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Heavy-quark pair production at two loops in QCD*

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We present updated predictions for the total cross section of top-quark pair production at Tevatron and LHC. For the LHC we also provide results at $\sqrt{s} = 10$ TeV, in view of the anticipated run in 2008 and quote numbers for the production of new heavy-quark pairs with mass in the range 0.5 – 2 TeV. Our two-loop results incorporate all logarithmically enhanced terms near threshold including Coulomb corrections as well as the exact dependence on the renormalization and factorization scale through next-to-next-to-leading order in QCD.

1. Introduction

Research on top-quark physics at hadron colliders has received great interest in the past years in view of the steadily improving measurements at Tevatron and the upcoming LHC (see Ref. [1] for a recent review). In this respect, the total cross section for top-quark pair production is a quantity of great importance for experimental analyses and even allows for measurements of the top-quark mass.

Moreover, on the theory side, the total cross section has been subject to numerous studies the motivation being improved predictions beyond the long-known next-to-leading order (NLO) corrections in QCD [2–4]. Recent work in this direction has aimed at completing the next-to-next-to-leading order (NNLO) QCD predictions [5–9], at resumming large Sudakov logarithms to next-to-next-to-leading logarithmic accuracy [10] and, at estimating bound state effects [11]. Also our knowledge on the parton distribution functions (PDFs) and the precision of the top-quark mass determination has continuously improved over the last years.

In order to study the impact of the various improvements on Tevatron and LHC predictions we build on the recent results of Ref. [10]. These approximate NNLO results for the total cross section are based on the complete logarithmic dependence on the heavy quark velocity $\beta = \sqrt{1 - 4m^2/s}$ near threshold $s \simeq 4m^2$. Moreover, they include the complete two-loop Coulomb corrections as well as the exact dependence on the renormalization and factorization scale at NNLO [12].

Recently, similar studies have appeared in Refs. [13–15]. While Ref. [13] largely follows our approach [10] to describe the total top-quark pair cross section at NNLO, Ref. [14] has limited itself to updating older predictions based on threshold resummation to next-to-leading logarithmic accuracy only. Thus, Ref. [14] necessarily arrives at larger theoretical uncertainties. The interesting study of Ref. [15] on the other hand applied consistently predictions to NLO accuracy in QCD. In doing so, it has investigated correlations of rates for top-quark pair production with many other cross sections at LHC to quantify a potential sensitivity to the gluon luminosity.

2. Total cross section

The total hadronic cross section for top-quark pair production depends on the hadronic center-of-mass energy squared s and the top-quark mass m_t. It is given by

$$\sigma(s, {m_t}^2) = \sum_{i,j=q,\bar{q},g} f_{i/p}\left(\mu_f^2\right) \otimes f_{j/p}\left(\mu_f^2\right) \otimes \hat{\sigma}({m_t}^2, \mu_f^2, \mu_r^2)\,, \qquad (1)$$

where $f_{i/p}$ are the PDFs of the proton. The partonic cross section is given by $\hat{\sigma}$ and $\otimes$ denotes the standard convolution (see e.g. Ref. [10]).

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The generally adopted procedure to estimate the theoretical uncertainty for σ in Eq. (1) exploits the residual dependence on the renormalization and factorization scale, μ_r and μ_f, which are identified throughout this article (i.e. $\mu_r = \mu_f = \mu$). The NLO QCD corrections for the parton cross section $\hat{\sigma}$ and the PDFs $f_{i/p}$ provide the first instance where a meaningful error can be determined in this way. We define the range as

$$\sigma(\mu = 2m_t) - \Delta\sigma_{PDF}(\mu = 2m_t) \le \sigma(\mu) \le \sigma(\mu = m_t/2) + \Delta\sigma_{PDF}(\mu = m_t/2)\,, \quad (2)$$

where $\Delta\sigma_{PDF}$ is computed from the variation of the cross section with respect to the parameters of the global fit (see e.g. Refs. [15–17]).

In this contribution we employ the approximate NNLO result [10] to predict cross sections (1) and the associated uncertainty ranges (2) at Tevatron and LHC. Let us therefore briefly comment on the anticipated accuracy. Our cross section $\sigma_{\mathrm{NNLO\ (approx)}}$ takes along all logarithmically enhanced terms $\ln^k \beta$, $k = 1, \ldots, 4$ as well as the complete Coulomb corrections ($\sim 1/\beta, 1/\beta^2$) at two loops for the dominant parton channels $q\bar{q}$ and gg and adds them on top of the exact NLO predictions. In this way, our predictions rely on exact expressions in the region of phase space $s \simeq 4m^2$, where perturbative corrections receive the largest weight from the convolution with the parton luminosities, cf. Eq. (1). The effect of new parton channels opening at NNLO (qq and $\bar{q}\bar{q}$) is expected to be small, cf. the qg and $\bar{q}g$ channels at NLO.

The region of large energies $s \gg 4m^2$ on the other hand is inaccessible within our approach [10]. However, it is expected to give only small contributions in a full NNLO calculation in line with the observed small corrections for top-quark pair production together with an additional jet at NLO [5] which are part of the full NNLO correction for top-quark pair production.

Moreover, we have also included the exact μ_r and μ_f scale dependence at NNLO [12] which can be constructed using renormalization group methods. For the time being, we have chosen a common value μ for the scales, and we will address the independent variation of μ_r and μ_f in a future publication. However, based on preliminary studies we do not expect large modifications here. In summary, we have accounted for all numerically dominant contributions and are confident that this provides a very good approximation to the unknown full NNLO result as experience from other reactions, e.g. Higgs-production in gluon fusion [18] shows.

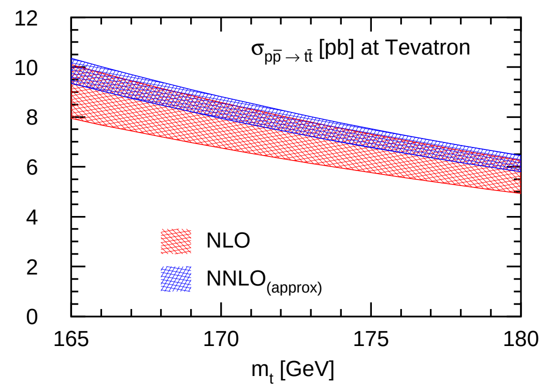

Figure 1. The NLO and NNLO (approx) QCD prediction for the $t\bar{t}$ total cross section at Tevatron for $\sqrt{s} = 1.96$ TeV. The bands denote the total uncertainty from PDF and scale variations for the MRST06nnlo set [16] according to Eq. (2).

Let us next present our results for Tevatron and LHC. In Figs. 1 and 2 we plot the uncertainty range (2) comparing NLO and NNLO accuracy. At Tevatron (Fig. 1) the central value at NNLO increases typically by 8% with respect to NLO. The residual scale dependence of $\sigma_{\mathrm{NNLO\ (approx)}}$ is 3%, which corresponds to a reduction by a factor of two compared to NLO. The overall uncertainty according to Eq. (2) is at NNLO (approx) about 8% for the CTEQ6.6 and 6% for the MRST06nnlo PDF set. At LHC (Fig. 2) our $\sigma_{\mathrm{NNLO\ (approx)}}$ leads only to a small shift of a few percent in the central value and the NNLO (approx) band is about 6% for CTEQ6.6 and about 4% for MRST06nnlo, which exhibits again a drastic reduction of the scale uncertainty as compared to the prediction based on NLO QCD.

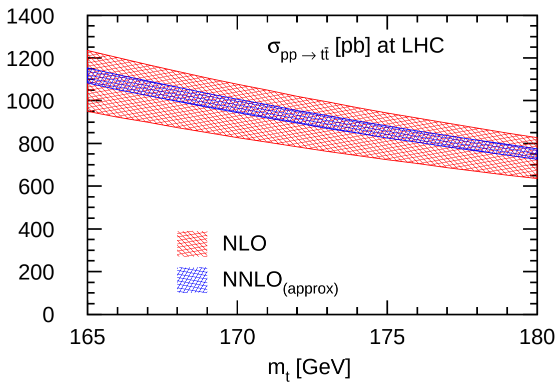

Figure 2. Same as Fig. 1 for LHC with $\sqrt{s} = 14$ TeV.

For phenomenological applications, the results of Eqs. (1), (2) are best presented by means of simple formulae for the mass dependence of the total cross section. To that end we make the ansatz following Ref. [14]

$$\sigma(m_t) = a + bx + cx^2 + dx^3 + ex^4 \,, \qquad (3)$$

where $x = (m_t/\text{GeV} - 171)$. The parameters a, b, c, d, e are fitted to reproduce σ in the mass range 150 GeV $\leq m_t \leq$ 190 GeV with a typical accuracy of better than 0.1 per mille. For Tevatron and LHC the respective results for various PDF sets are given in Tabs. 1–3. Note, that Eq.(3) uses a polynomial of degree four and also determines the parameter a for the central value $\sigma(m_t = 171$ GeV) from the fit.

Finally, we briefly quote some NNLO (approx) rates for the pair-production of new heavy quarks in the fundamental representation of the color $SU(3)$ gauge group at LHC with $\sqrt{s} = 14$ TeV (see also Ref. [14]). Such particles with a mass m_T appear in certain extensions of the Standard Model and we focus on a production model which is entirely dominated by QCD effects. Thus, our cross section $\sigma_{\text{NNLO (approx)}}$ provides a meaningful and accurate prediction because its numerical values arises largely from the threshold region where the logarithms $\ln^k \beta$ dominate.

In Tabs. 4, 5 we quote the corresponding numbers in the mass range 0.5 TeV $\leq m_T \leq$ 2 TeV (see Ref. [14] for results to NLO accuracy). We observe that the scale dependence at NNLO accuracy is rather small, showing the expected good stability of the perturbative prediction. The relative variation of σ with respect to the PDFs, though, is dominating by far. Note there is the usual factor of two between the PDF uncertainty quoted by MRST06nnlo [16] and the CTEQ6.5 [17] PDF sets due to the definition of the tolerance criteria in the respective fits. The reason for the large observed PDF uncertainty is the gluon PDF being poorly constrained in the relevant region of large momentum fraction $x \simeq 0.1 \dots 0.3$. This is a fact well-known to influence many searches for high-mass particles in gluon fusion channels (see e.g. Ref. [15] for the correlation of top-quark pair production rate with the high mass Higgs cross section).

3. Conclusion

We have presented updated predictions for cross sections of top-quark pair production based on the (approximate) NNLO results of Ref. [10]. These represent the best present estimates for hadro-production of top-quark pairs, both at Tevatron and LHC. We have argued that the neglected contributions (i.e. power suppressed terms away from threshold and new parton channels) are numerically small. We have found good convergence properties of the higher order corrections and greatly improved stability of the total cross section with respect to scale variations by our NNLO (approx) result. For applications, we have presented simple formulae (3) with 0.1 per mille accuracy for the mass dependence of the total cross section in the range 150 GeV $\leq m_t \leq$ 190 GeV. Finally, we have applied our results to estimate the pair-production rates of new quarks heavier than the top-quark in the range up to 2 TeV.

The results of Tabs. 1–3 for the fit of the mass dependence of σ have also been coded in a C-program, which is available from the authors upon request.

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Tevatron $\sqrt{s} = 1.96$TeV		a[pb]	b[pb]	c[pb] $\times 10^2$	d[pb] $\times 10^5$	e[pb] $\times 10^7$
CTEQ6.5	$\sigma(\mu = m_t)$	7.93923	- 0.247233	0.43143	- 6.09398	7.37166
	$\sigma(\mu = m_t) - \Delta\sigma_{PDF}$	7.49359	- 0.231436	0.39979	- 5.59529	6.73356
	$\sigma(\mu = m_t) + \Delta\sigma_{PDF}$	8.38488	- 0.263031	0.46307	- 6.59268	8.00976
	$\sigma(\mu = 2m_t)$	7.6572	- 0.239371	0.418793	- 5.92369	7.16791
	$\sigma(\mu = m_t/2)$	8.00918	- 0.247709	0.428962	- 6.00414	7.2013
CTEQ6.6	$\sigma(\mu = m_t)$	7.80984	- 0.243547	0.425424	- 6.01643	7.29114
	$\sigma(\mu = m_t) - \Delta\sigma_{PDF}$	7.36172	- 0.22749	0.39309	- 5.5075	6.6412
	$\sigma(\mu = m_t) + \Delta\sigma_{PDF}$	8.25797	- 0.259604	0.457757	- 6.52536	7.94108
	$\sigma(\mu = 2m_t)$	7.53093	- 0.235746	0.412856	- 5.84596	7.08203
	$\sigma(\mu = m_t/2)$	7.87933	- 0.244019	0.422956	- 5.92653	7.11881
MRST06nnlo	$\sigma(\mu = m_t)$	8.24268	- 0.262604	0.46727	- 6.70602	8.19512
	$\sigma(\mu = m_t) - \Delta\sigma_{PDF}$	8.02331	- 0.254847	0.451872	- 6.46408	7.88561
	$\sigma(\mu = m_t) + \Delta\sigma_{PDF}$	8.46205	- 0.27036	0.482669	- 6.94796	8.50462
	$\sigma(\mu = 2m_t)$	7.90318	- 0.252688	0.450793	- 6.47936	7.91184
	$\sigma(\mu = m_t/2)$	8.32601	- 0.263279	0.464701	- 6.60993	8.01315

Table 1
The coefficients of the parameterization (3) for the cross section $\sigma_{\rm NNLO\ (approx)}$ of Ref. [10] in pb at Tevatron ($\sqrt{s} = 1.96$ TeV) using the PDF sets CTEQ6.5 [17], CTEQ6.6 [15] and MRST06nnlo set [16].

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LHC $\sqrt{s} = 10$TeV		a[pb]	b[pb]	c[pb]	d[pb] $\times 10^2$	e[pb] $\times 10^5$
CTEQ6.5	$\sigma(\mu = m_t)$	418.588	- 11.8988	0.19815	- 0.267839	3.07947
	$\sigma(\mu = m_t) - \Delta\sigma_{PDF}$	399.508	- 11.4849	0.192657	- 0.261487	3.01046
	$\sigma(\mu = m_t) + \Delta\sigma_{PDF}$	437.668	- 12.3126	0.203644	- 0.274191	3.14848
	$\sigma(\mu = 2m_t)$	413.69	- 11.7539	0.195659	- 0.264393	3.03946
	$\sigma(\mu = m_t/2)$	400.724	- 11.3609	0.188716	- 0.254368	2.91524
CTEQ6.6	$\sigma(\mu = m_t)$	419.062	- 11.9351	0.199035	- 0.269327	3.09784
	$\sigma(\mu = m_t) - \Delta\sigma_{PDF}$	400.4	- 11.5275	0.193785	- 0.263554	3.03731
	$\sigma(\mu = m_t) + \Delta\sigma_{PDF}$	438.121	- 12.3427	0.204284	- 0.275101	3.15837
	$\sigma(\mu = 2m_t)$	413.944	- 11.7839	0.196439	- 0.265736	3.05688
	$\sigma(\mu = m_t/2)$	401.139	- 11.3946	0.18954	- 0.255779	2.93453
MRST06nnlo	$\sigma(\mu = m_t)$	449.93	- 12.6551	0.208761	- 0.279652	3.1917
	$\sigma(\mu = m_t) - \Delta\sigma_{PDF}$	441.788	- 12.4928	0.206866	- 0.277817	3.17472
	$\sigma(\mu = m_t) + \Delta\sigma_{PDF}$	458.073	- 12.8175	0.210656	- 0.281487	3.20868
	$\sigma(\mu = 2m_t)$	443.176	- 12.4618	0.205521	- 0.275247	3.14118
	$\sigma(\mu = m_t/2)$	431.685	- 12.111	0.199286	- 0.266217	3.03056

Table 2
Same as in Tab. 1 for the cross section $\sigma_{\text{NNLO (approx)}}$ at LHC with start-up energy $\sqrt{s} = 10$ TeV.

LHC $\sqrt{s} = 14$TeV		a[pb]	b[pb]	c[pb]	d[pb] $\times 10^2$	e[pb] $\times 10^5$
CTEQ6.5	$\sigma(\mu = m_t)$	917.522	- 24.8588	0.398405	- 0.520463	5.82374
	$\sigma(\mu = m_t) - \Delta\sigma_{PDF}$	887.489	- 24.2054	0.388932	- 0.507626	5.66159
	$\sigma(\mu = m_t) + \Delta\sigma_{PDF}$	947.556	- 25.5122	0.407878	- 0.5333	5.98589
	$\sigma(\mu = 2m_t)$	908.276	- 24.5978	0.394104	- 0.514815	5.76339
	$\sigma(\mu = m_t/2)$	878.259	- 23.7272	0.379206	- 0.493949	5.51362
CTEQ6.6	$\sigma(\mu = m_t)$	920.475	- 24.9757	0.400681	- 0.523783	5.85946
	$\sigma(\mu = m_t) - \Delta\sigma_{PDF}$	891.302	- 24.3671	0.392303	- 0.512828	5.72046
	$\sigma(\mu = m_t) + \Delta\sigma_{PDF}$	949.648	- 25.5842	0.409059	- 0.534738	5.99846
	$\sigma(\mu = 2m_t)$	910.798	- 24.7036	0.396235	- 0.517914	5.79166
	$\sigma(\mu = m_t/2)$	881.04	- 23.8377	0.381364	- 0.497136	5.54881
MRST06nnlo	$\sigma(\mu = m_t)$	969.128	- 25.9797	0.412622	- 0.534539	5.93782
	$\sigma(\mu = m_t) - \Delta\sigma_{PDF}$	957.674	- 25.7648	0.409906	- 0.530883	5.88547
	$\sigma(\mu = m_t) + \Delta\sigma_{PDF}$	980.582	- 26.1945	0.415337	- 0.538195	5.99016
	$\sigma(\mu = 2m_t)$	956.58	- 25.636	0.407062	- 0.527262	5.86177
	$\sigma(\mu = m_t/2)$	929.84	- 24.8578	0.393723	- 0.508485	5.63364

Table 3
Same as in Tab. 1 for the cross section $\sigma_{\text{NNLO (approx)}}$ at LHC with $\sqrt{s} = 14$ TeV.

m_T[TeV]	only scale uncertainty min	max	δ[%]	only pdf uncertainty min	max	δ[%]	total uncertainty min	max	δ[%]
0.5	4345.	4472.	1	4287.	4656.	4	4160.	4656.	6
0.6	1561.	1601.	1	1526.	1676.	5	1486.	1676.	6
0.7	634.1	649.2	1	616.0	682.5	5	600.8	682.5	6
0.8	282.3	288.5	1	272.6	304.4	6	266.4	304.4	7
0.9	134.5	137.2	1	129.3	145.1	6	126.6	145.1	7
1.0	67.64	68.94	1	64.81	73.08	6	63.50	73.08	7
1.1	35.45	36.17	1	33.93	38.41	6	33.22	38.41	7
1.2	19.23	19.65	1	18.38	20.91	6	17.97	20.91	8
1.3	10.74	10.99	1	10.26	11.72	7	10.01	11.72	8
1.4	6.147	6.301	1	5.862	6.741	7	5.708	6.741	8
1.5	3.589	3.687	1	3.417	3.957	7	3.319	3.957	9
1.6	2.130	2.192	1	2.021	2.363	8	1.959	2.363	9
1.7	1.282	1.322	2	1.212	1.432	8	1.172	1.432	10
1.8	0.781	0.806	2	0.735	0.878	9	0.710	0.878	11
1.9	0.480	0.497	2	0.450	0.544	9	0.433	0.544	11
2.0	0.298	0.309	2	0.277	0.340	10	0.266	0.340	12

Table 4

The NNLO (approx) cross section of Ref. [10] in fb for the pair-production of a (new) heavy quark with mass m_T at LHC ($\sqrt{s} = 14$ TeV) using the MRST06nnlo PDF set [16]. δ is the relative uncertainty with respect to the central value: $\delta = 100 \times (\max - \min)/(\max + \min)$.

m_T[TeV]	only scale uncertainty min	max	δ[%]	only pdf uncertainty min	max	δ[%]	total uncertainty min	max	δ[%]
0.5	3921.	4037.	1	3639.	4436.	10	3522.	4436.	11
0.6	1402.	1440.	1	1275.	1604.	11	1238.	1604.	13
0.7	568.0	582.5	1	508.6	656.5	13	494.1	656.5	14
0.8	252.4	258.2	1	222.5	294.0	14	216.6	294.0	15
0.9	120.3	122.8	1	104.5	141.0	15	102.0	141.0	16
1.0	60.48	61.66	1	51.94	71.37	16	50.76	71.37	17
1.1	31.78	32.30	1	26.95	37.64	17	26.44	37.64	17
1.2	17.25	17.57	1	14.50	20.57	17	14.21	20.57	18
1.3	9.626	9.802	1	8.023	11.58	18	7.847	11.58	19
1.4	5.503	5.614	1	4.555	6.673	19	4.444	6.673	20
1.5	3.208	3.277	1	2.630	3.925	20	2.560	3.925	21
1.6	1.902	1.946	1	1.545	2.347	21	1.502	2.347	22
1.7	1.144	1.173	1	0.921	1.426	22	0.892	1.426	23
1.8	0.696	0.715	1	0.554	0.876	23	0.535	0.876	24
1.9	0.428	0.441	1	0.337	0.544	24	0.324	0.544	25
2.0	0.265	0.274	2	0.206	0.342	25	0.198	0.342	27

Table 5

Same as in Tab. 4 using the CTEQ6.5 [17] PDF set.