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Inflation and late-time cosmic acceleration in non-minimal Maxwell-$F(R)$ gravity and the generation of large-scale magnetic fields

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Abstract

We study inflation and late-time acceleration in the expansion of the universe in non-minimal electromagnetism, in which the electromagnetic field couples to the scalar curvature function. It is shown that power-law inflation can be realized due to the non-minimal gravitational coupling of the electromagnetic field, and that large-scale magnetic fields can be generated due to the breaking of the conformal invariance of the electromagnetic field through its non-minimal gravitational coupling. Furthermore, it is demonstrated that both inflation and the late-time acceleration of the universe can be realized in a modified Maxwell-$F(R)$ gravity which is consistent with solar system tests and cosmological bounds and free of instabilities. At small curvature typical for current universe the standard Maxwell theory is recovered. We also consider classically equivalent form of non-minimal Maxwell-$F(R)$ gravity, and propose the origin of the non-minimal gravitational coupling function based on renormalization-group considerations.

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I. INTRODUCTION

It is observationally confirmed not only that inflation occurred in the early universe, but that the current expansion of the universe is accelerating [1, 2]. Although there exist various scenarios to account for the late-time acceleration in the expansion of the universe, the mechanism is not well established yet (for recent reviews, see [3–7]).

The scenarios to explain the late-time acceleration of the universe fall into two broad categories [6]. One is general relativistic approaches, i.e., dark energy. The other is modified gravity approaches, i.e., dark gravity. As the most promising one of the latter approaches, the modifications to the Einstein-Hilbert action, e.g., the addition of an arbitrary function of the scalar curvature to it, have been studied (for a review, see [7]). Such a modified theory is considered as an alternative gravitational theory, so that it must pass cosmological bounds and solar system tests.

Recently, Hu and Sawicki have proposed a very realistic modified gravitational theory that evade solar-system tests [8] (for related studies, see [9]). In this theory, an effective epoch described by the cold dark matter model with cosmological constant (ΛCDM), which explains high-precision observational data, is realized as in general relativity with cosmological constant (for a review of observational data confronted with modified gravity, see [10]). Although this theory is successful in explaining the late-time acceleration of the universe, the possibility of the realization of inflation has not been discussed in Ref. [8]. In Refs. [11–13], therefore, modified gravities in which both inflation and the late-time acceleration of the universe can be realized, following the previous inflation-acceleration unification proposal [14], have been presented and investigated. The classification of viable $F(R)$ gravities maybe suggested too [12]. Here, $F(R)$ is an arbitrary function of the scalar curvature R.

As another gravitational source of inflation and the late-time acceleration of the universe, a coupling between the scalar curvature and matter Lagrangian has been studied [15, 16]. Such a coupling may be applied for the realization of the dynamical cancellation of cosmological constant [17]. The criteria for the viability of such theories have been considered in Refs. [18–20]. Recently, as a simple case, a coupling between the scalar curvature function and the kinetic term of a massless scalar field in viable modified gravity has been considered [21].

On the other hand, it is known that the coupling between the scalar curvature and the Lagrangian of the electromagnetic field arises in curved spacetime due to one-loop vacuum-polarization effects in Quantum Electrodynamics (QED) [22]. Such a non-minimal gravitational coupling of the electromagnetic field breaks the conformal invariance of the electromagnetic field, so that electromagnetic quantum fluctuations can be generated at the inflationary stage even in the Friedmann-Robertson-Walker (FRW) spacetime, which is conformally flat [23–25]. They can appear as large-scale magnetic fields at the present time because their scale is made longer due to inflation. These large-scale magnetic fields can be the origin of the large-scale magnetic fields with the field strength 10^{-7}–10^{-6}G on 10kpc–1Mpc scale observed in clusters of galaxies [26] (for reviews of cosmic magnetic fields, see [27]).

In the present paper, we consider inflation and the late-time acceleration of the universe in non-minimal electromagnetism, in which the electromagnetic field couples to the function of scalar curvature. We show that power-law inflation can be realized due to the non-minimal gravitational coupling of the electromagnetic field, and that large-scale magnetic fields can be generated due to the breaking of the conformal invariance of the electromagnetic field through its non-minimal gravitational coupling[1]. The mechanism of inflation in this model is as follows. In the very early universe before inflation, electromagnetic quantum fluctuations are generated due to the breaking of the conformal invariance of the electromagnetic field and they act as a source for inflation. Furthermore, also during inflation electromagnetic quantum fluctuations are newly generated and the scale is stretched due to inflation, so that the scale can be larger than the Hubble horizon at that time, and they lead to the large-scale magnetic fields observed in galaxies and clusters of galaxies. This idea is based on the assumption that a given mode is excited quantum mechanically while it is subhorizon sized and then as it crosses outside the horizon "freezes in" as a classical fluctuation [23]. Furthermore, we demonstrate that both inflation and the late-time acceleration of the universe can be realized in a modified Maxwell-$F(R)$ gravity proposed in Ref. [13] which is consistent with solar-system tests and cosmological bounds and free of instabilities. We also consider classically equivalent form of non-minimal Maxwell-$F(R)$ gravity, and propose the origin of the non-minimal gravitational coupling function based on renormalization-group considerations.

This paper is organized as follows. In Sec. II we consider a non-minimal gravitational coupling of the electromagnetic field in general relativity. First, we describe our model and derive equations of motion from it. Next, we consider the evolution of the large-scale electric and magnetic fields. Furthermore, we analyze the gravitational field equation, and then show that power-law inflation can be realized. In Sec. III we consider a non-minimal gravitational coupling of the electromagnetic field in a modified gravitational theory proposed in Ref. [13]. We show that in this theory both inflation and the late-time acceleration of the universe can be realized. In Sec. IV we consider classically equivalent form of non-minimal Maxwell-$F(R)$ gravity. Finally, some summaries are given in Sec. V. In Appendix, we propose the origin of the non-minimal gravitational coupling function based on renormalization-group considerations.

We use units in which $k_{\mathrm{B}} = c = \hbar = 1$ and denote the gravitational constant $8\pi G$ by κ^2, so that $\kappa^2 \equiv 8\pi/{M_{\mathrm{Pl}}}^2$, where $M_{\mathrm{Pl}} = G^{-1/2} = 1.2 \times 10^{19}$GeV is the Planck mass. Moreover, in terms of electromagnetism we adopt Heaviside-Lorentz units.

II. INFLATION IN GENERAL RELATIVITY

In this section, we consider a non-minimal gravitational coupling of the electromagnetic field in general relativity.

A. Model

We consider the following model action:

$$S_{\mathrm{GR}} = \int d^4x \sqrt{-g} \left[\mathcal{L}_{\mathrm{EH}} + \mathcal{L}_{\mathrm{EM}} \right] , \tag{2.1}$$

$$\mathcal{L}_{\mathrm{EH}} = \frac{1}{2\kappa^2} R \, , \tag{2.2}$$

[1] In Ref. [28], gravitational-electromagnetic inflation from a 5-dimensional vacuum state has been considered.

$$\mathcal{L}_{\rm EM} = -\frac{1}{4} I(R) F_{\mu\nu} F^{\mu\nu} \,, \tag{2.3}$$

$$I(R) = 1 + f(R) \,, \tag{2.4}$$

where g is the determinant of the metric tensor $g_{\mu\nu}$, R is the scalar curvature arising from the spacetime metric tensor $g_{\mu\nu}$, and $\mathcal{L}_{\rm EH}$ is the Einstein-Hilbert action. Moreover, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field-strength tensor. Here, A_μ is the $U(1)$ gauge field. Furthermore, $f(R)$ is an arbitrary function of R.

The field equations can be derived by taking variations of the action Eq. (2.1) with respect to the metric $g_{\mu\nu}$ and the $U(1)$ gauge field A_μ as follows:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa^2 T^{(\rm EM)}_{\mu\nu} \,, \tag{2.5}$$

with

$$T^{(\rm EM)}_{\mu\nu} = I(R) \left(g^{\alpha\beta} F_{\mu\beta} F_{\nu\alpha} - \frac{1}{4} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \right)$$

$$+ \frac{1}{2} \left\{ f'(R) F_{\alpha\beta} F^{\alpha\beta} R_{\mu\nu} + g_{\mu\nu} \Box \left[f'(R) F_{\alpha\beta} F^{\alpha\beta} \right] - \nabla_\mu \nabla_\nu \left[f'(R) F_{\alpha\beta} F^{\alpha\beta} \right] \right\} , \tag{2.6}$$

and

$$-\frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} I(R) F^{\mu\nu} \right) = 0 \,, \tag{2.7}$$

where the prime denotes differentiation with respect to R, ∇_μ is the covariant derivative operator associated with $g_{\mu\nu}$, and $\Box \equiv g^{\mu\nu} \nabla_\mu \nabla_\nu$ is the covariant d'Alembertian for a scalar field. In addition, $R_{\mu\nu}$ is the Ricci curvature tensor, while $T^{(\rm EM)}_{\mu\nu}$ is the contribution to the energy-momentum tensor from the electromagnetic field.

We assume the spatially flat Friedmann-Robertson-Walker (FRW) spacetime with the metric

$$ds^2 = -dt^2 + a^2(t) d\boldsymbol{x}^2 = a^2(\eta)(-d\eta^2 + d\boldsymbol{x}^2) \,, \tag{2.8}$$

where a is the scale factor, and η is the conformal time. In this spacetime, $g_{\mu\nu} = \mathrm{diag}\left(-1, a^2(t), a^2(t), a^2(t)\right)$, and the components of $R_{\mu\nu}$ and R are given by

$$R_{00} = -3 \left(\dot{H} + H^2 \right) \,, \; R_{0i} = 0 \,, \; R_{ij} = \left(\dot{H} + 3H^2 \right) g_{ij} \,, \; R = 6 \left(\dot{H} + 2H^2 \right) \,, \tag{2.9}$$

where $H = \dot{a}/a$ is the Hubble parameter. Here, a dot denotes a time derivative, $\dot{} = \partial/\partial t$.

B. Evolution of large-scale electric and magnetic fields

First, we consider the evolution of the $U(1)$ gauge field in this background. Its equation of motion in the Coulomb gauge $\partial^j A_j(t, \boldsymbol{x}) = 0$ and the case of $A_0(t, \boldsymbol{x}) = 0$, reads

$$\ddot{A}_i(t, \boldsymbol{x}) + \left(H + \frac{\dot{I}}{I} \right) \dot{A}_i(t, \boldsymbol{x}) - \frac{1}{a^2} \overset{(3)}{\Delta} A_i(t, \boldsymbol{x}) = 0 \,, \tag{2.10}$$

where $\overset{(3)}{\Delta} = \partial^i \partial_i$ is the flat 3-dimensional Laplacian.

We shall quantize the $U(1)$ gauge field $A_\mu(t, \boldsymbol{x})$. It follows from the Lagrangian of the electromagnetic field (2.3) that the canonical momenta conjugate to $A_\mu(t, \boldsymbol{x})$ are given by

$$\pi_0 = 0\,, \quad \pi_i = I a(t) \dot{A}_i(t, \boldsymbol{x})\,. \tag{2.11}$$

We impose the canonical commutation relation between $A_i(t, \boldsymbol{x})$ and $\pi_j(t, \boldsymbol{x})$,

$$[A_i(t, \boldsymbol{x}), \pi_j(t, \boldsymbol{y})] = i \int \frac{d^3k}{(2\pi)^3} e^{i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y})} \left(\delta_{ij} - \frac{k_i k_j}{k^2} \right)\,, \tag{2.12}$$

where $\boldsymbol{k}$ is comoving wave number and $k = |\boldsymbol{k}|$. From this relation, we obtain the expression for $A_i(t, \boldsymbol{x})$ as

$$A_i(t, \boldsymbol{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} \sum_{\sigma=1,2} \left[\hat{b}(\boldsymbol{k}, \sigma) \epsilon_i(\boldsymbol{k}, \sigma) A(k, t) e^{i\boldsymbol{k}\cdot\boldsymbol{x}} + \hat{b}^\dagger(\boldsymbol{k}, \sigma) \epsilon_i^*(\boldsymbol{k}, \sigma) A^*(k, t) e^{-i\boldsymbol{k}\cdot\boldsymbol{x}} \right], \tag{2.13}$$

where $\epsilon_i(\boldsymbol{k}, \sigma)$ $(\sigma = 1, 2)$ are the two orthonormal transverse polarization vectors, and $\hat{b}(\boldsymbol{k}, \sigma)$ and $\hat{b}^\dagger(\boldsymbol{k}, \sigma)$ are the annihilation and creation operators which satisfy

$$\left[\hat{b}(\boldsymbol{k}, \sigma), \hat{b}^\dagger\left(\tilde{\boldsymbol{k}}, \tilde{\sigma}\right)\right] = \delta_{\sigma,\tilde{\sigma}} \delta^3\left(\boldsymbol{k} - \tilde{\boldsymbol{k}}\right), \; \left[\hat{b}(\boldsymbol{k}, \sigma), \hat{b}\left(\tilde{\boldsymbol{k}}, \tilde{\sigma}\right)\right] = \left[\hat{b}^\dagger(\boldsymbol{k}, \sigma), \hat{b}^\dagger\left(\tilde{\boldsymbol{k}}, \tilde{\sigma}\right)\right] = 0. \tag{2.14}$$

It follows from Eq. (2.10) that the mode function $A(k, t)$ satisfies the equation

$$\ddot{A}(k, t) + \left(H + \frac{\dot{I}}{I} \right) \dot{A}(k, t) + \frac{k^2}{a^2} A(k, t) = 0\,, \tag{2.15}$$

and that the normalization condition for $A(k, t)$ reads

$$A(k, t) \dot{A}^*(k, t) - \dot{A}(k, t) A^*(k, t) = \frac{i}{Ia}\,. \tag{2.16}$$

Replacing the independent variable t by η, we find that Eq. (2.15) becomes

$$\frac{\partial^2 A(k, \eta)}{\partial \eta^2} + \frac{1}{I(\eta)} \frac{dI(\eta)}{d\eta} \frac{\partial A(k, \eta)}{\partial \eta} + k^2 A(k, \eta) = 0\,. \tag{2.17}$$

We are not able to obtain the exact solution of Eq. (2.17) for the case in which I is given by a general function of η. In fact, however, we can obtain an approximate solution with sufficient accuracy by using the Wentzel-Kramers-Brillouin (WKB) approximation on subhorizon scales and the long-wavelength approximation on superhorizon scales, and matching these solutions at the horizon crossing [29, 30].

In the exact de Sitter background, we find $-k\eta = k/(aH)$. Moreover, at the horizon crossing, $H = k/a$ is satisfied, and hence $-k\eta_k = 1$ is satisfied. Here, η_k is the conformal time at the horizon-crossing. The subhorizon (superhorizon) scale corresponds to the region $k|\eta| \gg 1$ $(k|\eta| \ll 1)$. This is expected to be also a sufficiently good definition for the horizon crossing for power-law inflation $a \propto t^p$, where $p \gg 1$, which is almost equivalent to exponential inflation because in this case, $-k\eta = [p/(p-1)]\, k/(aH) \approx k/(aH)$.

The WKB subhorizon solution is given by

$$A_{\rm in}(k,\eta) = \frac{1}{\sqrt{2k}} I^{-1/2}(\eta) e^{-ik\eta}\,, \tag{2.18}$$

where we have assumed that the vacuum in the short-wavelength limit is the standard Minkowski vacuum.

On the other hand, the solution on superhorizon scales, $A_{\rm out}(k,\eta)$, can be obtained by using the long-wavelength expansion in terms of k^2 and matching this solution with the WKB subhorizon solution in Eq. (2.18) at the horizon crossing. The lowest order approximate solution of $A_{\rm out}(k,\eta)$ is given by [29]

$$A_{\rm out}(k,\eta) = C(k) + D(k)\int_{\eta}^{\eta_{\rm f}} \frac{1}{I\left(\tilde{\eta}\right)} d\tilde{\eta}\,, \tag{2.19}$$

where

$$C(k) = \frac{1}{\sqrt{2k}} I^{-1/2}(\eta)\left[1 - \left(\frac{1}{2}\frac{dI(\eta)}{d\eta} + ikI(\eta)\right)\int_{\eta}^{\eta_{\rm f}} \frac{1}{I\left(\tilde{\tilde{\eta}}\right)} d\tilde{\tilde{\eta}}\right] e^{-ik\eta}\Bigg|_{\eta=\eta_k}\,, \tag{2.20}$$

$$D(k) = \frac{1}{\sqrt{2k}} I^{-1/2}(\eta)\left(\frac{1}{2}\frac{dI(\eta)}{d\eta} + ikI(\eta)\right) e^{-ik\eta}\Bigg|_{\eta=\eta_k}\,. \tag{2.21}$$

Neglecting the decaying mode solution, from Eqs. (2.19) and (2.20) we find that $|A(k,\eta)|^2$ at late times is given by

$$|A(k,\eta)|^2 = |C(k)|^2 = \frac{1}{2kI(\eta_k)}\left|1 - \left[\frac{1}{2}\frac{1}{kI(\eta_k)}\frac{dI(\eta_k)}{d\eta} + i\right] e^{-ik\eta_k} k \int_{\eta_k}^{\eta_{\rm f}} \frac{I(\eta_k)}{I\left(\tilde{\tilde{\eta}}\right)} d\tilde{\tilde{\eta}}\right|^2\,, \tag{2.22}$$

where $\eta_{\rm f}$ is the conformal time at the end of inflation.

The proper electric and magnetic fields are given by

$$E_i^{\rm proper}(t,\boldsymbol{x}) = a^{-1} E_i(t,\boldsymbol{x}) = -a^{-1}\dot{A}_i(t,\boldsymbol{x})\,, \tag{2.23}$$

$$B_i^{\rm proper}(t,\boldsymbol{x}) = a^{-1} B_i(t,\boldsymbol{x}) = a^{-2}\epsilon_{ijk}\partial_j A_k(t,\boldsymbol{x})\,, \tag{2.24}$$

where $E_i(t,\boldsymbol{x})$ and $B_i(t,\boldsymbol{x})$ are the comoving electric and magnetic fields, and ϵ_{ijk} is the totally antisymmetric tensor ($\epsilon_{123} = 1$).

Using Eqs. (2.19) and (2.23), we find

$$|E^{\rm proper}(k,\eta)|^2 = 2\frac{1}{a^4}\left|\frac{\partial A(k,\eta)}{\partial\eta}\right|^2 = 2\frac{1}{a^4}\frac{|D(k)|^2}{|I(\eta)|^2}\,, \tag{2.25}$$

where the factor 2 comes from the two polarization degrees of freedom. Multiplying $|E^{\rm proper}(k,\eta)|^2$ in Eq. (2.25) by the phase-space density, $4\pi k^3/(2\pi)^3$, we obtain the amplitude of the proper electric fields in the position space

$$|E^{\rm proper}(L,\eta)|^2 = \frac{4\pi k^3}{(2\pi)^3}|E^{\rm proper}(k,\eta)|^2 = \frac{|D(k)|^2}{\pi^2 k}\frac{k^4}{a^4}\frac{1}{|I(\eta)|^2}\,, \tag{2.26}$$

on a comoving scale $L = 2\pi/k$. Furthermore, the energy density of the large-scale electric fields in the position space is given by

$$\rho_E(L,\eta) = \frac{1}{2}|E^{\rm proper}(L,\eta)|^2 I(\eta) = \frac{|D(k)|^2}{2\pi^2 k}\frac{k^4}{a^4}\frac{1}{I(\eta)}\,. \tag{2.27}$$

Similarly, using Eqs. (2.19) and (2.24), we find

$$|B^{\rm proper}(k,\eta)|^2 = 2\frac{k^2}{a^4}|A(k,\eta)|^2 = 2\frac{k^2}{a^4}|C(k)|^2\,, \tag{2.28}$$

where the factor 2 comes from the two polarization degrees of freedom. Multiplying $|B^{\rm proper}(k,\eta)|^2$ in Eq. (2.28) by the phase-space density, $4\pi k^3/(2\pi)^3$, we obtain the amplitude of the proper magnetic fields in the position space

$$|B^{\rm proper}(L,\eta)|^2 = \frac{4\pi k^3}{(2\pi)^3}|B^{\rm proper}(k,\eta)|^2 = \frac{k|C(k)|^2}{\pi^2}\frac{k^4}{a^4}\,, \tag{2.29}$$

on a comoving scale $L = 2\pi/k$. Furthermore, the energy density of the large-scale magnetic fields in the position space is given by

$$\rho_B(L,\eta) = \frac{1}{2}|B^{\rm proper}(L,\eta)|^2 I(\eta) = \frac{k|C(k)|^2}{2\pi^2}\frac{k^4}{a^4}I(\eta)\,. \tag{2.30}$$

Here we note the following point. As an example, if I is given by the following form: $I(\eta) = I_{\rm s}\left(\eta/\eta_{\rm s}\right)^{-\alpha}$, where $\eta_{\rm s}$ is some fiducial time during inflation, $I_{\rm s}$ is the value of $I(\eta)$ at $\eta = \eta_{\rm s}$, and α is a constant, from Eq. (2.20) we find $k|C(k)|^2 = \mathcal{C}/\left[2I(\eta_k)\right]$, where $\mathcal{C}$ is a constant of order unity [29, 30]. Hence, using this relation and Eq. (2.30), we find

$$\rho_B(L,\eta) = \frac{\mathcal{C}}{(2\pi)^2}\left(\frac{k}{a}\right)^4 \frac{I(\eta)}{I(\eta_k)}\,. \tag{2.31}$$

C. Power-law inflation

The $(\mu,\nu) = (0,0)$ component and the trace part of the $(\mu,\nu) = (i,j)$ component of Eq. (2.5), where i and j run from 1 to 3, read

$$\begin{aligned} H^2 + J_1 = \frac{\kappa^2}{3}\Bigg\{ & I(R)\left(g^{\alpha\beta}F_{0\beta}F_{0\alpha} - \frac{1}{4}g_{00}F_{\alpha\beta}F^{\alpha\beta}\right) \\ & + \frac{3}{2}\left[-f'(R)\left(\dot{H}+H^2\right) + 6f''(R)H\left(\ddot{H}+4H\dot{H}\right)\right]F_{\alpha\beta}F^{\alpha\beta} \\ & + \frac{3}{2}f'(R)H\left(F_{\alpha\beta}F^{\alpha\beta}\right)^{\bullet} - \frac{1}{2}f'(R)\frac{1}{a^2}\overset{(3)}{\Delta}\left(F_{\alpha\beta}F^{\alpha\beta}\right)\Bigg\}\,, \end{aligned} \tag{2.32}$$

$$J_1 = \frac{1}{6}F(R) - F'(R)\left(\dot{H}+H^2\right)\,, \tag{2.33}$$

and

$$
\begin{aligned}
&2\dot{H}+3H^2+J_2 \\
&\quad = \frac{\kappa^2}{2}\left\{\frac{1}{6}I(R)F_{\alpha\beta}F^{\alpha\beta}+\left[-f'(R)\left(\dot{H}+3H^2\right)\right.\right. \\
&\quad +6f''(R)\left(\dddot{H}+7H\ddot{H}+4\dot{H}^2+12H^2\dot{H}\right)+36f'''(R)\left.\left(\ddot{H}+4H\dot{H}\right)^2\right]F_{\alpha\beta}F^{\alpha\beta} \\
&\quad +3\left[f'(R)H+4f''(R)\left(\ddot{H}+4H\dot{H}\right)\right]\left(F_{\alpha\beta}F^{\alpha\beta}\right)^{\bullet}+f'(R)\left(F_{\alpha\beta}F^{\alpha\beta}\right)^{\bullet\bullet} \\
&\quad \left.-\frac{2}{3}f'(R)\frac{1}{a^2}\overset{(3)}{\Delta}\left(F_{\alpha\beta}F^{\alpha\beta}\right)\right\}, \qquad (2.34) \\
&J_2 = \frac{1}{2}F(R)-F'(R)\left(\dot{H}+3H^2\right)+6F''(R)\left[\dddot{H}+4\left(\dot{H}^2+H\ddot{H}\right)\right] \\
&\qquad +36F'''(R)\left(\ddot{H}+4H\dot{H}\right)^2, \qquad (2.35)
\end{aligned}
$$

where

$$
g^{\alpha\beta}F_{0\beta}F_{0\alpha}-\frac{1}{4}g_{00}F_{\alpha\beta}F^{\alpha\beta} = \frac{1}{2}\left(|E_i^{\mathrm{proper}}(t,\boldsymbol{x})|^2+|B_i^{\mathrm{proper}}(t,\boldsymbol{x})|^2\right), \qquad (2.36)
$$

$$
F_{\alpha\beta}F^{\alpha\beta} = 2\left(|B_i^{\mathrm{proper}}(t,\boldsymbol{x})|^2-|E_i^{\mathrm{proper}}(t,\boldsymbol{x})|^2\right), \qquad (2.37)
$$

respectively. Here, a large dot in terms of $F_{\alpha\beta}F^{\alpha\beta}$ denotes a time derivative, $\left(F_{\alpha\beta}F^{\alpha\beta}\right)^{\bullet} = \partial\left(F_{\alpha\beta}F^{\alpha\beta}\right)/\partial t$. Moreover, J_1 and J_2 are correction terms in a modified gravitational theory described by the action in Eq. (3.1) in the next section. Hence, because in this section we consider general relativity, i.e., the case $F(R)=0$ in the action in Eq. (3.2), here both J_1 and J_2 are zero. In deriving Eqs. (2.32) and (2.34), we have used equations in (2.9). Moreover, in deriving Eqs. (2.36) and (2.37), we have used Eqs. (2.23) and (2.24). Furthermore, applying Eqs. (2.26) and (2.29) to $|E_i^{\mathrm{proper}}(t,\boldsymbol{x})|^2$ and $|B_i^{\mathrm{proper}}(t,\boldsymbol{x})|^2$, respectively, we find

$$
\begin{aligned}
\left(F_{\alpha\beta}F^{\alpha\beta}\right)^{\bullet} = 8\Bigg\{&-H|B^{\mathrm{proper}}(L,\eta)|^2 \\
&+\left[H+3\frac{f'(R)}{1+f(R)}\left(\ddot{H}+4H\dot{H}\right)\right]|E^{\mathrm{proper}}(L,\eta)|^2\Bigg\}. \qquad (2.38)
\end{aligned}
$$

Here we consider the case in which magnetic fields are mainly generated rather than electric fields because we are interested in the generation of large-scale magnetic fields. It follows from Eqs. (2.27) and (2.30) that this situation is realized if I increases rapidly in time during inflation [30]. (Hence, from this point we neglect terms in electric fields.) Moreover, we consider the case in which $\overset{(3)}{\Delta}\left(F_{\alpha\beta}F^{\alpha\beta}\right)$ is very small because it corresponds to the second order spatial derivative of the quadratic quantity of electromagnetic quantum fluctuations, so that it can be neglected. In this case, using Eqs. (2.29) and (2.38), we find that Eqs. (2.32) and (2.34) are reduced to

$$
H^2 = \kappa^2\left[\frac{1}{6}I(R)-f'(R)\left(\dot{H}+5H^2\right)+6f''(R)H\left(\ddot{H}+4H\dot{H}\right)\right]\frac{k|C(k)|^2}{\pi^2}\frac{k^4}{a^4}, \qquad (2.39)
$$

and

$$2\dot{H} + 3H^2 = \kappa^2 \left[\frac{1}{6} I(R) + f'(R) \left(-5\dot{H} + H^2 \right) + 6 f''(R) \left(\dddot{H} - H\ddot{H} + 4\dot{H}^2 - 20H^2\dot{H} \right) \right.$$

$$\left. + 36 f'''(R) \left(\ddot{H} + 4H\dot{H} \right)^2 \right] \frac{k|C(k)|^2}{\pi^2} \frac{k^4}{a^4} , \tag{2.40}$$

respectively. Eliminating $I(R)$ from Eqs. (2.39) and (2.40), we obtain

$$\dot{H} + H^2 = \kappa^2 \left[f'(R) \left(-2\dot{H} + 3H^2 \right) + 3 f''(R) \left(\dddot{H} - 2H\ddot{H} + 4\dot{H}^2 - 24H^2\dot{H} \right) \right.$$

$$\left. + 18 f'''(R) \left(\ddot{H} + 4H\dot{H} \right)^2 \right] \frac{k|C(k)|^2}{\pi^2} \frac{k^4}{a^4} . \tag{2.41}$$

Here we consider the case in which $f(R)$ is given by the following form:

$$f(R) = f_{\mathrm{HS}}(R) \equiv \frac{c_1 \left(R/m^2 \right)^n}{c_2 \left(R/m^2 \right)^n + 1} , \tag{2.42}$$

which satisfies the conditions:

$$\lim_{R\to\infty} f_{\mathrm{HS}}(R) = \frac{c_1}{c_2} = \mathrm{const} , \tag{2.43}$$

$$\lim_{R\to 0} f_{\mathrm{HS}}(R) = 0 . \tag{2.44}$$

Here, c_1 and c_2 are dimensionless constants, n is a positive constant, and m denotes a mass scale. This form, $f_{\mathrm{HS}}(R)$, has been proposed by Hu and Sawicki [8]. The second condition (2.44) means that there could exist a flat spacetime solution. Hence, because in the late time universe the value of the scalar curvature becomes zero, the electromagnetic coupling I becomes unity, so that the ordinary Maxwell theory can be naturally recovered.

In order to show that power-law inflation can be realized, we consider the case in which the scale factor is given by $a(t) = \bar{a} \left(t/\bar{t} \right)^p$, where $\bar{t}$ is some fiducial time during inflation, $\bar{a}$ is the value of $a(t)$ at $t = \bar{t}$, and p is a positive constant. In this case, $H = p/t$, $\dot{H} = -p/t^2$, $\ddot{H} = 2p/t^3$, and $\dddot{H} = -6p/t^4$. Moreover, it follows from the fourth equation in (2.9) that $R = 6p(2p-1)/t^2$. At the inflationary stage, because $R/m^2 \gg 1$, we are able to use the following approximate relations:

$$f_{\mathrm{HS}}(R) \approx \frac{c_1}{c_2} \left[1 - \frac{1}{c_2} \left(\frac{R}{m^2} \right)^{-n} \right] , \tag{2.45}$$

$$f'_{\mathrm{HS}}(R) \approx \frac{n c_1}{{c_2}^2} \frac{1}{m^2} \left(\frac{R}{m^2} \right)^{-(n+1)} , \tag{2.46}$$

$$f''_{\mathrm{HS}}(R) \approx -\frac{n(n+1) c_1}{{c_2}^2} \frac{1}{m^4} \left(\frac{R}{m^2} \right)^{-(n+2)} , \tag{2.47}$$

$$f'''_{\mathrm{HS}}(R) \approx \frac{n(n+1)(n+2) c_1}{{c_2}^2} \frac{1}{m^6} \left(\frac{R}{m^2} \right)^{-(n+3)} . \tag{2.48}$$

Substituting the above relations in terms of a, H, and R and Eqs. (2.46)–(2.48) into Eq. (2.41), we find

$$p = \frac{n+1}{2}\,, \tag{2.49}$$

$$\frac{\bar{a}}{\bar{t}^p} = \left\{ \frac{1}{3^{n+1}\pi^2} \frac{1}{(n-1)\left[n(n+1)\right]^n} \frac{(-c_1)}{{c_2}^2} k|C(k)|^2 k^4 \kappa^2 m^{2n} \right\}^{1/4} . \tag{2.50}$$

Hence, if $n \gg 1$, p becomes much larger than unity, so that power-law inflation can be realized. Consequently, it follows form this result that the electromagnetic field with a non-minimal gravitational coupling in Eq. (2.3) can be a source of inflation.

Here we state the two following points. In this paper we consider only the case in which the values of the terms proportional to $f'(R)$, $f''(R)$ and $f'''(R)$ in the right-hand side of Eqs. (2.39) and (2.40) are dominant to the value of the term proportional to $I(R)$. Among the terms proportional to $f'(R)$, $f''(R)$ and $f'''(R)$, the term proportional to $f'(R)$ is dominant, and its value is order $f'(R)H^2 \approx n\left(c_1/{c_2}^2\right)\left(H^2/m^2\right)\left(R/m^2\right)^{-(n+1)}$, which follows from Eq. (2.46). Here, it follows from $H = p/t$ and $R = 6p(2p-1)/t^2$ that R is order $10H^2$. The condition that the term proportional to $f'(R)$ is dominant in the source term would be $I(R)/\left[f'(R)H^2\right] \sim 10c_2(R/m^2)^n/n \ll 1$. This would require extremely small c_2 because at the inflationary stage $R/m^2 \gg 1$ and $n \gg 1$. In such a case, the value of the right-hand side of Eq. (2.41), which is order $\kappa^2 f'(R)H^2\rho_B/I$ (this estimation is derived by using Eq. (2.30)), can be order H^2. Consequently, the right-hand side of Eq. (2.41) can balance with the left-hand side of Eq. (2.41), and hence Eq. (2.41) can be satisfied without contradiction to the result, i.e., power-law inflation in which p is much larger than unity can be realized. The reason why we consider the case in which the term proportional to $I(R)$ on the right-hand side of Eqs. (2.39) and (2.40) is so small in comparison with the term proportional to $f'(R)$ that it can be neglected is as follows: If the opposite case, namely, the term proportional to $I(R)$ is dominant to the term proportional to $f'(R)$, Eqs. (2.39) and (2.40) are approximately written as $H^2 \approx \kappa^2\rho_B/6$ and $2\dot{H} + 3H^2 \approx \kappa^2\rho_B/6$, respectively. Thus, in this case it follows from Eqs. (2.39) and (2.40) that H^2 and $2\dot{H} + 3H^2$ are the same order and their difference, $2\dot{H} + 2H^2$, must be much smaller than H^2. In fact, Eq. (2.41) implies that $\dot{H} + H^2$ balances with much smaller quantity than $\kappa^2\rho_B$. Now, $\left(\dot{H} + H^2\right)/H^2 = (p-1)/p$ and hence p must be very close to unity. Consequently, in this case power-law inflation cannot be realized.

Furthermore, when we consider the non-minimal electromagnetic theory described by Eq. (2.3), in the very early universe before the beginning of inflation electromagnetic quantum fluctuations can be generated due to the breaking of the conformal invariance of the electromagnetic field through its non-minimal gravitational coupling, This is because it is considered that in the very early universe before inflation (e.g., the grand unified theory (GUT) scale), there can exist quantum fluctuations of all physical quantities, as the quantum fluctuations of the inflaton field in the chaotic inflation scenario [31]. On the other hand, the non-minimal coupling between the electromagnetic field and the scalar curvature function $f(R)$ is purely classical. Furthermore, as explained above, in this paper we consider the case in which the term proportional to $f'(R)$ in the right-hand side of Eqs. (2.39) and (2.40) is dominant to the term proportional to $I(R)$. Hence, power-law inflation can be realized due to not the term proportional to $I(R)$, namely, the energy density of large-scale magnetic fields, but the term proportional to $f'(R)$, namely, a non-minimal electromagnetic coupling. Consequently, in this model we consider that inflation can be realized due to not purely quantum effects but semi-classical effects.

Finally, we note the following three points. In the present model, large-scale magnetic fields can be generated due to the breaking of the conformal invariance of the electromagnetic field through a coupling with the scalar curvature, $I(R)F_{\mu\nu}F^{\mu\nu}$, as is shown in the preceding subsection. If there does not exist such a coupling, i.e., $f(R) = 0$ and hence $I = 1$, in which the ordinary Maxwell theory is realized, electromagnetic quantum fluctuations cannot be generated in the FRW spacetime because this background spacetime is conformally flat. This result is also realized in the case of dilaton electromagnetism [32–36], in which the Lagrangian of the electromagnetic field is given by $\tilde{I}(\Phi)F_{\mu\nu}F^{\mu\nu}$ with $\tilde{I}(\Phi) = e^{\lambda\kappa\Phi}$ [35], where Φ is the dilaton field and λ is a dimensionless constant. (It is also realized in other scalar-field electromagnetism [37–39].)

Moreover, Bertolami and Páramos have recently considered constraints on a non-minimal gravitational coupling of matter, namely, for the present model, $f(R)$ in Eq. (2.4), from the observational data of the central temperature of the Sun [20]. They have studied the effect of a non-minimal gravitational coupling of matter on the hydrostatic equilibrium of the spherically symmetric system with a polytropic equation of state approximately describing the Sun with sufficient accuracy, assuming a perturbative regime to the usual Tolman-Oppenheimer-Volkoff (TOV) equation of hydrostatic equilibrium and taking into account the validity of the Newtonian regime in a theory with a non-minimal gravitational coupling of matter. According to them, there exists no strong constraints on a non-minimal gravitational coupling of matter obtained from the comparison of the predictions of the theoretical models and the current observational sensitivity to the central temperature of the Sun except for the relation of the perturbative approach, $|f(R)| \ll 1$. It follows from the relation $|f(R)| \ll 1$ that for the case $f(R) = f_{\mathrm{HS}}(R)$ in Eq. (2.42) with $m = m_e = 0.511\mathrm{MeV}$ [22], where m_e is the electron mass, using the maximum value of the central mass density of the Sun, $\rho_{\mathrm{c}} = 1.62 \times 10^2\,\mathrm{g\,cm^{-3}}$, and the expression of the scalar curvature in the Newtonian regime, $R \approx -8\pi G\rho_{\mathrm{c}}$, we find that the constraint on $f_{\mathrm{HS}}(R)$ is given by

$$|f_{\mathrm{HS}}(R)| \approx \left| \frac{c_1 \left(-8\pi G\rho_{\mathrm{c}}/m_e^2\right)^n}{c_2 \left(-8\pi G\rho_{\mathrm{c}}/m_e^2\right)^n + 1} \right| = \left| \frac{c_1 \left(-4.51 \times 10^{-46}\right)^n}{c_2 \left(-4.51 \times 10^{-46}\right)^n + 1} \right| \ll 1 \,. \tag{2.51}$$

Furthermore, the existence of the non-minimal gravitational coupling of the electromagnetic field $f(R)$ in Eq. (2.4) changes the value of the fine structure constant, i.e., the strength of the electromagnetic coupling. Hence, the deviation of the non-minimal electromagnetism from the usual Maxwell theory can be constrained from the observations of radio and optical quasar absorption lines [40], those of the anisotropy of the cosmic microwave background (CMB) radiation [41, 42], those of the absorption of CMB radiation at 21 cm hyperfine transition of the neutral atomic hydrogen [43], and big bang nucleosynthesis (BBN) [44, 45] as well as solar-system experiments [46] (for a recent review, see [47]).

III. INFLATION AND LATE-TIME COSMIC ACCELERATION IN MODIFIED GRAVITY

In this section, we consider a non-minimal gravitational coupling of the electromagnetic field in a modified gravitational theory proposed in Ref. [13].

A. Inflation

We consider the following model action:

$$S_{\rm MG} = \int d^4x\sqrt{-g}\,[\,\mathcal{L}_{\rm MG} + \mathcal{L}_{\rm EM}\,]\,, \tag{3.1}$$

$$\mathcal{L}_{\rm MG} = \frac{1}{2\kappa^2}\left[R + F(R)\right]\,, \tag{3.2}$$

where $F(R)$ is an arbitrary function of R. Here, $\mathcal{L}_{\rm EM}$ is given by Eq. (2.3). We note that $F(R)$ is the modified part of gravity, and hence $F(R)$ is completely different from the non-minimal gravitational coupling of the electromagnetic field $f(R)$ in Eq. (2.4).

Taking variations of the action Eq. (3.1) with respect to the metric $g_{\mu\nu}$, we find that the field equation of modified gravity is given by [13]

$$\left[1 + F'(R)\right] R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\left[R + F(R)\right] + g_{\mu\nu}\Box F'(R) - \nabla_\mu \nabla_\nu F'(R) = \kappa^2 T^{(\rm EM)}_{\mu\nu}\,. \tag{3.3}$$

The $(\mu, \nu) = (0, 0)$ component and the trace part of the $(\mu, \nu) = (i, j)$ component of Eq. (3.3), where i and j run from 1 to 3, are given by Eqs. (2.32) and (2.34), respectively. Similarly to the preceding section, we here consider the case in which terms in electric fields and $\overset{(3)}{\Delta}\left(F_{\alpha\beta}F^{\alpha\beta}\right)$ are negligible. In this case, eliminating $I(R)$ from Eqs. (2.32) and (2.34), we obtain

$$\begin{aligned}
&\dot{H} + H^2 + \left\{\frac{1}{6}F(R) - F'(R)H^2 + 3F''(R)\left[\dddot{H} + 4\left(\dot{H}^2 + H\ddot{H}\right)\right] + 18F'''(R)\left(\ddot{H} + 4H\dot{H}\right)^2\right\} \\
&\quad = \kappa^2\Bigg[f'(R)\left(-2\dot{H} + 3H^2\right) + 3f''(R)\left(\dddot{H} - 2H\ddot{H} + 4\dot{H}^2 - 24H^2\dot{H}\right) \\
&\quad + 18f'''(R)\left(\ddot{H} + 4H\dot{H}\right)^2\Bigg]\frac{k|C(k)|^2}{\pi^2}\frac{k^4}{a^4}\,.
\end{aligned} \tag{3.4}$$

Here we consider the case in which $F(R)$ is given by

$$F(R) = -M^2\frac{\left[(R/M^2) - (R_0/M^2)\right]^{2l+1} + (R_0/M^2)^{2l+1}}{c_3 + c_4\left\{\left[(R/M^2) - (R_0/M^2)\right]^{2l+1} + (R_0/M^2)^{2l+1}\right\}}\,, \tag{3.5}$$

which satisfies the following conditions: $\lim_{R\to\infty} F(R) = -M^2/c_4 = \text{const}$, $\lim_{R\to 0} F(R) = 0$. Here, c_3 and c_4 are dimensionless constants, l is a positive integer, and M denotes a mass scale. We consider that in the limit $R \to \infty$, i.e., at the very early stage of the universe, $F(R)$ becomes an effective cosmological constant, and that at the present time $F(R)$ becomes a small constant, namely,

$$\lim_{R\to\infty} F(R) = -M^2\frac{1}{c_4} = -2\Lambda_{\rm i}\,, \tag{3.6}$$

$$F(R_0) = -M^2\frac{(R_0/M^2)^{2l+1}}{c_3 + c_4(R_0/M^2)^{2l+1}} = -2R_0\,, \tag{3.7}$$

where $\Lambda_{\mathrm{i}} \left(\gg {H_0}^2 \right)$ is an effective cosmological constant in the very early universe and $R_0 \left(\approx {H_0}^2 \right)$ is current curvature. Here, H_0 is the Hubble constant at the present time [48]: $H_0 = 100h \, \mathrm{km \, s^{-1} \, Mpc^{-1}} = 2.1h \times 10^{-42} \mathrm{GeV} \approx 1.5 \times 10^{-33} \mathrm{eV}$, where we have used $h = 0.70$ [49]. From Eqs. (3.6) and (3.7), we find

$$c_3 = \frac{1}{2} \left(\frac{R_0}{M^2} \right)^{2l} \left(1 - \frac{R_0}{\Lambda_{\mathrm{i}}} \right) \approx \frac{1}{2} \left(\frac{R_0}{M^2} \right)^{2l} , \tag{3.8}$$

$$c_4 = \frac{1}{2} \frac{M^2}{\Lambda_{\mathrm{i}}} , \tag{3.9}$$

where the last approximate equality in Eq. (3.8) follows from $(R_0/\Lambda_{\mathrm{i}}) \ll 1$.

Furthermore, we consider the case in which $f(R)$ is given by the following form:

$$f(R) = f_{\mathrm{NO}}(R) \equiv \frac{\left[(R/M^2) - (R_0/M^2) \right]^{2q+1} + (R_0/M^2)^{2q+1}}{c_5 + c_6 \left\{ \left[(R/M^2) - (R_0/M^2) \right]^{2q+1} + (R_0/M^2)^{2q+1} \right\}} , \tag{3.10}$$

which satisfies the following conditions: $\lim_{R \to \infty} f_{\mathrm{NO}}(R) = 1/c_6 = \mathrm{const}$, $\lim_{R \to 0} f_{\mathrm{NO}}(R) = 0$. Here, c_5 and c_6 are dimensionless constants, and q is a positive integer. The form of $F(R)$ in Eq. (3.5) and $f_{\mathrm{NO}}(R)$ in Eq. (3.10) is taken from Ref. [13]. This form corresponds to the extension of the form of $f_{\mathrm{HS}}(R)$ in Eq. (2.42). It has been shown in Ref. [13] that modified gravitational theories described by the action (3.2) with $F(R)$ in Eq. (3.5) successfully pass the solar-system tests as well as cosmological bounds and they are free of instabilities.

At the inflationary stage, because $R/M^2 \gg 1$ and $R/M^2 \gg R_0/M^2$, we are able to use the following approximate relations:

$$F(R) \approx -M^2 \frac{1}{c_4} \left[1 - \frac{c_3}{c_4} \left(\frac{R}{M^2} \right)^{-(2l+1)} \right] , \tag{3.11}$$

and

$$f_{\mathrm{NO}}(R) \approx \frac{1}{c_6} \left[1 - \frac{c_5}{c_6} \left(\frac{R}{M^2} \right)^{-(2q+1)} \right] . \tag{3.12}$$

At the very early stage of the universe, because $R \to \infty$, it follows from Eq. (3.6) and the condition, $\lim_{R \to \infty} f_{\mathrm{NO}}(R) = 1/c_6 = \mathrm{const}$, that Eq. (3.4) are reduced to

$$\dot{H} + H^2 = \frac{\Lambda_{\mathrm{i}}}{3} . \tag{3.13}$$

From this equation, we obtain

$$a(t) \propto \exp \left(\sqrt{\frac{\Lambda_{\mathrm{i}}}{3}} t \right) . \tag{3.14}$$

Hence exponential inflation can be realized. Thus, we see that the terms in $F(R)$ on the left-hand side of Eq. (3.4), i.e., the part of the braces $\{\}$, can be a source of inflation, in addition to the terms in $f(R)$ on the right-hand side of Eq. (3.4). In fact, if there do not exist any terms in $F(R)$, in which the theory is general relativity, or the contribution of the terms in $F(R)$ to inflation is much smaller than those in $f(R)$, Eq. (3.4) is equivalent to Eq. (2.41). In such a case, similarly to the consideration in Sec. II C, substituting $a(t) \propto t^{\tilde{p}}$, where $\tilde{p}$ is a positive constant, the approximate expressions of $f'_{\mathrm{NO}}(R)$, $f''_{\mathrm{NO}}(R)$ and $f'''_{\mathrm{NO}}(R)$ derived from Eq. (3.12) into Eq. (3.4), we find $\tilde{p} = q + 1$. Hence, if $q \gg 1$, $\tilde{p}$ becomes much larger than unity, so that power-law inflation can be realized. Consequently, in the present model there exist two sources of inflation, one from the modified part of gravity, $F(R)$, and the other from the non-minimal gravitational coupling of the electromagnetic field, $f(R)$. We here note that even if the value of Λ_{i} is so small that the modification of gravity cannot contribute to inflation, inflation can be realized due to the non-minimal gravitational coupling of the electromagnetic field, namely, the change of the value of $f(R)$ in terms of R, and the generation of magnetic fields. This is an important feature of the present model.

B. Late-time cosmic acceleration

Next, we consider the late-time acceleration of the universe. As shown above, at the early stage of the universe, at which the curvature is very large, inflation can be realized due to the terms in $F(R)$ and/or those in $f(R)$. As curvature becomes small, the contribution of these terms to inflation becomes small, namely, the values of these terms in Eq. (3.4) become small, and then inflation ends. After inflation, radiation becomes dominant, and subsequently matter becomes dominant. When the energy density of matter becomes small and the value of curvature becomes R_0, there appears the small effective cosmological constant at the present time as seen in Eq. (3.7). Hence, the current cosmic acceleration can be realized. It has been shown in Ref. [13] that both inflation and the late-time acceleration of the universe can be realized in modified gravitational theories described by the action (3.2) with $F(R)$ in Eq. (3.5) for the case without the non-minimal gravitational coupling of the electromagnetic field $f(R)$ in Eq. (2.4). In this subsection, we confirm that also in this theory with the non-minimal electromagnetic coupling $f(R)$, the late-time acceleration of the universe can be realized. (Incidentally, it has been shown in Ref. [21] that in this theory with a non-minimal coupling with the kinetic term of a massless scalar field, the late-time acceleration of the universe can be realized.)

In the limit $R \to R_0$, i.e., the present time, because $R/M^2 - R_0/M^2 \ll 1$, we are able to use the following approximate relations:

$$F(R) \approx -M^2 \frac{c_3}{\left[c_3 + c_4 \left(R_0/M^2\right)^{2l+1}\right]^2} \times \left\{ \left(\frac{R}{M^2} - \frac{R_0}{M^2}\right)^{2l+1} + \left[\frac{c_3 + c_4 \left(R_0/M^2\right)^{2l+1}}{c_3}\right] \left(\frac{R_0}{M^2}\right)^{2l+1} \right\}, \quad (3.15)$$

and

$$f_{\mathrm{NO}}(R) \approx \frac{c_5}{\left[c_5 + c_6 \left(R_0/M^2\right)^{2q+1}\right]^2} \times \left\{ \left(\frac{R}{M^2} - \frac{R_0}{M^2}\right)^{2q+1} + \left[\frac{c_5 + c_6 \left(R_0/M^2\right)^{2q+1}}{c_5}\right] \left(\frac{R_0}{M^2}\right)^{2q+1} \right\}. \quad (3.16)$$

From Eqs. (3.4), (3.15), the approximate expressions of $F'(R)$, $F''(R)$ and $F'''(R)$ derived from Eq. (3.15), and the approximate expressions of $f'_{\mathrm{NO}}(R)$, $f''_{\mathrm{NO}}(R)$ and $f'''_{\mathrm{NO}}(R)$ derived from Eq. (3.16), we see that if $q > l$, $f_{\mathrm{NO}}(R)$ becomes constant more rapidly than $F(R)$ in the limit $R \to R_0$. As a result, the electrodynamics looks as purely minimal theory at the current universe. For such a case, in the limit $R \to R_0$, Eqs. (3.4) are reduced to

$$\dot{H} + H^2 = \frac{R_0}{3} . \tag{3.17}$$

From this equation, we obtain

$$a(t) \propto \exp\left(\sqrt{\frac{R_0}{3}} t\right) , \tag{3.18}$$

so that

$$\frac{\ddot{a}(t)}{a(t)} = \frac{R_0}{3} > 0 . \tag{3.19}$$

Thus, the late-time acceleration of the universe can be realized.

Finally, we note the following point: In this model, even if the value of R_0 is so small that the modification of gravity cannot contribute to the late-time acceleration of the universe, the late-time acceleration can be realized due to due to the non-minimal gravitational coupling of the electromagnetic field and the generation of magnetic fields. This is also an important feature of the present model.

IV. CLASSICALLY EQUIVALENT FORM OF NON-MINIMAL MAXWELL-$F(R)$ GRAVITY

In this section, we consider classically equivalent form of non-minimal Maxwell-$F(R)$ gravity.

The action (3.1) can be rewritten by using auxiliary fields. Introducing two scalar fields ζ and ξ, we can rewrite the action (3.1) to the following form [15, 21]:

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2\kappa^2} \left[\zeta + F(\zeta) \right] + I(\zeta) \mathcal{L}_{\mathrm{M}} + \xi \left(R - \zeta \right) \right\} , \tag{4.1}$$

$$\mathcal{L}_{\mathrm{M}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} , \tag{4.2}$$

where $\mathcal{L}_{\mathrm{M}}$ is the Lagrangian describing the ordinary Maxwell theory. The form in Eq. (4.1) is reduced to the original form in Eq. (3.1) by using the equation $\zeta = R$, which is derived by taking variation of the action (4.1) with respect to one auxiliary field ξ. Moreover, taking variation of the form in Eq. (4.1) with respect to the other auxiliary field ζ, we find

$$\xi = \frac{1}{2\kappa^2} \left[1 + F'(\zeta) \right] + I'(\zeta) \mathcal{L}_{\mathrm{M}} , \tag{4.3}$$

where the prime denotes differentiation with respect to ζ. Substituting Eq. (4.3) into Eq. (4.1) and eliminating ξ from Eq. (4.1), we find

$$S = \int d^4x\sqrt{-g}\Big\{ \frac{1}{2\kappa^2}\left[1+F'(\zeta)\right]R + \left[I(\zeta)+I'(\zeta)\left(R-\zeta\right)\right]\mathcal{L}_{\mathrm{M}} + \frac{1}{2\kappa^2}\left[F(\zeta)-F'(\zeta)\zeta\right]\Big\}. \tag{4.4}$$

We make the following conformal transformation of the action given by Eq. (4.4):

$$g_{\mu\nu} \to \hat{g}_{\mu\nu} = e^{\varphi} g_{\mu\nu}\,, \tag{4.5}$$

with

$$e^{\varphi} = 1 + F'(\zeta)\,, \tag{4.6}$$

where φ is a scalar field. Here, the hat denotes quantities in a new conformal frame in which the term in the coupling between $F'(\zeta)$ and the scalar curvature in the first term on the right-hand side of Eq. (4.4) disappears. Consequently, the action in the new conformal frame is given by [50]

$$\begin{aligned} S_{\mathrm{N}} = \int d^4x\sqrt{-\hat{g}}\Bigg[&\frac{1}{2\kappa^2}\left(\hat{R} - \frac{3}{2}\hat{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\right) \\ &+ \left(e^{-2\varphi}\left\{I\left[\zeta(\varphi)\right] - I'\left[\zeta(\varphi)\right]\zeta(\varphi)\right\} + e^{-\varphi}I'\left[\zeta(\varphi)\right]\left(\hat{R} + 3\hat{\Box}\varphi - \frac{3}{2}\hat{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\right)\right)\hat{\mathcal{L}}_{\mathrm{M}} \\ &+ \frac{1}{2\kappa^2}e^{-2\varphi}\left\{F\left[\zeta(\varphi)\right] - \left(e^{\varphi}-1\right)\zeta(\varphi)\right\}\Bigg]\,, \end{aligned} \tag{4.7}$$

where

$$\hat{\Box}\varphi = \frac{1}{\sqrt{-\hat{g}}}\partial_\mu\left(\sqrt{-\hat{g}}\hat{g}^{\mu\nu}\partial_\nu\varphi\right)\,, \tag{4.8}$$

and $\hat{g}$ is the determinant of $\hat{g}^{\mu\nu}$. In deriving Eq. (4.7), we have used Eq. (4.6). Moreover, $\zeta(\varphi)$ in Eq. (4.7) is obtained by solving Eq. (4.6) with respect to ζ as $\zeta = \zeta(\varphi)$. Hence, the action in the new conformal frame (4.7) includes the Brans-Dicke type scalar field φ [51]. From the term proportional to $\hat{\mathcal{L}}_{\mathrm{M}}$ on the right-hand side of Eq. (4.7), we see that the form of the Lagrangian in terms of the electromagnetic field in Eq. (4.7) is close to that of the electromagnetic field with the coupling to the dilaton, which has been explained in Sec. II C. In other words, the Lagrangian of non-minimal Maxwell-$F(R)$ gravity is qualitatively similar to Lagrangian describing dilaton electromagnetism (except for the term in the coupling between the scalar curvature and the electromagnetic field). As explained in Refs. [52, 53], however, this fact does not mean the physical equivalence between them.

V. CONCLUSION

In the present paper, we have considered inflation and the late-time acceleration in the expansion of the universe in non-minimal electromagnetism, in which the electromagnetic field couples to the scalar curvature function. As a result, we have shown that power-law inflation can be realized due to the non-minimal gravitational coupling of the electromagnetic field, and that large-scale magnetic fields can be generated due to the breaking of the conformal invariance of the electromagnetic field through its non-minimal gravitational coupling. Furthermore, we have demonstrated that both inflation and the late-time acceleration of the universe can be realized in a modified Maxwell-$F(R)$ gravity proposed in Ref. [13] which is consistent with solar-system tests and cosmological bounds and free of instabilities. We have also considered classically equivalent form of non-minimal Maxwell-$F(R)$ gravity.

Finally, we make a remark about the observational deviation of a non-minimal electromagnetic theory from the ordinary Maxwell theory. It follows from the fourth equation in Eq. (2.9) that in exponential inflation the scalar curvature is proportional to the square of the Hubble parameter. Moreover, it is known that the root-mean-square (rms) amplitude of curvature perturbations is also proportional to the square of the Hubble parameter. In a non-minimal electromagnetic theory, because magnetic fields couple to the scalar curvature, there can exist the cross correlations between magnetic fields and curvature perturbations through the Hubble parameter. Hence, if the primordial large-scale magnetic fields are detected [54, 55] by future experiments such as PLANCK [56], SPIDERS (post-PLANCK) [57] and Inflation Probe (CMBPol mission) in the Beyond Einstein program of NASA [58] on the anisotropy of CMB radiation, and if there exist (do not exist) the cross correlations between the primordial large-scale magnetic fields and curvature perturbations, it is observationally suggested that at the inflationary stage there should exist a non-minimal gravitational coupling of the electromagnetic field (the strength and/or the form of non-minimal coupling of the electromagnetic field may be observationally restricted).

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APPENDIX: ASYMPTOTIC FREEDOM VERSUS NON-MINIMAL COUPLING

It is very interesting that one can generalize the discussion of this work for interacting theories: scalar/spinor electrodynamics and non-Abelian gauge theory. As a simple example, let us consider the $SU(2)$ gauge theory with the Lagrangian: $\mathcal{L} = -\,(1/4)\, G^a_{\mu\nu} G^{a\mu\nu}$, where $G^a_{\mu\nu}$ is the $SU(2)$ field strength. The effective renormalization-group improved Lagrangian for such a theory in matter sector has been found in Ref. [59] for a de Sitter background as

$$\mathcal{L}_{SU(2)} = -\frac{1}{4}\frac{\tilde{g}^2}{\tilde{g}^2(\tilde{t})} G^a_{\mu\nu} G^{a\mu\nu}\,, \tag{A.1}$$

with

$$\tilde{g}^2(\tilde{t}) = \frac{\tilde{g}^2}{1 + 11\tilde{g}^2\tilde{t}/\left(12\pi^2\right)}, \tag{A.2}$$

where $\tilde{g}(\tilde{t})$ is the running $SU(2)$ gauge coupling constant, $\tilde{g}$ is the value of $\tilde{g}(\tilde{t})$ in the case $\tilde{t} = 0$, and $\tilde{t}$ is a renormalization-group parameter. Note that the running gauge coupling constant typically shows asymptotically free behavior: it goes to zero at very high energy. For the covariantly constant gauge background with $G^a_{\mu\nu}G^{a\mu\nu}/2 = \tilde{H}^2$, where $\tilde{H}$ corresponds to the magnetic field in the $SU(2)$ gauge theory, it has been proposed in Ref. [59] that $\tilde{t}$ is given by

$$\tilde{t} = \frac{1}{2}\ln\frac{R/4 + \tilde{g}\tilde{H}}{\mu^2}, \tag{A.3}$$

where μ is a mass parameter.

It is clear that with the decrease of the energy scale (namely, as the universe expands), $\tilde{t}$ is decreasing, as $\tilde{t}$ is very large at the early universe. Taking into account the results of this work, one can try to relate the asymptotic freedom in a non-Abelian gauge theory with non-minimal Maxwell-modified gravity. In this way, using the proposal of Eq. (2.42) in Sec. II for non-minimal $f(R)$ in front of $G^a_{\mu\nu}G^{a\mu\nu}$, one gets

$$\frac{c_1\left(R/m^2\right)^n}{c_2\left(R/m^2\right)^n + 1} = \frac{11\tilde{g}^2}{12\pi^2}\tilde{t}. \tag{A.4}$$

Hence, according to this assumption, at a very large curvature ($R/m^2 \gg 1$), $\tilde{t} \approx \left[12\pi^2/\left(11\tilde{g}^2\right)\right](c_1/c_2)$, while at the current universe ($R \to 0$), $\tilde{t} \to 0$. Thus, asymptotic freedom induces the appearance of the non-minimal gravitational gauge coupling in (non-) Abelian gauge theories at high energy.

Generally speaking, such a scenario is universal and it works not only for asymptotically free theories. For instance, for scalar QED one can easily write the renormalization-group improved effective Lagrangian in curved spacetime. In the matter sector (zero scalar field background) it has qualitatively the same form as Eq. (A.1), the only sign of $\tilde{t}$ is different in the expression for the running gauge coupling constant. As a result, such an effective Lagrangian again induces the non-minimal gravitational coupling of the electromagnetic sector.

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