Tetra-maximal Neutrino Mixing and Its Implications on Neutrino Oscillations and Collider Signatures

Zhi-zhong Xing *

Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China

Abstract

We propose a novel neutrino mixing pattern in terms of only two small integers 1 and 2 together with their square roots and the imaginary number i. This ansatz is referred to as the "tetra-maximal" mixing because it can be expressed as a product of four rotation matrices, whose mixing angles are all $\pi/4$ in the complex plane. It predicts $\theta_{12} = \arctan(2 - \sqrt{2}) \approx 30.4^\circ$, $\theta_{13} = \arcsin[(\sqrt{2} - 1)/(2\sqrt{2})] \approx 8.4^\circ$, $\theta_{23} = 45^\circ$ and $\delta = 90^\circ$ in the standard parametrization, and the Jarlskog invariant of leptonic CP violation is found to be $\mathcal{J} = 1/32$. These results are compatible with current data and can soon be tested in a variety of neutrino oscillation experiments. Implications of the tetra-maximal neutrino mixing on the decays of doubly-charged Higgs bosons $H^{\pm\pm} \to l^\pm_\alpha l^\pm_\beta$ (for $\alpha, \beta = e, \mu, \tau$) are also discussed in the triplet seesaw mechanism at the TeV scale, which will be explored at the upcoming LHC.

PACS number(s): 14.60.Pq, 13.10.+q, 25.30.Pt

*E-mail: xingzz@ihep.ac.cn

$\boxed{1}$ Recent solar [1], atmospheric [2], reactor [3] and accelerator [4] neutrino experiments have convincingly verified the hypothesis of neutrino oscillation, a pure quantum phenomenon which can naturally occur if neutrinos are massive and lepton flavors are mixed. The mixing of lepton flavors is described by a 3×3 unitary matrix V, whose nine elements are commonly parametrized in terms of three rotation angles and three CP-violating phases. Defining three unitary rotation matrices in the complex (1,2), (1,3) and (2,3) planes as

$$
\begin{aligned}
O_{12}(\theta_{12}, \delta_{12}) &= \begin{pmatrix} c_{12} & \hat{s}^*_{12} & 0 \\ -\hat{s}_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} , \\
O_{13}(\theta_{13}, \delta_{13}) &= \begin{pmatrix} c_{13} & 0 & \hat{s}^*_{13} \\ 0 & 1 & 0 \\ -\hat{s}_{13} & 0 & c_{13} \end{pmatrix} , \\
O_{23}(\theta_{23}, \delta_{23}) &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & \hat{s}^*_{23} \\ 0 & -\hat{s}_{23} & c_{23} \end{pmatrix} ,
\end{aligned}
\tag{1}
$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $\hat{s}_{ij} \equiv e^{i\delta_{ij}} \sin\theta_{ij}$ (for $1 \leq i < j \leq 3$), we can write out the standard parametrization of V advocated by the Particle Data Group [5] and in Ref. [6]:

$$
\begin{aligned}
V &= O_{23}(\theta_{23}, 0) \otimes O_{13}(\theta_{13}, \delta) \otimes O_{12}(\theta_{12}, 0) \otimes P_\nu \\
&= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} P_\nu ,
\end{aligned}
\tag{2}
$$

in which $P_\nu = \text{Diag}\{e^{i\rho}, e^{i\sigma}, 1\}$ is a diagonal phase matrix which contains two non-trivial Majorana phases of CP violation. A global analysis of current neutrino oscillation data yields $30^\circ < \theta_{12} < 38^\circ$, $36^\circ < \theta_{23} < 54^\circ$ and $\theta_{13} < 10^\circ$ at the 99% confidence level [7], but three phases of V remain entirely unconstrained. The on-going and forthcoming neutrino oscillation experiments will measure θ_{13} and δ. On the other hand, the neutrinoless double-beta decay experiments will help to probe or constrain ρ and σ.

The observed pattern of neutrino flavor mixing is certainly far beyond the imagination of many people. For instance, the tri-maximal neutrino mixing proposed by Cabibbo [8],

$$
V_{\rm C} = \sqrt{\frac{1}{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}
\tag{3}
$$

with $\omega = e^{i2\pi/3}$ being a complex cube-root of unity (i.e., $\omega^3 = 1$), used to be a vivid ansatz in illustration of both large flavor mixing and maximal CP violation in the lepton sector; but it has been ruled out by current experimental data on neutrino oscillations. A simple modification of $V_{\rm C}$,

$$
\begin{aligned}
V_{\rm HPS} &= V_{\rm C} \otimes O_{13}(\pi/4, \pi) \\
&= Q_l \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix} Q_\nu
\end{aligned}
\tag{4}
$$

with $Q_l = {\rm Diag}\{1, \omega, \omega^2\}$ and $Q_\nu = {\rm Diag}\{1, 1, i\}$, which has been proposed by Harrison, Perkins and Scott [9] and referred to as the tri-bimaximal neutrino mixing matrix [1], turns out to be favored in today's neutrino phenomenology. To generate the non-vanishing mixing angle θ_{13} and non-trivial CP-violating phases, however, slight corrections to $V_{\rm HPS}$ have to be introduced [11]. So far a lot of interest has been paid to the tri-bimaximal mixing pattern and its viable variations, which can be realized in a number of neutrino mass models incorporated with certain flavor symmetries and (or) seesaw mechanisms [12].

The salient feature of $V_{\rm HPS}$ is that its entries are all formed from small integers (0, 1, 2 and 3) and their square roots, which are often suggestive of discrete flavor symmetries in the language of group theories. Then a natural question is whether one can construct a different but viable neutrino mixing pattern with fewer small integers. We find that the answer to this phenomenologically interesting question is affirmative: we may just use two small integers 1 and 2 together with their square roots and the imaginary number i to build a neutrino mixing matrix which is compatible with current neutrino oscillation data. This new pattern, which will be referred to as the "tetra-maximal" neutrino mixing, predicts

$$\begin{aligned}
\theta_{12} &= \arctan\left[2\left(1-\sqrt{\frac{1}{2}}\right)\right] \approx 30.4^\circ \; , \\
\theta_{13} &= \arcsin\left[\frac{1}{2}\left(1-\sqrt{\frac{1}{2}}\right)\right] \approx 8.4^\circ \; , \\
\theta_{23} &= 45^\circ \; ,
\end{aligned} \tag{5}$$

and $\delta = 90^\circ$ together with $\rho = \sigma = -90^\circ$. Since θ_{13} is large and δ is maximal, the Jarlskog invariant of leptonic CP violation [13] turns out to be $\mathcal{J} = 1/32$, which can give rise to appreciable effects of CP or T violation in long-baseline neutrino oscillations. Thus the tetra-maximal neutrino mixing scenario is easily testable in a variety of neutrino oscillation experiments in the near future.

2 Now let us describe how to construct the new neutrino mixing matrix in terms of 1, 2 and i. We notice that the tri-maximal mixing pattern $V_{\rm C}$ can be decomposed as

$$V_{\rm C} \;=\; P^\prime_l \otimes O_{23}(\pi/4, \pi/2) \otimes O_{13}(\theta^\prime_{13}, 0) \otimes O_{12}(\pi/4, 0) \; , \tag{6}$$

where $P^\prime_l = {\rm Diag}\{1, -i\omega^2, \omega\}$ and $\theta^\prime_{13} = \arctan(\sqrt{1/2}) \approx 35.3^\circ$. Therefore, the tri-bimaximal neutrino mixing matrix $V_{\rm HPS}$ arises from a product of four rotation matrices in the complex plane: three of them involve the rotation angle $\pi/4$, and the fourth involves the rotation angle $\theta^\prime_{13} \neq \pi/4$. The unique value of $\theta^\prime_{13}$ given above is crucial to assure that Eq. (6) can successfully reproduce the form of $V_{\rm C}$ in Eq. (3) and then the form of $V_{\rm HPS}$ in Eq. (4). Indeed, one happens to obtain $\theta_{12} = \theta^\prime_{13}$ from $V_{\rm HPS}$. This mysterious angle has a simple

---

[1] Note that this pattern is quite similar to the democratic neutrino mixing pattern [10], although their consequences on θ_{12} and θ_{23} are quite different.

geometric explanation [14]: it corresponds to the angle formed by two unequal diagonals from the same vertex of a cube.

But here we consider the possibility of $\theta'_{13} = \pi/4$. In this case, we construct a new neutrino mixing pattern in terms of four rotation matrices whose mixing angles are all $\pi/4$:

$$
\begin{aligned}
V &= P_l \otimes O_{23}(\pi/4, \pi/2) \otimes O_{13}(\pi/4, 0) \otimes O_{12}(\pi/4, 0) \otimes O_{13}(\pi/4, \pi) \\
&= \frac{1}{2} \begin{pmatrix} 1+\sqrt{\frac{1}{2}} & 1 & 1-\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{2}}\left[1-i\left(1-\sqrt{\frac{1}{2}}\right)\right] & 1-i\sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}}\left[1+i\left(1+\sqrt{\frac{1}{2}}\right)\right] \\ -\sqrt{\frac{1}{2}}\left[1+i\left(1-\sqrt{\frac{1}{2}}\right)\right] & 1+i\sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}}\left[1-i\left(1+\sqrt{\frac{1}{2}}\right)\right] \end{pmatrix} ,
\end{aligned} \tag{7}
$$

where $P_l = \text{Diag}\{1, 1, i\}$. It is clear that V only contains two small integers 1 and 2 together with their square roots and the imaginary number i. Because the mixing angle in each of the four rotation matrices of V is $\pi/4$, this neutrino mixing matrix can be referred to as the "tetra-maximal" neutrino mixing pattern. Some discussions about the phenomenological consequence of V are in order.

1. Comparing between Eqs. (2) and (7), we can easily obtain the values of three neutrino mixing angles as already listed in Eq. (5). It is also straightforward to calculate the Jarlskog invariant of CP violation from Eq. (7):

   $$\mathcal{J} \ = \ \text{Im}\left(V_{e2} V_{\mu 3} V^*_{e3} V^*_{\mu 2}\right) \ = \ \frac{1}{32} \ . \tag{8}$$

   On the other hand, we obtain $\mathcal{J} = c_{12} s_{12} c^2_{13} s_{13} c_{23} s_{23} \sin\delta = \sin\delta/32$ from Eq. (2) with the help of Eq. (5). We are therefore left with $\sin\delta = 1$ or equivalently $\delta = \pi/2$. Note that the maximal value of $\mathcal{J}$ can only be achieved from the *unrealistic* tri-maximal neutrino mixing pattern $V_{\rm C}$; i.e., $\mathcal{J}_{\rm max} = 1/(6\sqrt{3})$. We find that leptonic CP violation in the tetra-maximal mixing case is about one third of $\mathcal{J}_{\rm max}$ (namely, $\mathcal{J}/\mathcal{J}_{\rm max} = 3\sqrt{3}/16 \approx 32.5\%$).

2. To figure out two Majorana CP-violating phases ρ and σ, we may redefine the phases of three charged-lepton fields and three neutrino fields such that V_{e1}, V_{e2}, $V_{\mu 3}$ and $V_{\tau 3}$ in Eq. (7) become real and positive while $\delta = \pi/2$ properly appears in the other five elements of V. This exercise will yield $\rho = \sigma = -\pi/2$. A more straightforward way to determine ρ and σ is to calculate the effective mass of the neutrinoless double-beta decay by using Eqs. (2) and (5),

   $$
   \begin{aligned}
   \langle m\rangle_{\beta\beta} &= \left|m_1 c^2_{12} c^2_{13} e^{2i\rho} + m_2 s^2_{12} c^2_{13} e^{2i\sigma} + m_3 s^2_{13} e^{-2i\delta}\right| \\
   &= \frac{1}{4}\left| m_1 \left(1+\sqrt{\frac{1}{2}}\right)^2 e^{2i\rho} + m_2 e^{2i\sigma} + m_3 \left(1+\sqrt{\frac{1}{2}}\right)^2 e^{-2i\delta} \right| ,
   \end{aligned} \tag{9}
   $$

   and compare this result with the one which can be directly obtained from Eq. (7). We see no interference or cancellation in the latter procedure, and thus we simply arrive at $\rho = \sigma = -\delta$ from Eq. (9). Namely, $\rho = \sigma = -\pi/2$.

3. The off-diagonal asymmetries of V, which may serve as a simple description of the geometric structure of V [15], are found to be

$$\mathcal{A}_1 \equiv |V_{e2}|^2 - |V_{\mu 1}|^2 = |V_{\mu 3}|^2 - |V_{\tau 2}|^2 = |V_{\tau 1}|^2 - |V_{e3}|^2 \;=\; -\frac{1}{4}\left(\frac{1}{4} + \sqrt{\frac{1}{2}}\right) \;,$$
$$\mathcal{A}_2 \equiv |V_{e2}|^2 - |V_{\mu 3}|^2 = |V_{\mu 1}|^2 - |V_{\tau 2}|^2 = |V_{\tau 3}|^2 - |V_{e1}|^2 \;=\; -\frac{1}{4}\left(\frac{1}{4} - \sqrt{\frac{1}{2}}\right) \;, \qquad (10)$$

which are about V_{e1}-$V_{\mu 2}$-$V_{\tau 3}$ and V_{e3}-$V_{\mu 2}$-$V_{\tau 1}$ axes of V, respectively. More explicitly, $\mathcal{A}_1 \approx -0.24$ and $\mathcal{A}_2 \approx +0.11$. Hence V looks more symmetric about its V_{e3}-$V_{\mu 2}$-$V_{\tau 1}$ axis. The fact of $\mathcal{A}_1 \neq \mathcal{A}_2 \neq 0$ in the tetra-maximal mixing case implies that all the six unitarity triangles of V in the complex plane are different from one another, although their areas are all equal to $\mathcal{J}/2 = 1/64$.

4. The tetra-maximal neutrino mixing pattern shows an apparent μ-τ flavor symmetry, $|V_{\mu i}| = |V_{\tau i}|$ (for $i = 1, 2, 3$), as one can directly see from Eq. (7). This result has an interesting implication on the flavor distribution of ultrahigh-energy cosmic neutrinos at neutrino telescopes. Given the canonical source of cosmic neutrinos, where the neutrino flavor composition is

$$\phi_e : \phi_\mu : \phi_\tau \;=\; 1 : 2 : 0 \qquad (11)$$

due to the pion-muon decay chain arising from energetic pp or $p\gamma$ collisions [16], the condition of $\theta_{23} = \pi/4$ and $\delta = \pi/2$ will lead to an exact neutrino flavor democracy at a terrestrial neutrino telescope [17]:

$$\phi_e^{\rm T} : \phi_\mu^{\rm T} : \phi_\tau^{\rm T} \;=\; 1 : 1 : 1 \;. \qquad (12)$$

Note that such a result can also be obtained from the tri-bimaximal neutrino mixing pattern $V_{\rm HPS}$, which provides the condition of $\theta_{13} = 0$ and $\theta_{23} = \pi/4$ [18].

In short, the relatively large values of θ_{13} and $\mathcal{J}$ predicted by this tetra-maximal neutrino mixing scenario makes it easily testable in the forthcoming long-baseline (reactor and accelerator) neutrino oscillation experiments.

3 In the basis where the mass eigenstates of three charged leptons coincide with their flavor eigenstates, one may reconstruct the Majorana neutrino mass matrix M by using the neutrino mixing matrix V and three neutrino masses m_i (for $i = 1, 2, 3$):

$$M \;=\; V \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} V^T \;. \qquad (13)$$

Taking account of the tetra-maximal neutrino mixing pattern given in Eq. (7), we find that $M_{e\tau} = M^*_{e\mu}$ and $M_{\tau\tau} = M^*_{\mu\mu}$ hold. Namely,

$$M \;=\; \begin{pmatrix} M_{ee} & M_{e\mu} & M^*_{e\mu} \\ M_{e\mu} & M_{\mu\mu} & M_{\mu\tau} \\ M^*_{e\mu} & M_{\mu\tau} & M^*_{\mu\mu} \end{pmatrix} \; . \tag{14}$$

Such a specific texture of M, which can give rise to the maximal CP-violating phase in neutrino oscillations (i.e., $\delta = \pm\pi/2$), is possible to result from a certain flavor symmetry and its breaking mechanism [19]. Here we focus our interest on the magnitudes of $|M_{\alpha\beta}|^2$, because they can in principle be determined from a number of lepton-number-violating processes in a given model. After a straightforward calculation, we obtain

$$\begin{aligned} |M_{ee}|^2 &= \frac{1}{16}\left[\frac{17+12\sqrt{2}}{4}m_1^2 + m_2^2 + \frac{17-12\sqrt{2}}{4}m_3^2 \right. \\ &\quad \left. + \left(3+2\sqrt{2}\right)m_1 m_2 + \frac{1}{2}m_1 m_3 + \left(3-2\sqrt{2}\right)m_2 m_3\right] \; , \\ |M_{e\mu}|^2 &= \frac{1}{16}\left[\frac{7+4\sqrt{2}}{8}m_1^2 + \frac{3}{2}m_2^2 + \frac{7-4\sqrt{2}}{8}m_3^2 \right. \\ &\quad \left. - \frac{3+2\sqrt{2}}{2}m_1 m_2 - \frac{1}{4}m_1 m_3 - \frac{3-2\sqrt{2}}{2}m_2 m_3\right] \; , \\ |M_{\mu\mu}|^2 &= \frac{1}{16}\left[\frac{33-20\sqrt{2}}{16}m_1^2 + \frac{9}{4}m_2^2 + \frac{33+20\sqrt{2}}{16}m_3^2 \right. \\ &\quad \left. - \frac{9-10\sqrt{2}}{4}m_1 m_2 - \frac{15}{8}m_1 m_3 - \frac{9+10\sqrt{2}}{4}m_2 m_3\right] \; , \\ |M_{\mu\tau}|^2 &= \frac{1}{16}\left[\frac{33-20\sqrt{2}}{16}m_1^2 + \frac{9}{4}m_2^2 + \frac{33+20\sqrt{2}}{16}m_3^2 \right. \\ &\quad \left. + \frac{15-6\sqrt{2}}{4}m_1 m_2 + \frac{17}{8}m_1 m_3 + \frac{15+6\sqrt{2}}{4}m_2 m_3\right] \; . \end{aligned} \tag{15}$$

It is then easy to verify

$$\sum_{\alpha} |M_{\alpha\alpha}|^2 + \sum_{\alpha\neq\beta} |M_{\alpha\beta}|^2 \;=\; \sum_{i=1}^{3} m_i^2 \; , \tag{16}$$

where α and β run over e, μ and τ. Since the absolute mass scale of m_i is unknown, we consider three special patterns of the neutrino mass spectrum allowed by current neutrino oscillation data: (1) normal hierarchy with $m_1 \approx 0$; (2) inverted hierarchy with $m_3 \approx 0$; and (3) near degeneracy with $m_1 \approx m_2 \approx m_3$. Taking $\Delta m^2_{21} = 8.0 \times 10^{-5}$ eV^2 and $|\Delta m^2_{32}| = 2.5 \times 10^{-3}$ eV^2 [7] as typical inputs, we are then able to calculate $|M_{\alpha\beta}|^2$ for three different neutrino mass hierarchies by using Eq. (15). Our numerical results for $|M_{\alpha\beta}|^2$ are listed in TABLE I. Note that $\langle m\rangle_{\beta\beta} \equiv |M_{ee}|$, the effective mass of the neutrinoless double-beta decay, is found to be 3.3×10^{-3} eV (normal hierarchy), 4.8×10^{-2} eV (inverted hierarchy) or m_1 (near degeneracy) in this tetra-maximal neutrino mixing ansatz.

The origin of M is of course model-dependent. For simplicity, we assume that M results from the triplet seesaw mechanism [20]. By introducing an $SU(2)_{\rm L}$ Higgs triplet Δ into the standard model, we can write out the following renormalizable Yukawa interaction term:

$$-\mathcal{L}_\Delta \; = \; \frac{1}{2}\overline{l_{\rm L}} Y_\Delta \Delta i\sigma_2 l^c_{\rm L} + {\rm h.c.} \; , \tag{17}$$

where

$$\Delta \equiv \left(\begin{matrix} H^- & -\sqrt{2} \; H^0 \cr \sqrt{2} \; H^{--} & -H^- \end{matrix} \right) \; . \tag{18}$$

Note that Δ can also couple to the standard-model Higgs doublet H and thus violate lepton number by two units [21]. When the neutral components of H and Δ acquire their vacuum expectation values $\langle H \rangle \equiv v/\sqrt{2}$ and $\langle \Delta \rangle \equiv v_\Delta$, respectively, the electroweak gauge symmetry is spontaneously broken and the resultant Majorana neutrino mass matrix reads $M = Y_\Delta v_\Delta$. A clear signature of the triplet seesaw mechanism is the existence of doubly-charged Higgs bosons $H^{\pm\pm}$. If the mass scale of Δ is of $\mathcal{O}(1)$ TeV, then $H^{\pm\pm}$ can be produced at the LHC via the Drell-Yan process $q\bar{q} \to \gamma^*, Z^* \to H^{++}H^{--}$ or through the charged-current process $q\bar{q}' \to W^* \to H^{\pm\pm}H^{\mp}$. Note that the masses of $H^{\pm\pm}$ and $H^\pm$ are expected to be nearly degenerate in a class of triplet seesaw models [20–22], and thus only $H^{\pm\pm} \to l^\pm_\alpha l^\pm_\beta$ (for $\alpha, \beta = e, \mu, \tau$) and $H^{\pm\pm} \to W^\pm W^\pm$ decay modes are kinematically open. Note also that the leptonic channel $H^{\pm\pm} \to l^\pm_\alpha l^\pm_\beta$ becomes dominant when $v_\Delta < 1$ MeV is taken [22]. Therefore, we concentrate on the same-sign dilepton events of $H^{\pm\pm}$, which signify the lepton number violation and serve for the cleanest collider signatures of new physics [23]. The branching ratio of $H^{--} \to l^-_\alpha l^-_\beta$ turns out to be

$$\mathcal{B}(H^{--} \to l^-_\alpha l^-_\beta) \; = \; \frac{2}{1+\delta_{\alpha\beta}} \cdot \frac{|M_{\alpha\beta}|^2}{\displaystyle\sum_{i=1}^{3} m^2_i} \; , \tag{19}$$

which is completely determined by the values of m_i and V. Taking account of Eq. (15), we can estimate the magnitude of $\mathcal{B}(H^{--} \to l^-_\alpha l^-_\beta)$ for three special patterns of the neutrino mass spectrum chosen above. Our numerical results are listed in TABLE II. The measurement of these lepton-number-violating decay modes at the LHC will help test the tetra-maximal neutrino mixing scenario and distinguish it from other neutrino mixing patterns [24] in the TeV-scale triplet seesaw mechanism.

4 Motivated by the principle of simplicity, we have proposed a novel neutrino mixing pattern in terms of only two small integers 1 and 2 together with their square roots and the imaginary number i. Different from the tri-bimaximal mixing scenario, our tetra-maximal mixing scenario can accommodate both non-vanishing θ_{13} and large CP violation. Its explicit predictions include $\theta_{12} = \arctan(2-\sqrt{2}) \approx 30.4^\circ$, $\theta_{13} = \arcsin[(\sqrt{2}-1)/(2\sqrt{2})] \approx 8.4^\circ$, $\theta_{23} = 45^\circ$, $\delta = 90^\circ$, $\rho = \sigma = -90^\circ$ and $\mathcal{J} = 1/32$, which are compatible with current data and can soon be tested in a variety of neutrino oscillation experiments. We have also illustrated possible implications of the tetra-maximal neutrino mixing on collider signatures by taking account of the TeV-scale triplet seesaw mechanism. In particular, the branching ratios of leptonic decays of doubly-charged Higgs bosons $H^{\pm\pm} \to l^\pm_\alpha l^\pm_\beta$ (for $\alpha, \beta = e, \mu, \tau$) have been calculated for three special patterns of the neutrino mass matrix. The results are found to be encouraging and interesting.

The flavor symmetry behind the tetra-maximal neutrino mixing pattern has to be seen. It is always possible to build a specific neutrino mass model from which such a flavor mixing pattern can be derived, although this kind of model building usually relies on some natural or contrived assumptions. All in all, the tetra-maximal neutrino mixing can shortly be confronted with a number of precision neutrino experiments and even the LHC. A test of its many phenomenological consequences is therefore close at hand.

Acknowledgments: This work was supported in part by the National Natural Science Foundation of China.

REFERENCES

[1] SNO Collaboration, Q.R. Ahmad *et al.*, Phys. Rev. Lett. **89**, 011301 (2002).

[2] For a review, see: C.K. Jung *et al.*, Ann. Rev. Nucl. Part. Sci. **51**, 451 (2001).

[3] KamLAND Collaboration, K. Eguchi *et al.*, Phys. Rev. Lett. **90**, 021802 (2003); CHOOZ Collaboration, M. Apollonio *et al.*, Phys. Lett. B **420**, 397 (1998); Palo Verde Collaboration, F. Boehm *et al.*, Phys. Rev. Lett. **84**, 3764 (2000).

[4] K2K Collaboration, M.H. Ahn *et al.*, Phys. Rev. Lett. **90**, 041801 (2003).

[5] Particle Data Group, W.M. Yao *et al.*, J. Phys. G **33**, 1 (2006).

[6] H. Fritzsch and Z.Z. Xing, Phys. Lett. B **517**, 363 (2001); Z.Z. Xing, Int. J. Mod. Phys. A **19**, 1 (2004).

[7] A. Strumia and F. Vissani, hep-ph/0606054.

[8] N. Cabibbo, Phys. Lett. B **72**, 333 (1978).

[9] P.F. Harrison, D.H. Perkins, and W.G. Scott, Phys. Lett. B **530**, 167 (2002); P.F. Harrison and W.G. Scott, Phys. Lett. B **535**.

[10] H. Fritzsch and Z.Z. Xing, Phys. Lett. B **372**, 265 (1996); Phys. Lett. B **440**, 313 (1998); Phys. Rev. D **61**, 073016 (2000).

[11] Z.Z. Xing, Phys. Lett. B **533**, 85 (2002).

[12] For recent reviews with extensive references, see: H. Fritzsch and Z.Z. Xing, Prog. Part. Nucl. Phys. **45**, 1 (2000); Altarelli and F. Feruglio, New J. Phys. **6**, 106 (2004); R.N. Mohapatra and A.Yu. Smirnov, Ann. Rev. Nucl. Part. Sci. **56**, 569 (2006); A. Strumia and F. Vissani, in Ref. [7].

[13] C. Jarlskog, Phys. Rev. Lett. **55**, 1039 (1985); H. Fritzsch and Z.Z. Xing, Nucl. Phys. B **556**, 49 (1999).

[14] T.D. Lee, "New Insights to Old Problems", hep-ph/0605017.

[15] Z.Z. Xing, Phys. Rev. D **65**, 113010 (2002).

[16] See, e.g., Z.Z. Xing, Phys. Rev. D **74**, 013009 (2006); Z.Z. Xing and S. Zhou, Phys. Rev. D **74**, 013010 (2006); Z.Z. Xing, Nucl. Phys. B (Proc. Suppl.) **168**, 274 (2007); Nucl. Phys. B (Proc. Suppl.) **175-176**, 421 (2008); and references therein.

[17] Z.Z. Xing and S. Zhou, arXiv:0804.3512 [hep-ph].

[18] J.G. Learned and S. Pakvasa, Astropart. Phys. **3**, 267 (1995).

[19] T. Kitabayashi and M. Yasue, Phys. Lett. B **621**, 133 (2005); I. Aizawa, T. Kitabayashi, and M. Yasue, Phys. Rev. D **72**, 055014 (2005); Nucl. Phys. B **728**, 220 (2005); Z.Z. Xing, H. Zhang, and S. Zhou, Phys. Lett. B **641**, 189 (2006); T. Baba and M. Yasue, Phys. Rev. D **77**, 075008 (2008).

[20] M. Magg and C. Wetterich, Phys. Lett. B **94**, 61 (1980); J. Schechter and J.W.F. Valle, Phys. Rev. D **22**, 2227 (1980); T.P. Cheng and L.F. Li, Phys. Rev. D **22**, 2860 (1980); R.N. Mohapatra and G. Senjanovic, Phys. Rev. D **23**, 165 (1981).

[21] W. Chao, Z. Si, Z.Z. Xing, and S. Zhou, arXiv:0804.1265 [hep-ph].

[22] See, e.g., K. Huitu, J. Maalampi, A. Pietila, and M. Raidal, Nucl. Phys. B **487**, 27 (1997); B. Dion *et al.*, Phys. Rev. D **59**, 075006 (1999); E.J. Chun, K.Y. Lee, and S.C. Park, Phys. Lett. B **566**, 142 (2003); A.G. Akeroyd and M. Aoki, Phys. Rev. D **72**, 035011 (2005); A. Hektor *et al.*, Nucl. Phys. B **787**, 198 (2007); T. Han, B. Mukhopadhyaya, Z. Si, and K. Wang, Phys. Rev. D **76**, 075013 (2007); C.S. Chen, C.Q. Geng, and D.V. Zhuridov, arXiv:0801.2011.

[23] W.Y. Keung and G. Senjanovic, Phys. Rev. Lett. **50**, 1427 (1983).

[24] See, e.g., J. Garayoa and T. Schwetz, arXiv:0712.1453; M. Kadastik, M. Raidal, and L. Rebane, arXiv:0712.3912; A.G. Akeroyd, M. Aoki, and H. Sugiyama, arXiv:0712.4019; P. Fileviez Pérez, T. Han, G.Y. Huang, T. Li, and K. Wang, arXiv:0803.3450; P. Ren and Z.Z. Xing, arXiv:0805.4292.

TABLES

TABLE I. The values of $|M_{\alpha\beta}|^2$ (for $\alpha, \beta = e, \mu, \tau$) for three special patterns of the neutrino mass spectrum: (1) normal hierarchy with $m_0 \approx 0$; (2) inverted hierarchy with $m_3 \approx 0$; and (3) near degeneracy with $m_1 \approx m_2 \approx m_3$, where $\Delta m^2_{21} = 8.0 \times 10^{-5}$ eV^2 and $|\Delta m^2_{32}| = 2.5 \times 10^{-3}$ eV^2 have typically been input. Note that $|M_{e\tau}|^2 = |M_{e\mu}|^2$ and $|M_{\tau\tau}|^2 = |M_{\mu\mu}|^2$ hold for the tetra-maximal neutrino mixing matrix under discussion.

	Neutrino mass hierarchy		
	$m_1 \approx 0$	$m_3 \approx 0$	$m_1 \approx m_2 \approx m_3$
$\|M_{ee}\|^2$ (eV^2)	1.11×10^{-5}	2.34×10^{-3}	m_1^2
$\|M_{e\mu}\|^2$ (eV^2)	3.21×10^{-5}	2.56×10^{-5}	0
$\|M_{\mu\mu}\|^2$ (eV^2)	4.64×10^{-4}	5.94×10^{-4}	0
$\|M_{\mu\tau}\|^2$ (eV^2)	7.96×10^{-4}	6.46×10^{-4}	m_1^2
$\sum_{i=1}^{3} m_i^2$ (eV^2)	2.66×10^{-3}	4.92×10^{-3}	$3m_1^2$

TABLE II. The results of $\mathcal{B}(H^{--} \to l^-_\alpha l^-_\beta)$ (for $\alpha, \beta = e, \mu, \tau$) for three special patterns of the neutrino mass spectrum: (1) normal hierarchy with $m_0 \approx 0$; (2) inverted hierarchy with $m_3 \approx 0$; and (3) near degeneracy with $m_1 \approx m_2 \approx m_3$, where $\Delta m^2_{21} = 8.0 \times 10^{-5}$ eV^2, $|\Delta m^2_{32}| = 2.5 \times 10^{-3}$ eV^2 and the tetra-maximal mixing parameters have typically been input.

	Neutrino mass hierarchy		
	$m_1 \approx 0$	$m_3 \approx 0$	$m_1 \approx m_2 \approx m_3$
$\mathcal{B}(H^{--} \to e^- e^-)$	0.42%	47.56%	33.33%
$\mathcal{B}(H^{--} \to e^- \mu^-)$	2.41%	1.04%	0
$\mathcal{B}(H^{--} \to e^- \tau^-)$	2.41%	1.04%	0
$\mathcal{B}(H^{--} \to \mu^- \mu^-)$	17.44%	12.07%	0
$\mathcal{B}(H^{--} \to \mu^- \tau^-)$	59.85%	26.26%	66.67%
$\mathcal{B}(H^{--} \to \tau^- \tau^-)$	17.44%	12.07%	0