

MSSM Higgs Couplings to Bottom Quarks: Two-Loop Corrections

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We present the two-loop SUSY-QCD corrections to the effective bottom Yukawa couplings within the minimal supersymmetric extension of the Standard Model. The effective Yukawa couplings include the resummation of the non-decoupling corrections Δm_b for large values of $\text{tg}\beta$. We have derived the two-loop SUSY-QCD corrections to the leading SUSY-QCD and top-induced SUSY-electroweak contributions to Δm_b . The scale dependence of the resummed Yukawa couplings is reduced from $\mathcal{O}(10\%)$ to the per-cent level. These results reduce the theoretical uncertainties of the MSSM Higgs branching ratios to the accuracy which can be achieved at a future linear e^+e^- collider.

The Higgs mechanism [1] is a cornerstone of the Standard Model (SM) and its supersymmetric extensions. The masses of the fundamental particles, electroweak gauge bosons, leptons, and quarks, are generated by interactions with Higgs fields. The search for Higgs bosons is thus one of the most important endeavors in high-energy physics and is being pursued at the upgraded proton-antiproton collider Tevatron [2] with a centre-of-mass (CM) energy of 1.96 TeV, followed in the near future by the proton-proton collider LHC [3] with 14 TeV CM energy.

The minimal supersymmetric extension of the Standard Model (MSSM) requires the introduction of two Higgs doublets. After electroweak symmetry breaking there are five elementary Higgs particles, two CP-even (h, H), one CP-odd (A) and two charged ones (H^\pm). At lowest order all couplings and masses of the MSSM Higgs sector are fixed by two independent input parameters, which are generally chosen as $\text{tg}\beta = v_2/v_1$, the ratio of the two vacuum expectation values $v_{1,2}$, and the pseudoscalar Higgs mass M_A . Including the one-loop and dominant two-loop corrections the upper bound on the light scalar Higgs mass is $M_h \lesssim 135$ GeV [4]. The couplings of the various Higgs bosons to fermions and gauge bosons depend on mixing angles α and β , which are defined by diagonalizing the neutral and charged Higgs mass matrices.

The negative direct searches at LEP2 yield lower bounds of $M_{h,H} > 92.8$ GeV and $M_A > 93.4$ GeV. The range $0.7 < \text{tg}\beta < 2.0$ in the MSSM is excluded by the Higgs searches for a SUSY scale $M_{SUSY} = 1$ TeV at the LEP2 experiments [5].

The dominant genuine SUSY-QCD and SUSY-electroweak corrections to bottom-Yukawa-coupling induced processes, as e.g. Higgs boson decays to $b\bar{b}$ pairs and Higgs radiation off bottom quarks, can be derived from

the effective Lagrangian [6, 7, 8]

$$\mathcal{L}_{eff} = -\frac{m_b \bar{b}}{v} [\tilde{g}_b^h h + \tilde{g}_b^H H - \tilde{g}_b^A i\gamma_5 A] b \quad (1)$$

with the resummed Yukawa coupling factors

$$\begin{aligned} \tilde{g}_b^h &= \frac{g_b^h}{1 + \Delta m_b} \left(1 - \frac{\Delta m_b}{\text{tg}\alpha \text{tg}\beta} \right) \\ \tilde{g}_b^H &= \frac{g_b^H}{1 + \Delta m_b} \left(1 + \Delta m_b \frac{\text{tg}\alpha}{\text{tg}\beta} \right) \\ \tilde{g}_b^A &= \frac{g_b^A}{1 + \Delta m_b} \left(1 - \frac{\Delta m_b}{\text{tg}^2\beta} \right) \end{aligned} \quad (2)$$

where Δm_b determines the relative corrections to the bottom Yukawa couplings. The Higgs couplings are given by

$$g_b^h = -\frac{\sin\alpha}{\cos\beta}, \quad g_b^H = \frac{\cos\alpha}{\cos\beta}, \quad g_b^A = \text{tg}\beta \quad (3)$$

The leading one-loop corrections Δm_b to these effective couplings can be cast into the form

$$\begin{aligned} \Delta m_b &= \Delta m_b^{QCD(1)} + \Delta m_b^{elw(1)} \\ \Delta m_b^{QCD(1)} &= \frac{2}{3} \frac{\alpha_s(\mu_R)}{\pi} m_{\bar{g}} \mu \text{tg}\beta I(m_{\bar{b}_1}^2, m_{\bar{b}_2}^2, m_{\bar{g}}^2) \\ \Delta m_b^{elw(1)} &= \frac{\lambda_t^2(\mu_R)}{(4\pi)^2} A_t \mu \text{tg}\beta I(m_{\bar{t}_1}^2, m_{\bar{t}_2}^2, \mu^2) \end{aligned} \quad (4)$$

with the scalar function

$$I(a, b, c) = \frac{ab \log \frac{a}{b} + bc \log \frac{b}{c} + ca \log \frac{c}{a}}{(a-b)(b-c)(a-c)}$$

The parameter $v = \sqrt{v_1^2 + v_2^2} = \sqrt{1/\sqrt{2}G_F}$ is related to the Fermi constant G_F and $\lambda_t = \sqrt{2}m_t/v_2$ denotes the

top Yukawa coupling. The SUSY–QCD and top-induced SUSY–electroweak corrections turn out to be significant for large values of $\text{tg}\beta$ and moderate or large μ and A_t values. In order to improve the perturbative result all terms of $\mathcal{O}[(\alpha_s \mu \text{tg}\beta/M_{SUSY})^n]$ and $\mathcal{O}[(\lambda_t^2 A_t \text{tg}\beta/M_{SUSY})^n]$ have been resummed in Eq. (2) [7, 8]. The correction Δm_b is *non-decoupling* in the sense that scaling all SUSY parameters $m_{\tilde{b}_{1,2}}, m_{\tilde{g}}, \mu$ in Eq. (2) leaves Δm_b invariant. However, its contribution develops decoupling properties [9]. The corrections Δm_b contain the strong coupling $\alpha_s(\mu_R)$ and the top Yukawa coupling $\lambda_t(\mu_R)$ with significant renormalization scale dependences. This leads to theoretical uncertainties in e.g. the MSSM Higgs boson decay widths and branching ratios of up to $\mathcal{O}(10\%)$ [8] which are larger than the achievable accuracy at a future e^+e^- linear collider (ILC) [10]. In this letter we present the two-loop SUSY–QCD corrections to the contributions Δm_b of Eq. (4) in order to reduce the theoretical uncertainties to the per-cent level.

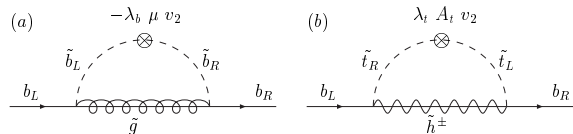


FIG. 1: One-loop diagrams of (a) the SUSY–QCD and (b) the top-induced SUSY–electroweak contributions to the bottom self-energy with mass insertions corresponding to the corrections Δm_b of the bottom Yukawa couplings. The virtual particles involve bottom quarks b , sbottom \tilde{b} and stop \tilde{t} squarks, gluinos \tilde{g} and charged Higgsinos \tilde{h}^\pm .

Typical SUSY–QCD and SUSY–electroweak diagrams that contribute to the bottom self-energy at one-loop order are displayed in Fig. 1. The leading Δm_b corrections can be obtained from the diagrams in Fig. 1 with off-diagonal mass insertions in the virtual sbottom and stop propagators in the chiral squark basis [21]. These mass insertions yield a factor $\lambda_b \mu v_2$ for the sbottom propagators and $\lambda_t A_t v_2$ in the stop case. One obtains the one-loop results of Eq. (4) by replacing $v_2 \rightarrow \sqrt{2}\phi_2^{0*}$ and expressing the neutral Higgs component ϕ_2^{0*} of the second Higgs doublet by the mass eigenstates h, H, A [8]. These replacements lead to the exact interactions with non-propagating Higgs fields, i.e. in the low-energy limit of small Higgs momentum [11]. This method will be applied to the leading two-loop diagrams within SUSY–QCD. A typical sample of two-loop diagrams contributing to the bottom self-energy is shown in Figs. 2a,b [a mass insertion has to be included in all possible ways in the sbottom/stop propagators].

Dimensional regularization has been adopted for isolating the ultraviolet singularities. The bottom momentum and its mass have been put to zero while keeping the bottom Yukawa coupling λ_b finite in the mass insertions. All supersymmetric particles as well as the top quark have been treated with full mass dependence. The two-loop vacuum integrals have been reduced to the two-loop

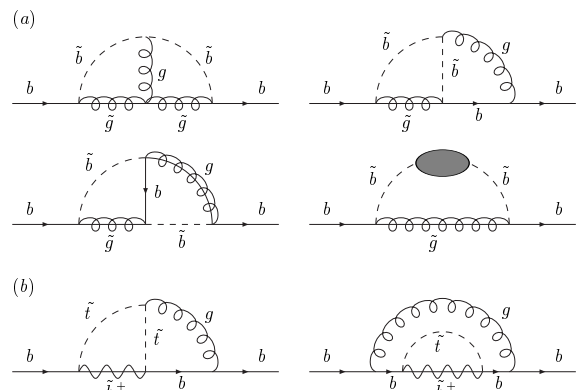


FIG. 2: Typical two-loop diagrams of (a) the SUSY–QCD and (b) the top-induced SUSY–electroweak contributions to the bottom self-energy involving bottom quarks b , sbottom \tilde{b} and stop \tilde{t} squarks, gluons g , gluinos \tilde{g} and charged Higgsinos \tilde{h}^\pm .

master integral $T_{134}(m_1, m_3, m_4)$ [12] and one-loop one-point functions $A_0(m)$ by standard reduction methods [13]. After adding all two-loop diagrams linear ultraviolet divergences are left over which are absorbed by the renormalization of all masses and couplings appearing at one-loop order. The heavy masses $m_{\tilde{b}_i}, m_{\tilde{t}_i}, m_{\tilde{g}}$ appearing in the propagators have been renormalized on-shell. The trilinear coupling A_t has been treated in the on-shell scheme, too. The strong coupling α_s and the top Yukawa coupling λ_t have been defined in the $\overline{\text{MS}}$ scheme with 5 active flavors, i.e. the top quark and the supersymmetric particles have been decoupled from the scale dependence of the strong coupling $\alpha_s(\mu_R)$. Care has to be taken to include only the desired order, i.e. $\mathcal{O}(\alpha_s^2 \mu \text{tg}\beta/M_{SUSY})$ for Δm_b^{QCD} and $\mathcal{O}(\alpha_s \lambda_t^2 A_t \text{tg}\beta/M_{SUSY})$ for Δm_b^{elw} . In this order the trilinear coupling A_t only receives a finite renormalization at NLO. More details of our calculation will be published in [14].

Since dimensional regularization violates supersymmetry by e.g. attributing $(n-2)$ degrees of freedom to the gluons but two degrees of freedom to its supersymmetric gluino partner, anomalous counter terms have to be added in order to restore the supersymmetric relations between the corresponding couplings. Since the strong coupling factors at one-loop correspond to the Yukawa couplings between gluino, sbottom and bottom quark states an anomalous counter term has to be introduced in order to express all strong coupling factors in terms of the conventional $\overline{\text{MS}}$ QCD coupling [15]. Moreover, the bottom Yukawa coupling λ_b in Fig. 1a determines the strength of the Higgs coupling to sbottom states at one-loop order which differs from the $\overline{\text{MS}}$ bottom Yukawa coupling by a finite amount [15]. The total anomalous counter term for Δm_b^{QCD} is given by [$C_A = 3, C_F = 4/3$]

$$\delta \Delta m_b^{QCD, \text{anom}} = \left(\frac{C_A}{3} - \frac{C_F}{2} \right) \frac{\alpha_s}{\pi} \Delta m_b^{QCD(1)}$$

For Δm_b^{elw} the situation is similar. The one-loop order

involves the bottom and top Yukawa couplings of the Higgsino-bottom-stop vertices as well as the top Yukawa coupling of the Higgs-stop-stop vertex. Both are shifted from the $\overline{\text{MS}}$ couplings by finite amounts [15]. The sum of anomalous counter terms for Δm_b^{elw} is given by

$$\delta\Delta m_{b;anom}^{elw} = -C_F \frac{\alpha_s}{\pi} \Delta m_b^{elw(1)}$$

The final results have been included in the program HDECAY [16] which calculates the masses and couplings of the MSSM Higgs bosons as well as their decay widths and branching ratios.

The numerical analysis of the corrections Δm_b and their impact on neutral Higgs boson decays is performed for the “small α_{eff} ” MSSM scenario [17] as a representative case:

$$\begin{aligned} \text{tg}\beta &= 30, \quad M_{\tilde{Q}} = 800 \text{ GeV}, \quad M_{\tilde{g}} = 500 \text{ GeV} \\ M_2 &= 500 \text{ GeV}, \quad A_b = A_t = -1.133 \text{ TeV}, \quad \mu = 2 \text{ TeV} \end{aligned} \quad (5)$$

For the Higgs masses and couplings [22] we use the RG-improved two-loop expressions of Ref. [18]. The bottom quark pole mass has been chosen to be $M_b = 4.60 \text{ GeV}$, which corresponds to a $\overline{\text{MS}}$ mass $\overline{m}_b(\overline{m}_b) = 4.26 \text{ GeV}$. The strong coupling constant has been normalized to $\alpha_s(M_Z) = 0.118$.

The scale dependences of the corrections Δm_b^{QCD} and Δm_b^{elw} are displayed in Fig. 3 at one- and two-loop order. The central scale of the SUSY-QCD part Δm_b^{QCD} is chosen as the average of the SUSY-particle masses contributing at one loop, i.e. $\mu_0 = (m_{\tilde{b}_1} + m_{\tilde{b}_2} + m_{\tilde{g}})/3$, and as $\mu_0 = (m_{\tilde{t}_1} + m_{\tilde{t}_2} + \mu)/3$ for the SUSY-electroweak part Δm_b^{elw} . We obtain a significant reduction of the scale dependence at two-loop order and thus a large reduction of the theoretical uncertainty. Moreover a broad maximum/minimum develops at scales of about 1/4 to 1/3 of the chosen central scale in contrast to the monotonous scale dependences at one-loop order. In the “small α_{eff} ” scenario the SUSY-QCD corrections are large and positive, while the SUSY-electroweak corrections are of moderate negative size. However, the sign and size of the corrections depends on the chosen MSSM scenario. The two-loop corrections amount to $\mathcal{O}(10\%)$ in Δm_b^{QCD} and a few per cent in Δm_b^{elw} for the central scale choices.

The branching ratios of the neutral MSSM Higgs bosons are depicted in Figs. 4a-c. The bands at one-loop order (dashed blue curves) and two-loop order (full red curves) are defined by varying the renormalization scale between 1/3 and 3 times the corresponding central scale of the SUSY-QCD and SUSY-electroweak parts. We only show the two dominant decay modes into $b\bar{b}$ and $\tau^+\tau^-$ pairs. The uncertainties of the branching ratios reduce from $\mathcal{O}(10\%)$ at one-loop order to the per-cent level at two-loop order. The per-cent accuracy now matches the expected experimental accuracies at a future linear e^+e^- collider.

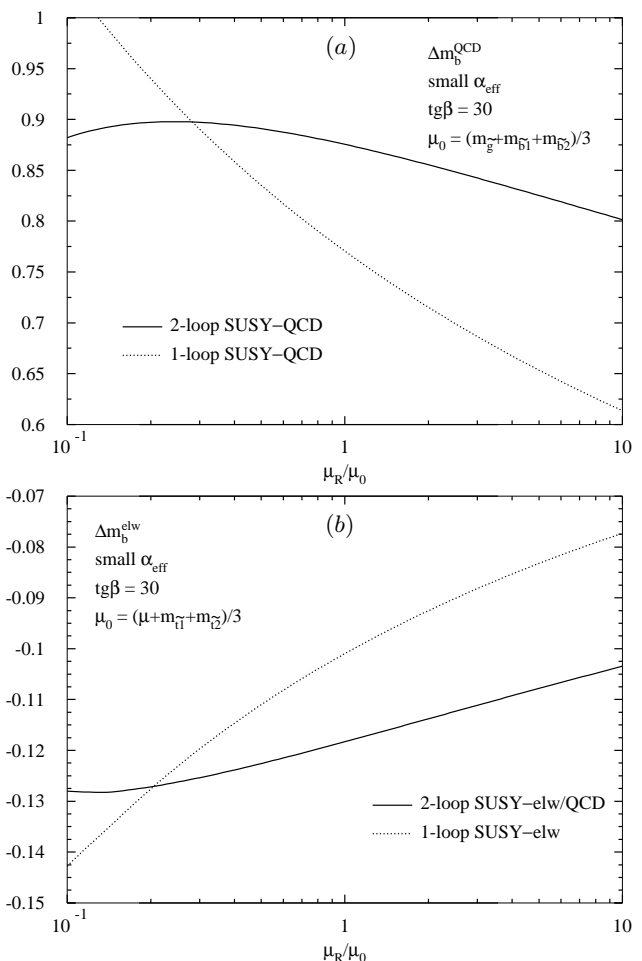


FIG. 3: Scale dependence of the corrections Δm_b at one- and two-loop order: (a) the SUSY-QCD part Δm_b^{QCD} and (b) the SUSY-electroweak part Δm_b^{elw} in the “small α_{eff} ” scenario.

Since we have determined the effective resummed Yukawa coupling at two-loop order the results will also affect all other processes which are significantly induced by bottom Yukawa couplings, as e.g. MSSM Higgs radiation off bottom quarks at e^+e^- colliders [19] and hadron colliders [20]. The two-loop corrections can easily be included in the corresponding numerical programs.

In summary, the significant scale dependence of $\mathcal{O}(10\%)$ of the NLO predictions for processes involving the bottom quark Yukawa couplings of supersymmetric Higgs bosons requires the inclusion of NNLO corrections. For the corrected Yukawa couplings, we find a reduction of the scale dependence to the per-cent level at NNLO. The improved NNLO predictions for the bottom Yukawa couplings can thus be taken as a base for experimental analyses at the Tevatron and the LHC as well as the ILC.

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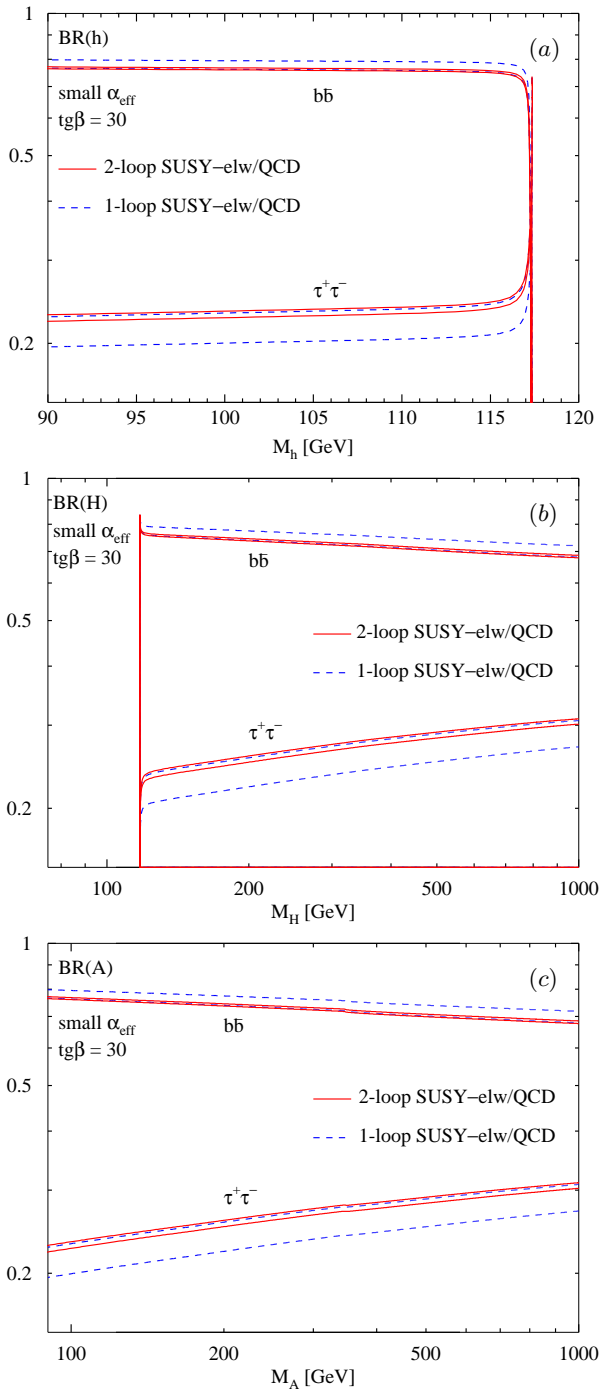


FIG. 4: Branching ratios of (a) the light scalar, (b) the heavy scalar and (c) the pseudoscalar Higgs boson in the “small α_{eff} ” scenario. The dashed blue bands indicate the scale dependence at one-loop order and the full red bands at two-loop order by varying the renormalization scales between 1/3 and 3 times the central scales given by the corresponding average SUSY particle masses.

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