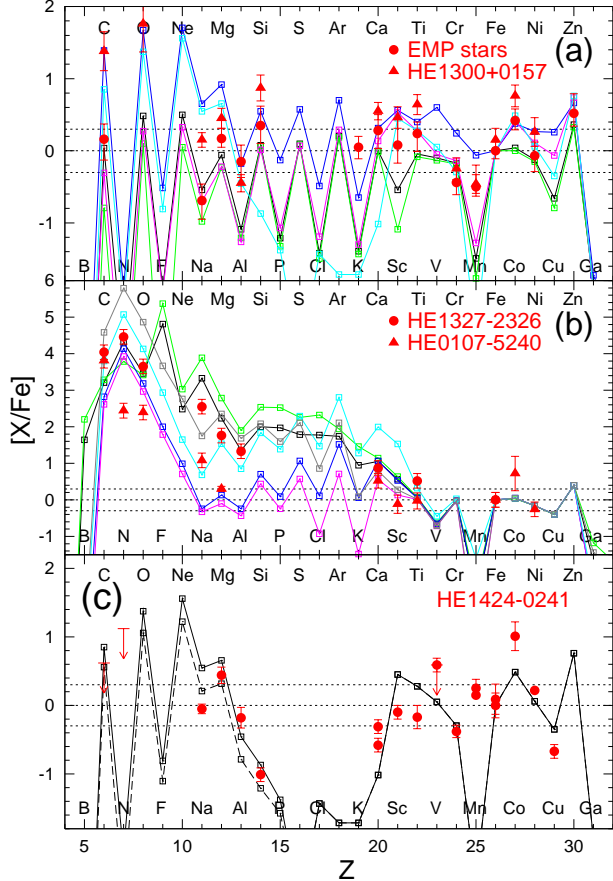


FIG. 5.— Comparisons between the abundance patterns of the angle-integrated yield (black), the EMP, CEMP, and HMP stars (red filled marks, Cayrel et al. 2004; Frebel et al. 2007; Christlieb et al. 2002; Bessell & Christlieb 2005; Collet et al. 2006; Frebel et al. 2005, 2006), and the angle-delimited yields (colored lines) for (a) model A and (b) model B. The color of the lines represent the yields integrated over $0^\circ \leq \theta < 10^\circ$ (green line), $10^\circ \leq \theta < 20^\circ$ (blue line), $20^\circ \leq \theta < 30^\circ$ (magenta line), $30^\circ \leq \theta < 40^\circ$ (cyan line), and $40^\circ \leq \theta < 50^\circ$ (gray line). (c) Comparison between the abundance pattern of HE 1424–0241 (red filled circles, Cohen et al. 2007) and the angle-delimited yields of model A for $30^\circ \leq \theta < 40^\circ$ (solid line) and $30^\circ \leq \theta < 35^\circ$ (dashed line).

respectively. Intriguingly, the region with $30^\circ \leq \theta < 40^\circ$ includes the Fe mass elements in the outer layer and the O+Mg and O+C mass elements in the inner layer but not the Si mass elements. Thus, the yield for $30^\circ \leq \theta < 40^\circ$ shows a Si-deficient abundance pattern. Furthermore, the Fe and Si mass elements at $15^\circ < \theta < 35^\circ$ initially located along the jet axis ($\theta < 10^\circ$), and thus the high-entropy environment is realized in these mass elements and $[\text{Sc}/\text{Fe}]$, $[\text{Ti}/\text{Fe}]$, $[\text{Co}/\text{Fe}]$, and $[\text{Zn}/\text{Fe}]$ are enhanced in the yields for $10^\circ \leq \theta < 40^\circ$. On the other hand, $[\text{Sc}/\text{Fe}]$ is low in the yield for $0^\circ \leq \theta < 10^\circ$ because the matter initially located at $10^\circ \leq \theta$.

Figure 5b shows the angle-delimited yields of model B. The all angle-delimited yields of model B show high $[\text{C}/\text{Fe}]$. This is the same feature as the angle-integrated yield. Since most of the heavy elements locate at $\theta < 10^\circ$, the abundance pattern of the yield for $0^\circ \leq \theta < 10^\circ$ is similar to that of the angle-integrated yield, except for N that is mostly contained in the H mass elements. On the other hand, the yields for $10^\circ \leq \theta < 50^\circ$ consist of the Fe mass elements injected as a jet and the O+C mass elements. In the yields for $10^\circ \leq \theta < 50^\circ$, $[\text{C}/\text{Mg}]$ is

almost similar and the differences of $[\text{C}/\text{Fe}]$ and $[\text{Mg}/\text{Fe}]$ mainly stem from the different amount of Fe.

3.2.2. Abundance patterns of the metal-poor stars

A very peculiar, Si-deficient, metal-poor star HE 1424–0241 was observed (Cohen et al. 2007). Its abundance pattern with high $[\text{Mg}/\text{Si}]$ (~ 1.4) and normal $[\text{Mg}/\text{Fe}]$ (~ 0.4) is difficult to be reproduced by previous SN models. This is because $\log\{[X(\text{Mg})/X(\text{Si})]/[X(\text{Mg})/X(\text{Si})]_\odot\} \lesssim 1.6$ is realized in the O+Mg layer at the presupernova stage (e.g., Woosley & Weaver 1995; Umeda & Nomoto 2005). Thus, in order to reproduce the abundance pattern of HE 1424–0241, the SN yield is required to include explosively-synthesized Fe but not explosively-synthesized Si.

The angle-delimited yield may possibly explain the high $[\text{Mg}/\text{Si}]$ and normal $[\text{Mg}/\text{Fe}]$ (Fig. 5c). Figure 5c shows that the yields integrated over $30^\circ \leq \theta < 40^\circ$ and $30^\circ \leq \theta < 35^\circ$ of model A reproduce the abundance pattern of HE 1424–0241. The yield consist of Mg in the inner region and Fe in the outer region (Fig. 4a). Although there are some elements to be improved, the elusive feature of HE 1424–0241 could be explained by taking into account the angular dependence of the yield. The high $[\text{Mg}/\text{Si}]$ and normal $[\text{Mg}/\text{Fe}]$ can be realized with an appropriate integration range if the Fe mass elements penetrate the stellar mantle (i.e., the duration of the jet injection is long) and if the O+Mg mass elements are ejected in all directions (i.e., \dot{E}_{dep} is high).

If the yield depends on the direction, the abundance patterns of the angle-delimited yields have a large scatter (Fig. 5a). Although the diversities of $[\text{C}/\text{Fe}]$ and $[\text{O}/\text{Fe}]$ in the angle-delimited yields are similar to those in the observations of the EMP and CEMP stars, the observations of the EMP stars provide abundance ratios with comparatively small scatters (e.g., $[\text{Sc}/\text{Fe}]$, Cayrel et al. 2004). This implies that SNe contributing to most of the EMP stars experienced strong mixing of the ejecta, e.g., due to an interaction with interstellar medium (ISM), so that the yield no longer depends on the direction. Actually, it was suggested that the interaction between the SN ejecta and ISM induces further mixing of the abundances if ISM has an inhomogeneous structure (e.g., Nakasato & Shigeyama 2000). According to structure formation calculations (e.g., Yoshida et al. 2007), the Pop III stars are surrounded by the inhomogeneous ISM. Therefore, the mixing of the SN ejecta is likely to take place. On the other hand, the abundance pattern of HE 1424–0241 implies weak mixing of the SN ejecta. Such a different star formation process might cause the peculiar abundance pattern of HE 1424–0241. In order to conclude the origin of HE 1424–0241, it is necessary to calculate three-dimensional evolution of a supernova remnant in the inhomogeneous ISM.

3.3. Comparison with the spherical supernova model

The calculations of the jet-induced explosions show that the ejection of the inner matter is compatible with the fallback of the outer matter (Figs. 3ab). This is consistent with the two-dimensional illustration of the mixing-fallback model (Fig. 12b in Tominaga et al. 2007b).

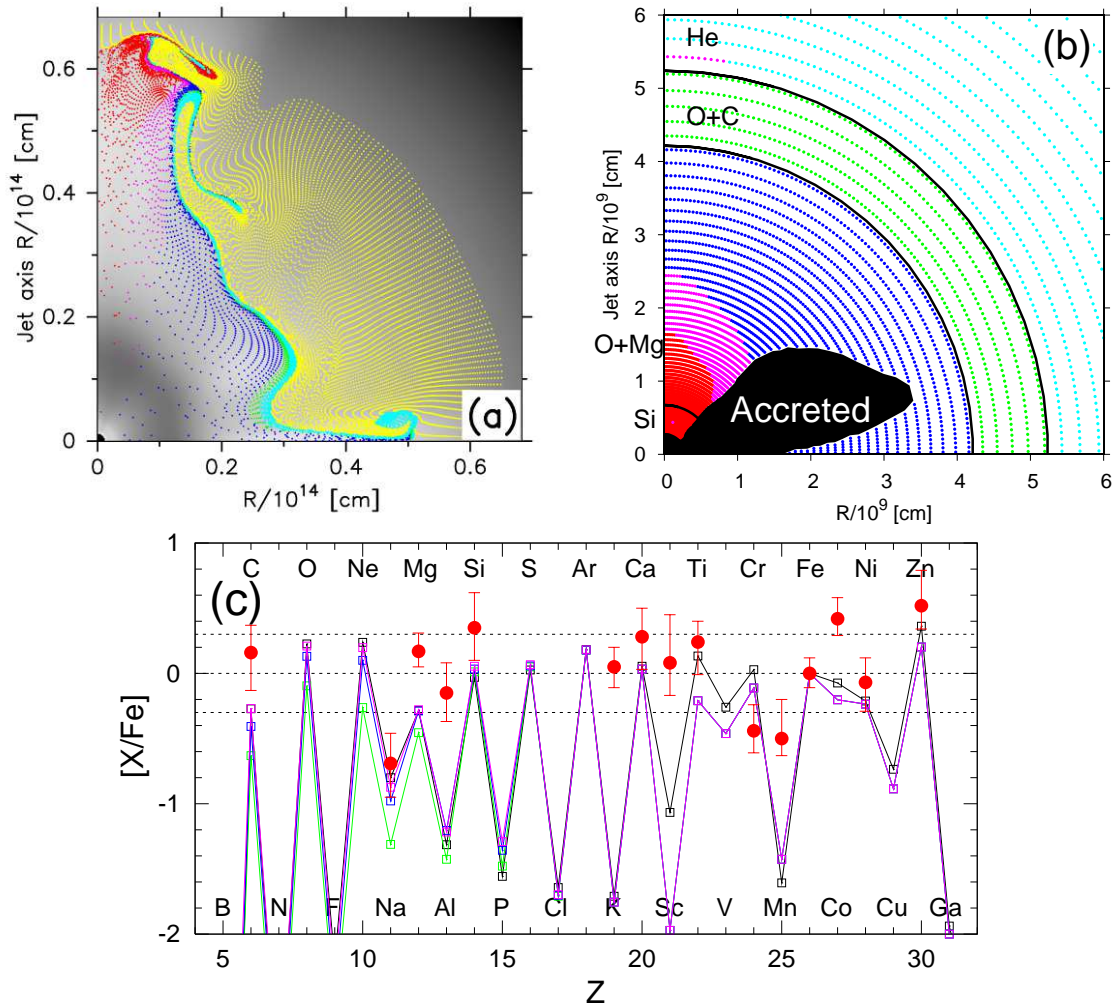


FIG. 6.— (a) Density structure (back ground gray scale) and the positions of the mass elements at $t = 10^5$ s for model C. The color and size of the marks represent the same as Figs. 4ab. (b) Initial locations of the mass elements which are finally accreted (black) and ejected (colored points) for model C. The color of the mass elements represents the same as Figs. 4ab. (c) Comparison of the abundance patterns of model C (black line), the EMP stars (red circles, Cayrel et al. 2004), and the mixing-fallback models with (1) $M_{\text{cut}}(\text{ini}) = 2.3M_{\odot}$, $M_{\text{mix}}(\text{out}) = 12.2M_{\odot}$ and $f = 0.41$ (green line), (2) $M_{\text{cut}}(\text{ini}) = 2.3M_{\odot}$, $M_{\text{mix}}(\text{out}) = 10.3M_{\odot}$ and $f = 0.27$ (blue line), and (3) $M_{\text{cut}}(\text{ini}) = 2.3M_{\odot}$, $M_{\text{mix}}(\text{out}) = 10.8M_{\odot}$ and $f = 0.19$ (magenta line).

In this subsection, I clarify the relation between the mixing-fallback model and the jet-induced explosion model by comparing the yields. The mixing-fallback model has three parameters; initial mass cut [$M_{\text{cut}}(\text{ini})$], outer boundary of the mixing region [$M_{\text{mix}}(\text{out})$], and a fraction of matter ejected from the mixing region (f). The remnant mass is written as

$$M_{\text{rem}} = M_{\text{cut}}(\text{ini}) + (1 - f)[M_{\text{mix}}(\text{out}) - M_{\text{cut}}(\text{ini})]. \quad (1)$$

The three parameters would relate to the hydrodynamical properties of the jet-induced explosion models, e.g., the inner boundary (M_0), the outer edge of the accreted region ($M_{\text{acc,out}}$), and the width between the edge of the accreted region and the jet axis.

I apply model C for the comparison. The inner boundary, the outer edge of the accreted region, and the central remnant mass of model C are $M_0 = 2.3M_{\odot}$, $M_{\text{acc,out}} = 12.2M_{\odot}$, and $M_{\text{rem}} = 8.1M_{\odot}$. The abundance distribution of model C at $t = 10^5$ s is shown in Figure 6a.

Figure 6b shows the initial positions of the ejected mass

elements in model C. Color of the marks represents the composition of the mass elements. Explosive nucleosynthesis takes place in the mass elements between the edge of the accreted region and the jet axis. Therefore, the amounts of the explosive nucleosynthesis products are reduced relative to the spherical explosions. The shock collision at the equatorial plane realizes the mass ejection from the deep O+Mg layer along the equatorial plane. Although the mass ejection could be due to the reflective boundary condition on the equatorial plane, this does not affect the following discussion because the amount of the mass ejection is small.

The angle-integrated yield of model C is compared with the yields of the spherical SN model with $M_{\text{ms}} = 40M_{\odot}$ and $E_{\text{dep}} = 3 \times 10^{52}$ ergs. For the spherical SN model, the explosion energy is deposited instantaneously as a thermal bomb. Here, I set $M_{\text{cut}}(\text{ini})$ to be the same as M_0 [i.e., $M_{\text{cut}}(\text{ini}) = 2.3M_{\odot}$] and apply three sets of parameters, $M_{\text{mix}}(\text{out})$ and f , as follows:

(1) $M_{\text{mix}}(\text{out}) = 12.2M_{\odot}$ and $f = 0.41$. $M_{\text{mix}}(\text{out})$ corresponds to $M_{\text{acc,out}}$ and f is the ratio of the ejected

mass and the total mass between M_0 and $M_{\text{acc,out}}$.

(2) $M_{\text{mix}}(\text{out}) = 10.3M_{\odot}$ and $f = 0.27$. f is the fraction of the solid angle of the region where the ejected mass elements located at $r \sim 10^3$ km (the Si-burning region, hereafter the fraction is written as f_{Si}). $M_{\text{mix}}(\text{out})$ is set to yield the same M_{rem} as model C.

(3) $M_{\text{mix}}(\text{out}) = 10.8M_{\odot}$ and $f = 0.19$. f and $M_{\text{mix}}(\text{out})$ are set to yield the most resemblant abundance pattern to model C. The resultant M_{rem} ($= 9.2M_{\odot}$) is slightly larger than M_{rem} of model C.

Figure 6c shows that the angle-integrated abundance pattern of model C is roughly reproduced by the mixing-fallback model with the parameter sets of (2) and (3). Thus, I conclude that the jet-induced explosion is approximated by the mixing-fallback model reasonably. M_0 , f_{Si} , and M_{rem} in the jet-induced explosion model are represented by $M_{\text{cut}}(\text{ini})$, f , and M_{rem} in the mixing-fallback model, respectively.

There are some elements showing differences, Sc, Ti, V, Cr, Co, and Zn. The enhancements of [Sc/Fe] and [Ti/Fe] improve agreements with the observations. The differences stem from the high-entropy explosion due to the concentration of the energy injection in the jet-induced explosion (e.g., Maeda & Nomoto 2003). In particular, Sc is the most sensitive element to the entropy and [Sc/Fe] is more enhanced for the deeper M_0 because of the weaker lateral expansion (Fig. 5a). Such thermodynamical features of the jet-induced explosion model cannot be reproduced by the mixing-fallback model exactly but a ‘‘low-density’’ modification might mimic the high-entropy environment (e.g., Umeda & Nomoto 2005; Tominaga et al. 2007b).

4. CONCLUSIONS AND DISCUSSION

I perform two-dimensional hydrodynamical and nucleosynthesis calculations of the jet-induced explosions of a Pop III $40M_{\odot}$ star. Here I test three jet-induced explosion models A, B, and C as summarized in Table 1 and conclude as follows.

(1) **Fallback:** The dynamics and the abundance distributions depend sensitively on the energy deposition rate \dot{E}_{dep} . The explosion with lower \dot{E}_{dep} leads to a larger amount of fallback, and consequently smaller $M(\text{Fe})$ and higher [C/O], [C/Mg], and [C/Fe]. Such dependences of [C/Fe] and $M(\text{Fe})$ on \dot{E}_{dep} predict that higher [C/Fe] tends to be realized for lower [Fe/H]. This is consistent with the observations (e.g., Beers & Christlieb 2005). Note, however, the formation of star with low [C/Fe] and [Fe/H] is possible because [Fe/H] depends on the swept-up H mass, i.e., the interaction between the SN ejecta and ISM (e.g., Cioffi et al. 1988).

(2) **Abundance distribution and angular dependence:** I present the aspherical abundance distributions and investigate the angular dependence of the yield. After the explosion, the elements ejected even along the equatorial plane locate in the all directions, while the elements ejected only along the jet axis expand laterally but tend to be distributed along the jet axis. The abundance distributions in the jet-induced SN ejecta could be examined by spatially-resolved observations of supernova remnants (e.g., Hwang et al. 2004; Keohane et al. 2007).

The angle-delimited yield could reproduce the extremely peculiar abundance pattern of HE 1424–0241. However, the angle-delimited yields of model A have a

large scatter that may be inconsistent with the relatively small scatter in the abundance ratios of the EMP stars. This implies that the angular dependence of the yield in most SNe is diluted by the strong mixing of the SN ejecta. On the other hand, the angle-delimited yields of model B show high [C/Fe] that is the same feature as the angle-integrated yield.

The angle-delimited yield strongly depends on which mass elements are included into the integration. This would be determined by the abundance mixing in the SN ejecta and by the region where the next-generation star takes in the metal-enriched gas. To investigate this issue further, it is required to calculate three-dimensional evolution of the supernova remnant in the ISM.

(3) **Comparison with the spherical explosion:** The angle-integrated yield of the jet-induced explosion is well reproduced by a spherical SN model applied the mixing-fallback model. This confirms that the mixing-fallback model approximates the jet-induced explosion well and that the mixing and fallback in hypernovae assumed in the mixing-fallback model are actually achieved in aspherical explosions. The abundance ratios between elements synthesized in different regions (e.g., C, O, Mg, and Fe) depend on the hydrodynamical structure of the explosion, e.g., the fallback. Thus, such macroscopic properties of the jet-induced explosion are represented by the mixing-fallback model. In particular, M_0 , f_{Si} , and M_{rem} in the jet-induced explosion model are represented by $M_{\text{cut}}(\text{ini})$, f , and M_{rem} in the mixing-fallback model, respectively.

On the other hand, the ratios between the explosively-synthesized elements depend on the thermodynamical properties of the explosion. In particular, [Sc/Fe], [Ti/Fe], [V/Fe], [Cr/Fe], [Co/Fe] and [Zn/Fe] are enhanced by the high-entropy environment in the jet-induced explosion, thus showing differences from the mixing-fallback model. The enhancement of [Sc/Fe] and [Ti/Fe] improve the agreement with the observations. Note, the enhancement of [Ti/Fe] relative to the spherical SN model in this paper is larger than in Maeda & Nomoto (2003). This might be because the relativistic jet suppresses the lateral expansion and enhances the energy concentration.

The enhancement of [Sc/Fe] is also suggested to be obtained by nucleosynthesis in the p -rich ejecta expelled from the innermost region (Pruet et al. 2004, 2005; Fröhlich et al. 2006; Iwamoto et al. 2006). However, it seems difficult to enhance [Zn/Fe] (~ 0.5) and [Sc/Fe] (~ 0) only with nucleosynthesis in the p -rich ejecta (see § 5.2 in Tominaga et al. 2007b). On the other hand, a jet-induced explosion enhances [Sc/Fe], [Ti/Fe], and [Zn/Fe] simultaneously and realizes the mixing and fallback in an energetic explosion. Therefore, I propose that the jet-induced explosion is a presumable origin of the enhancement of [Sc/Fe]. The origin would be concluded by future quantitative investigations including various elements.

Although the models with same θ_{jet} are shown in this paper, the asphericity is likely to be different for each SN. For example, the observations of supernova remnants show various aspherical structures (e.g., Cassiopeia A, Hwang et al. 2004; Fesen et al. 2006; and W49B, Miceli et al. 2006; Keohane et al. 2007). Furthermore, recent observations of the nebular spectra of SNe sug-

gested that all core-collapse SNe are aspherical explosions (Maeda et al. 2008; Modjaz et al. 2008). The degree of asphericity in SNe in the early universe will be revealed by detailed comparisons of abundance ratios between the aspherical SN models with different θ_{jet} and the metal-poor stars which represent the hydrodynamical and/or thermodynamical properties.

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APPENDIX

SPECIAL RELATIVISTIC HYDRODYNAMIC CODE

A two-dimensional special relativistic Eulerian hydrodynamic code is developed with Marquina's flux formula (Donat et al. 1998) and with a conversion method from the observer frame to the proper frame (Martí & Müller 1996). The code applies third order Runge-Kutta method in time of Shu & Osher (1988, see also Aloy et al. 1999) and second order piecewise hyperbolic method (PHM) in space of Marquina (1994).

The equations of special relativistic hydrodynamics are described in terms of a four-velocity vector field and an energy momentum tensor (Martí & Müller 1994). Physical quantities in a rest frame (relativistic rest-mass density: D , the i -th components of momentum densities: S_i , and energy density: τ) and a comoving frame are related as follows:

$$\begin{aligned} D &= \rho\Gamma, \\ S_i &= \rho h\Gamma^2 v_i, \\ \tau &= \rho h\Gamma^2 - p - \rho\Gamma, \end{aligned}$$

where the light velocity is set to $c = 1$, ρ is the proper rest-mass density, v_i are the i -th components of velocities, $\Gamma = 1/\sqrt{1 - \sum_i v_i^2}$ is the Lorentz factor of the fluid element with respect to the rest frame, p is the proper pressure, and $h = 1 + e_{\text{pr,in}}/\rho + p/\rho$ is the specific enthalpy (here, $e_{\text{pr,in}}$ is the proper internal energy per unit volume).

The basic equations in the special relativistic hydrodynamics are written in spherical polar coordinates (r, θ) as

$$\frac{\partial D}{\partial t} + \frac{\partial(r^2 D v_r)}{\partial(r^3/3)} + \frac{1}{r} \frac{\partial(\sin\theta D v_\theta)}{\partial(-\cos\theta)} = 0, \quad (\text{A1})$$

$$\frac{\partial S_r}{\partial t} + \frac{\partial(S_r v_r + p)}{\partial r} + \frac{1}{r} \frac{\partial(\sin\theta S_r v_\theta)}{\partial(-\cos\theta)} = -2 \frac{S_r v_r}{r} + \frac{S_\theta v_\theta}{r} + g_r \rho, \quad (\text{A2})$$

$$\frac{\partial S_\theta}{\partial t} + \frac{\partial(r^2 S_\theta v_r)}{\partial(r^3/3)} + \frac{1}{r} \frac{\partial(S_\theta v_\theta + p)}{\partial\theta} = -\frac{1}{r} \frac{\cos\theta}{\sin\theta} - \frac{S_\theta v_r}{r} + g_\theta \rho, \quad (\text{A3})$$

$$\frac{\partial \tau}{\partial t} + \frac{\partial\{r^2(S_r - D v_r)\}}{\partial(r^3/3)} + \frac{1}{r} \frac{\partial\{\sin\theta(S_\theta - D v_\theta)\}}{\partial(-\cos\theta)} = (g_r v_r + g_\theta v_\theta)\rho, \quad (\text{A4})$$

where g_i ($i = r, \theta$) are the i -th gravitational acceleration components. The equations are an equation of continuity, momentum conservation equations, an energy conservation equation. The gravitational potential includes the contributions of the self-gravity and the central remnant. I test two methods to include the self-gravity as follows: (1) the Poisson equation is approximated with an integration of spherical harmonics (Hachisu 1986) and (2) the sparse banded matrix resulting from differencing the Poisson equation is solved with a bi-conjugate gradient stabilized (BiCGSTAB) method (Barrett et al. 1994)⁷ with modified incomplete LU (MILU) factorization. Since the results are consistent, I apply the former method in the calculations.

One-dimensional Riemann problems

The code is tested with one-dimensional shock tube problems (e.g., Martí & Müller 1994). I set the computational region $0 \leq x \leq 1$ where $c = 1$ that is divided into two regions, i.e., left and right regions, at $x = 0.5$. The density (ρ), pressure (p), and velocity (v) of left and right regions are represented by L and R subscripts, respectively. The time variation is derived by the analytical solution of the Riemann problem (Martí & Müller 1994). I test three problems with 10^3 uniform meshes ($\Delta x = 1/10^3$) and a constant adiabatic index. The initial states are as follows:

Problem (a) $\gamma = 5/3$

$$\begin{aligned} (\rho_L, p_L, v_L) &= (1, 10^3, 0) \text{ at } 0.0 \leq x \leq 0.5, \\ (\rho_R, p_R, v_R) &= (1, 0.01, 0) \text{ at } 0.5 \leq x \leq 1.0. \end{aligned}$$

Problem (b) $\gamma = 4/3$

$$\begin{aligned} (\rho_L, p_L, \Gamma_L) &= (1, 1, 10^3) \text{ at } 0.0 \leq x \leq 0.5, \\ (\rho_R, p_R, \Gamma_R) &= (1, 10^6, 1) \text{ at } 0.5 \leq x \leq 1.0. \end{aligned}$$

⁷ Subroutines are available at Netlib (<http://www.netlib.org/>).

Problem (c) $\gamma = 4/3$

$$(\rho_L, p_L, v_L) = (1, 10, -0.9) \text{ at } 0.0 \leq x \leq 0.5,$$

$$(\rho_R, p_R, v_R) = (10, 100, 0.9) \text{ at } 0.5 \leq x \leq 1.0.$$

The analytical solutions are reproduced by the calculations (Figs. A1abc). The agreements confirm that the code correctly solves left- and right-oriented rarefaction and shock waves.

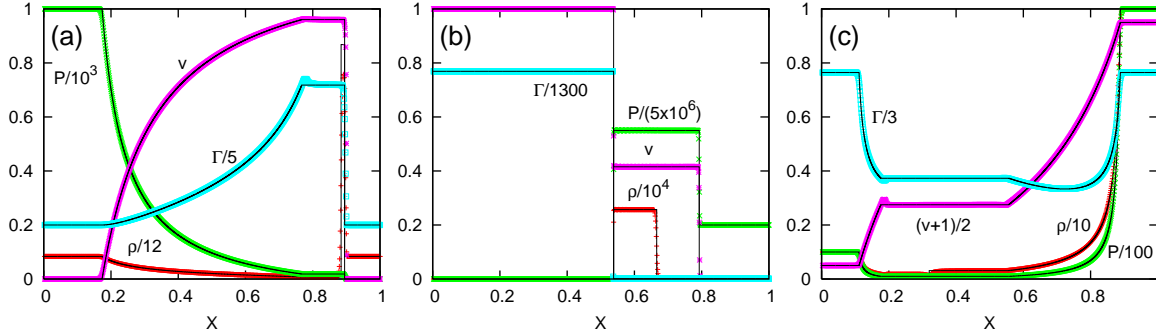


FIG. A1.— Comparison between the analytical solutions of the Riemann problems (*black solid line*) and the results of the special relativistic hydrodynamics calculations (density: *red marks*, pressure: *green marks*, velocity: *magenta marks*, Lorentz factor: *cyan marks*).

Two-dimensional shock tube problem

The two-dimensional shock tube problem was proposed by Del Zanna & Bucciantini (2002) and repeated by subsequent studies (e.g., Lucas-Serrano et al. 2004; Zhang & MacFadyen 2006; Mizuta et al. 2006). I test the problem with $10^3 \times 10^3$ square uniform meshes. In this problem, the two-dimensional region ($0 \leq x \leq 1, 0 \leq y \leq 1$) are divided into the following four regions:

$$(\rho, p, v_x, v_y) = (0.50, 1.00, 0.00, 0.00) \text{ at } 0.0 \leq x \leq 0.5, 0.0 \leq y \leq 0.5,$$

$$(\rho, p, v_x, v_y) = (0.10, 1.00, 0.99, 0.00) \text{ at } 0.0 \leq x \leq 0.5, 0.5 \leq y \leq 1.0,$$

$$(\rho, p, v_x, v_y) = (0.10, 0.01, 0.00, 0.00) \text{ at } 0.5 \leq x \leq 1.0, 0.0 \leq y \leq 0.5,$$

$$(\rho, p, v_x, v_y) = (0.10, 1.00, 0.00, 0.99) \text{ at } 0.5 \leq x \leq 1.0, 0.5 \leq y \leq 1.0,$$

where v_i ($i = x, y$) are the i -th components of velocities and the adiabatic index is $\gamma = 5/3$. Figure A2 shows color contours of $\log(\rho)$ at $t = 0.4$.

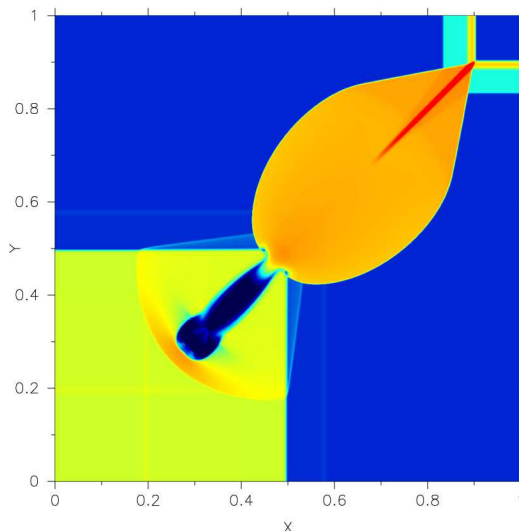


FIG. A2.— Two-dimensional shock tube problem at $t = 0.4$ with $10^3 \times 10^3$ square uniform meshes. Color contours of the logarithm of the rest mass density are plotted.

Double Mach reflection problem

Woodward & Colella (1984) introduced a double Mach reflection problem of a strong shock in the Newtonian case and the problem was extended to the special relativistic case (Zhang & MacFadyen 2006). I test the code with the same initial conditions as in the previous studies (Zhang & MacFadyen 2006; Mizuta et al. 2006). The computational region is a $0 \leq x \leq 4, 0 \leq y \leq 1$ rectangle captured by 512×128 uniform square meshes. The density and pressure of the unshocked gas, the adiabatic index, and the classical Mach number ($M_c \equiv V_S/c_s$, where V_S is a shock velocity and c_s is a sound speed) are set to be $\rho = 1.4$, $p = 0.0025$, $\gamma = 1.4$, and $M_c = 10$ respectively. Thus, the shock velocity is $V_S = 0.4984$ and the density, pressure, and velocity of the post-shock gas are $\rho = 8.564$, $p = 0.3804$, and $v = 0.4247$. The shock front is initially set to cross the x axis at $x = 1/6$ with an angle of 60° . The initial conditions are as follows:

$$\begin{aligned} (\rho, p, v_x, v_y) &= (1.4, 0.0025, 0.00, 0.00) \text{ at } x - y \tan 60^\circ < 1/6, \\ (\rho, p, v_x, v_y) &= (8.564, 0.3804, 0.4247 \sin 60^\circ, 0.4247 \cos 60^\circ) \text{ at } x - y \tan 60^\circ > 1/6. \end{aligned}$$

The boundary conditions are inflow of the post-shock gas at $x = 0$, at $0 \leq x \leq 1/6$ on $y = 0$ axis, and at $0 \leq x \leq (1/6 + \tan 60^\circ + V_S t / \sin 60^\circ)$ on $y = 1$ axis, reflective boundaries at $1/6 < x < 4$ on $y = 0$ axis, and freely inflow and outflow conditions at the other boundaries. Figure A3 shows thirty-level iso-surfaces of $\log(\rho)$ at $t = 4.0$.

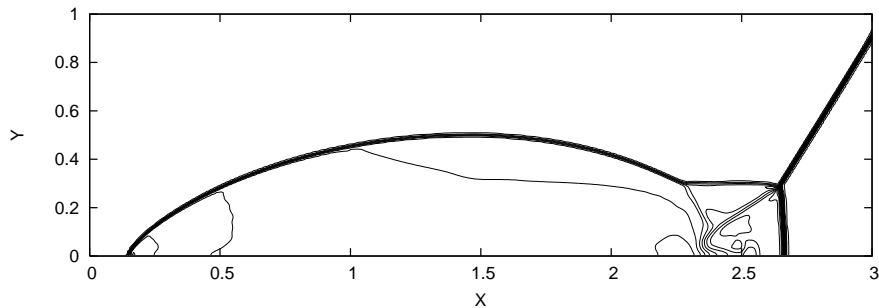


FIG. A3.— Double-Mach reflection problem at $t = 4.0$ with 512×128 uniform square meshes. Thirty equally-spaced contours of the logarithm of the rest mass density are plotted.

Emery step problem

A wind tunnel problem with a reflecting step, called the Emery step problem, was introduced by Emery (1968) and repeated by subsequent studies (Zhang & MacFadyen 2006; Mizuta et al. 2006). The computational domain is a $0 \leq x \leq 3, 0 \leq y \leq 1$ rectangle captured by 240×80 uniform square meshes. The domain is filled by a gas with $\rho = 1.4$, $p = 0.1534$, and $v = 0.999$. The pressure is set so that $M_c = 3$. The wind tunnel has the reflecting step locating at $0.6 \leq x$ and $y \leq 0.2$. The following boundary conditions are applied: the upper and lower boundaries are reflective and the boundaries at $x = 0$ and $x = 3$ are open. Figure A4 shows thirty-level iso-surfaces of $\log(\rho)$ at $t = 4.0$.

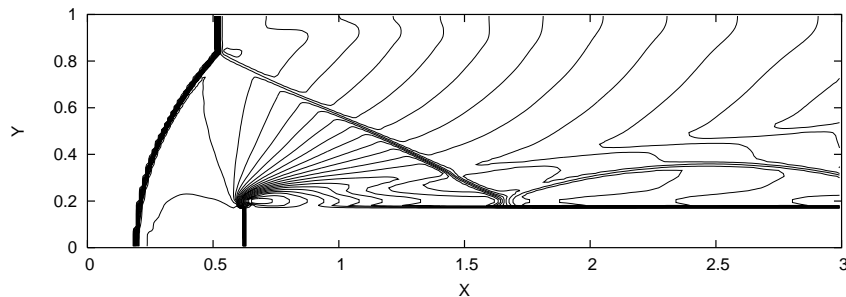


FIG. A4.— Emery step problem at $t = 4.0$ with 240×80 uniform square meshes. Thirty equally-spaced contours of the logarithm of the rest mass density are plotted.

NUCLEOSYNTHESIS CALCULATION

The two-dimensional special relativistic hydrodynamical calculation does not include nuclear energy releases and applies a constant adiabatic index $\gamma = 4/3$. The proper internal energy is written as $e_{\text{pr,in}} = p/(\gamma - 1)$. Temperature T is derived with an analytical equation of state including radiation and e^-e^+ pair (e.g., Freiburghaus et al. 1999) as follows:

$$e_{\text{pr,in}} = aT^4 \left\{ 1 + \frac{7}{4} \cdot \frac{T_9^2}{T_9^2 + 5.3} \right\}. \quad (\text{B1})$$

where $a = 7.57 \times 10^{-15}$ ergs cm^{-3} K^{-4} is the radiation-density constant and $T_9 = T/10^9$ K.

I calculate explosive nucleosynthesis in a non-relativistic spherical explosion of a $40M_{\odot}$ star with 3×10^{52} ergs. The post-processing calculations are performed with the thermodynamic histories obtained by the two-dimensional special relativistic Eulerian hydrodynamics code and a one-dimensional Lagrangian hydrodynamics code used in Umeda & Nomoto (2002, 2005) and Tominaga et al. (2007b). The one-dimensional Lagrangian hydrodynamical calculation includes nuclear energy releases from the α -network and the equation of state includes the gas, radiation, e^-e^+ pair (Sugimoto & Nomoto 1975), Coulomb interactions between ions and electrons, and phase transition (Nomoto et al. 1982; Nomoto & Hashimoto 1988). The abundance distributions derived by both calculations are consistent (Figs. B1ab). This consistency justifies the results of the two-dimensional special relativistic hydrodynamics and nucleosynthesis calculations even with the simple assumptions.

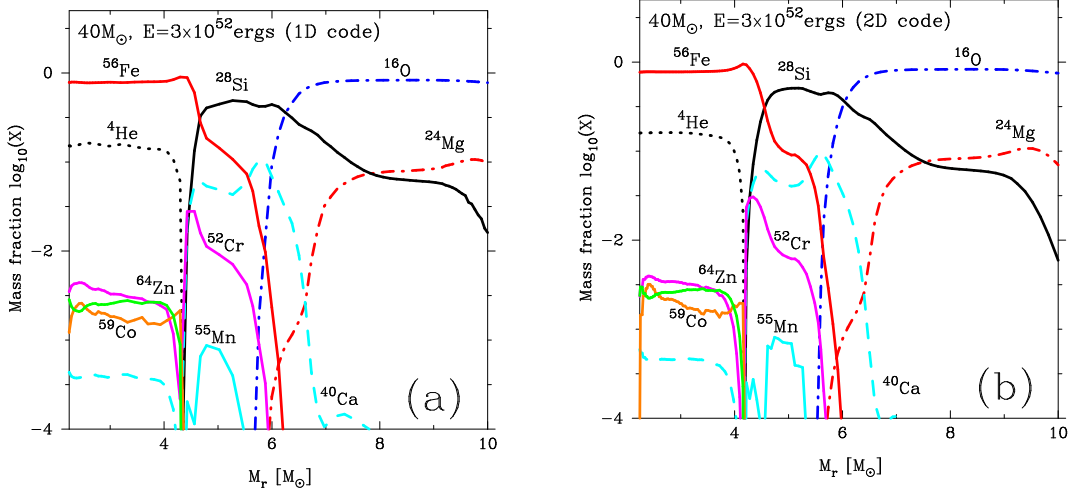


FIG. B1.— Abundance distributions after the non-relativistic spherical explosion of a $40M_{\odot}$ star with 3×10^{52} ergs. The post-processing calculations are performed with the thermodynamic histories obtained by (a) the one-dimensional spherical hydrodynamics code and (b) the two-dimensional special relativistic hydrodynamical code.

JET INJECTION

The jet characterized by the five parameters, \dot{E}_{dep} , E_{dep} , θ_{jet} , Γ_{jet} , and f_{th} , is injected from the inner boundary at R_0 . The jet injection is performed by putting the velocities ($v_{r,\text{jet}}$, $v_{\theta,\text{jet}}$), pressure (p_{jet}), and density (ρ_{jet}) of the jet at the inner boundary as the boundary condition. $v_{r,\text{jet}}$, $v_{\theta,\text{jet}}$, p_{jet} , and ρ_{jet} are derived from the five parameters.

The superficial area of the jet injection is $A_{\text{jet}} = 4\pi R_0^2(1 - \cos\theta_{\text{jet}})$ and thus the energy deposition rate is $\dot{E}_{\text{dep}} = A_{\text{jet}}f_{e,\text{jet}}$, where $f_{e,\text{jet}}$ is an energy density flux of the jet. $f_{e,\text{jet}}$ is written with the jet properties as $f_{e,\text{jet}} = S_{r,\text{jet}} - D_{\text{jet}}v_{r,\text{jet}}$, where $S_{r,\text{jet}} = \rho_{\text{jet}}h_{\text{jet}}\Gamma_{\text{jet}}^2v_{r,\text{jet}}$, $D_{\text{jet}} = \rho_{\text{jet}}\Gamma_{\text{jet}}$, and h_{jet} is the specific enthalpy of the jet. Although the energy density flux at the inner boundary (f_e) depends on the properties of the jet and the infalling matter and changes with time, f_e is equivalent to $f_{e,\text{jet}}$ when the jet is injected freely.

On the other hand, the energy density per unit volume is $e_{\text{jet}} = \tau_{\text{jet}} = \rho_{\text{jet}}h_{\text{jet}}\Gamma_{\text{jet}}^2 - p_{\text{jet}} - \rho_{\text{jet}}\Gamma_{\text{jet}}$. The energy density consists of the thermal energy density ($e_{\text{th,jet}}$) and the kinetic energy density ($e_{\text{kin,jet}}$). Applying $h_{\text{jet}} = 1 + e_{\text{pr,in,jet}}/\rho_{\text{jet}} + p_{\text{jet}}/\rho_{\text{jet}}$ and $e_{\text{pr,in,jet}} = p_{\text{jet}}/(\gamma - 1)$, $e_{\text{th,jet}}$ and $e_{\text{kin,jet}}$ are written by the density ρ_{jet} , pressure p_{jet} , Lorentz factor Γ_{jet} , and adiabatic index γ of the jet as follows:

$$e_{\text{kin,jet}} = (1 - f_{\text{th}})e_{\text{jet}} = \rho_{\text{jet}}(\Gamma_{\text{jet}}^2 - \Gamma_{\text{jet}}), \quad (\text{C1})$$

$$e_{\text{th,jet}} = f_{\text{th}}e_{\text{jet}} = p_{\text{jet}}\left(\frac{\gamma}{\gamma - 1}\Gamma_{\text{jet}}^2 - 1\right). \quad (\text{C2})$$

Since $e_{\text{jet}} = f_{e,\text{jet}}/v_{r,\text{jet}} - p_{\text{jet}}$, the velocities, pressure, and density of the jet are written as follows:

$$v_r = \sqrt{1 - 1/\Gamma_{\text{jet}}^2}, \quad (\text{C3})$$

$$v_{\theta} = 0, \quad (\text{C4})$$

$$p_{\text{jet}} = \frac{f_{\text{th}}\dot{E}_{\text{dep}}}{v_r A_{\text{jet}}\left(\frac{\gamma}{\gamma - 1}\Gamma_{\text{jet}}^2 - 1 + f_{\text{th}}\right)}, \quad (\text{C5})$$

$$\rho_{\text{jet}} = \frac{(1 - f_{\text{th}})\left(\frac{\dot{E}_{\text{dep}}}{v_r A_{\text{jet}}} - p_{\text{jet}}\right)}{\Gamma_{\text{jet}}^2 - \Gamma_{\text{jet}}}. \quad (\text{C6})$$

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