Small amplitude effects in $B^0 \to D^+D^-$ and related decays

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Intrigued by a recent Belle result for a large direct CP asymmetry in $B^0 \to D^+D^-$, we study the effects of a $\bar{b} \to \bar{u}u\bar{d}$ quark transition by combining the asymmetry information with rates and asymmetries in isospin-related decays. Arguing for a hierarchy among several contributions to these decays, including an exchange amplitude which we estimate, we present tests for factorization of the leading terms, and obtain an upper bound on the ratio of $\bar{b} \to \bar{u}u\bar{d}$ and $\bar{b} \to \bar{c}c\bar{d}$ amplitudes. We prove an approximate $\Delta I = 1/2$ amplitude relation for $B \to D\bar{D}$, and an approximate equality between CP asymmetries in $B^0 \to D^+D^-$ and $B^+ \to D^+\bar{D}^0$. Violations of these relations by Belle measurements, at 1.8σ and 3.6σ respectively, if confirmed, would indicate a possible New Physics contribution in $\bar{b} \to \bar{u}u\bar{d}$. Applying flavor SU(3), we extend this study to a total of ten processes including $\Delta S = 0$ decays involving final D_s and initial B_s mesons, and $\Delta S = 1$ decays of B and B_s mesons into pairs of charmed pseudoscalar mesons. The decays $B_s \to D\bar{D}$ provide useful information about a small exchange amplitude, responsible for a decay rate difference between $B^+ \to D^+\bar{D}^0$ and $B^0 \to D^+D^-$. A method for determining the weak phase γ, based on CP asymmetries in $B^0(t) \to D^+D^-$ and the decay rate for $B_s \to D_s^+ D_s^-$ or $B^{+(0)} \to D_s^+ \bar{D}^0(D^-)$, is shown to involve high sensitivity to SU(3) breaking.

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1 Introduction

Accurate measurements of the weak phase $\beta \equiv \arg(V_{tb}^* V_{td}/V_{cb}^* V_{cd})$, $\sin 2\beta = 0.680 \pm 0.025$, $\cos 2\beta > 0$ [1], have provided a precision test for the Cabibbo-Kobayashi-Maskawa [2,3] framework and for the Kobayashi-Maskawa mechansim of CP violation. The accuracy of this test relies on the pure dominance by a single weak phase of a few $\bar{b} \to \bar{c}c\bar{s}$

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Decay mode		BaBar	Belle	Average
$B^0 \to D^+D^-$	$\mathcal{B}$	$2.8 \pm 0.4 \pm 0.5$	$1.97 \pm 0.20 \pm 0.20$	2.11 ± 0.31
	A_{CP}	$-0.11 \pm 0.22 \pm 0.07$	$0.91 \pm 0.23 \pm 0.06$	0.37 ± 0.17
	S	$-0.54 \pm 0.34 \pm 0.06$	$-1.13 \pm 0.37 \pm 0.09$	-0.75 ± 0.26
$B^+ \to D^+D^0$	$\mathcal{B}$	$3.8 \pm 0.6 \pm 0.5$	$3.85 \pm 0.31 \pm 0.38$	3.84 ± 0.42
	A_{CP}	$-0.13 \pm 0.14 \pm 0.02$	$0.00 \pm 0.08 \pm 0.02$	-0.03 ± 0.07
$B^0 \to D^0D^0$	$\mathcal{B}$	< 0.6 (90% c. l.)	< 0.43 (90% c. l.)	< 0.43 (90% c. l.)
$B^0 \to D_s^+D_s^-$	$\mathcal{B}$	< 1.0 (90% c. l.)	< 0.36 (90% c. l.)	< 0.36 (90% c. l.)

Table I: Charge-averaged branching ratios $\mathcal{B}$ in units of 10^{-4} and CP asymmetries A_{CP}, S in $B \to D\bar{D}$, from Refs. [10–13]. Also included are upper limits on $\mathcal{B}(B^0 \to D_s^+D_s^-)$ [14, 15].

processes including $B^0 \to J/\psi K_S$ [4, 5]. This implies a mixing-induced asymmetry, $S = \sin 2\beta$, and a vanishingly small direct CP asymmetry, as confirmed experimentally, $A_{CP} = -0.012 \pm 0.020$ [1].

The decay $B^0 \to D^+D^-$ is dominated by $\bar{b} \to \bar{c}c\bar{d}$, but involves a smaller non-negligible amplitude from $\bar{b} \to \bar{u}u\bar{d}$ carrying a different weak phase. The second amplitude introduces hadronic uncertainties in predictions for the asymmetries S and A_{CP} in this process. Early model-independent estimates of the ratio of the two amplitudes contributing to this process vary from a few percent to upper bounds of about 0.2 [4,6] or 0.3 [7]. A more recent model-dependent calculation finds 0.03 [8]. Values larger than 0.3 may be obtained in extensions of the Standard Model [9]. The two asymmetries depend also on the strong phase difference between the $\bar{b} \to \bar{c}c\bar{d}$ and $\bar{b} \to \bar{u}u\bar{d}$ amplitudes [4].

Asymmetries in $B^0 \to D^+D^-$, measured by the BaBar and Belle collaborations, are quoted in the upper part of Table I. The table also includes branching ratios for $B^0 \to D^+D^-, B^+ \to D^+\bar{D}^0, B^0 \to D^0\bar{D}^0$, a direct CP asymmetry measured for $B^+ \to D^+\bar{D}^0$ [10–13], and upper limits on $\mathcal{B}(B^0 \to D_s^+D_s^-)$ measured by BaBar [14] and Belle [15]. The BaBar asymmetries in $B^0 \to D^+D^-$ [10] are consistent with $A_{CP} = 0, S = -\sin 2\beta$, showing no evidence for a $\bar{b} \to \bar{u}u\bar{d}$ term in the decay amplitude. In contrast, the Belle asymmetry measurements [11], which fluctuate outside the physical region, $A_{CP}^2 + S^2 \leq 1$, deviate substantially from the above nominal values, indicating a sizable $\bar{b} \to \bar{u}u\bar{d}$ amplitude. The direct asymmetry A_{CP} measured by Belle is nonzero at a level higher than 3σ. Its central value indicates the possibility of a second amplitude larger than permitted in the Cabibbo-Kobayashi-Maskawa (CKM) framework.

A major goal of this paper, largely intrigued by the Belle results, is to study carefully the dynamics and CKM structure of the $B^0 \to D^+D^-$ decay amplitude and of decay amplitudes for the two isospin-related processes, $B^+ \to D^+\bar{D}^0$ and $B^0 \to D^0\bar{D}^0$. In references [8] and [16] these processes have been stated to originate in a $\Delta I = 1/2$ effective Hamiltonian implying an isospin triangle relation among the three amplitudes. It will be shown that, while $\Delta I = 1/2$ is not a property of the effective Hamiltonian, an approximate $\Delta I = 1/2$ rule is expected to hold for the three decay amplitudes and should be tested experimentally. Applying flavor SU(3) to the above processes, we will

extend our study to include strangeness-conserving decays involving final D_s and initial B_s mesons, and strangeness-changing decays of B and B_s mesons into pairs of charmed pseudoscalar mesons.

In Section 2 we study the asymmetries measured by BaBar and Belle in $B^0 \to D^+D^-$ in terms of two parameters, the ratio r of $\bar{b} \to \bar{u}u\bar{d}$ and $\bar{b} \to \bar{c}c\bar{d}$ amplitudes and their relative strong phase δ. Section 3 introduces expressions for the amplitudes of the three processes $B^0 \to D^+D^-, B^0 \to D^0\bar{D}^0$ and $B^+ \to D^+\bar{D}^0$ in terms of isospin amplitudes. We identify circumstances under which an isospin triangle relation between these amplitudes can be violated by a (small) $\Delta I = 3/2$ contribution. Section 4 studies $B \to D\bar{D}$ decays and two other SU(3) related $\Delta S = 0$ decays of B^0 and B_s in terms of graphical contributions, while Section 5 extends this study to corresponding strangeness changing decays of B^0, B^+ and B_s. Section 6 discusses a hierarchy among graphical amplitudes, presenting tests of factorization for the dominant terms. In Section 7 we discuss briefly consequences of this hierarchy on a theoretical upper limit on r, illuminating an inconsistency between the CP asymmetries measured by Belle in $B^0 \to D^+D^-$ and $B^+ \to D^+\bar{D}^0$. Section 8 discusses a way for determining the weak phase γ by combining information from asymmetries in $B^0 \to D^+D^-$ and decay rates of corresponding $\Delta S = 1$ decays to charm-anticharm, while Section 9 concludes. An Appendix provides a dictionary between graphical amplitudes and SU(3) reduced matrix elements of four-quark operators appearing in the effective Hamiltonian.

2 Ratio of $\bar{b} \to \bar{u}u\bar{d}$ and $\bar{b} \to \bar{c}c\bar{d}$ terms in $B^0 \to D^+D^-$

We start our discussion by translating the $B^0 \to D^+D^-$ asymmetries, measured separately by BaBar and Belle, into values of the ratio r of $\bar{b} \to \bar{u}u\bar{d}$ and $\bar{b} \to \bar{c}c\bar{d}$ amplitudes and the relative strong phase δ between these amplitudes. Denoting

$$
\begin{aligned}
A(B^0 \to D^+D^-) &= A_c + A_u\,e^{i(\delta+\gamma)} = A_c\left[1 + r\,e^{i(\delta+\gamma)}\right] , \qquad (r \equiv A_u/A_c) , \\
A(\bar{B}^0 \to D^+D^-) &= A_c + A_u\,e^{i(\delta-\gamma)} = A_c\left[1 + r\,e^{i(\delta-\gamma)}\right] , \\
\lambda_{D^+D^-} &\equiv e^{-2i\beta}\frac{A(\bar{B}^0 \to D^+D^-)}{A(B^0 \to D^+D^-)} ,
\end{aligned}
\tag{1}
$$

one has [4]

$$
\begin{aligned}
S(D^+D^-) &\equiv \frac{2\mathrm{Im}(\lambda_{D^+D^-})}{1 + |\lambda_{D^+D^-}|^2} = -\frac{\sin 2\beta + 2r\cos\delta\sin(2\beta+\gamma) + r^2\sin 2(\beta+\gamma)}{1 + 2r\cos\delta\cos\gamma + r^2} , \\
A_{CP}(D^+D^-) &\equiv \frac{|\lambda_{D^+D^-}|^2 - 1}{|\lambda_{D^+D^-}|^2 + 1} = \frac{2r\sin\delta\sin\gamma}{1 + 2r\cos\delta\cos\gamma + r^2} .
\end{aligned}
\tag{2}
$$

Keeping only linear terms in r,

$$
\begin{aligned}
S(D^+D^-) &\simeq -\sin 2\beta - 2r\cos 2\beta\cos\delta\sin\gamma , \\
A_{CP}(D^+D^-) &\simeq 2r\sin\delta\sin\gamma ,
\end{aligned}
\tag{3}
$$

3
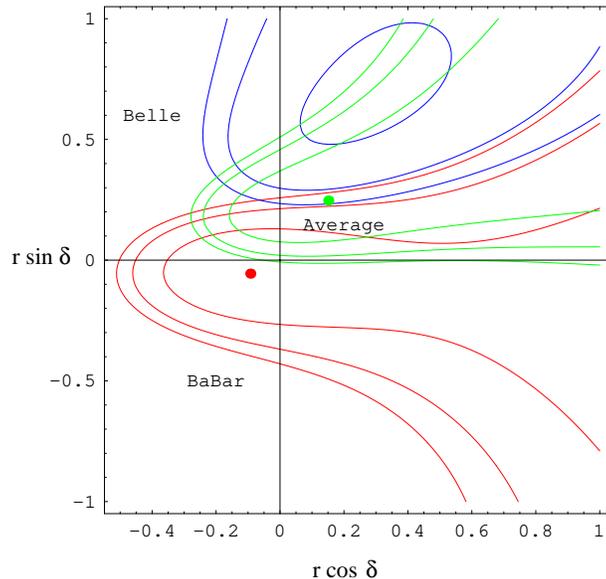
Figure 1: χ^2 plots in the $(r\cos\delta, r\sin\delta)$ plane assuming $\beta = 21.5°, \gamma = 68°$. The red, blue and green curves show constraints following from $B^0 \to D^+D^-$ asymmetries measured by BaBar, Belle and their averages. In each case the most inside, intermediate and most outside curves represent bounds at $68\%, 90\%$ and 95% confidence levels. Red and green points describe solutions corresponding to central values of the BaBar asymmetries and the averaged asymmetries.

implies

$$r \approx \frac{\sqrt{[(S + \sin 2\beta)/\cos 2\beta]^2 + A_{CP}^2}}{2\sin\gamma} \ . \tag{4}$$

Consider the measured asymmetries and the current values of β and γ, $\beta = (21.5 \pm 1.0)°$ [1], $\gamma = (67.6^{+2.8}_{-4.5})°$ [17] (see also [18,19]). Using this information, the approximation (3) and (4), or the precise expressions (2), determine r and δ. The resulting errors in r and δ are dominated by the errors in the measured asymmetries. Taking central values for the asymmetries and values $\beta = 21.5°, \gamma = 68°$, Eq. (4) implies central values around $r = 0.1$ (BaBar) and $r = 0.6$ (Belle). In both cases the error in r is about 0.2. The central value of r for Belle, for which the linear approximations (3) involve non-negligible quadratic corrections, should be considered with care because this value of r is based on non-physical values of the asymmetries obeying $A_{CP}^2 + S^2 > 1$.

In order to study the implications of the asymmetries on the pair of parameters (r, δ), we have performed a two dimensional χ^2 analysis for these two parameters using the asymmetry measurements and assuming $\beta = 21.5°, \gamma = 68°$. In Fig. 1 we plot the resulting contours in the $(r\cos\delta, r\sin\delta)$ plane for $\chi^2 = 2.30, 4.61, 5.99$, corresponding to $68\%, 90\%, 95\%$ confidence levels. The red, blue and green curves show constraints following from the asymmetries measured by BaBar, Belle and their averages. In each case the innermost, intermediate and outermost curves describe bounds at $68\%, 90\%$ and 95% confidence levels. The red and green points are solutions corresponding to central values of the BaBar asymmetries and the averaged asymmetries. We do not show a

4

central Belle point because the central values of the Belle asymmetries lie outside the physical region.

The Belle two-parameter boundary curve for 90% confidence level contains a point with closest distance to the origin, $r = 0.29, \delta = 82°$. The point on this curve with smallest δ has $\delta = 37°, r = 0.87$. (Note that the values $r = 0.29$ and $\delta = 37°$ are lower than the 90% confidence level lower limits on these separate single variables.) Thus, the Belle data alone would provide evidence for a sizable r and for a large strong phase difference δ. We note, however, that the BaBar and Belle regions of 90% confidence level do not overlap. In order to draw firm conclusions about r and δ one should therefore wait for better agreement between the asymmetries measured by the two collaborations.

3 Isospin amplitudes in $B \to D\bar{D}$

The low energy effective Hamiltonian governing $B^0 \to D^+D^-$, $B^0 \to D^0\bar{D}^0$ and $B^+ \to D^+\bar{D}^0$ involves two CKM factors $V_{cb}^*V_{cd}$ and $V_{ub}^*V_{ud}$, both of order λ^3 ($\lambda = |V_{us}| = 0.2258 \pm 0.0010$ [17]). Each of these factors multiplies a combination of four quark operators with coefficients given by calculable Wilson coefficients C_i [20],

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[V_{cb}^*V_{cd} \sum_{i=1}^{2} C_i \mathcal{O}_i^c + V_{ub}^*V_{ud} \sum_{i=1}^{2} C_i \mathcal{O}_i^u + (V_{cb}^*V_{cd} + V_{ub}^*V_{ud}) \sum_{k=3}^{10} C_k \mathcal{O}_k \right] . \quad (5)$$

The current-current operators $\mathcal{O}_i^c$ and $\mathcal{O}_i^u$ ($i = 1, 2$) have flavor dependence $(\bar{b}c)(\bar{c}d)$ and $(\bar{b}u)(\bar{u}d)$, respectively. Thus, while the first pair of operators are pure $\Delta I = \frac{1}{2}$, the second pair involves both $\Delta I = \frac{1}{2}$ and $\Delta I = \frac{3}{2}$. The QCD penguin operators $\mathcal{O}_k$ ($k = 3 - 6$) with flavor structure $(\bar{b}d)$ are pure $\Delta I = 1/2$, while the electroweak penguin operators $\mathcal{O}_k \sim (\bar{b}d) \sum_q e_q(\bar{q}q)$ ($k = 7 - 10$, $q = u, d, s, c$), which depend on the quark charges e_q, involve both $\Delta I = 1/2$ and $\Delta I = 3/2$. Thus, in contrast to statements made in Refs. [8] and [16], the effective Hamiltonian underlying $B \to D\bar{D}$ decays is not pure $\Delta I = 1/2$. It contains an additional $\Delta I = 3/2$ component, also when neglecting electroweak penguin contributions which have very small Wilson coefficients [20].

The final $D\bar{D}$ states consist of $I = 0$ and $I = 1$. This implies a total of three isospin amplitudes, $A_{0,\frac{1}{2}}, A_{1,\frac{1}{2}}$ and $A_{1,\frac{3}{2}}$, where the two subscripts denote $I(D\bar{D})$ and ΔI, respectively. Neglecting very small electroweak penguin contributions, the $\Delta I = 3/2$ amplitude $A_{1,\frac{3}{2}}$ occurs in association with a CKM factor $V_{ub}^*V_{ud}$ but not with $V_{cb}^*V_{cd}$.

The three physical $B \to D\bar{D}$ decay amplitudes can be written in terms of the three isospin amplitudes,

$$\begin{aligned}
A^{+-} &\equiv A(B^0 \to D^+D^-) = \frac{1}{2} A_{0,\frac{1}{2}} + \frac{1}{2} A_{1,\frac{1}{2}} + \frac{1}{2} A_{1,\frac{3}{2}} , \\
A^{00} &\equiv A(B^0 \to D^0\bar{D}^0) = -\frac{1}{2} A_{0,\frac{1}{2}} + \frac{1}{2} A_{1,\frac{1}{2}} + \frac{1}{2} A_{1,\frac{3}{2}} , \\
A^{+0} &\equiv A(B^+ \to D^+\bar{D}^0) = A_{1,\frac{1}{2}} - \frac{1}{2} A_{1,\frac{3}{2}} .
\end{aligned} \quad (6)$$

These relations can be inverted,

$$A_{0,\frac{1}{2}} = A^{+-} - A^{00} ,$$

$$A_{1,\frac{1}{2}} = \frac{1}{3}(A^{+-} + A^{00} + 2A^{+0}) \,,$$

$$A_{1,\frac{3}{2}} = \frac{2}{3}(A^{+-} + A^{00} - A^{+0}) \,. \tag{7}$$

While $\Delta I = 1/2$ is not a property of the low energy effective Hamiltonian, we will argue below that $|A_{1,\frac{3}{2}}| \ll |A_{0,\frac{1}{2}}|, |A_{1,\frac{1}{2}}|$ is a reasonable assumption which should be tested experimentally. Neglecting the $\Delta I = 3/2$ amplitude, one obtains an approximate triangle relation [8, 16],

$$A^{+-} + A^{00} = A^{+0} \,. \tag{8}$$

A potential proof for a nonzero $\Delta I = 3/2$ amplitude would be a violation of (8). This happens when the amplitude triangle does not close, for instance when $|A^{+-}| + |A^{00}| < |A^{+0}|$.

In order to illustrate such a possibility consider the Belle measurements for $B \to D\bar{D}$. We define

$$|A^{+-}| = 10^2 \sqrt{\mathcal{B}(B^0 \to D^+D^-)\left[1 - A_{CP}(B^0 \to D^+D^-)\right]} \,,$$

$$|A^{+0}| = 10^2 \sqrt{\mathcal{B}(B^+ \to D^+\bar{D}^0)\left[1 - A_{CP}(B^+ \to D^+\bar{D}^0)\right](\tau_0/\tau_+)} \,,$$

$$|A^{00}| = 10^2 \sqrt{\mathcal{B}(B^0 \to D^0\bar{D}^0)\left[1 - A_{CP}(B^0 \to D^0\bar{D}^0)\right]} \,. \tag{9}$$

Using the Belle values for CP-averaged branching ratios and CP asymmetries given in Table I, and a ratio of B^+ and B^0 lifetimes [1] $\tau_+/\tau_0 = 1.071 \pm 0.009$, we find

$$|A^{+-}| = 0.42 \pm 0.56 \,, \quad |A^{+0}| = 1.90 \pm 0.14 \,, \quad |A^{00}| < 0.57 \,(1\sigma) \,. \tag{10}$$

The small magnitude of A^{+-} follows from the large positive CP asymmetry measured by Belle, implying observing mostly $\bar{B}^0$ decays into D^+D^- with only a few B^0 decays into this final state. The upper bound on $|A^{00}|$ is obtained from a 1σ upper limit on $\mathcal{B}(B^0 \to D^0\bar{D}^0)$. We have assumed that the CP asymmetry in $B^0 \to D^0\bar{D}^0$ is not large and negative (in Section 7 we will argue for a vanishingly small asymmetry), and we have neglected possible correlations between errors in branching ratio and asymmetry measurements in the other two modes. We note that the triangle (8) does not close for central values of the measured amplitudes (10), and allowing for deviations from these values up to 1.8σ .

In contrast, the triangle relation holds well when using the central values of the BaBar measurements. A critical test for the closure of the amplitude triangle requires a reduction in errors and a better agreement between BaBar and Belle.

4 $\Delta S = 0$ decays into $D\bar{D}$ using graphical amplitudes

Useful expressions for amplitudes in $B \to D\bar{D}$ using flavor SU(3), which can be generalized to $B^0 \to D_s^+D_s^-$ and $B_s \to D^+D_s^-$, are in terms of graphical contributions representing the flow of isospin and flavor SU(3) in these decays [21]. This includes

a (color-favored) tree amplitude T involving $V_{cb}^* V_{cd}$, penguin and penguin-annihilation amplitudes P and PA with u, c and t-quark loops, exchange amplitudes E_c and E_u involving $V_{cb}^* V_{cd}$ and $V_{ub}^* V_{ud}$, and an annihilation amplitude A_u involving $V_{ub}^* V_{ud}$. The amplitude E_c is associated with popping a pair of $u\bar{u}$ or $d\bar{d}$ out of the vacuum, while E_u and A_u involve $c\bar{c}$ popping. At this point we neglect very small electroweak penguin contributions to which we return in the next section.

The graphical amplitudes have well-defined isospin properties. The two graphical amplitudes E_u and A_u involve both $\Delta I = 1/2$ and $\Delta I = 3/2$, while all other amplitudes are pure $\Delta I = 1/2$ by construction. We will show below that the $\Delta I = 3/2$ amplitude consists solely of the combination $E_u + A_u$. This combination is thus responsible for a potential violation of the amplitude triangle relation (8).

We denote

$$P = V_{ub}^* V_{ud}\, p_u + V_{cb}^* V_{cd}\, p_c + V_{tb}^* V_{td} p_t = V_{cb}^* V_{cd}\, p_{ct} + V_{ub}^* V_{ud}\, p_{ut}\ , \quad (p_{ij} \equiv p_i - p_j)\ , \quad (11)$$

absorbing the first term in the tree amplitude by defining,

$$V_{cb}^* V_{cd}\, t_c \equiv T + V_{cb}^* V_{cd}\, p_{ct}\ . \tag{12}$$

Similarly, writing

$$PA = V_{ub}^* V_{ud}\, pa_u + V_{cb}^* V_{cd}\, pa_c + V_{tb}^* V_{td} pa_t = V_{cb}^* V_{cd}\, pa_{ct} + V_{ub}^* V_{ud}\, pa_{ut}\ , \quad (pa_{ij} \equiv pa_i - pa_j)\ , \tag{13}$$

the first term is absorbed in E_c,

$$V_{cb}^* V_{cd}\, e_c \equiv E_c + V_{cb}^* V_{cd}\, pa_{ct}\ . \tag{14}$$

Other terms involving the CKM factor $V_{ub}^* V_{ud}$ are

$$E_u \equiv V_{ub}^* V_{ud}\, e_u\ , \quad A_u \equiv V_{ub}^* V_{ud}\, a_u\ . \tag{15}$$

Using these definitions with a shorthand notation, $p_u \equiv p_{ut}, pa_u \equiv pa_{ut}$, we find:

$$\begin{aligned}
a. \quad A(B^0 \to D^+ D^-) &= V_{cb}^* V_{cd}\, (t_c + e_c) + V_{ub}^* V_{ud}\, (p_u + pa_u)\ , \\
b. \quad A(B^0 \to D^0 \bar{D}^0) &= V_{cb}^* V_{cd}\, (-e_c) + V_{ub}^* V_{ud}\, (-pa_u - e_u)\ , \\
c. \quad A(B^+ \to D^+ \bar{D}^0) &= V_{cb}^* V_{cd}\, (t_c) + V_{ub}^* V_{ud}\, (p_u + a_u)\ , \\
d. \quad A(B^0 \to D_s^+ D_s^-) &= V_{cb}^* V_{cd}\, (e_c) + V_{ub}^* V_{ud}\, (pa_u)\ , \\
e. \quad A(B_s \to D^+ D_s^-) &= V_{cb}^* V_{cd}\, (t_c) + V_{ub}^* V_{ud}\, (p_u)\ .
\end{aligned} \tag{16}$$

The minus sign in the amplitude involving a D^0 follows from our convention, $D^0 \equiv -c\bar{u}$ [21].

The five physical amplitudes depend on the four combinations, $V_{cb}^* V_{cd}\, t_c + V_{ub}^* V_{ud}\, p_u$, $V_{cb}^* V_{cd}\, e_c + V_{ub}^* V_{ud}\, pa_u, V_{ub}^* V_{ud}\, e_u, V_{ub}^* V_{ud}\, a_u$. This equals the number of independent SU(3) reduced matrix elements describing $\Delta S = 0$ B and B_s decays to pairs of charmed pseudoscalar mesons, $\langle 1\|\overline{\mathbf{3}}\|3\rangle, \langle 8\|\overline{\mathbf{3}}\|3\rangle, \langle 8\|\mathbf{6}\|3\rangle, \langle 8\|\overline{\mathbf{15}}\|3\rangle$ (see Appendix A). Consequently

the five decay amplitudes are not mutually independent. They obey one triangle relation in the SU(3) symmetry limit,

$$A(B^0 \to D_s^+ D_s^-) + A(B_s \to D^+ D_s^-) = A(B^0 \to D^+ D^-) . \tag{17}$$

The parameters r and δ studied in Section 2 can be expressed in terms of graphical amplitudes,

$$r = \frac{|V_{ub}^* V_{ud}|}{|V_{cb}^* V_{cd}|} \frac{|p_u + pa_u|}{|t_c + e_c|} , \qquad \delta = \arg\left(-\frac{p_u + pa_u}{t_c + e_c}\right) , \tag{18}$$

and the isospin amplitudes defined in section 3 are given by

$$
\begin{aligned}
A_{0,\frac{1}{2}} &= V_{cb}^* V_{cd} \left(t_c + 2e_c\right) + V_{ub}^* V_{ud} \left(p_u + 2pa_u + e_u\right) , \\
A_{1,\frac{1}{2}} &= V_{cb}^* V_{cd}\, t_c + \frac{1}{3} V_{ub}^* V_{ud} \left(3p_u - e_u + 2a_u\right) , \\
A_{1,\frac{3}{2}} &= -\frac{2}{3} V_{ub}^* V_{ud} \left(e_u + a_u\right) .
\end{aligned}
\tag{19}
$$

The last relation confirms our statement above that the $\Delta I = 3/2$ amplitude involves only the combination $E_u + A_u$.

5 $\Delta S = 1$ B and B_s decays into charm-anticharm

The parametrization (16) of $\Delta S = 0$ decays into charm-anticharm in terms of flavor SU(3) graphical amplitudes can be extended to $\Delta S = 1$ CKM-favored decays of B and B_s mesons, which are governed by $\bar{b} \to \bar{c} c \bar{s}$. U-spin reflection symmetry $d \leftrightarrow s$ implies expressions similar to Eqs. (16) for corresponding U-spin related amplitudes [22], in which one replaces $V_{cb}^* V_{cd}$ by $V_{cb}^* V_{cs}$ and $V_{ub}^* V_{ud}$ by $V_{ub}^* V_{us}$. Since U-spin transforms $B^0 \leftrightarrow B_s$, $D^\pm \leftrightarrow D_s^\pm$ while keeping B^+, D^0 and $\bar{D}^0$ invariant, one has:

$$
\begin{aligned}
a. \quad & A(B_s \to D_s^+ D_s^-) &&= V_{cb}^* V_{cs} \left(t_c + e_c\right) + V_{ub}^* V_{us} \left(p_u + pa_u\right) , \\
b. \quad & A(B_s \to D^0 \bar{D}^0) &&= V_{cb}^* V_{cs} \left(-e_c\right) + V_{ub}^* V_{us} \left(-pa_u - e_u\right) , \\
c. \quad & A(B^+ \to D_s^+ \bar{D}^0) &&= V_{cb}^* V_{cs} \left(t_c\right) + V_{ub}^* V_{us} \left(p_u + a_u\right) , \\
d. \quad & A(B_s \to D^+ D^-) &&= V_{cb}^* V_{cs} \left(e_c\right) + V_{ub}^* V_{us} \left(pa_u\right) , \\
e. \quad & A(B^0 \to D_s^+ D^-) &&= V_{cb}^* V_{cs} \left(t_c\right) + V_{ub}^* V_{us} \left(p_u\right) .
\end{aligned}
\tag{20}
$$

We note that while both $V_{cb}^* V_{cd}$ and $V_{ub}^* V_{ud}$ in (16) are of order λ^3, the factors $V_{cb}^* V_{cs}$ and $V_{ub}^* V_{us}$ in (20) are order λ^2 and λ^4 respectively. More precisely, one has

$$-\frac{V_{cb}^* V_{cd}}{V_{cb}^* V_{cs}} = \frac{V_{ub}^* V_{us}}{V_{ub}^* V_{ud}} = \frac{\lambda}{1 - \lambda^2/2} . \tag{21}$$

This implies that in the U-spin symmetry limit CP rate differences in the five U-spin pairs of corresponding $\Delta S = 0$ and $\Delta S = 1$ decays, such as $B^0 \to D^+ D^-$ and $B_s \to D_s^+ D_s^-$, have equal magnitudes but opposite signs [22],

$$\Gamma(\bar{B}_s \to D_s^+ D_s^-) - \Gamma(B_s \to D_s^+ D_s^-) = -[\Gamma(\bar{B}^0 \to D^+ D^-) - \Gamma(B^0 \to D^+ D^-)] . \tag{22}$$

Decay mode:	$B^0 \to D_s^+ D^-$	$B^+ \to D_s^+ \bar{D}^0$	$B_s \to D_s^+ D_s^-$
Branching ratio:	79 ± 7 [15, 24, 25]	109 ± 18 [24, 25]	94^{+44}_{-42} [26]

Table II: Charged averaged branching ratios in units of 10^{-4} for $\Delta S = 1$ B and B_s decays into charm-anticharm.

Since the rates of $\Delta S = 1$ decays are about λ^{-2} larger than those of $\Delta S = 0$ decays, the CP asymmetries in the former are expected to be about λ^2 smaller than in the latter. The case of Eq. (22) has been discussed in Ref. [23]. Similar CP rate difference relations hold between the other four pairs of processes. An SU(3) triangle relation analogous to (17) holds for $\Delta S = 1$ decays,

$$A(B_s \to D^+ D^-) + A(B^0 \to D_s^+ D^-) = A(B_s \to D_s^+ D_s^-) \ . \tag{23}$$

So far we have neglected electroweak penguin contributions in both $\Delta S = 0$ and $\Delta S = 1$ decays. This is justifiable in the first case by very small Wilson coefficients associated with electroweak penguin operators [20] multiplying a CKM factor $V_{tb}^* V_{td}$ of the usual order λ^3. However, electroweak penguin terms in strangeness-changing decays involve a CKM factor $V_{tb}^* V_{ts} \sim \mathcal{O}(\lambda^2)$, much larger than terms involving $V_{ub}^* V_{us} \sim \mathcal{O}(\lambda^4)$ which are kept in (20). Thus, for consistency one must keep also electroweak penguin terms in $\Delta S = 1$ decays.

In general, there are four types of diagrams describing electroweak penguin (EWP) contributions, corresponding to the above-mentioned four SU(3) reduced matrix elements. The four diagrams may be associated with a color-suppressed EWP amplitude P_{EW}^C, an EWP-exchange amplitude $PE_{EW}(c)$ associated with $c\bar{c}$ popping, and two EWP-annihilation amplitudes, PA_{EW} and $PA_{EW}(c)$, associated with $u\bar{u}, d\bar{d}, s\bar{s}$ popping and $c\bar{c}$ popping, respectively. We neglect the two EWP amplitudes involving $c\bar{c}$ popping for a reason discussed in Section 6. Using $V_{tb}^* V_{td(s)} = -V_{cb}^* V_{cd(s)} - V_{ub}^* V_{ud(s)}$, the remaining two EWP amplitudes, P_{EW}^C and PA_{EW}, may be absorbed in the following way in definitions of four amplitudes occurring in (16) and (20) without changing these ten equations:

$$t_c - \frac{2}{3} P_{EW}^C \to t_c \ , \quad e_c - \frac{2}{3} PA_{EW} \to e_c \ , \, p_u - \frac{2}{3} P_{EW}^C \to p_u \ , \quad pa_u - \frac{2}{3} PA_{EW} \to pa_u \ . \tag{24}$$

We conclude this section by quoting in Table II branching ratios for three of the five processes occurring in Eqs. (20), including a very recent measurement of $\mathcal{B}(B_s \to D_s^+ D_s^-)$ by the CDF Collaboration [26].

6 Expected hierarchy among graphical amplitudes

6.1 $\Delta S = 1$ decays

6.1.1 Ratio of two CKM factors

Consider first the five $\Delta S = 1$ amplitudes (20) for B and B_s decays into pairs of charmed pseudoscalar mesons. Each amplitude involves a dominant term with a CKM

factor $V_{cb}^* V_{cs}$ and a much smaller term involving $V_{ub}^* V_{us}$. The CKM suppression of the second amplitude, which is often being neglected, is [17]:

$$\frac{|V_{ub}^* V_{us}|}{|V_{cb}^* V_{cs}|} = (0.40 \pm 0.05)\lambda^2/(1 - \lambda^2) = 0.021 \pm 0.003 \ . \tag{25}$$

6.1.2 The amplitudes t_c and e_c

The large CKM factor $V_{cb}^* V_{cs}$ multiplies two amplitudes, a dominant term t_c and a smaller exchange contribution e_c accompanied by an EWP-annihilation contribution PA_{EW} [see (24)]. The latter two terms are suppressed by $\Lambda_{\rm QCD}/m_B$ as they involve the interaction of a light spectator quark. Thus, we expect the two decay modes $B_s \to D^+ D^-$ and $B_s \to D^0 \bar{D}^0$ governed by e_c to have branching ratios much smaller [27] than those quoted in Table II for $B^0 \to D_s^+ D^-, B^+ \to D_s^+ \bar{D}^0$ and $B_s \to D_s^+ D_s^-$ which are dominated by t_c. The corresponding small ratios of branching ratios, e.g. $\mathcal{B}(B_s \to D^+ D^-)/\mathcal{B}(B^0 \to D_s^+ D^-)$, would determine $|e_c/t_c|^2$.

A reasonable although not precise estimate for $|e_c/t_c|$ may proceed as follows. Consider the two processes $B^0 \to D^- \pi^+$ and $B^0 \to D_s^- K^+$, both of which originate in the quark subprocess $\bar{b} \to \bar{c} u \bar{d}$. While the first decay is governed by a color-favored tree amplitude with a small exchange contribution, the second one obtains only a contribution from an exchange amplitude. Drawing a parallel between these amplitudes and the corresponding amplitudes in $\bar{b} \to \bar{c} c \bar{s}$, and using [25] $\mathcal{B}(B^0 \to D^- \pi^+) = (26.8 \pm 1.3) \times 10^{-4}, \mathcal{B}(B^0 \to D_s^- K^+) = (2.8 \pm 0.5) \times 10^{-5}$, we estimate

$$\frac{|e_c|}{|t_c + e_c|} \sim \sqrt{\frac{\mathcal{B}(B^0 \to D_s^- K^+)}{\mathcal{B}(B^0 \to D^- \pi^+)}} = 0.102 \pm 0.009 \ . \tag{26}$$

In this crude approximation this would imply

$$0.093 \pm 0.008 \le \frac{|e_c|}{|t_c|} \le 0.114 \pm 0.010 \ . \tag{27}$$

We note three corrections which may affect this estimate:

1. The amplitude e_c includes by definition a term PA_{EW} (both terms require an interaction of the spectator quark), while no such term contributes to $B^0 \to D_s^- K^+$.

2. The exchange amplitude in $B^0 \to D_s^- K^+$ involves $s\bar{s}$ popping in a $\bar{c} u$ system, whereas e_c is described by $u\bar{u}$ or $d\bar{d}$ popping in a $\bar{c} c$ system.

3. $B^0 \to D^- \pi^+$ is dominated by a purely a color-favored tree amplitude, while t_c involves also a smaller penguin term p_{ct}. See discussion below.

These three differences are expected to affect (27) by a factor which is hard to estimate.

In our discussion below we will make a conservative assumption based on measurements for $\Delta S = 0$ decays,

$$\frac{|e_c|}{|t_c|} \approx \sqrt{\frac{\mathcal{B}(B^0 \to D^0 \bar{D}^0)}{\mathcal{B}(B^+ \to D^+ \bar{D}^0)} \frac{\tau_+}{\tau_0}} \le 0.3 \ . \tag{28}$$

This upper bound is obtained from the branching ratios quoted in Table I for $B^0 \to D^0\bar{D}^0$ and $B^+ \to D^+\bar{D}^0$, for which 1σ upper and lower limits are used. The two processes $B^0 \to D^0\bar{D}^0$ and $B^+ \to D^+\bar{D}^0$ are dominated by e_c and t_c, respectively, while smaller contributions involving $V_{ub}^*V_{ud}$ have been neglected. (See discussion below.) It would be useful to compare the bound (28) and the crude estimate (27) with direct measurements of $|e_c/t_c|$ in ratios of branching ratios for $\Delta S = 1$ decays including $\mathcal{B}(B_s \to D^+D^-)/\mathcal{B}(B^+ \to D_s^+\bar{D}^0)$ and $\mathcal{B}(B_s \to D^0\bar{D}^0)/\mathcal{B}(B^0 \to D_s^+D^-)$. Using the averaged measured $\mathcal{B}(B^+ \to D^+\bar{D}^0)$ in Table I to normalize $|t_c|^2$, Eq. (27) would imply $\mathcal{B}(B^0 \to D_s^+D_s^-) = (4.0^{+1.8}_{-1.4}) \times 10^{-6}$, about an order of magnitude smaller than the current upper limit on this branching ratio (see Table I.)

6.1.3 The ratio $|V_{cb}^*V_{cd}p_{ct}/T|$ in t_c

The amplitude t_c consists of a combination of a genuine color-favored tree amplitude and a smaller loop-suppressed penguin amplitude, p_{ct}, with t and c quarks in the loop [see Eq. (12)]. It is difficult to obtain a good estimate for the ratio of these two amplitudes, the sum of which contributes to both $\Delta S = 0$ and $\Delta S = 1$ decays. A QCD loop factor $[\alpha_s(m_b)/12\pi]\ln(m_t^2/m_c^2)$ characterizing the suppression of $V_{cb}^*V_{cd}p_{ct}$ relative to T (or a typical Wilson coefficient for penguin operators [20]) is of order five percent. A dynamical enhancement by a factor of four to six relative to a QCD loop factor has been measured for the penguin-to-tree ratio in $B^0 \to \pi^+\pi^-$ [28,29]. We will permit a similar enhancement in $B^0 \to D^+D^-$, thus allowing $|V_{cb}^*V_{cd}p_{ct}|$ to be at most as large as $0.3|T|$,

$$\frac{|V_{cb}^*V_{cd}p_{ct}|}{|T|} \leq 0.3 . \tag{29}$$

6.1.4 Factorization of $(V_{cs}/V_{cd})T$ in $B^0 \to D_s^+D^-$ and $B^+ \to D_s^+\bar{D}^0$

The tree amplitude is expected to factorize within a reasonable approximation into a product of a $B \to D$ form factor and the D_s meson decay constant. While this approximation cannot be justified by the heavy b quark limit of QCD (which can only be applied when a B meson decays into two energetic mesons [30,31]), it holds to leading order in $1/N_c$ in the large N_c limit [32]. Early factorization tests of this kind, implicitly neglecting a p_{ct} contribution, have been proposed and studied in Ref. [33,34].

We will now update a factorization test for $(V_{cs}/V_{cd})T$ in $B^0 \to D_s^+D^-$ and $B^+ \to D_s^+\bar{D}^0$ by relating the branching ratios for these two processes given in Table II to the above quoted branching ratio for $B^0 \to D^-\pi^+$. The latter has been accounted rather well by factorization [31], up to a small contribution from an exchange amplitude. The dominant contributions of the isosinglet amplitudes $(V_{cs}/V_{cd})T$ and $V_{cb}^*V_{cs}p_{ct}$ to the decay rates of $B^+ \to D_s^+\bar{D}^0$ and $B^0 \to D_s^+D^-$ are expected to be equal in the isospin symmetry limit. Thus we will use the weighted average of these two branching ratios, correcting $\mathcal{B}(B^+ \to D_s^+\bar{D}^0)$ by the lifetime ratio τ_0/τ_+:

$$\tilde{\mathcal{B}}(B \to D_s^+\bar{D}) = (82.4 \pm 6.5) \times 10^{-4} . \tag{30}$$

This implies the following ratio of measured branching ratios,

$$\frac{\tilde{\mathcal{B}}(B \to D_s^+ \bar{D})}{\mathcal{B}(B^0 \to D^- \pi^+)} = 3.07 \pm 0.28 \ . \tag{31}$$

Assuming factorization for the processes in the numerator and denominator and taking $V_{cs}/V_{ud} = 1$, one would expect this ratio to be given by [34]

$$\frac{\tilde{\mathcal{B}}(B \to D_s^+ \bar{D})}{\mathcal{B}(B^0 \to D^- \pi^+)} = \frac{f_{D_s}^2}{f_\pi^2} \frac{F_V^2(\omega_{D_s})}{F_V^2(\omega_\pi)} \frac{[(1 + \sqrt{\zeta_D})^2 - \zeta_{D_s}]^2}{[(1 + \sqrt{\zeta_D})^2 - \zeta_\pi]^2} \frac{p_{D_s}}{p_\pi} \ , \tag{32}$$

where

$$\omega_x \equiv \frac{m_B^2 + m_D^2 - m_x^2}{2 m_B m_D} \ , \quad \zeta_x \equiv \frac{m_x^2}{m_B^2} \ . \tag{33}$$

Here f_{D_s}, f_π and p_{D_s}, p_π are corresponding decay constants and momenta in the B meson rest frame, while F_V is the $B \to D$ vector form factor. Taking a linear form factor [35], $F_V(\omega) = F_V(1)[1 - (0.69 \pm 0.14)(\omega - 1)]$, and using [36] $f_\pi = 130.4 \pm 0.2$ MeV, $f_{D_s} = 273 \pm 10$ MeV, with meson masses and momenta in the B rest frame listed in Ref. [25], one finds

$$\frac{\tilde{\mathcal{B}}(B \to D_s^+ \bar{D})}{\mathcal{B}(B^0 \to D^- \pi^+)} = 4.42^{+0.74}_{-0.57} \ . \tag{34}$$

Comparing the factorization calculation with the experimental ratio (31) we note that the factorization result is on the high side, showing a discrepancy of 2.1σ relative to experiment. We do not expect a very good agreement here because of two corrections, each of which may be about 30% in amplitude:

1. A penguin term $V_{cb}^* V_{cs} p_{ct}$ contributing to the numerator but not to the denominator. As mentioned, this term could be as large as $0.3T$ and could interfere destructively with T, leading to a ratio smaller than (34) by up to a factor of two.

2. Nonfactorizable $1/N_c$ corrections to T contributing to both numerator and denominator. These terms are also expected to lead to corrections around 30% in the amplitude.

6.2 $\Delta S = 0$ decays

6.2.1 Ratio of two CKM factors

We now turn to the $\Delta S = 0$ decay amplitudes given in Eqs. (16). These amplitudes involve the two CKM factors $V_{cb}^* V_{cd}$ and $V_{ub}^* V_{ud}$ which are of comparable order λ^3, with a ratio [17]

$$\frac{|V_{ub}^* V_{ud}|}{|V_{cb}^* V_{cd}|} = 0.40 \pm 0.05 \ . \tag{35}$$

In the U-spin symmetry limit these CKM factors multiply the same hadronic amplitudes occurring in $\Delta S = 1$ decays. The somewhat larger CKM factor $V_{cb}^* V_{cd}$ multiplies a dominant term t_c and an exchange contribution e_c which is expected to be much smaller. This leads to a large suppression of $\mathcal{B}(B^0 \to D^0 \bar{D}^0)$ and $\mathcal{B}(B^0 \to D_s^+ D_s^-)$ relative to $\mathcal{B}(B^0 \to D^+ D^-), \mathcal{B}(B^+ \to D^+ \bar{D}^0)$ and $\mathcal{B}(B_s \to D^+ D_s^-)$ [27, 37].

6.2.2 Factorization of T in $B^0 \to D^+D^-$ and $B^+ \to D^+\bar{D}^0$

The hadronic amplitude $V_{cb}^* V_{cd} t_c$ consists of a dominant term T, which is factorizable in the large N_c limit, and a sub-dominant term $V_{cb}^* V_{cd} p_{ct}$, which may be as large as about $0.3T$. Using notations as above, factorization of T implies that the contribution of this amplitude to the rates for $B^0 \to D^+D^-$ and $B^+ \to D^+\bar{D}^0$ are equal and are given by

$$\frac{\mathcal{B}(B^0 \to D^+D^-)_T}{\mathcal{B}(B^0 \to D^-\pi^+)} = \frac{\mathcal{B}(B^+ \to D^+\bar{D}^0)_T}{\mathcal{B}(B^0 \to D^-\pi^+)}$$
$$= \frac{\lambda^2}{1-\lambda^2} \frac{f_D^2}{f_\pi^2} \frac{F_V^2(\omega_D)}{F_V^2(\omega_\pi)} \frac{[(1+\sqrt{\zeta_D})^2 - \zeta_D]^2}{[(1+\sqrt{\zeta_D})^2 - \zeta_\pi]^2} \frac{p_D}{p_\pi} = 0.136^{+0.022}_{-0.018} \ . \tag{36}$$

We have used a value [36] $f_D = 205.8 \pm 8.9$ MeV. Using the measured branching ratio, $\mathcal{B}(B^0 \to D^-\pi^+) = (26.8 \pm 1.3) \times 10^{-4}$, this implies

$$\mathcal{B}(B^0 \to D^+D^-)_T = \mathcal{B}(B^+ \to D^+\bar{D}^0)_T = (3.64^{+0.62}_{-0.52}) \times 10^{-4} \ . \tag{37}$$

This result is in agreement, well within 1σ, with the branching ratios measured by BaBar and Belle for $B^+ \to D^+\bar{D}^0$. It deviates from the BaBar and Belle measurements of $\mathcal{B}(B^0 \to D^+D^-)$ by 1.0σ and 2.8σ, respectively. (See Table I.) As we mentioned, deviations at this level are expected due to a term $V_{cb}^* V_{cd} p_{ct}$ and $1/N_c$ corrections.

6.2.3 The amplitude p_u and the smaller terms, pa_u, e_u, a_u

The CKM factor $V_{ub}^* V_{ud}$ in $\Delta S = 0$ decay amplitudes multiplies four hadronic terms, p_u, pa_u, e_u and a_u. The QCD penguin amplitude $p_u \equiv p_{ut}$, involving t and u quarks in the loop, is expected to have a magnitude comparable to that of p_{ct} multiplying $V_{cb}^* V_{cd}$. Since we anticipate $|V_{cb}^* V_{cd} p_{ct}| \leq 0.3|T|$ Eq. (35) implies

$$\frac{|V_{ub}^* V_{ud} p_u|}{|V_{cb}^* V_{cd} t_c|} \simeq \frac{|V_{cb}^* V_{cd} p_{ct}|}{|T + V_{cb}^* V_{cd} p_{ct}|} \frac{|V_{ub}^* V_{ud}|}{|V_{cb}^* V_{cd}|} \leq \frac{0.3}{1-0.3} \frac{|V_{ub}^* V_{ud}|}{|V_{cb}^* V_{cd}|} = 0.2 \ . \tag{38}$$

This upper bound assumes a worst-case scenario of destructive interference between T and $V_{cb}^* V_{cd} p_{ct}$ in t_c, as indicated by the factorization prediction (34) which is larger than the corresponding experimental ratio (31).

The other three amplitudes, pa_u, e_u and a_u, involving an interaction of a spectator quark, are expected to be suppressed by $\Lambda_{\rm QCD}/m_B$. For instance, the weak annihilation amplitude a_u factorizes at leading order in $1/N_c$,

$$a_u = \frac{G_F}{\sqrt{2}}(C_1 + \frac{C_2}{N_c}) \langle 0|\bar{b}\gamma_\mu(1-\gamma_5)u|B^+\rangle \langle D^+\bar{D}^0|\bar{u}\gamma_\mu d|0\rangle + O(1/N_c^2) \ . \tag{39}$$

The $|0\rangle \to |D^+\bar{D}^0\rangle$ matrix element of the $I = 1$ vector current is parameterized by one form factor,

$$\langle D^+(p_D)\bar{D}^0(p_{\bar{D}})|\bar{u}\gamma_\mu d|0\rangle = f_+^{(I=1)}(q^2)(p_D - p_{\bar{D}})_\mu \ . \tag{40}$$

Combining the two factors in Eq. (39), one finds that the leading term in a_u is proportional to the isospin breaking D meson mass difference which is negligibly small,

$$a_u = \frac{G_F}{\sqrt{2}}(C_1 + \frac{C_2}{N_c})f_B f_+^{(I=1)}(m_B^2)(m_{D^+}^2 - m_{D^0}^2) + O(1/N_c^2) \ . \tag{41}$$

This implies that a_u is dominated by nonfactorizable contributions including initial state gluon emission.

Since pa_u is also suppressed by a QCD loop factor, and e_u and a_u are suppressed by requiring a popping of a heavy $c\bar{c}$ pair, we will assume

$$|pa_u|, |e_u|, |a_u| \ll |p_u| \ . \tag{42}$$

Thus, in the subsequent analysis we will neglect these very small amplitudes. [For the same argument, the interaction of a spectator quark and $c\bar{c}$ popping, we have neglected in Section 5 the two EWP contributions, $PE_{EW}(c)$ and $PA_{EW}(c)$.]

7 Reiterating $B \to D\bar{D}$ decays and a bound on r

Neglecting the very small amplitudes in (42), Eqs. (16) (18) and the last of Eqs. (19) become:

$$
\begin{aligned}
a. \quad & A(B^0 \to D^+D^-) && = && V_{cb}^* V_{cd}\,(t_c + e_c) + V_{ub}^* V_{ud}\,p_u \ , \\
b. \quad & A(B^0 \to D^0\bar{D}^0) && = && -V_{cb}^* V_{cd}\,e_c \ , \\
c. \quad & A(B^+ \to D^+\bar{D}^0) && = && V_{cb}^* V_{cd}\,t_c + V_{ub}^* V_{ud}\,p_u \ , \\
d. \quad & A(B^0 \to D_s^+ D_s^-) && = && V_{cb}^* V_{cd}\,e_c \ , \\
e. \quad & A(B_s \to D^+ D_s^-) && = && V_{cb}^* V_{cd}\,t_c + V_{ub}^* V_{ud}\,p_u \ ,
\end{aligned}
\tag{43}
$$

$$r = \frac{|V_{ub}^* V_{ud}|}{|V_{cb}^* V_{cd}|}\frac{|p_u|}{|t_c + e_c|} \ , \qquad \delta = \arg\left(-\frac{p_u}{t_c + e_c}\right) \ , \qquad A_{1,\frac{3}{2}} = 0 \ . \tag{44}$$

Using the two upper limits (28) and (38) we find

$$r \le 0.3 \ . \tag{45}$$

We consider this a conservative upper bound, as it allows for a worst-case scenario of two destructive interference terms in the denominator of r and for a large enhancement of the penguin amplitude in its numerator.

We will now discuss the rate and asymmetry measurements in $B \to D\bar{D}$ in light of the expressions (43), which are expected to hold to a very good approximation. First, we note that CP asymmetries in $B^0 \to D^0\bar{D}^0$ and $B^0 \to D_s^+ D_s^-$ vanish in this approximation because the amplitudes for these processes involve a single CKM factor $V_{cb}^* V_{cd}$. [This justifies the discussion below Eq. (10).] The decay rates for these two processes, which are dominated by an amplitude e_c, are equal in the SU(3) limit. A small decay rate difference between $B^0 \to D^0\bar{D}^0$ and $B^0 \to D_s^+ D_s^-$ is expected due to two effects working in opposite directions, $u\bar{u}$ versus $s\bar{s}$ popping on the one hand and exclusive production of $D^0\bar{D}^0$ versus $D_s^+ D_s^-$ on the other.

Second, as mentioned, neglecting e_u and a_u leads to $\Delta I = 1/2$ in $B \to D\bar{D}$ implying the triangle amplitude relation (8). This relation was shown to be violated at 1.8σ by branching ratios and asymmetries measured by Belle. The Standard Model $\Delta I = 3/2$ amplitude $e_u + a_u$ is too small to account for an observable violation.

The Belle asymmetries by themselves also show two unexpected features (see Table I), which are not shared by the BaBar measurements:

1. The above upper bound on r and the second Eq. (2) imply a theoretical upper limit on the direct asymmetry in $B^0 \to D^+D^-$,

$$|A_{CP}(B^0 \to D^+D^-)| \leq \frac{2r}{1+r^2} \leq 0.55 . \tag{46}$$

The value measured by Belle is larger than this upper limit by 1.5σ.

2. The asymmetries in $B^0 \to D^+D^-$ and $B^+ \to D^+\bar{D}^0$ are expected to be equal, up to second order corrections from an interference of the small amplitudes e_c and p_u. In contrast to this expectation, the Belle asymmetry in $B^0 \to D^+D^-$ is positive and large while the one in $B^+ \to D^+\bar{D}^0$ is very small. The difference between the two asymmetries involves a statistical significance of 3.6σ. The Standard Model interference of e_c and p_u and the amplitude pa_u, which we neglected in (43a), are too small to account for this large difference between the two CP asymmetries.

8   Determining γ

The amplitude for $B^0 \to D^+D^-$ given in (16a) or (43a) and a suitably chosen $\Delta S = 1$ amplitude in (20) provide a sufficient number of observables for determining the weak phase γ in the flavor SU(3) limit. This method has been proposed in Refs. [23] and [37]. Here we wish to recapitulate this method, showing that the determination of γ in this way is very sensitive to uncertainties in SU(3) breaking.

The two asymmetries $S(D^+D^-)$ and $A_{CP}(D^+D^-)$ are given in Eqs. (2) in terms of the three parameters r, δ and γ. We are assuming $\beta = (21.5 \pm 1.0)°$. A third equation for these parameters is provided by the ratio of CP-averaged decay rates for $B^0 \to D^+D^-$ and its U spin counterpart $B_s \to D_s^+D_s^-$. Alternatively, one may use instead of the latter process the decay mode $B^0 \to D_s^+D^-$ or $B^+ \to D_s^+\bar{D}^0$. In this case one would have to estimate the effect of the exchange amplitude e_c contributing to $B^0 \to D^+D^-$ but not to the latter two $\Delta S = 1$ decay processes. [See discussion above including the upper bound (28).]

Focusing our attention on the U spin pair $B^0 \to D^+D^-$ and $B_s \to D_s^+D_s^-$, we define an experimentally measured ratio of CP-averaged decay rates,

$$R \equiv \left(\frac{V_{cs}}{V_{cd}}\right)^2 \frac{\bar{\Gamma}(B^0 \to D^+D^-)}{\bar{\Gamma}(B_s \to D_s^+D_s^-)} = \frac{1-\lambda^2}{\lambda^2} \frac{\mathcal{B}(B^0 \to D^+D^-)}{\mathcal{B}(B_s \to D_s^+D_s^-)} \frac{\tau_s}{\tau_0} , \tag{47}$$

where [1] $\tau_s/\tau_0 = 0.939 \pm 0.021$ is the ratio of B_s and B^0 lifetimes. Neglecting the second term in (20a) suppressed by the tiny CKM factor (25), and introducing an SU(3)

breaking parameter ξ for the ratio of t_c+e_c amplitudes in $B_s \to D_s^+ D_s^-$ and $B^0 \to D^+ D^-$, one obtains:

$$\xi^2 R = 1 + 2r \cos \delta \cos \gamma + r^2 . \tag{48}$$

Thus, for a given value of ξ, the three observables $S(D^+D^-), A_{CP}(D^+D^-)$ and R in Eqs. (2) and (48) enable a determination of r, δ and γ up to discrete ambiguities.

In order to obtain an analytic solution for γ, and to overcome two of its four discrete ambiguities, it is convenient to introduce another observable in $B^0 \to D^+D^-$ [38] (see also [37]),

$$D \equiv \frac{2\mathrm{Re}(\lambda_{D^+D^-})}{1 + |\lambda_{D^+D^-}|^2} , \tag{49}$$

obeying with the two asymmetries,

$$A_{CP}^2 + S^2 + D^2 = 1 . \tag{50}$$

Expressing the new observable in terms of r, δ and γ,

$$D = \frac{\cos 2\beta + 2r \cos \delta \cos(2\beta + \gamma) + r^2 \cos 2(\beta + \gamma)}{1 + 2r \cos \delta \cos \gamma + r^2} , \tag{51}$$

we note that this quantity is positive for $\beta = (21.5 \pm 1.0)°, r \leq 0.3$ [as required by (45)], and for arbitrary values of δ and γ. This information on the sign of D, which remains undetermined by the two asymmetries using (50), avoids two discrete ambiguities in γ.

Eqs. (2) and (48) and the positivity of D imply the following equation for γ in terms of A_{CP}, S and $\xi^2 R$ [37]:

$$\frac{+\sqrt{1 - A_{CP}^2 - S^2} \cos 2(\beta + \gamma) - S \sin 2(\beta + \gamma) - 1}{\cos 2\gamma - 1} = \frac{1}{\xi^2 R} . \tag{52}$$

The plus sign in front of the square root follows from the positivity of D.

In order to demonstrate the high sensitivity of determining γ to the value of the SU(3) breaking parameter ξ, we plot in Fig. 2(a) the dependence of γ on ξ for two sets of values for A_{CP}, S and R:
(1) BaBar central values, $A_{CP} = -0.11, S = -0.54, R = 0.52$ (solid curve).
(2) Central values of the averages of BaBar and Belle, $A_{CP} = 0.37, S = -0.75, R = 0.39$ (dashed curve).
We do not use the Belle central values because they are non-physical, violating the inequality $S^2 + A_{CP}^2 \leq 1$. Also shown in the plot is a band describing the currently allowed 1σ range for γ [17], $\gamma = (67.6^{+2.8}_{-4.5})°$. Fig. 2(b) shows values of r corresponding to solutions for γ for case (1) (solid curve) and (2) (dashed curve). A solution obtained with too large values of r, $r > 1.5$ in case (1) and $r > 0.6$ in case (2), is not shown.

The two curves of γ for case (1) and (2) cross the allowed band for γ at $\xi = 1.3$ and $\xi = 1.7$, respectively. This difference in the values of the SU(3) breaking parameter follows from the large experimental errors in the $B^0 \to D^+D^-$ measurements. The slopes of the two curves at the above points are quite steep, implying in both cases a high theoretical sensitivity of the determined value of γ to the assumed value of ξ. For
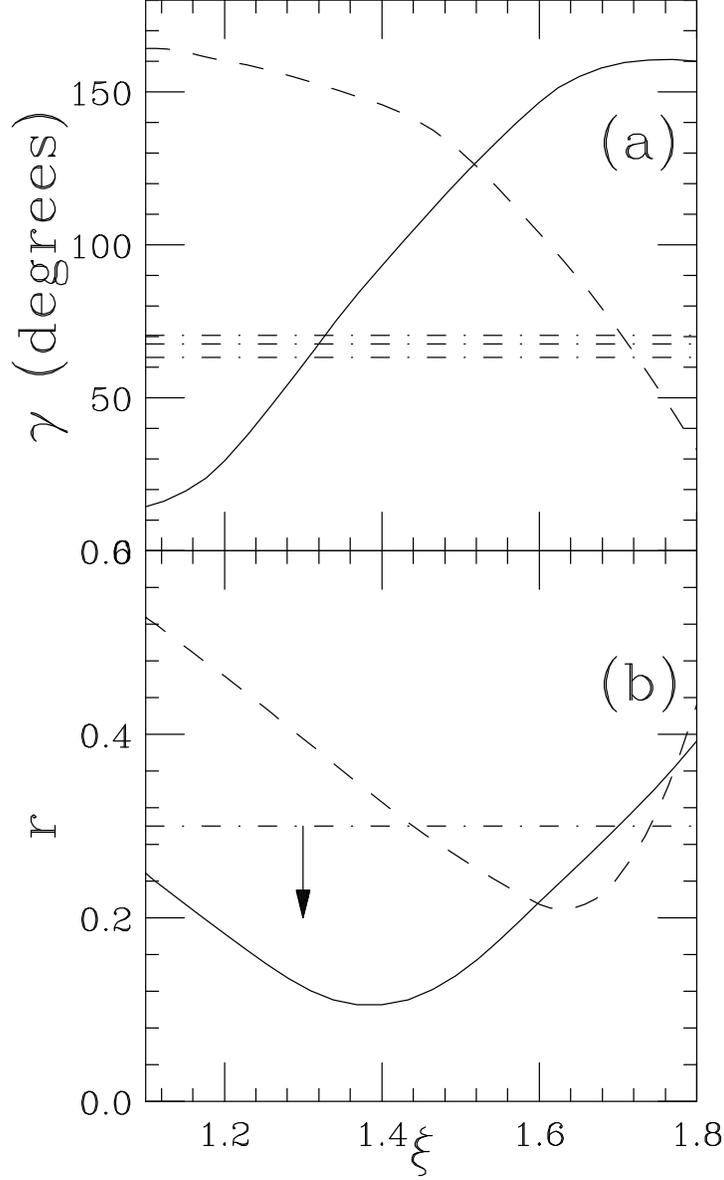
Figure 2: Behavior of solutions as a function of parameter ξ describing SU(3) violation. Solid curves denote values obtained using BaBar central values as input; dashed curves based on BaBar–Belle average central values. (a) Weak phase γ. Horizontal dashed lines denote HFAG central and $\pm 1\sigma$ values of γ. (b) Parameter $r = A_u/A_c$ describing ratio of amplitudes. Horizontal dot-dashed line denotes upper bound $r \leq 0.3$ described in text.

instance, the dashed curve increases approximately linearly from $\gamma = 65°$ at $\xi = 1.7$ to $\gamma = 105°$ at $\xi = 1.6$. Thus, assuming perfect measurements for the branching ratio and asymmetries in $B^0 \to D^+D^-$, an uncertainty of $10°$ in γ would require knowing ξ to better than 2%. A somewhat lower sensitivity has been noted in the second paper in Ref. [23].

The parameter ξ, representing SU(3) breaking in the ratio of $t_c + e_c$ amplitudes in

$B_s \to D_s^+ D_s^-$ and $B^0 \to D^+ D^-$ including a small calculable phase space effect, involves theoretical uncertainties from several sources:

- As mentioned, t_c include a dominant tree amplitude, for which the leading term in a $1/N_c$ expansion factorizes. SU(3) breaking in the factorizable term is given by a product of a measured ratio of decay constants [36], $f_{D_s}/f_D = 1.33 \pm 0.07$, and a ratio of form factors $F_{B_s \to D_s}(m_{D_s}^2)/F_{B \to D}(m_D^2)$. The latter ratio is estimated at ~ 1.05 in a chiral SU(3) perturbation expansion, at leading order in the heavy b and c quark masses m_H [39]. However, a complete analysis of all $\mathcal{O}(1/m_H)$ terms shows that simultaneous violation of both chiral and heavy quark symmetries can be as large as 30% [40]. The form factor ratio can eventually be determined from $B^0 \to D^- \ell^+ \nu$ measured in $e^+ e^-$ collisions on the $\Upsilon(4S)$ [35] and $B_s \to D_s^- \ell^+ \nu$ accessible to the LHCb collaboration working at the LHC. Thus, SU(3) breaking in the tree amplitude involves nonfactorizable $1/N_c$ corrections and uncertainties in ratios of form factors and decay constants combining to a total of at least 30%.

- A penguin contribution $V_{cb}^* V_{cd} p_{ct}$ in t_c is not expected to factorize, which introduces an uncontrollable SU(3) breaking correction in this contribution at a level of 30%.

- We have already discussed uncertainties in the magnitude of e_c. 30% uncertainties due to SU(3) breaking corrections in this amplitude and in p_{ct} mentioned above translate through (28) and (29) into two uncertainties in SU(3) breaking in $t_c + e_c$, each of which is at a level of 10%.

In view of these combined uncertainties, a precision of 10% in ξ is unachievable. This implies a very large intrinsic theoretical uncertainty of at least $50°$ in determining γ as demonstrated in Fig. 2(a).

The origin of the high sensitivity to SU(3) breaking can be traced back to the way flavor SU(3) symmetry is being applied here for a determination of γ. In this method an assumption of SU(3) symmetry is used to normalize the *dominant* amplitude in $B^0 \to D^+ D^-$ in terms of the measured amplitude for $B_s \to D_s^+ D_s^-$. In contrast, the error in γ introduced by an uncertainty in SU(3) breaking is expected to be small in cases where a *small* penguin amplitude in $\Delta S = 0$ decays is normalized by a measurable SU(3) related $\Delta S = 1$ decay amplitude. Two cases, where this has been demonstrated, are $B^0 \to \pi^+ \pi^-$ [41], where the penguin amplitude is subdominant, and most prominently $B^0 \to \rho^+ \rho^-$ [42] in which the penguin amplitude is very small.

9   Conclusion

We have studied the effect of a small $\bar{b} \to \bar{u} u \bar{d}$ amplitude in $B \to D\bar{D}$ decays. While this amplitude violates $\Delta I = 1/2$, we have argued that this violation is expected to be very small. Considering the Belle measurements, we pointed out a violation at a level of 1.8σ of a $\Delta I = 1/2$ amplitude relation, and an inconsistency at a level of 3.6σ between CP asymmetries in $B^0 \to D^+ D^-$ and $B^+ \to D^+ \bar{D}^0$. No such inconsistency has been observed by the BaBar collaboration. If these discrepancies persist they would have to be associated with a New Physics contribution to $\bar{b} \to \bar{u} u \bar{d}$.

Using conservative considerations within the CKM framework, we obtained a model-independent upper bound, $r \equiv A_u/A_c \leq 0.3$, for the ratio of $\bar{b} \to \bar{u}u\bar{d}$ and $\bar{b} \to \bar{c}c\bar{d}$ amplitudes in $B^0 \to D^+D^-$. This implies an upper limit, $|A_{CP}| \leq 0.55$, for the direct CP asymmetry in this process. The Belle asymmetry is 1.5σ larger than this bound, while the BaBar measurement is well below this upper limit.

U spin symmetry has been applied for obtaining relations between amplitudes and CP asymmetries for $\Delta S = 0$ decays of B and B_s to pairs of charm-anticharm mesons and amplitudes and asymmetries in corresponding $\Delta S = 1$ decays. We have shown that, while these relations are not useful for a precise determination of γ in $B^0 \to D^+D^-$ in the presence of small uncertainties in SU(3) breaking, they may provide important information about small contributions to the latter process.

For instance, the amplitude e_c may account for a difference between the decay rates of $B^0 \to D^+D^-$ and $B^+ \to D^+\bar{D}^0$, and for a small difference between CP asymmetries in these decays. This amplitude dominates the $\Delta S = 0$ decays, $B^0 \to D^0\bar{D}^0$, $B^0 \to D_s^+D_s^-$, and the $\Delta S = 1$ decays $B_s \to D^+D^-$, $B_s \to D^0\bar{D}^0$, and may be extracted from branching ratios of these processes. Using Eq. (27) one would predict $\mathcal{B}(B^0 \to D_s^+D_s^-) = (4.0^{+1.8}_{-1.4}) \times 10^{-6}$, whereas (28) only implies $\mathcal{B}(B^0 \to D_s^+D_s^-) \leq 3 \times 10^{-5}$, close to the upper limit in Table I [15]. Similarly, using (30) $\tilde{\mathcal{B}}(B \to D_s^+\bar{D}) = (82.4 \pm 6.5) \times 10^{-4}$ Eq. (27) would imply $\mathcal{B}(B_s \to D\bar{D}) = (0.9^{+0.4}_{-0.3}) \times 10^{-4}$, while the conservative upper bound (28) leads to a more modest prediction $\mathcal{B}(B_s \to D\bar{D}) \leq 7 \times 10^{-4}$.

Thus, a sensitivity of 1×10^{-4} in $\mathcal{B}(B_s \to D^+D^-)$ and $\mathcal{B}(B_s \to D^0\bar{D}^0)$, in comparison with $\tilde{\mathcal{B}}(B \to D_s^+\bar{D}) = (82.4 \pm 6.5) \times 10^{-4}$, can be used for obtaining precise information on the ratio $|e_c/t_c|$ at a level of 0.1. This precision is considerably more powerful than the current upper bound (28) obtained from the λ^2 suppressed $\mathcal{B}(B^0 \to D^0\bar{D}^0)$. The above sensitivity may be achieved at experiments carried by the LHCb collaboration working at the Large Hadron Collider. More precise measurement of the Cabibbo-favored decay branching ratio $\mathcal{B}(B_s \to D_s^+D_s^-)$ than currently available may soon be achieved at the Tevatron. This would lead to useful information on corrections to U spin symmetry relating this process and $B^0 \to D^+D^-$.

A SU(3) operator analysis

In this Appendix we consider relations between the graphical amplitudes defined in Sections 4 and 5 and a flavor SU(3) analysis for the operators appearing in the low energy effective Hamiltonian (5). These operators have the following transformation properties under flavor SU(3). Current-current operators: $\mathcal{O}^c_{1,2} \sim \bar{\mathbf{3}}$, $\mathcal{O}^u_{1,2} \sim \bar{\mathbf{3}}, \mathbf{6}, \overline{\mathbf{15}}$, QCD penguin operators: $\mathcal{O}_{3-6} \sim \bar{\mathbf{3}}$, electroweak penguin operators: $\mathcal{O}_{7-10} \sim \bar{\mathbf{3}}, \mathbf{6}, \overline{\mathbf{15}}$. An explicit SU(3) decomposition of the Hamiltonian can be found e.g. in [43].

In $\bar{B} \to D\bar{D}$ decays, the initial and final states transform like $\mathbf{3}$ and $\mathbf{1}, \mathbf{8}$, respectively, which implies that all these decay amplitudes can be expressed in terms of four SU(3) reduced matrix elements $\langle \mathbf{1}\|\bar{\mathbf{3}}\|\mathbf{3}\rangle, \langle \mathbf{8}\|\bar{\mathbf{3}}\|\mathbf{3}\rangle, \langle \mathbf{8}\|\mathbf{6}\|\mathbf{3}\rangle, \langle \mathbf{8}\|\overline{\mathbf{15}}\|\mathbf{3}\rangle$. This agrees with the counting of independent amplitudes performed in Section 4 in terms of graphical amplitudes.

The explicit expansion of all ten $\Delta S = 0$ and $\Delta S = 1$ amplitudes in terms of reduced SU(3) matrix elements can be found, for example, in [44]. The $\Delta S = 0$ amplitudes are

$$
\begin{pmatrix}
A(B^0 \to D^+ D^-) \\
A(B^0 \to D^0 \bar{D}^0) \\
A(B^+ \to D^+ \bar{D}^0) \\
A(B^0 \to D_s^+ D_s^-) \\
A(B_s \to D^+ D_s^-)
\end{pmatrix}
=
\begin{pmatrix}
\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{6}} & 0 & -\frac{1}{2} \\
-\frac{1}{\sqrt{3}} & -\frac{1}{2\sqrt{6}} & -\frac{1}{2} & -\frac{3}{4} \\
0 & -\frac{1}{2}\sqrt{\frac{3}{2}} & -\frac{1}{2} & \frac{3}{4} \\
\frac{1}{\sqrt{3}} & \frac{1}{2\sqrt{6}} & -\frac{1}{2} & -\frac{1}{4} \\
0 & -\frac{1}{2}\sqrt{\frac{3}{2}} & \frac{1}{2} & -\frac{1}{4}
\end{pmatrix}
\begin{pmatrix}
\langle \mathbf{1}\|\bar{\mathbf{3}}\|\mathbf{3}\rangle_d \\
\langle \mathbf{8}\|\bar{\mathbf{3}}\|\mathbf{3}\rangle_d \\
\langle \mathbf{8}\|\mathbf{6}\|\mathbf{3}\rangle_d \\
\langle \mathbf{8}\|\overline{\mathbf{15}}\|\mathbf{3}\rangle_d
\end{pmatrix} . \tag{53}
$$

The corresponding $\Delta S = 1$ amplitudes are given by the same transformation matrix,

$$
\begin{pmatrix}
A(B_s \to D_s^+ D_s^-) \\
A(B_s^0 \to D^0 \bar{D}^0) \\
A(B^+ \to D_s^+ \bar{D}^0) \\
A(B_s^0 \to D^+ D^-) \\
A(B^0 \to D_s^+ D^-)
\end{pmatrix}
=
\begin{pmatrix}
\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{6}} & 0 & -\frac{1}{2} \\
-\frac{1}{\sqrt{3}} & -\frac{1}{2\sqrt{6}} & -\frac{1}{2} & -\frac{3}{4} \\
0 & -\frac{1}{2}\sqrt{\frac{3}{2}} & -\frac{1}{2} & \frac{3}{4} \\
\frac{1}{\sqrt{3}} & \frac{1}{2\sqrt{6}} & -\frac{1}{2} & -\frac{1}{4} \\
0 & -\frac{1}{2}\sqrt{\frac{3}{2}} & \frac{1}{2} & -\frac{1}{4}
\end{pmatrix}
\begin{pmatrix}
\langle \mathbf{1}\|\bar{\mathbf{3}}\|\mathbf{3}\rangle_s \\
\langle \mathbf{8}\|\bar{\mathbf{3}}\|\mathbf{3}\rangle_s \\
\langle \mathbf{8}\|\mathbf{6}\|\mathbf{3}\rangle_s \\
\langle \mathbf{8}\|\overline{\mathbf{15}}\|\mathbf{3}\rangle_s
\end{pmatrix} . \tag{54}
$$

The subscript $q = d, s$ on the reduced matrix elements is a reminder that they differ for $\Delta S = 0, 1$ weak Hamiltonians through their dependence on CKM matrix elements.

Comparing these expressions with the graphical expansions (16) and (20), we find the following relations between SU(3) reduced matrix elements and graphical amplitudes (for $q = d, s$):

$$
\begin{aligned}
\langle \mathbf{1}\|\bar{\mathbf{3}}\|\mathbf{3}\rangle_q &= V_{cb}^* V_{cq} \frac{1}{\sqrt{3}}(t_c + 3e_c) + V_{ub}^* V_{uq} \frac{1}{\sqrt{3}}(p_u + 3pa_u + e_u) , \\
\langle \mathbf{8}\|\bar{\mathbf{3}}\|\mathbf{3}\rangle_q &= -V_{cb}^* V_{cq} \frac{1}{2\sqrt{6}}8t_c - V_{ub}^* V_{uq} \frac{1}{2\sqrt{6}}(8p_u - e_u + 3a_u) , \\
\langle \mathbf{8}\|\mathbf{6}\|\mathbf{3}\rangle_q &= V_{ub}^* V_{uq} \frac{1}{2}(e_u - a_u) , \\
\langle \mathbf{8}\|\overline{\mathbf{15}}\|\mathbf{3}\rangle_q &= V_{ub}^* V_{uq} \frac{1}{2}(e_u + a_u) .
\end{aligned} \tag{55}
$$

As mentioned in Section 5, electroweak penguin (EWP) contributions introduce four new graphical amplitudes. This agrees with the above counting of SU(3) reduced matrix elements. A complete expansion of EWP terms in the ten processes (16) and (20) in terms of graphical amplitudes defined in Section 5 is:

$$
\begin{aligned}
P_{EW}(B^0 \to D^+ D^-) &= V_{tb}^* V_{td}\left(\frac{2}{3}P_{EW}^C - \frac{1}{3}PE_{EW(c)} + \frac{2}{3}PA_{EW} - \frac{1}{3}PA_{EW(c)}\right) , \\
P_{EW}(B^0 \to D^0 \bar{D}^0) &= V_{tb}^* V_{td}\left(-\frac{2}{3}PA_{EW} - \frac{2}{3}PA_{EW(c)}\right) ,
\end{aligned}
$$

$$P_{EW}(B^+ \to D^+ \bar{D}^0) = V_{tb}^* V_{td} (\frac{2}{3} P_{EW}^C + \frac{2}{3} PE_{EW(c)}) ,$$

$$P_{EW}(B^0 \to D_s^+ D_s^-) = V_{tb}^* V_{td} (\frac{2}{3} PA_{EW} - \frac{1}{3} PA_{EW(c)}) ,$$

$$P_{EW}(B_s^0 \to D^+ D_s^-) = V_{tb}^* V_{td} (\frac{2}{3} P_{EW}^C - \frac{1}{3} PE_{EW(c)}) . \tag{56}$$

$$P_{EW}(B_s \to D_s^+ D_s^-) = V_{tb}^* V_{ts} (\frac{2}{3} P_{EW}^C - \frac{1}{3} PE_{EW(c)} + \frac{2}{3} PA_{EW} - \frac{1}{3} PA_{EW(c)}) ,$$

$$P_{EW}(B_s \to D^0 \bar{D}^0) = V_{tb}^* V_{ts} (-\frac{2}{3} PA_{EW} - \frac{2}{3} PA_{EW(c)}) ,$$

$$P_{EW}(B^+ \to D_s^+ \bar{D}^0) = V_{tb}^* V_{ts} (\frac{2}{3} P_{EW}^C + \frac{2}{3} PE_{EW(c)}) .$$

$$P_{EW}(B_s \to D^+ D^-) = V_{tb}^* V_{ts} (\frac{2}{3} PA_{EW} - \frac{1}{3} PA_{EW(c)}) ,$$

$$P_{EW}(B^0 \to D_s^+ D^-) = V_{tb}^* V_{ts} (\frac{2}{3} P_{EW}^C - \frac{1}{3} PE_{EW(c)}) . \tag{57}$$

Two of the EWP amplitudes can be related within a very good approximation to the amplitudes e_u and a_u appearing in Eqs. (16) and (20). Neglecting the EWP operators $\mathcal{O}_{7,8}$ which have negligibly small Wilson coefficients, SU(3) symmetry implies,

$$PE_{EW(c)} + PA_{EW(c)} = -\frac{3}{2} \frac{C_9 + C_{10}}{C_1 + C_2} (a_u + e_u) ,$$

$$PE_{EW(c)} - PA_{EW(c)} = \frac{3}{2} \frac{C_9 - C_{10}}{C_1 - C_2} (a_u - e_u) . \tag{58}$$

The proof of these relations is based on operator relations between **6** and $\overline{\mathbf{15}}$ components of the EWP part of the effective Hamiltonian and corresponding components of the tree part $(q = d, s)$ [43, 45],

$$\mathcal{H}_{EWP}^q(\overline{\mathbf{15}}) = -\frac{3}{2} \frac{C_9 + C_{10}}{C_1 + C_2} \frac{V_{tb}^* V_{tq}}{V_{ub}^* V_{uq}} \mathcal{H}_T^q(\overline{\mathbf{15}}) , \quad \mathcal{H}_{EWP}^q(\mathbf{6}) = \frac{3}{2} \frac{C_9 - C_{10}}{C_1 - C_2} \frac{V_{tb}^* V_{tq}}{V_{ub}^* V_{uq}} \mathcal{H}_T^q(\mathbf{6}) . \tag{59}$$

The first operator relation implies for $q = s$ a relation between $\Delta I = 1$ EWP and tree amplitudes in $B \to K\pi$ decays [46].

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