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Renormalizable A_4 Model for Lepton Sector

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Abstract

We study flavor symmetry for the lepton sector with minimal Higgs sector, namely one A_4-triplet SU(2)-doublet scalar. To increase predictivity even further, we impose the constraint of renormalizability. A geometric interpretation of an A_4-triplet aids our understanding of tribimaximal mixing. We investigate the neutrino mass hierarchy in such a minimal A_4 model and find there are two solutions: one with $m_2 \gg m_1 = m_3$ is phenomenologically unacceptable; the other with $m_3 \gg m_1 = m_2$ is a normal hierarchy. An inverted hierarchy is impossible without addition of more parameters by either more Higgs scalars or higher-order irrelevant operators.

Introduction.

In particle theory phenomenology, model building fashions vary with time and because the present lack of data (soon to be compensated by the Large Hadron Collider) does not allow discrimination between models some fashions develop a life of their own. In the present Letter we take the apparently retrogressive step of imposing the requirement of renormalizability[#1], as holds for quantum electrodynamics (QED), quantum chromodynamics (QCD) and the standard electroweak model, to show that non-abelian flavor symmetry becomes then much more restrictive and predictive. In a specific model we show that a normal neutrino mass hierarchy is strongly favored over an inverted hierarchy.

For several years now there has been keen interest in the use of A_4 [1, 2] as a finite flavor symmetry in the lepton sector, especially neutrino mixing; other approaches are in [3]. In particular, the empirically approximate tribimaximal mixing [4] of the three neutrinos can be predicted. It is usually stated that either normal or inverted neutrino mass spectrum can be predicted.

In the present Letter we revisit these two questions in a minimal A_4 framework with only one A_4-$\mathbf{3}$ of Higgs doublets coupling to neutrinos and permitting only renormalizable couplings. For such a minimal model there is more predicitivity regarding neutrino masses.

Although the standard model was originally discovered using the criterion of renormalizability, it is sometimes espoused [5] that renormalizability is not prerequisite in an effective lagrangian. Nevertheless, imposing renormalizability in the present case is more sensible because it does render the model far more predictive by avoiding the many additional parameters associated with higher-order irrelevant operators. Our choice of Higgs sector also minimizes the number of free parameters.

We shall first discuss some geometry of A_4 symmetry as follows.

Geometry of A_4 symmetry

The group A_4 is the order g=12 symmetry of a regular tetrahedron T and is a subgroup of the rotation group $SO(3)$. A_4 has irreducible representations

[#1] The model we discuss has an anomaly which can be cancelled in a more complete T' model incorporating quarks without affecting the results for leptons discussed in the present article.

which are three singlets $1_1, 1_2, 1_3$ and a triplet 3. In the embedding $A_4 \subset SO(3)$ the **3** of A_4 is identified with the adjoint **3** of $SO(3)$.

Since the only Higgs doublets coupling to neutrinos in our model are in a **3** of A_4, it is very useful to understand geometrically the three components of a **3**.

A regular tetrahedron has four vertices, four faces and six edges. Straight lines joining the midpoints of opposite edges pass through the centroid and form a set of three orthogonal axes. Regarding the regular tetrahedron as the result of cutting off the four odd corners from a cube, these axes are parallel to the sides of the cube (see Fig. 1). With respect to the regular tetrahedron, a vacuum expectation value (VEV) of the **3** such as $< \mathbf{3} >= v(1, 1, -2)$, as will be used,clearly breaks SO(3) to U(1) and correspondingly A_4 to Z_2, since it requires a rotation by π about the 3-axis to restore the tetrahedron.

At the same time, we can understand the appearance of tribimaximal mixing [6] with matrix [4]

$$U_{TBM} = \begin{pmatrix} -\sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{6}} & \sqrt{\frac{2}{3}} \\ \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{3}} \\ \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} & 0 \end{pmatrix}, \tag{1}$$

and our definitions are such that the ordering $\nu_{1,2,3}$ and $\nu_{\tau,\mu,e}$ satisfy

$$\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = U_{TBM} \begin{pmatrix} \nu_\tau \\ \nu_\mu \\ \nu_e \end{pmatrix} \tag{2}$$

With respect to a side of the aforementioned cube, a face diagonal is at angle $\pi/4$ and the body diagonal is at angle $\tan^{-1}(\sqrt{\frac{1}{2}})$ with respect to the body diagonal (see Fig. 2). These two angles, together with $\theta_{13} = 0$ are the three corresponding to the matrix of Eq.(1). As we shall show this mixing occurs naturally for VEVs $< \mathbf{3} >\propto (1, 1, 1)$ and $< \mathbf{3} >\propto (1, 1, -2)$.

Assuming no CP violation, the Majorana matrix M_{nu} is real and symmetric and therefore of the form

$$M_\nu = \begin{pmatrix} A & B & C \\ B & D & F \\ C & F & E \end{pmatrix} \tag{3}$$

and is related to the diagonlized form by

$$M_{diag} = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} = U_{TBM} M_\nu U_{TBM}^T. \tag{4}$$

Substituting Eq.(1) into Eq.(4) shows that M_ν must be of the general form in terms of real parameters A, B, C:

$$M_\nu = \begin{pmatrix} A & B & C \\ B & A & C \\ C & C & A+B-C \end{pmatrix}, \tag{5}$$

which has eigenvalues

$$\begin{aligned} m_1 &= (A+B-2C) \\ m_2 &= (A+B+C) \\ m_3 &= (A-B). \end{aligned} \tag{6}$$

The observed mass spectrum corresponds approximately to $|m_1| = |m_2|$ which requires either $C = 0$ or $C = 2(A+B)$. For a normal hierarchy, $(A+B) = 0$ and $C = 0$. For an inverted hierarchy $A = B$ and $C = 0$ or $C = 4A$.

Now we study our minimal A_4 model to examine the occurrence of the Majorana matrix Eq.(5) and the eigenvalues Eq.(6).

Minimal A_4 model

We assign the leptons to (A_4, Z_2) irreps as follows [#2]

$$\left.\begin{matrix} \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L \\ \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \\ \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \end{matrix}\right\} L_L(3,+1) \quad \begin{matrix} \tau_R^- \; (1_1,-1) & N_R^{(1)} \; (1_1,+1) \\ \mu_R^- \; (1_2,-1) & N_R^{(2)} \; (1_2,+1) \\ e_R^- \; (1_3,-1) & N_R^{(3)} \; (1_3,+1). \end{matrix} \tag{7}$$

The lagrangian is

[#2] These assignments differ from Ma-Rajasekharan [1] and from Altarelli-Feruglio [2]. The lagrangian, Eq. (8) is, however, the most general dimension-4 $(A_4 \times Z_2)$ invariant with our assignments.

$$
\begin{aligned}
\mathcal{L}_Y = &\frac{1}{2} M_1 N_R^{(1)} N_R^{(1)} + M_{23} N_R^{(2)} N_R^{(3)} \\
&+ \left\{ Y_1 \left(L_L N_R^{(1)} H_3 \right) + Y_2 \left(L_L N_R^{(2)} H_3 \right) + Y_3 \left(L_L N_R^{(3)} H_3 \right) \right. \\
&\left. + Y_\tau \left(L_L \tau_R H_3' \right) + Y_\mu \left(L_L \mu_R H_3' \right) + Y_e \left(L_L e_R H_3' \right) \right\} + \text{h.c.}
\end{aligned} \tag{8}
$$

where $SU(2)$-doublet Higgs scalars are in $H_3(3, +1)$ and $H_3^{'}(3, -1)$.

The charged lepton masses originate [#3] from $< H_3^{'} >= (\frac{m_\tau}{Y_\tau}, \frac{m_\mu}{Y_\mu}, \frac{m_e}{Y_e})$ and are, to leading order, disconnected from the neutrino masses if we choose a flavor basis where the charged leptons are mass eigenstates. The N_R^i masses break the $L_\tau \times L_\mu \times L_e$ symmetry but change the charged lepton masses only by very small amounts $\propto Y^2 m_i / M_N$ at one-loop level.

The right-handed neutrinos have mass matrix

$$
M_N = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & 0 & M_{23} \\ 0 & M_{23} & 0 \end{pmatrix}. \tag{9}
$$

We take the VEV of the scalar H_3 to be

$$
< H_3 >= (V_1, V_2, V_3), \tag{10}
$$

whereupon the Dirac matrix is [#4]

$$
M_D = \begin{pmatrix} Y_1 V_1 & Y_2 V_3 & Y_3 V_2 \\ Y_1 V_3 & Y_2 V_2 & Y_3 V_1 \\ Y_1 V_2 & Y_2 V_1 & Y_3 V_3 \end{pmatrix}. \tag{11}
$$

[#3] The VEV of $H_3^{'}$ is chosen to give charged leptons their masses and is not directly related to TBM mixing angles for neutrinos.

[#4] The form of Eq.(11) follows from the lagrangian of Eq.(8) with general $< H_3 >= (V_1, V_2, V_3)$ and is not specialized to either a Ma-Rajasekaran or an Altarelli-Feruglio basis.

The Majorana mass matrix M_ν is given by

$$M_\nu = M_D M_N^{-1} M_D^T. \tag{12}$$

Defining $x_1 \equiv Y_1^2/M_1$ and $x_{23} \equiv Y_2 Y_3/M_{23}$ we find the symmetric form

$$\begin{pmatrix} x_1V_1^2 + 2x_{23}V_2V_3 & x_1V_1V_3 + x_{23}(V_2^2 + V_1V_3) & x_1V_1V_2 + x_{23}(V_3^2 + V_1V_2) \\ & x_1V_3^2 + 2x_{23}V_1V_2 & x_1V_2V_3 + x_{23}(V_1^2 + V_2V_3) \\ & & x_1V_2^2 + 2x_{23}V_1V_3 \end{pmatrix}. \tag{13}$$

To ensure the texture of Eq.(13) coincides with Eq.(5), and hence gives tribimaximal mixing we find the three equations corresponding to the mixing angles

$$x_1V_1^2 + 2x_{23}V_2V_3 = x_1V_3^2 + 2x_{23}V_1V_2 \tag{14}$$

$$x_1V_1V_2 + x_{23}(V_3^2 + V_1V_2) = x_1V_2V_3 + x_{23}(V_1^2 + V_2V_3) \tag{15}$$

$$x_1(V_1^2+V_1V_3-V_1V_2)+x_{23}(2V_2V_3+V_2^2+V_1V_3-V_3^2-V_1V_2) = x_1V_2^2+2x_{23}V_1V_3. \tag{16}$$

We find no solutions of Eqs.(14,15,16) with any of $x_1, x_{23}, V_1, V_2, V_3$ vanishing. From 1-3 symmetry, Eq. (14) and Eq.(15) are both satisfied only if $V_1 = V_3$. Solution of Eq.(16) further requires $(2V_1 + V_2)(V_1 - V_2) = 0$ since it can be shown that $x_1 = x_{23}$ is not possible for any hierachy consistent with exeperiment.

In the minimal A_4 model, therefore, only two VEVs of H_3 give tribimaximal mixing.

The first is [#5]

$$< H_3 >= (V, V, V) \tag{17}$$

studied in [6]. But careful comparison with the mass eigenstates reveals that $m_2 \gg m_1 = m_3 = 0$ so the wrong mass eigenstate (ν_2) is selected for the observed hierarchy, and so Eq.(17) is an unacceptable VEV for $< H_3 >$.

The only other VEV for the A_4-**3** is therefore [#6]

[#5] This VEV, $< H_3 >\propto (1, 1, 1)$, can be transformed to $< H_3 >\propto (0, 0, 1)$ by an A_4 transformation. These possibilities have been called in the literature the Ma-Rajaskaran and Altarelli-Feruglio bases respectively.

[#6] Because $< H_3 >\propto (1, 1, 1)$ could be made consistent with the neutrino masses in most previous A_4 models which had more parameters, the alternative $< H_3 >\propto (1, -2, 1)$ seems not to have been previously studied.

$$< H_3 >= (V, -2V, V), \tag{18}$$

which also gives tribimaximal mixing and the mass spectrum $m_3 \gg m_1 = m_2$ corresponding to a normal hierarchy. Thus Eq.(18) provides the only allowed VEV for $< H_3 >$.

An inverted hierarchy with $m_1 = m_2 \gg m_3$ is not possible within a minimal A_4 renormalizable model and so is disfavored. This means, for example, that neutrinoless double β-decay will require higher precision experiments.

Our conclusion is that the A_4 model in a minimal form does favor the normal hierarchy. We have considered a more restrictive model based on A_4 than previously considered[#7]. The theory has been required to be renormalizable and the Higgs scalar content is the minimum possible.

We have required that the neutrino mixing matrix be of the tribimaximal form. We then find that the masses for the neutrinos are highly constrained and can be in a normal, not inverted hierarchy.

Most, if not all, previous A_4 models [2] in the literature permit higher-order irrelevant non-renormalizable operators and their concomitant proliferation of parameters and hence allow a wide variety if possiblities for the neutrino masses. We believe the renormalizabilty condition is sensible for these flavor symmetries because of the higher predictivity.

The next step, currently under intense investigation, is whether the present renornmalizable A_4 model can be extended to a renormalizable $T^{'}$ model [7,8]. It is necessary but not sufficent condition for this that a successful renormalizable A_4 model, as presented here, exists.

Our principle conclusion is that the constraint of renormalizability which is respected by successful theories like QED and QCD gives sharper predictivity to flavor symmetry. For example, in the model discussed in the present Letter, a normal neutrino mass hierarchy is strongly favored over an inverted neutrino mass hierarchy. Such a conclusion cannot be reached without invoking renormalizability as a working principle.

[#7]Previous analysis [2] has included scalars called "flavons" which are standard model singlets with vacuum alignments different from those of the present model in which scalars $H_3(Z_2 = +1)$, $H_3^{'}(Z_2 = -1)$ are electroweak doublets.

Acknowledgements

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Figure captions

Figure 1. Geometry of triplet of A_4.

Figure 2. Geometry of tribimaximal mixing.

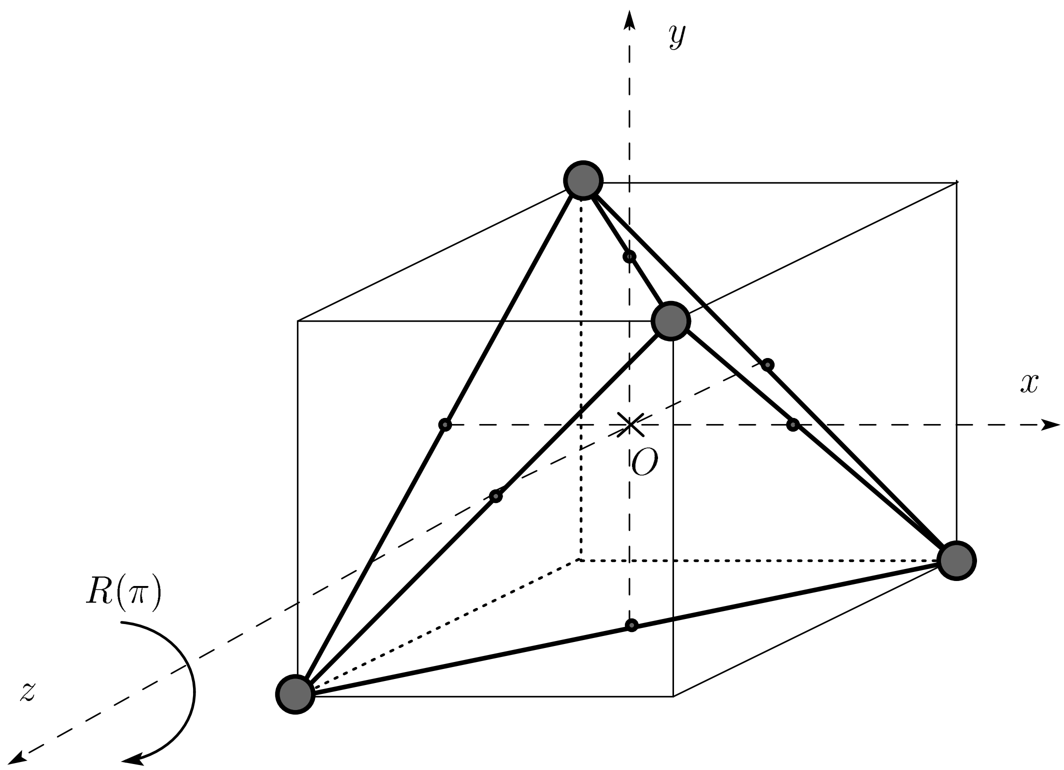

Figure 1.

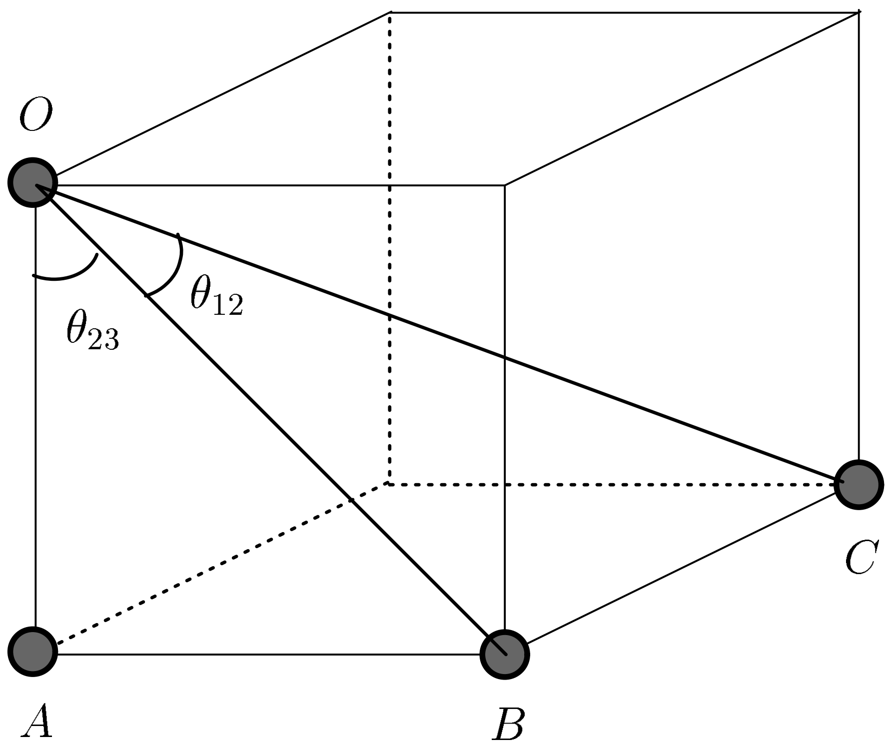

Figure 2.