

Fast Estimator of Primordial Non-Gaussianity from Temperature and Polarization Anisotropies in the Cosmic Microwave Background II: Partial Sky Coverage and Inhomogeneous Noise

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ABSTRACT

In our recent paper (Yadav et al. 2007) we described a fast cubic (bispectrum) estimator of the amplitude of primordial non-Gaussianity of local type, f_{NL} , from a combined analysis of the Cosmic Microwave Background (CMB) temperature and E-polarization observations. In this paper we generalize the estimator to deal with a partial sky coverage as well as inhomogeneous noise. Our generalized estimator is still computationally efficient, scaling as $O(N_{pix}^{3/2})$ compared to the $O(N_{pix}^{5/2})$ scaling of the brute force bispectrum calculation for sky maps with N_{pix} pixels. Upcoming CMB experiments are expected to yield high-sensitivity temperature and E-polarization data. Our generalized estimator will allow us to optimally utilize the combined CMB temperature and E-polarization information from these realistic experiments, and to constrain primordial non-Gaussianity.

Subject headings: cosmic microwave background, early universe, inflation, non-Gaussianity.

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1. Introduction

Non-Gaussianity from the simplest inflation models, that are based on a slowly rolling scalar field, is very small (Salopek & Bond 1990, 1991; Falk et al. 1993; Gangui et al. 1994; Acquaviva et al. 2003; Maldacena 2003); however, a very large class of more general models, e.g., models with multiple scalar fields, features in inflation potential, non-adiabatic fluctuations, non-canonical kinetic terms, deviations from the Bunch-Davies vacuum, among others, predict substantially higher level of primordial non-Gaussianity (Bartolo et al. 2004, for a review and references therein).

Primordial non-Gaussianity can be described in terms of the 3-point correlation function of Bardeen’s curvature perturbations, $\Phi(k)$, in Fourier space:

$$\langle \Phi(\mathbf{k}_1)\Phi(\mathbf{k}_2)\Phi(\mathbf{k}_3) \rangle = (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) F(k_1, k_2, k_3). \quad (1)$$

Depending on the shape of the 3-point function, i.e., $F(k_1, k_2, k_3)$, non-Gaussianity can be broadly classified into two classes (Babich et al. 2004). First, the local, “squeezed,” non-Gaussianity where $F(k_1, k_2, k_3)$ is large for the configurations in which $k_1 \ll k_2, k_3$. Second, the non-local, “equilateral,” non-Gaussianity where $F(k_1, k_2, k_3)$ is large for the configuration when $k_1 \sim k_2 \sim k_3$.

The local form arises from a non-linear relation between inflaton and curvature perturbations (Salopek & Bond 1990, 1991; Gangui et al. 1994), curvaton models (Lyth et al. 2003), or the New Ekpyrotic models (Koyama et al. 2007; Buchbinder et al. 2007). The equilateral form arises from non-canonical kinetic terms such as the Dirac-Born-Infeld (DBI) action (Alishahiha et al. 2004), the ghost condensation (Arkani-Hamed et al. 2004), or any other single-field models in which the scalar field acquires a low speed of sound (Chen et al. 2007; Cheung et al. 2007). While we focus on the local form in this paper, it is straightforward to repeat our analysis for the equilateral form.

The local form of non-Gaussianity may be parametrized in real space as (Gangui et al. 1994; Verde et al. 2000; Komatsu & Spergel 2001):

$$\Phi(\mathbf{r}) = \Phi_L(\mathbf{r}) + f_{NL} (\Phi_L^2(\mathbf{r}) - \langle \Phi_L^2(\mathbf{r}) \rangle) \quad (2)$$

where f_{NL} characterizes the amplitude of primordial non-Gaussianity. Different inflationary models predict different amounts of f_{NL} , starting from $O(1)$ to $f_{NL} \sim 100$, beyond which values have been excluded by the Cosmic Microwave Background (CMB) bispectrum of WMAP temperature data, $-36 < f_{NL} < 100$, at the 2σ level (Komatsu et al. 2003; Creminelli et al. 2006b; Spergel et al. 2006).

So far all the constraints on primordial non-Gaussianity use only temperature information of the CMB. By also having the E-polarization information together with CMB

temperature information, one can improve the sensitivity to the primordial fluctuations (Babich & Zaldarriaga 2004; Yadav & Wandelt 2005; Yadav et al. 2007). Although the experiments have already started characterizing E-polarization anisotropies (Kovac et al. 2002; Kogut et al. 2003; Page et al. 2007; Montroy et al. 2006), the errors are large in comparison to temperature anisotropy. The upcoming experiments such as Planck satellite will characterize E-polarization anisotropy to a higher accuracy. It is very timely to develop the tools which can optimally utilize the combined CMB temperature and E-polarization information to constrain models of the early universe.

Throughout this paper we use the standard Lambda CDM cosmology with the following cosmological parameters: $\Omega_b = 0.042$, $\Omega_{cdm} = 0.239$, $\Omega_L = 0.719$, $h = 0.73$, $n = 1$, and $\tau = 0.09$. For all of our simulations we used HEALPix maps with $N_{pix} \approx 3 \times 10^6$ pixels.

1.1. Generalized Bispectrum Estimator of Primordial Non-Gaussianity

In our recent paper (Yadav et al. 2007) we described a fast cubic (bispectrum) estimator of f_{NL} , using a combined analysis of the temperature and E-polarization observations. The estimator was optimal for homogeneous noise, where optimality was defined by saturation of the Fisher matrix bound.

In this paper we generalize our previous estimator of f_{NL} to deal more optimally with a partial sky coverage and the inhomogeneous noise. The generalization is done in an analogous way to how Creminelli et al. (2006a) generalized the temperature only estimator developed by Komatsu et al. (2005); however, the final result of Creminelli et al. (2006a) (their Eq. (30)) is off by a factor of two, which results in the error in f_{NL} that is much larger than the Fisher matrix prediction, as we shall show below.

The fast bispectrum estimator of f_{NL} from the combined CMB temperature and E-polarization data can be written as $\hat{f}_{NL} = \frac{\hat{S}_{prim}}{N}$, where (Yadav et al. 2007)

$$\hat{S}_{prim} = \frac{1}{f_{sky}} \int r^2 dr \int d^2\hat{n} B^2(\hat{n}, r) A(\hat{n}, r), \quad (3)$$

$$N = \sum_{ijkpqr} \sum_{2 \leq \ell_1 \leq \ell_2 \leq \ell_3} \frac{1}{\Delta_{\ell_1 \ell_2 \ell_3}} B_{\ell_1 \ell_2 \ell_3}^{pqr, prim} (C^{-1})_{\ell_1}^{ip} (C^{-1})_{\ell_2}^{jq} (C^{-1})_{\ell_3}^{kr} B_{\ell_1 \ell_2 \ell_3}^{ijk, prim}, \quad (4)$$

$$B(\hat{n}, r) \equiv \sum_{ip} \sum_{lm} (C^{-1})^{ip} a_{\ell m}^i \beta_{\ell}^p(r) Y_{\ell m}(\hat{n}), \quad (5)$$

$$A(\hat{n}, r) \equiv \sum_{ip} \sum_{lm} (C^{-1})^{ip} a_{\ell m}^i \alpha_\ell^p(r) Y_{\ell m}(\hat{n}), \quad (6)$$

$$\beta_\ell^i(r) = \frac{2b_\ell^i}{\pi} \int k^2 dk P_\phi(k) g_\ell^i(k) j_\ell(kr), \quad (7)$$

$$\alpha_\ell^i(r) = \frac{2b_\ell^i}{\pi} \int k^2 dk g_\ell^i(k) j_\ell(kr), \quad (8)$$

and f_{sky} is a fraction of the sky observed. Indices i, j, k, p, q and r can either be T or E . Here, $\Delta_{\ell_1 \ell_2 \ell_3}$ is 1 when $\ell_1 \neq \ell_2 \neq \ell_3$, 6 when $\ell_1 = \ell_2 = \ell_3$, and 2 otherwise, $B_{\ell_1 \ell_2 \ell_3}^{pqr, prim}$ is the theoretical bispectrum for $f_{NL} = 1$ (Yadav et al. 2007), $P_\Phi(k)$ is the power spectrum of the primordial curvature perturbations, and $g_\ell^i(r)$ is the radiation transfer function of adiabatic perturbations.

It has been shown that the above mentioned estimator is optimal for the full sky coverage and homogeneous noise (Yadav et al. 2007). To be able to deal with the realistic data, the estimator has to be able to deal with the inhomogeneous noise and foreground masks.

The estimator can be generalized to deal with a partial sky coverage and the inhomogeneous noise by adding a linear term to \hat{S}_{prim} : $\hat{S}_{prim} \rightarrow \hat{S}_{prim} + \hat{S}_{prim}^{linear}$. For the temperature only case, this has been done in Creminelli et al. (2006a). Following the same argument, we find that the linear term for the combined analysis of CMB temperature and polarization data is given by

$$\hat{S}_{prim}^{linear} = \frac{-1}{f_{sky}} \int r^2 dr \int d^2 \hat{n} \{ 2B(\hat{n}, r) \langle A_{sim}(\hat{n}, r) B_{sim}(\hat{n}, r) \rangle_{MC} + A(\hat{n}, r) \langle B_{sim}^2(\hat{n}, r) \rangle_{MC} \}, \quad (9)$$

where $A_{sim}(\hat{n}, r)$ and $B_{sim}(\hat{n}, r)$ are the A and B maps generated from Monte Carlo simulations that contain signal and noise, and $\langle \dots \rangle$ denotes the average over the Monte Carlo simulations.

The generalized estimator is given by

$$\hat{f}_{NL} = \frac{\hat{S}_{prim} + \hat{S}_{prim}^{linear}}{N}, \quad (10)$$

which is the main result of this paper. Note that $\langle \hat{S}_{prim}^{linear} \rangle_{MC} = -\langle \hat{S}_{prim} \rangle_{MC}$, and this relation also holds for the equilateral shape. Therefore, it is straightforward to find the generalized estimator for the equilateral shape: first, find the cubic estimator of the equilateral shape,

$\hat{S}_{equilateral}$, and take the Monte Carlo average, $\langle \hat{S}_{equilateral} \rangle_{MC}$. Let us suppose that $\hat{S}_{equilateral}$ contains terms in the form of ABC , where A , B , and C are some filtered maps. Use the Wick's theorem to re-write the average of a cubic product as $\langle ABC \rangle_{MC} = \langle A \rangle_{MC} \langle BC \rangle_{MC} + \langle B \rangle_{MC} \langle AC \rangle_{MC} + \langle C \rangle_{MC} \langle AB \rangle_{MC}$. Finally, remove the MC average from single maps, and replace maps in the product with the simulated maps: $\langle A \rangle_{MC} \langle BC \rangle_{MC} + \langle B \rangle_{MC} \langle AC \rangle_{MC} + \langle C \rangle_{MC} \langle AB \rangle_{MC} \rightarrow A \langle B_{sim} C_{sim} \rangle_{MC} + B \langle A_{sim} C_{sim} \rangle_{MC} + C \langle A_{sim} B_{sim} \rangle_{MC}$. This operation gives the correct expression for the linear term, both for the local form and the equilateral form.

One can find the estimator of f_{NL} from the temperature data only by setting $i = j = k = p = q = r \equiv T$. We have compared our formula in the temperature-only limit with the original formula derived by Creminelli et al. (2006a) (their Eq. (30)), and found a discrepancy. To see the discrepancy, let us re-write the estimator as: $\hat{f}_{NL} = \frac{\hat{S}_{prim} + x \hat{S}_{prim}^{linear}}{N}$. Our formula gives $x = 1$, while Eq. (30) of Creminelli et al. (2006a) gives $x = 0.5$.¹

To make sure that our normalization gives the minimum variance estimator, we have done Monte Carlo simulations with varying x . We find that $x = 1$ minimizes the variance (as shown in Fig. 1). We conclude that the analysis given in Creminelli et al. (2006a) resulted in the larger-than-expected uncertainty in f_{NL} because of this error in their normalization of the linear term.

The main contribution to the linear term comes from the inhomogeneous noise and sky cut. For the temperature only case, most of the contribution to the linear term comes from the inhomogeneous noise, and the partial sky coverage does not contribute much to the linear term. This is because the sky-cut induces a monopole contribution outside the mask. In the analysis one subtracts the monopole from outside the mask before measuring \hat{S}_{prim} , which makes the linear contribution from the mask small (Creminelli et al. 2006a). For a combined analysis of the temperature and polarization maps, however, the linear term does get a significant contribution from a partial sky coverage (see the right panel of Fig. 2). Subtraction of the monopole outside of the mask is of no help for polarization, as the monopole does not exist in the polarization maps by definition. (The lowest relevant multipole for polarization is $l = 2$.)

The estimator is still computationally efficient, taking only $N_{pix}^{3/2}$ (times the r sampling, which is of order 100) operations in comparison to the full bispectrum calculation which takes $N_{pix}^{5/2}$ operations. Here N_{pix} refers to the total number of pixels. For Planck, $N_{pix} \sim 5 \times 10^7$,

¹Equation (30) in Creminelli et al. (2006a) is off by a factor of 2. Since we used $\sum_{\ell_1 \ell_2 \ell_3} = 6 \sum_{\ell_1 \leq \ell_2 \leq \ell_3} \frac{1}{\Delta_{\ell_1 \ell_2 \ell_3}}$, to compare our normalization factor x with Creminelli et al. (2006a) one needs to divide the normalization in Creminelli et al. (2006a) by a factor of 6.

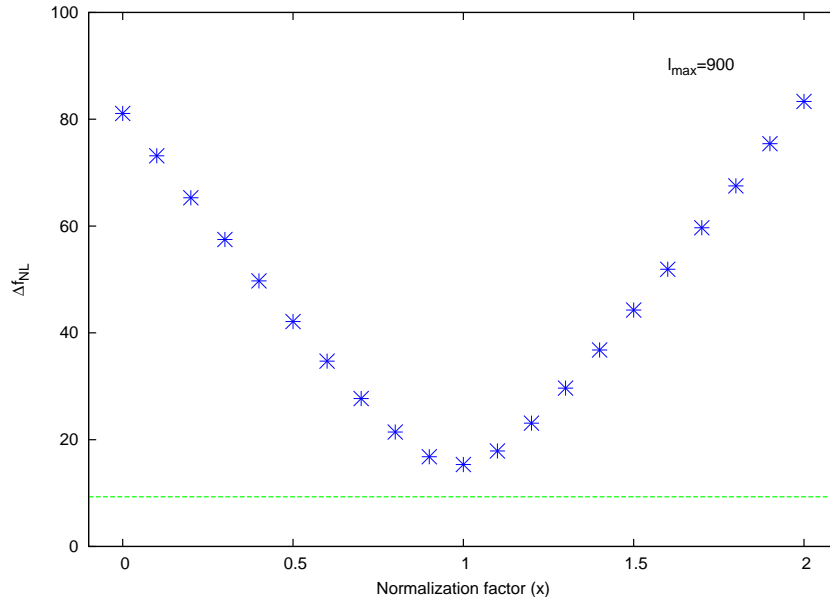


Fig. 1.— Testing normalization of the linear term in the estimator of f_{NL} . The symbols show the standard deviation of f_{NL} derived from the Monte Carlo simulations using the estimator for a given normalization, x . The horizontal line shows the Fisher matrix prediction. Our formula gives $x = 1$, while Creminelli et al. (2006a) give $x = 0.5$ (their Eq. (30)). We have used simulated polarized Gaussian CMB maps with the Planck inhomogeneous noise as well as the WMAP Kp0 and P06 mask for temperature and polarization, respectively.

and so the full bispectrum analysis is not feasible while our analysis is.

2. Results

In the left panel of figure 2, we show the variance of f_{NL} using the estimator (with and without the linear term) for the Gaussian CMB simulations in the presence of inhomogeneous noise and partial sky coverage. For this analysis we use the noise properties that are expected for the Planck satellite, assuming the cycloidal scanning strategy (Dupac & Tauber 2005). Inhomogeneous nature of the noise is depicted in the lower map of figure 4 where we show the number of observations (N_{obs}) for the different pixels in the sky. As for the foreground masks, we use WMAP Kp0 intensity mask and P06 polarization mask. We find that, with the inclusion of the linear term, the variance reduces by more than a factor of 5. The linear term greatly reduces the variance approaching the Fisher matrix bound. However, the estimator is close to, but not exactly the same as the Fisher variance prediction in the noise

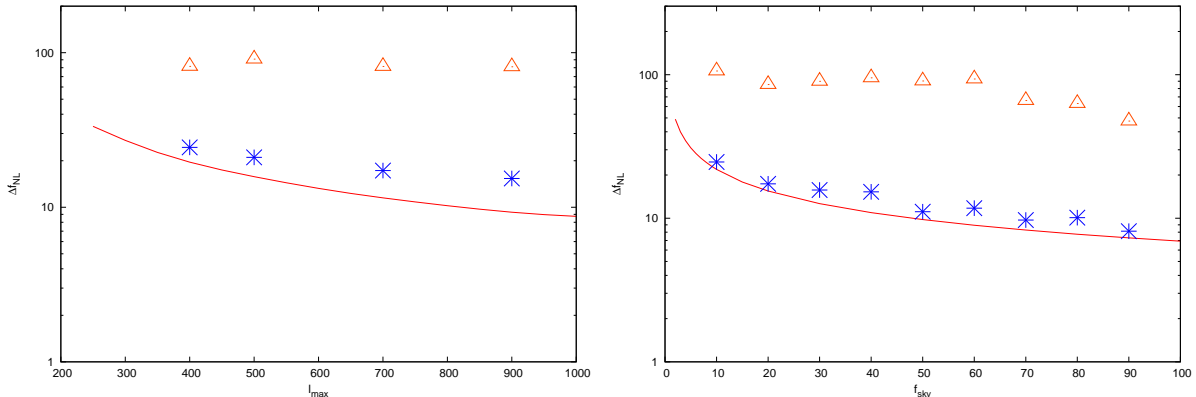


Fig. 2.— Optimality of the generalized estimator. The solid lines show the Fisher matrix prediction for the standard deviation of f_{NL} , the triangles show the standard deviation derived from the Monte Carlo simulations using the estimator without the linear term, and the stars show the standard deviation derived from the Monte Carlo simulations using the generalized estimator (i.e., with the linear term). **Left panel:** The uncertainty vs the maximum multipole that is used in the analysis, l_{max} . The simulations contain the Gaussian CMB signal, inhomogeneous noise (which simulates the Planck satellite), WMAP Kp0 and P06 masks. **Right panel:** The uncertainty vs a fraction of the sky observed, f_{sky} , for $l_{max} = 500$. The simulations include the Gaussian CMB signal, and flat sky-cut (which is azimuthally symmetric in the Galactic coordinates), while they do not include instrumental noise. This figure therefore shows that the sky cut contributes significantly to the linear term of polarization.

dominated regime.

Nevertheless, we do not observe the increase of variance at higher l_{max} : the variance becomes smaller as we include more multipoles. This result is in contradiction with the result of Creminelli et al. (2006a) and Creminelli et al. (2006b). We attribute this discrepancy to the error in the normalization of linear term in their formula.

In the right panel of figure 2, we show the variance of f_{NL} again using Gaussian simulations, but now in the presence of flat sky cut and in the *absence* of any noise. The purpose of the plot is to demonstrate (as pointed out in the previous section) that for the combined CMB temperature and polarization analysis, sky-cut does contribute significantly to the linear term. We find that the generalized estimator does a very good job in reducing the variance excess, and the simulated variance of f_{NL} does accurately saturate the Fisher matrix bound.

Noise	Sky-cut	$\langle f_{NL} \rangle$	f_{NL}^{input}	σ_{sim}
No	flat cut, $f_{\text{sky}} = 0.8$	103.2	100	10.1
Inhomogeneous	WMAP Kp0 and P06 masks	108.7	100	21.04

Table 1: Unbiasedness of the generalized estimator. Non-Gaussian CMB maps with $f_{NL}^{\text{input}} = 100$ are used for $\ell_{\text{max}} = 500$. The standard deviation of f_{NL} , σ_{sim} , was obtained using Gaussian simulations.

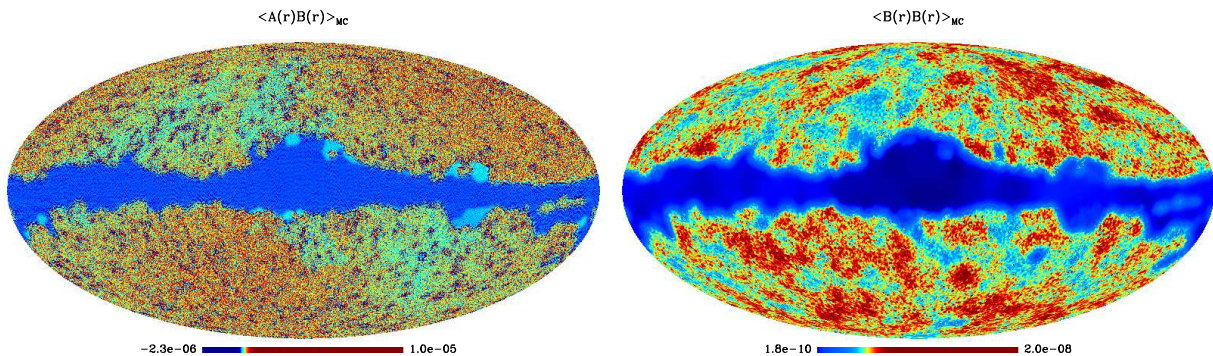


Fig. 3.— The $\langle A_{\text{sim}}(\hat{n}, r)B_{\text{sim}}(\hat{n}, r) \rangle_{MC}$ and $\langle B_{\text{sim}}^2(\hat{n}, r) \rangle_{MC}$ maps in dimension-less units for a slice near the surface of last scattering. These maps are calculated from Monte Carlo simulations with the Gaussian signal, Planck inhomogeneous noise, and WMAP Kp0 and P06 masks.

Can our estimator recover the correct f_{NL} , i.e., is our estimator unbiased? We have tested our estimator against simulated non-Gaussian CMB temperature and E-polarization maps. The non-Gaussian CMB temperature and E-polarization maps were generated using the method described in Liguori et al. (2007). We find that our estimator is unbiased, i.e., we can recover the f_{NL} value which was used to generate the non-Gaussian CMB maps. The results for the unbiasedness of the estimator are shown in Table 1. The analysis also shows the unbiasedness of the estimator described in Yadav et al. (2007).

Figures 3 and 4 show the maps $\langle A_{\text{sim}}(\hat{n}, r)B_{\text{sim}}(\hat{n}, r) \rangle_{MC}$ and $\langle B_{\text{sim}}^2(\hat{n}, r) \rangle_{MC}$, which appear in the linear term (Eq. 9) of the estimator. These maps are calculated using 100 Monte Carlo simulations of the data. Since the linear term contributes only in the presence of inhomogeneities, we also show these maps calculated with noise-only simulations (i.e. no signal). Notice how these maps correlate with the inhomogeneous noise (as shown in the lower map of figure 4).

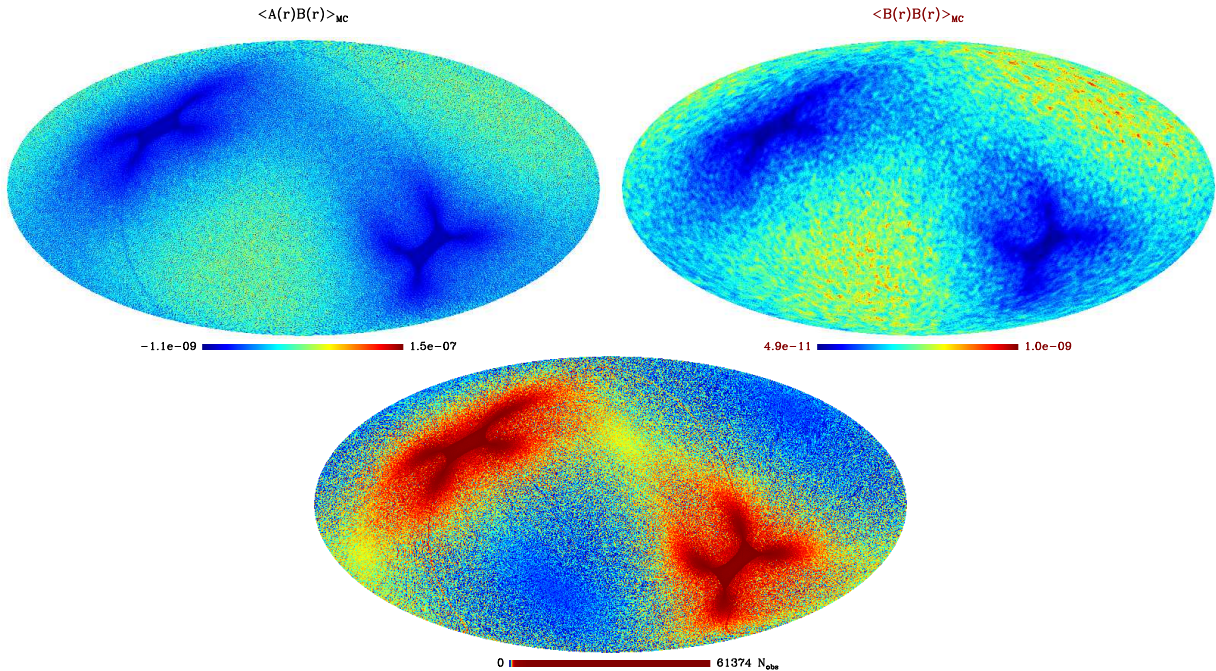


Fig. 4.— The top panels show $\langle A_{sim}(\hat{n}, r)B_{sim}(\hat{n}, r) \rangle_{MC}$ and $\langle B_{sim}^2(\hat{n}, r) \rangle_{MC}$ maps for the noise only analysis (i.e. no CMB signal or mask). The maps are in dimension-less units and are shown for a slice near the surface of last scattering. The Lower map shows the number of observations per pixel (N_{obs}) at the resolution of $N_{pix} = 12582912$.

3. Conclusion and discussion

Upcoming CMB experiments will provide a wealth of information about the CMB polarization anisotropies together with temperature anisotropies. The combined information from the CMB temperature and polarization data improves the sensitivity to primordial non-Gaussianity (Babich & Zaldarriaga 2004; Yadav & Wandelt 2005; Yadav et al. 2007). The promise of learning about the early universe by constraining the amplitude of primordial non-Gaussianity is now well established. In this paper we have generalized the bispectrum estimator of non-Gaussianity described in Yadav et al. (2007), to deal with the inhomogeneous nature of noise and incomplete sky coverage.

The generalization from Yadav et al. (2007) enables us to increase optimality of the estimator significantly, without compromising the computational efficiency of the estimator: the estimator is still computationally efficient, scaling as $O(N_{pix}^{3/2})$ compared to the $O(N_{pix}^{5/2})$ scaling of the full bispectrum (Babich & Zaldarriaga 2004) calculation for sky maps with N_{pix} pixels. For the Planck satellite, this translates into a speed-up by factors of millions, reducing

the required computing time from thousands of years to just hours and thus making f_{NL} estimation feasible. The speed of our estimator allows us to study its statistical properties using Monte Carlo simulations.

We have used Gaussian and non-Gaussian simulations to characterize the estimator. We have shown that the generalized fast estimator is able to deal with the partial sky coverage very well and in fact the variance of f_{NL} saturates the Fisher matrix bound. In the presence of both the realistic noise and galactic mask, we find that the generalized estimator greatly reduces the variance in comparison to the Yadav et al. (2007) estimator of non-Gaussianity using combined CMB temperature and polarization data.

Since the estimator is able to deal with the partial sky coverage very effectively, the estimator can also be used to constrain primordial non-Gaussianity using the data from ground and balloon based CMB experiments which observe only a small fraction of the sky. The estimator also solves the problem (Yadav et al. 2007) of non-trivial polarization mode coupling due to foreground masks. Earlier this issue was dealt with by removing the most contaminated ℓ modes from the analysis (usually $\ell < 30$).

The naive approach of using galactic masks to deal with the polarization contamination is to be refined. Both temperature and polarization foregrounds are expected to produce non-Gaussian signals. Some sources of non-primordial non-Gaussianity are CMB lensing, point sources, and the Sunyaev Zel’dovich effect. Understanding the non-Gaussianity from the polarization foreground sources and refining the estimator to be able to deal with it will be the subject of our future work.

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