Duality cascade of softly broken supersymmetric theories

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Abstract

We study the duality cascade of softly broken supersymmetric theories. We investigate the renormalization group (RG) flow of SUSY breaking terms as well as supersymmetric couplings. It is found that the magnitudes of SUSY breaking terms are suppressed in most regimes of the RG flow through the duality cascade. At one stage of cascading, the gaugino mass of the strongly coupled sector and scalar masses converge to certain values, which are determined by the gauge coupling and the gaugino mass of the weakly coupled sector. At the next stage, the strongly and weakly coupled sectors are interchanged with each other. We also show the possibility that cascading would be terminated by the gauge symmetry breaking, which is induced by the so-called B-term.

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1 Introduction

Conformal dynamics plays important roles in various aspects of (supersymmetric) field theories and particle phenomenology. Conformal fixed points and conformal field theories (CFTs) are essential in Seiberg duality [1, 2]. That leads to more complicated and interesting renormalization group (RG) flows of dual field theories, that is, the duality cascade [3, 4], which is a successive chain of the dualities from the ultraviolate (UV) region to the infrared (IR) region and reduces the rank of gauge groups one after another. Furthermore, the AdS/CFT (gravity/gauge) correspondence [5] suggests that the cascading theories would be realized in supergravity theory with a warped background, that is, the Klebanov-Strassler warp throat. In the supergravity description, the energy scale of the field theory corresponds to the distance from a tip of the throat. The duality cascade process means that the charges of D-branes disappear as the prove brane gets closer to the tip. The investigation of the duality cascade from the string/supergravity viewpoint is highly non-trivial check for the gravity/gauge correspondence.

Superconformal dynamics is also important in applications to particle phenomenology. For example, conformal dynamics may generate realistic hierarchies of quark and lepton masses [6, 7, 8]. Conformal dynamics has significant effects on supersymmetry (SUSY) breaking terms, too. In simple gauge theories, soft SUSY breaking terms, i.e. the gaugino mass and soft scalar masses, are exponentially suppressed toward the IR attractive fixed point [9, 7, 10]. Thus, sfermion masses are exponentially suppressed in the above models with CFT-induced Yukawa hierarchy [6, 7, 8].[1] Another aspect of conformal dynamics relevant to SUSY breaking terms is conformal sequestering [12, 13, 14, 15, 16, 17]. Conformal dynamics may suppress flavor-dependent SUSY breaking terms and make flavor-blind contributions dominant. Conformal dynamics may be important to realize a SUSY breaking model [18].

Thus, superconformal dynamics is important in particle physics. Here we study more about the duality cascade. Recently, several models have been proposed to realize supersymmetric standard models as well as their extensions at the bottom of the cascade [19, 20]. If we would like to realize the gauge theories in Type IIA/IIB string theory, it admits high ranks of the gauge groups since there are configurations with the various number of

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[1] Five-dimensional warped theory with the same behavior was studied e.g. in [11].

the D-branes. To explain how we obtain the standard model like theories with fewer ranks from the infinitely many string vacua with large ranks, those models are quite interesting and have opened possible candidates for high energy theories. Those models are exactly supersymmetric. At any rate, supersymmetry is broken in Nature even if supersymmetric theory is realized at high energy. Thus, if the cascading theories are relevant to the particle physics at the weak scale, supersymmetry should be broken at a certain stage, e.g. at the top or bottom of the cascade (high or low energy) or between them (intermediate energy). Here we assume that SUSY is softly broken at the beginning of the cascade. Then, we study RG flows of SUSY breaking terms as well as supersymmetric couplings.

This paper is organized as follows. In section 2, we review briefly the RG flow of supersymmetric couplings in the duality cascade. In section 3, we study RG flows of SUSY breaking terms. In section 4, we study symmetry breaking due to the B-term by using illustrative examples. Section 5 is devoted to conclusion and discussion.

2 RG flow in duality cascade of rigid supersymmetric theories

Here, we give a brief review on the RG flow in the duality cascade of rigid supersymmetric theories [3, 4]. We consider the gauge group $SU(kN) \times SU((k-1)N)$ and we denote their gauge couplings, g_k and g_{k-1}. Also, our model has two chiral multiplets Q_r ($r = 1, 2$) in the bifundamental representation of $SU(kN) \times SU((k-1)N)$, i.e. the fundamental representation for $SU(kN)$ and the anti-fundamental representation for $SU((k-1)N)$, and two chiral multiplets $\bar{Q}_s$ ($s = 1, 2$) in the anti-bifundamental representation. Then we introduce the following superpotential,

$$W = h \operatorname{tr} \det_{r,s}(Q_r \bar{Q}_s) = h \left[(Q_1)_a^\alpha (\bar{Q}_1)_\beta^a (Q_2)_b^\beta (\bar{Q}_2)_\alpha^b - (Q_1)_a^\alpha (\bar{Q}_2)_\beta^a (Q_2)_b^\beta (\bar{Q}_1)_\alpha^b \right],$$

(1)

where the indices α and β are group indices for $SU(kN)$ and the indices a and b are group indices for $SU((k-1)N)$.

Now, we study the RG flow of gauge couplings g_k and g_{k-1} and the quartic coupling h and their fixed points. The fields Q_r and $\bar{Q}_s$ have the same anomalous dimension, which we denote by γ_Q. In the NSVZ scheme [21],

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beta-function of the gauge coupling g in generic gauge theory is written as

$$\mu \frac{d\alpha}{d\mu} = \beta_\alpha = -F(\alpha)[3T_G - \sum_i T_i(1 - \gamma_i)], \tag{2}$$

where $\alpha = g^2/(8\pi^2)$ and

$$F(\alpha) = \frac{\alpha^2}{1 - T_G\alpha}. \tag{3}$$

Here, T_i and γ_i denote Dynkin indices and anomalous dimensions of the chiral matter fields, while T_G denotes the Dynkin index of the adjoint representation. For example, we have $T_G = N$ for the $SU(N)$ gauge group and $T_i = 1/2$ for the fundamental representation of the $SU(N)$ gauge group. Using this scheme, beta-functions of the gauge couplings g_k and g_{k-1} are written as

$$\beta_{\alpha_k} = -F(\alpha_k)N[k + 2 + 2(k-1)\gamma_Q], \tag{4}$$
$$\beta_{\alpha_{k-1}} = -F(\alpha_{k-1})N[k - 3 + 2k\gamma_Q]. \tag{5}$$

In addition, we can write the beta-function of $\eta = h\mu$ as

$$\beta_\eta = \eta(1 + 2\gamma_Q). \tag{6}$$

Suppose that both $SU(kN)$ and $SU((k-1)N)$ sectors are within the conformal window [1], i.e. $3k/2 \le 2(k-1) \le 3k$ and $3(k-1)/2 \le 2k \le 3(k-1)$. Then, we have two fixed points [23, 1, 2],

$$A: \quad k - 3 + 2k\gamma_Q = 0, \qquad \alpha_k = \eta = 0, \tag{7}$$

and

$$B: \quad k + 2 + 2(k-1)\gamma_Q = 0, \qquad \alpha_{k-1} = \eta = 0. \tag{8}$$

The anomalous dimension γ_Q is a function of the couplings. We represent a value of the gauge coupling g_{k-1} (g_k) at the first (second) fixed point by g_{k-1}^* (g_k^*).

At the vicinity of the first fixed point A given by (7) with $g_{k-1} \approx g_{k-1}^*$ and $0 < \alpha_k, \eta \ll 1$ (region I), it is found that $\beta_{\alpha_{k-1}} \approx 0$, $\beta_{\alpha_k} < 0$ and $\beta_\eta > 0$, that is, α_k increases and η decreases toward the IR direction. Thus, the theory would flow to the other fixed point B given by (8) toward the IR direction. On the other hand, around the fixed point B with $g_k \approx g_k^*$ and $0 < \alpha_{k-1}, \eta \ll 1$ (region II), it is found that $\beta_{\alpha_k} \approx 0$, $\beta_{\alpha_{k-1}} > 0$ and $\beta_\eta < 0$.

Hence, the quartic operator $h \operatorname{tr} \det_{r,s}(Q_r \bar{Q}_s)$ is relevant and the coupling η increases toward the IR, while α_{k-1} shrinks.

We could examine the RG flows of the gauge couplings α_k and α_{k-1}, if we admit using the anomalous dimension γ_Q obtained in the 1-loop level. For a sufficiently large N, the anomalous dimension γ_Q is given as

$$\gamma_Q = -N(k\alpha_k + (k-1)\alpha_{k-1}). \tag{9}$$

In Fig. 1, the RG flows in the coupling space (α_k, α_{k-1}) obtained in the NSVZ scheme are shown in the case of $k = 5$. Here we rescale the couplings as $N\alpha \to \alpha$. The points A $(0, 0.05)$ and B $(0.175, 0)$ represent the fixed points. The renormalized trajectory (R.T.) connecting these fixed points is shown by the bold line.

The flows in the region I are subject to the conformal dynamics around the UV fixed point A, while the flows in the region II are subject to that around the IR fixed point B. The convergence in the region I is not strong, since the fixed point coupling α_{k-1}^* is not so strong in the case of $k = 5$. It is seen in Fig. 1 that the R.T. bends on the way and the behavior of the R.T. changes quickly there. Thus the RG property on the R.T. in Fig. 1 may be characterized well as that in the region I or II.
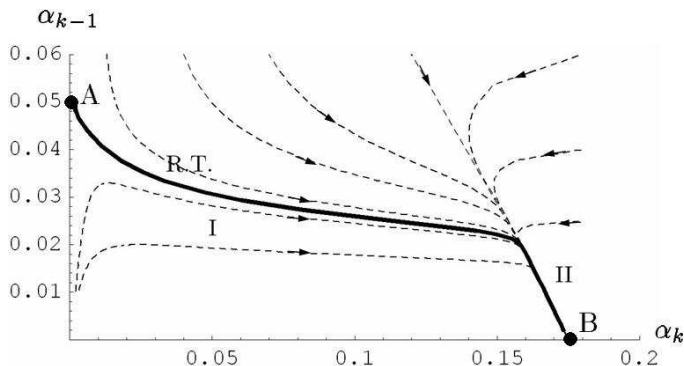

Figure 1: RG flows in the coupling space (α_k, α_{k-1}) in the case of $k = 5$. The points A and B represents the UV and IR fixed points respectively. The renormalized trajectory connecting these fixed points is shown by the bold line.

The theory around the fixed point B is strongly coupled and would be well-described by its Seiberg dual [1, 2], which has the gauge group $SU((k -$

$1)N) \times SU((k-2)N)$ and two bifundamental chiral multiplets q_r and two anti-bifundamental chiral multiplets $\bar{q}_s$ and another kind of chiral multiplets M_{rs}, which correspond to $Q_r\bar{Q}_s$ and belong to the adjoint representation for $SU((k-1)N)$ [2] and singlet for $SU((k-2)N)$. This dual theory has the following superpotential,

$$W = y \text{ tr } \bar{q}_r M_{rs} q_s + m \text{ tr} \det_{r,s} M_{rs}. \tag{10}$$

The second term is the mass term of M_{rs}, which corresponds to $h \text{ tr } \det_{r,s}(Q_r\bar{Q}_s)$. The mass m would be related with the coupling h as

$$h(\Lambda_k)\Lambda_k \sim \frac{m(\Lambda_k)}{\Lambda_k}, \tag{11}$$

where Λ_k is a typical energy scale of $SU(kN)$ gauge theory such as the energy scale, where the theory enters the conformal regime, i.e. $g_k(\Lambda_k) \approx g_k^*$. The β-function of α_{k-2} is written in a way similar to those of α_{k-1} and α_k. In addition, the β-function of y is written as

$$\beta_y = \frac{y}{2}(\gamma_M + 2\gamma_q), \tag{12}$$

where γ_M is the anomalous dimension of M_{rs}. The dual theory has a non-trivial fixed point, $g_{k-2} = g_{k-2}^*$ and $y = y^*$ when $g_{k-1} = 0$, where $g_{k-2}^*, y^* \neq 0$. At the fixed point, it is satisfied that $\gamma_M = -2\gamma_q$, that is M_{rs} has the same conformal dimension as $Q_r\bar{Q}_s$. Thus, at the vicinity of the fixed point, $g_{k-2} = g_{k-2}^*$, $y = y^*$ and $g_{k-1} = 0$, both operators, $h \text{ tr } \det_{r,s}(Q_r\bar{Q}_s)$ and $m \text{ tr} \det_{r,s} M_{rs}$ are relevant, and the mass m/μ and coupling $h\mu$ increase towards the IR direction. Because the fields M_{rs} become heavy, we integrate out them and the effective superpotential results in [4] [3]

$$W = \tilde{h} \text{ tr} \det_{rs} q_r\bar{q}_s, \tag{13}$$

where $\tilde{h} = -y^2/m$. The operator $\tilde{h} \text{ tr } \det_{rs} q_r\bar{q}_s$ is irrelevant and the coupling $\tilde{h}$ decreases towards the IR direction. Thus, the low energy effective theory is the same as the starting theory except replacing the gauge group $SU(kN) \times$

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[2] In the followings, we will ignore the irrelevant mesons M_{rs}^0 which are singlet for $SU((k-1)N)$.

[3] See also [22].

$SU((k-1)N)$ by $SU((k-1)N) \times SU((k-2)N)$. This RG flow would continue and the low-energy effective theory would become the $SU((k-n)N) \times SU((k-n-1)N)$ gauge theory with the quartic superpotential $W = \tilde{h} \operatorname{tr} \det_{rs} q_r \bar{q}_s$ until the theory becomes outside of the conformal window. The RG flow toward the IR is illustrated as

$$
\begin{array}{ccc}
(g_k \approx 0, g_{k-1} \approx g_{k-1}^*, \eta \approx 0) & & \\
\downarrow & & \\
(g_k \approx g_k^*, g_{k-1} \approx 0, \eta \approx 0) & \leftrightarrow & (g_{k-2} \approx g_{k-2}^*, g_{k-1} \approx 0, y \approx y^*, m/\mu \approx 0) \\
\downarrow & \text{dual} & \downarrow \\
(g_k \approx g_k^*, g_{k-1} \approx 0, \eta \gg 1) & \leftrightarrow & (g_{k-2} \approx g_{k-2}^*, g_{k-1} \approx 0, y \approx y^*, m/\mu \gg 1) \\
& & \downarrow \text{integrating out } M_{rs} \\
& & (g_{k-2} \approx g_{k-2}^*, g_{k-1} \approx 0, \tilde{\eta} \approx 0).
\end{array}
$$

At the end of cascade we would obtain the $SU(2N) \times SU(N)$ gauge theory. The $SU(2N)$ gauge sector has the $2N$ flavors and the quantum deformed moduli space [24, 2], $\Delta W = X(\det_{\text{all}} M_{rs} - B\bar{B} - \Lambda^{4N})$, where X is a Lagrange multiplier superfield, B and $\bar{B}$ are baryon and anti-baryon superfields, which are singlets under $SU(N)$. If we assume that only B and $\bar{B}$ develop their vacuum expectation values (VEVs), then baryons and mesons become massive. Thus the effective theory becomes the pure $SU(N)$ supersymmetric Yang-Mills theory and finally the theory is confined.

3 RG flow of soft SUSY breaking terms

Here, we study the RG flow of SUSY breaking terms in softly broken supersymmetric theories. It is convenient to use the spurion method [25, 26, 27, 28, 29, 10] to derive RG equations of soft SUSY breaking terms from those for supersymmetric couplings.

3.1 A simple theory with conformal fixed point

Here, we give a brief review on the spurion method and apply for a simple gauge theory with a conformal fixed point. We consider a generic gauge theory with the gauge coupling g, the gaugino mass $M_{1/2}$, Yukawa couplings y_{ijk}, corresponding A-terms a_{ijk} and soft scalar masses m_i. We define the following superfield couplings

$$
\tilde{\alpha} = \alpha \left(1 + M_{1/2}\theta^2 + \bar{M}_{1/2}\bar{\theta}^2 + (2|M_{1/2}|^2 + \Delta_g)\theta^2\bar{\theta}^2\right), \qquad (14)
$$

$$\tilde{y}_{ijk} = y_{ijk} - a_{ijk}\theta^2 + \frac{1}{2}(m_i^2 + m_j^2 + m_k^2)y_{ijk}\theta^2\bar{\theta}^2, \tag{15}$$

where Δ_g is written as [28]

$$\Delta_g = -\frac{F(\alpha)}{\alpha}\left[\sum_i T_i m_i^2 - T_G|M_{1/2}|^2\right]. \tag{16}$$

Then, beta-functions of superfields $\tilde{\alpha}$ and $\tilde{y}_{ijk}$ ($\tilde{\bar{y}}_{ijk}$) including soft SUSY breaking terms are obtained from those of α and y_{ijk} ($\bar{y}_{ijk}$), $\beta_\alpha(\alpha, y_{ijk}, \bar{y}_{ijk})$ and $\beta_{y_{ijk}}(\alpha, y_{ijk}, \bar{y}_{ijk})$ by replacing α and y_{ijk} ($\bar{y}_{ijk}$) by $\tilde{\alpha}$, $\tilde{y}_{ijk}$ ($\tilde{\bar{y}}_{ijk}$), i.e.,

$$\mu\frac{d\tilde{\alpha}}{d\mu} = \beta_\alpha(\tilde{\alpha}, \tilde{y}_{ijk}, \tilde{\bar{y}}_{ijk}), \qquad \mu\frac{d\tilde{y}_{ijk}}{d\mu} = \beta_{y_{ijk}}(\tilde{\alpha}, \tilde{y}_{ijk}, \tilde{\bar{y}}_{ijk}). \tag{17}$$

That implies that the beta-function of the gaugino mass $M_{1/2}$ is obtained as

$$\mu\frac{dM_{1/2}}{d\mu} = \left(M_{1/2}\alpha\frac{\partial}{\partial\alpha} - a_{ijk}\frac{\partial}{\partial y_{ijk}}\right)\left(\frac{\beta_\alpha}{\alpha}\right) \equiv D_1\left(\frac{\beta_\alpha}{\alpha}\right). \tag{18}$$

The RG equation for the soft scalar mass m_i of a chiral superfield ϕ_i is also easily obtained as

$$\mu\frac{dm_i^2}{d\mu} = \gamma_i(\tilde{\alpha}, \tilde{y}_{ijk}, \tilde{\bar{y}}_{ijk})\big|_{\theta^2\bar{\theta}^2}. \tag{19}$$

These equations are found to be consistent with the equations for the $\theta^2\bar{\theta}^2$ components of Eqs. (17). Explicitly, the RG equations are written down as

$$\mu\frac{dm_i^2}{d\mu} = D_2\gamma_i , \tag{20}$$

$$D_2 = D_1\bar{D}_1 + (|M_{1/2}|^2 + \Delta_g)\alpha\frac{\partial}{\partial\alpha}$$
$$+ \frac{1}{2}(m_i^2 + m_j^2 + m_k^2)\left(y_{ijk}\frac{\partial}{\partial y_{ijk}} + \bar{y}_{ijk}\frac{\partial}{\partial\bar{y}_{ijk}}\right). \tag{21}$$

It is found that these RG equations lead to very interesting properties of the soft SUSY breaking parameters at the vicinity of an IR attractive fixed point [9, 7, 10]. Deviations of the gauge coupling and the Yukawa coupling

from their fixed point values, $\delta\alpha = \alpha - \alpha^*$ and $\delta y_{ijk} = y_{ijk} - y_{ijk}^*$, decrease exponentially. Then the spurion method tells that both of

$$\delta\tilde{\alpha} = \alpha^* M_{1/2}\theta^2 - F(\alpha^*)\sum_i T_i m_i^2 \theta^2 \bar{\theta}^2, \tag{22}$$

$$\delta\tilde{y}_{ijk} = -a_{ijk}\theta^2 + \frac{1}{2}(m_i^2 + m_j^2 + m_k^2)y_{ijk}^*\theta^2\bar{\theta}^2, \tag{23}$$

also decrease exponentially towards the IR direction. Therefore, the gaugino mass $M_{1/2}$ and the A-term a_{ijk} are found to be suppressed[4] and the soft scalar masses satisfy the IR sum rules given by $\sum_i T_i m_i^2 = 0$ and $m_i^2 + m_j^2 + m_k^2 = 0$.

It is easy to see the above mentioned behavior in the case with a single gauge coupling only. We consider the perturbation around the fixed point as $\alpha = \alpha^* + \delta\alpha$, where $\delta\alpha \ll 1$. The beta-function of $\delta\alpha$ around the fixed point is written as

$$\mu\frac{d\delta\alpha}{d\mu} = \left(\frac{\partial\beta_\alpha}{\partial\alpha}\right)_{\alpha=\alpha^*}\delta\alpha \equiv \Gamma\delta\alpha. \tag{24}$$

Because this fixed point is the IR attractive, that leads to $\Gamma > 0$. Then, the spurion method leads immediately to the RG flow of the gaugino mass, that is, the gaugino mass is renormalized as

$$M_{1/2}(\mu) = M_{1/2}(\mu_0)\left(\frac{\mu}{\mu_0}\right)^\Gamma. \tag{25}$$

Thus the gaugino mass $M_{1/2}$ is found to be exponentially suppressed around the IR fixed point. Similarly, we can show that the sum $\sum_i T_i m_i^2$ is exponentially suppressed in this theory. Furthermore, it is straightforward to extend this discussion to the theory with a gauge coupling and Yukawa couplings and to show that the gaugino mass $M_{1/2}$ and the A-term a_{ijk} as well as the sums $\sum_i T_i m_i^2$ and $m_i^2 + m_j^2 + m_k^2$ are exponentially suppressed.

For the dual gauge theory with the dual quarks q and $\bar{q}$ and the meson field M, the superpotential is given by $y\bar{q}Mq$. Therefore the second sum rule is reduced to be $m_q^2 + m_{\bar{q}}^2 + m_M^2 = 0$. We may also understand this as follows. For example, when we use the one-loop anomalous dimensions, we can show that at the fixed point the gauge coupling and Yukawa coupling are related as $y^* = Cg^*$, where C is a constant determined by group-theoretical factors [30].

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[4]That implies that the ratio a_{ijk}/y_{ijk} is also suppressed exponentially, because the Yukawa coupling y_{ijk} has a fixed point.

At the fixed point, this relation is realized as the relation between superfield couplings as $|\tilde{y}|^2/(8\pi^2) = C^2\tilde{\alpha}$, and their $\theta^2\bar{\theta}^2$-terms lead to [31, 22]

$$m_q^2 + m_{\bar{q}}^2 + m_M^2 = |M_{1/2}|^2. \tag{26}$$

Since the gaugino mass $M_{1/2}$ is exponentially damping toward the conformal fixed point, the sum $m_q^2 + m_{\bar{q}}^2 + m_M^2$ is also exponentially damping as mentioned above.

3.2 Cascading theory

Applying the above spurion method to the cascading theory, we investigate the RG flow of soft SUSY breaking terms through the duality cascade. We consider the $SU(kN) \times SU((k-1)N)$ gauge theory with two pairs of chiral matter fields Q_r and $\bar{Q}_s$ and their superpotential (1). The beta-functions of their gaugino masses, $M_{1/2}^{(k)}$ and $M_{1/2}^{(k-1)}$, are written as

$$\begin{aligned}
\mu\frac{dM_{1/2}^{(k)}}{d\mu} &= -N(k+2+2(k-1)\gamma_Q)H'(\alpha_k)\alpha_k M_{1/2}^{(k)} \\
&\quad -2(k-1)NH(\alpha_k)\frac{\partial\gamma_Q}{\partial\alpha_k}\alpha_k M_{1/2}^{(k)} \\
&\quad -2(k-1)NH(\alpha_k)\frac{\partial\gamma_Q}{\partial\alpha_{k-1}}\alpha_{k-1}M_{1/2}^{(k-1)}, \tag{27}
\end{aligned}$$

$$\begin{aligned}
\mu\frac{dM_{1/2}^{(k-1)}}{d\mu} &= -N(k-3+2k\gamma_Q)H'(\alpha_{k-1})\alpha_{k-1}M_{1/2}^{(k-1)} \\
&\quad -2kNH(\alpha_{k-1})\frac{\partial\gamma_Q}{\partial\alpha_{k-1}}\alpha_{k-1}M_{1/2}^{(k-1)} \\
&\quad -2kNH(\alpha_{k-1})\frac{\partial\gamma_Q}{\partial\alpha_k}\alpha_k M_{1/2}^{(k)}, \tag{28}
\end{aligned}$$

where $H(\alpha) = F(\alpha)/\alpha \approx \alpha$ and $H'(\alpha) = dH/d\alpha$.

As in Section 2, we start the RG flow at the energy scale Λ from the vicinity of the fixed point A, i.e. $(g_k, g_{k-1}, \eta) = (0, g_{k-1}^*, 0)$. Around the fixed point A, we have $k - 3 + 2k\gamma_Q \approx 0$. As long as g_{k-1} is large and stable, the second term in (28) reduces the gaugino mass $M_{1/2}^{(k-1)}$ exponentially as the energy scale μ decreases. On the other hand, we find $\beta_{M_{1/2}^{(k)}} < 0$ because $k + 2 + 2(k-1)\gamma_Q > 0$ and $H(\alpha_k) \approx \alpha_k \approx 0$. Thus, the gaugino mass $M_{1/2}^{(k)}$

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increases as the energy scale μ decreases. However, such increase of $M_{1/2}^{(k)}$ is not drastic during the weak coupling region of α_k.

Next, the theory moves from the vicinity of the fixed point $(g_k, g_{k-1}, \eta) = (0, g_{k-1}^*, 0)$ towards another fixed point, $(g_k, g_{k-1}, \eta) = (g_k^*, 0, 0)$, where $k + 2 + 2(k-1)\gamma_Q \approx 0$. Around the latter fixed point, we find $\beta_{M_{1/2}^{(k-1)}} > 0$ because $k - 3 + 2k\gamma_Q < 0$, $H(\alpha_{k-1}) \approx 0$ and $M_{1/2}^{(k)}$ becomes irrelevant as below. That is, the gaugino mass $M_{1/2}^{(k-1)}$ decreases perturbatively as the energy scale μ decreases.

On the other hand, the gaugino mass $M_{1/2}^{(k)}$ would be suppressed exponentially in turn due to the second term in (27), as going towards the IR fixed point. However we note that the third term cannot be neglected, when $\alpha_k M_{1/2}^{(k)}$ is reduced to be comparable with $\alpha_{k-1} M_{1/2}^{(k-1)}$. Then the gaugino mass $M_{1/2}^{(k)}$ does not follow a simple exponential suppression. Rather it converges to a certain value determined by α_{k-1} and $M_{1/2}^{(k-1)}$ obtained at the renormalized scale.

If we admit using the one-loop anomalous dimension, then the RG behavior discussed above could be explicitly examined. Here we shall look into the theory on the renormalized trajectory given in Fig. 1. In Fig. 2, the gaugino masses $M_{1/2}^{(k-1)}(\mu)$ and $M_{1/2}^{(k)}(\mu)$ of the theory with $k = 5$ are plotted with respect to the scale parameter $\ln(\mu/\mu_0)$. At the scale μ_0, the gauge couplings are chosen as $(\alpha_k, \alpha_{k-1}) = (0.0128, 0.04)$, which is a point on the renormalized trajectory rather close to the fixed point A in Fig. 1. The initial values at $\mu = \mu_0$ are taken to be 1.0 for both gaugino masses.

It is seen that $M_{1/2}^{(k-1)}$ is reduced as discussed, but turns to be negative due to the third term in (28), since $M_{1/2}^{(k)}$ glows slightly first. In the region II, the gaugino mass $M_{1/2}^{(k)}$ turns out to be suppressed strongly, while $M_{1/2}^{(k-1)}$ changes perturbatively. In Fig. 3, the log-plot of the gaugino mass $M_{1/2}^{(k)}$ is shown by the bold line. It is also seen that the suppression behavior deviates from the exponential one in the end and $M_{1/2}^{(k)}$ converges to a line. Indeed, the convergence point of $\alpha_k M_{1/2}^{(k)}$ could be estimated as

$$\alpha_k M_{1/2}^{(k)} \sim -\alpha_{k-1} M_{1/2}^{(k-1)}. \tag{29}$$

Similarly, we examine the RG running of the soft mass squared m_Q^2. At the vicinity of the fixed points, m_Q^2 is also expected to be exponentially sup-
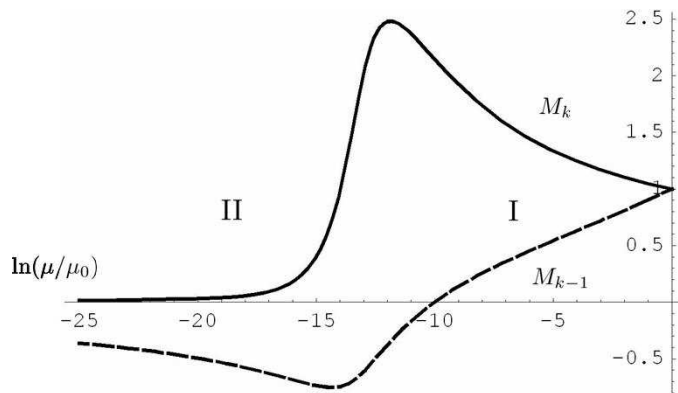
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Figure 2: RG running of the gaugino masses $M_{1/2}^{(k-1)}(\mu)$ and $M_{1/2}^{(k)}(\mu)$ with respect to $\ln(\mu/\mu_0)$. The gauge couplings are given at $\mu = \mu_0$ as $(\alpha_k, \alpha_{k-1})=(0.0128, 0.04)$ and run along the renormalized trajectory.

pressed as discussed in section 3.1. However existence of two gauge couplings makes the situation more complicated. The RG equation for m_Q^2 is given as

$$\mu \frac{dm_Q^2}{d\mu} = \gamma_Q(\tilde{\alpha}_k, \tilde{\alpha}_{k-1})|_{\theta^2\bar{\theta}^2} . \tag{30}$$

Here, let us use the one-loop anomalous dimension given by (9). Then the RG equation is reduced to be

$$\mu \frac{dm_Q^2}{d\mu} = -k\alpha_k(2|M_{1/2}^{(k)}|^2 + \Delta_k) - (k-1)\alpha_{k-1}(2|M_{1/2}^{(k-1)}|^2 + \Delta_{k-1}), \tag{31}$$

where

$$\Delta_k = H(\alpha_k)\left[3k|M_{1/2}^{(k)}|^2 - 2(k-1)m_Q^2\right], \tag{32}$$

$$\Delta_{k-1} = H(\alpha_{k-1})\left[3(k-1)|M_{1/2}^{(k-1)}|^2 - 2km_Q^2\right]. \tag{33}$$

In Fig. 3, the RG evolution of m_Q^2 of the same theory on the renormalized trajectory is shown by dotted lines. The initial values are taken as $\ln m_Q^2 = 0, 2.5, 5.0$ just for the illustration. In the region I, we may neglect subleading terms of α_k and also M_{k-1}, since it is suppressed. Then, Eq. (31)
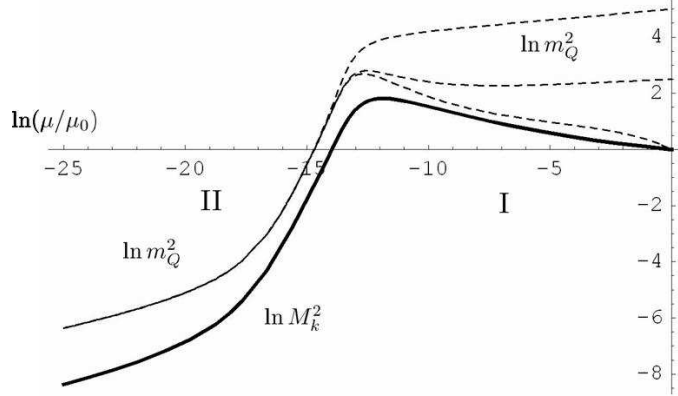
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Figure 3: RG running behaviors of the scalar mass $\ln m_Q^2$ and the gaugino mass $2\ln M_{1/2}^{(k)}$ are shown by dotted lines and the bold line, respectively.

is approximated to be

$$\mu\frac{dm_Q^2}{d\mu} \simeq 2k(k-1)(\alpha_{k-1}^*)^2 m_Q^2 - 2k\alpha_k|M_{1/2}^{(k)}|^2. \tag{34}$$

This equation tells us that m_Q^2 is not just suppressed but converges as

$$m_Q^2 \to \frac{1}{(k-1)(\alpha_{k-1}^*)^2}\alpha_k|M_{1/2}^{(k)}|^2, \tag{35}$$

since running of $\alpha_k|M_{1/2}^{(k)}|^2$ is rather slow. In the case of $k=5$, the fixed point coupling g_{k-1}^* is not large and the convergence is not so strong. In the region II, running of $M_{1/2}^{(k)}$ changes to exponential suppression. However, similarly to the behavior in the region I, it converges in the IR limit as

$$m_Q^2 \to \frac{1}{k(\alpha_k^*)^2}\alpha_{k-1}|M_{1/2}^{(k-1)}|^2. \tag{36}$$

We summarize the RG flow of the gaugino masses and soft scalar masses as the theory moves from the fixed point $(g_k, g_{k-1}, \eta) = (0, g_{k-1}^*, 0)$ toward the fixed point $(g_k, g_{k-1}, \eta) = (g_k^*, 0, 0)$. At the first stage, i.e. the perturbative regime of α_k, the gaugino mass $M_{1/2}^{(k-1)}$ is suppressed, while $M_{1/2}^{(k)}$ and the soft scalar mass squared m_Q^2 increase perturbatively. In entering the conformal

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regime of α_k, both $M_{1/2}^{(k)}$ and m_Q^2 begin exponential damping, while $M_{1/2}^{(k-1)}$ runs perturbatively. In the IR limit, the gaugino mass $M_{1/2}^{(k)}$ and the soft scalar mass squared m_Q^2 are found to converge to certain values determined by α_{k-1} and $M_{1/2}^{(k-1)}$. Hence, these parameters evolute to be of the same order and are fixed in the IR limit irrespectively of their initial values.

In addition to the gaugino masses $M_{1/2}^{(k)}$, $M_{1/2}^{(k-1)}$ and scalar mass m_Q, the SUSY breaking terms corresponding to the superpotential (1) may be important, that is,

$$W = h(1 - A_h \theta^2) \operatorname{tr} \det_{r,s}(Q_r \bar{Q}_s). \tag{37}$$

The RG flow behavior of the coupling $\mu h A_h$ is drastic following the anomalous dimensions of Q_r and $\bar{Q}_s$. Both RG flows of μh and $\mu h A_h$ are almost the same. That implies that their ratio A_h does not change drastically[5].

The theory around the fixed point $(g_k, g_{k-1}, \eta) = (g_k^*, 0, 0)$ would be well-described by its dual theory with the gauge coupling g_{k-2} and the Yukawa coupling y. The dual theory has the gaugino mass $M_{1/2}^{(k-2)}$, soft scalar masses of q_r, $\bar{q}_s$ and M_{rs} as m_q and m_M, the A-term a and the B-term b. The latter two terms are associated with the superpotential (10) and are written as

$$W = y(1 - A_y \theta^2)\operatorname{tr} \bar{q}_r M_{rs} q_s + m(1 - B\theta^2)\operatorname{tr} \det_{r,s} M_{rs}. \tag{38}$$

Here, we denote $a = y A_y$ and $b = mB$. The exact matching relations of soft terms between dual theories are not clear, but we assume that $M_{1/2}^{(k)}(\Lambda_k) \sim M_{1/2}^{(k-2)}(\Lambda_k)$ and all of soft scalar masses are of the same order at Λ_k. Furthermore, we assume that all of A_h, A_y and B are of the same order at Λ_k.

When the gauge coupling g_{k-2} approaches toward its non-trivial fixed point, the gaugino mass $M_{1/2}^{(k-2)}$ and soft scalar mass squared m_q^2 are also exponentially suppressed. This behavior is similar to that of $M_{1/2}^{(k)}$ and m_Q^2 discussed previously. Moreover, in the dual theory the Yukawa coupling y approaches to the fixed point y^*. In this case, a small deviation $\delta y = y - y^*$

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[5]Note that the RG flow of η has no fixed point with a finite value of η. In our case, the RG flow of A_h will be ruled by gauge couplings and gaugino masses which can be finite values. In the region II, A_h will be affected by mainly $\alpha_{k-1}^n M_{1/2}^{(k-1)}$ in the dimensionful parameters.

is exponentially damping as (24). The spurion method leads that the A-term coupling A_y is also suppressed exponentially. On the other hand, the RG behavior of B is rather similar to one of A_h. It is found that both RG flow behaviors of the mass m/μ and b/μ are almost the same and they are determined by large anomalous dimensions of M_{rs}. However, their ratio B does not change drastically[6].

In the dual theory, not only m_q^2 but also the sum of soft scalar masses squared, $m_q^2 + m_{\bar{q}}^2 + m_M^2$, is also suppressed in the conformal region. That implies that each of m_q^2 and m_M^2 is suppressed when $m_q^2 = m_{\bar{q}}^2$, which is the relation we are assuming. However, we cannot neglect the effects through $SU(N(k-1))$ gauge interaction such as the discussions of convergence points, (29) and (36), in the original $SU(Nk) \times SU(N(k-1))$ theory.

The gaugino mass $M_{1/2}^{(k-2)}$, the A-parameter A_y and the scalar masses squared m_q^2 and m_M^2 in the dual theory are not just suppressed out, rather converge to certain values given by α_{k-1} and $M_{1/2}^{(k-1)}$ in the IR limit again. It is straightforward to solve the RG equations, if we admit using the one-loop anomalous dimensions of q and M just as performed above. However, we shall avoid to present a similar analysis here. It may be explicitly seen that both m_q^2 and m_M^2 as well as $M_{1/2}^{(k-2)}$ and $|A_y|^2$ converge the values of the same order given by $\alpha_{k-1}|M_{1/2}^{(k-1)}|^2$. The meson field M belongs to the adjoint representation of $SU(N(k-1))$ group and suffers from the effects through $SU(N(k-1))$ gauge interaction more. Therefore, m_M^2 is found to be positive and larger than m_q^2 in the IR [7].

When the supersymmetric mass m of the chiral fields M_{rs} becomes large, we integrate out these fields. Then, the low energy theory becomes the $SU((k-1)N) \times SU((k-2)N)$ gauge theory with two pairs of bifundamental and anti-bifundamental fields and the quartic superpotential, $W = \tilde{h}\mathrm{tr}\det q_r \bar{q}_s$. The theory has soft SUSY breaking terms, i.e. the gaugino masses, $M_{1/2}^{(k-1)}$ and $M_{1/2}^{(k-2)}$, and soft scalar mass m_q. In addition, we have the SUSY breaking term corresponding to the superpotential $W = \tilde{h}\mathrm{tr}\det q_r \bar{q}_s$, that is,

$$W = \tilde{h}(1 - \theta^2 A_{\tilde{h}})\mathrm{tr}\det q_r \bar{q}_s. \tag{39}$$

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[6]Note that the RG flow of m/μ has no fixed point with its fine value. In this IR region, B will be affected by mainly $\alpha_{k-1}^n M_{1/2}^{(k-1)}$ in the dimensionful paramters.

[7]Soft masses for singlet mesons M_{rs}^0 may be driven to be negative because of the Yukawa couplings.

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The size of $A_{\tilde{h}}$ may be of the order of B or A_y at the decoupling scale of M_{rs}. If these SUSY breaking terms are smaller than other mass scales such as the energy scale μ and the supersymmetric mass m, the above cascade continues as rigid SUSY theory in Section 2. Through the cascade, the gaugino masses and soft scalar masses are damping except the perturbative regime, where the theory moves from the fixed point $(g_k, g_{k-1}, \eta) = (0, g_{k-1}^*, 0)$ toward the fixed point $(g_k, g_{k-1}, \eta) = (g_k^*, 0, 0)$. When we integrate out M_{rs}, which are charged under the $SU((k-1)N)$ gauge group, threshold corrections would appear. For example, the gaugino mass $M_{1/2}^{(k-1)}$ would receive such threshold corrections $\Delta M_{1/2}^{(k-1)}$, which would be estimated by $\Delta M_{1/2}^{(k-1)} = \mathcal{O}(\alpha_{k-1}B)$. That would be small, because α_{k-1} is small. At any rate, if the cascade continues, the total gaugino mass $M_{1/2}^{(k-1)}$ would be suppressed at the next stage such as the gaugino mass $M_{1/2}^{(k)}$ is suppressed at the stage discussed above.

As discussed above, the cascade would continue unless SUSY breaking terms are comparable with other mass scales such as the energy scale μ and the supersymmetric mass m. Gaugino masses and SUSY breaking scalar masses would be suppressed through the cascade except the regime I, where the gaugino mass $M_{1/2}^{(k)}$ would increase. On the other hand, the SUSY breaking parameters, B and A_h, would not be suppressed like the others. Note that the B-term corresponds to the off-diagonal entries of mass squared matrix of the fields M_{rs}, that is, eigenvalues of mass squared would be written by $|m|^2 + m_M^2 \pm |mB|$. A large value of $|B|$ would induce a tachyonic mode. Then, the scalar component of superfields M_{rs} may develop their VEVs and the gauge symmetry $SU((k-1)N)$ may be broken. Also, through this symmetry breaking, the matter fields q_r and $\bar{q}_s$ may gain mass terms due to the Yukawa coupling with M_{rs}. Then, the duality cascade would be terminated when mass parameters, $|m|^2$, $|mB|$ and m_M^2, are comparable [8]. This type of ending of the duality cascade could happen only in the softly broken supersymmetric theories and such symmetry breaking would be important. Thus,

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[8]Similarly, the singlet meson fields M_{rs}^0 may develop their VEVs depending on values of their various mass terms. Their VEVs induce mass terms of dual quarks. If such masses are large enough, the dual quarks would decouple and the flavor number would reduce to be outside of the conformal window. Then, the cascade could end. In addition, scalar components of q_r and $\bar{q}_s$ may develop their VEVs depending on values the A-terms and their soft scalar masses as well as other parameters in the scalar potential. Their VEVs break gauge symmetry and the cascade would end.

we will study such breaking more explicitly in the next section. Similar symmetry breaking would be realized not only in the "magnetic dual theory", but also in the original "electric theory" with the quartic A-term (37). If the quartic A-term is comparable with SUSY breaking scalar masses m_Q, the origin of scalar potential of Q would be unstable and similar symmetry breaking would happen. Such gauge symmetry breaking with reducing the flavor number may correspond to the symmetry breaking by VEVs of M_{rs} with inducing dual quark masses.

Whether M_{rs} include tachyonic modes depends on values of $|m|^2 + m_M^2 \pm |mB|$, i.e. their initial conditions as well as matching conditions. In a certain parameter region, the scalar fields M_{rs} may include tachyonic modes and symmetry breaking may happen. In the other parameter regions, the cascade would continue like the rigid supersymmetric theory. For example, when the magnitude of SUSY breaking terms is much smaller than the supersymmetric mass m and the energy scale μ, the cascade would continue in almost the same way as the rigid supersymmetric theory. Then, it would end with the pure $SU(N)$ supersymmetric Yang-Mills theory with non-vanishing gaugino mass.

4   Symmetry breaking and illustrative model

4.1   Symmetry breaking

In the previous section, we have pointed out the possibility that a tachyonic mode in the meson fields M_{rs} would appear because of soft SUSY breaking terms and its VEV would break the symmetry. Here, we study this aspect more explicitly.

4.1.1 $SU(kN) \times SU((k-1)N)$ model

First, we study the $SU((k-2)N) \times SU((k-1)N)$ theory, which is dual to the $SU(N) \times SU((k-1)N)$ theory. As discussed in the previous section, the dual theory includes the meson fields M_{rs}, which have the supersymmetric mass m, the SUSY breaking soft scalar masses m_M and the B-term mB. That is, their scalar potential V is written by

$$V = (|m|^2 + m_M^2) \sum_{rs} |M_{rs}|^2 + (mB(M_{11}M_{22} - M_{12}M_{21}) + h.c.) + V_D^{(k-1)},$$
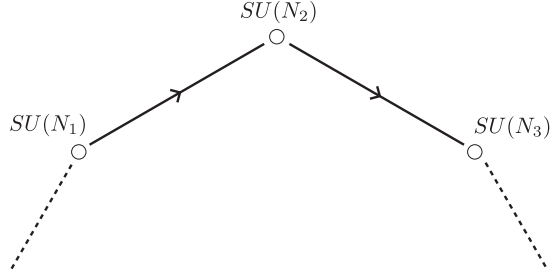
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Figure 4: $\prod_i SU(N_i)$ quiver model

$$V_D^{(k-1)} \;\;=\;\; \frac{1}{2}g_{k-1}^2 D_{(k-1)}^2, \tag{40}$$

where $D_{(k-1)}$ denotes the D-term of the $SU((k-1)N)$ vector multiplet. Here, we have assumed the $SU(2)$ invariance for the (r,s) indices of M_{rs}. The eigenvalues of mass squared matrix are given by

$$|m|^2 + m_M^2 \pm |mB|. \tag{41}$$

If $|m|^2 \gg |m_M^2|, |mB|$, the theory is almost supersymmetric and the duality cascade would continue. (Note that m is the supersymmetric mass and the others are masses induced by SUSY breaking.) However, if the masses (41) include a negative eigenvalue, there appears a tachyonic mode at the origin of the field space M_{rs}. Note that the D-flat direction corresponds to the VEV direction, where M_{rs} are written by diagonal elements, that is, Cartan elements. That implies that when a negative eigenvalue is included in (41), such a direction would be unbounded from below in the tree-level scalar potential. Thus, the meson fields M_{rs} would develop their VEVs, whose order would be equal to the cut-off scale of the $SU((k-2)N) \times SU((k-1)N)$ theory, i.e. Λ_k. The VEVs of adjoint fields M_{rs} break the gauge group $SU((k-1)N)$ to a smaller group and induce mass terms of q_r and $\bar{q}_s$ through the Yukawa couplings $y q_r M_{rs} \bar{q}_s$.

4.1.2 $\prod_i SU(N_i)$ quiver model

The above analysis can be extended to the $\prod_i SU(N_i)$ quiver gauge theory with their bifundamental matter fields. We consider a subsector of the quiver theory, that is, the $SU(N_1) \times SU(N_2) \times SU(N_3)$ gauge theory with

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bifundamental matter fields, $(N_1, \bar{N}_2, 1)$ and $(1, N_2, \bar{N}_3)$ as shown in Fig. 4. The $SU(N_1)$ and $SU(N_3)$ sectors would have other types of bifundamental matter fields, but we neglect them [9]. In addition, for simplicity we consider the case with $N_1 = N_3$. Here, we dualize the $SU(N_2)$ sector. Then, there appear the dual matter fields q and $\bar{q}$ with the representations $(\bar{N}_1, \tilde{N}_2, 1)$ and $(1, \tilde{\bar{N}}_2, N_3)$, where $\tilde{N}_2 = N_1 - N_2$. In addition, the meson field M with the representation $(N_1, 1, \bar{N}_3)$ appears and has Yukawa couplings among q and $\bar{q}$. The supersymmetric mass term of the meson field in the superpotential is not allowed, i.e. $m = mB = 0$. In this case, only the SUSY breaking soft scalar mass m_M as well as the D-term potentials appears in the scalar potential of the meson field M. Thus, the scalar potential is simple. The scalar mass squared m_M^2 tends to converge to a positive value as discussed in the previous section. Thus, the symmetry breaking may not happen by the VEV of M in this theory.

When $N_1 = N_3 = 2$, supersymmetric mass terms of meson fields in the superpotential would be allowed. Alternatively, when the model includes anti-meson fields $\bar{M}$, the supersymmetric mass term would be allowed in the superpotential. In these models, the corresponding B-terms would also be allowed. Furthermore, in the latter model, there are D-flat directions, i.e. $M = \pm \bar{M}$. In this case, the scalar potential would be written by

$$
\begin{aligned}
V &= (|m|^2 + m_M^2)|M|^2 + (|m|^2 + m_{\bar{M}}^2)|\bar{M}|^2 + (mBM\bar{M} + h.c.) \\
&\quad + V_D^{(N_1)} + V_D^{(N_3)},
\end{aligned} \tag{42}
$$

where $V_D^{(N_1)}$ and $V_D^{(N_3)}$ are D-term scalar potentials for the $SU(N_1)$ and $SU(N_3)$ vector multiplets. This potential at the tree level is unbounded from below along the D-flat direction $M = \pm \bar{M}$ if

$$
2|m|^2 + m_M^2 + m_{\bar{M}}^2 < 2|mB|. \tag{43}
$$

In addition, the meson fields include a tachyonic mode when

$$
(|m|^2 + m_M^2)(|m|^2 + m_{\bar{M}}^2) < |mB|^2, \tag{44}
$$

or

$$
(|m|^2 + m_M^2)(|m|^2 + m_{\bar{M}}^2) > |mB|^2 \quad \text{and} \quad 2|m|^2 + m_M^2 + m_{\bar{M}}^2 < 0. \tag{45}
$$

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[9]In each non-abelian gauge group, for example, we need vector-like matter fields in order to cancel anomaly. However, we assume that the theory is anomaly-free at every stage.
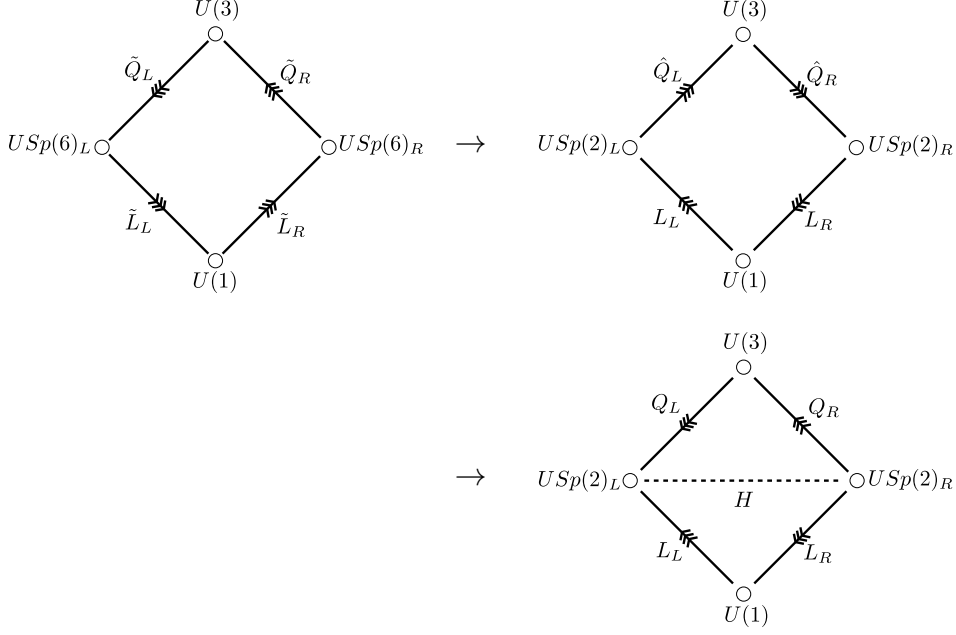
Figure 5: Quiver diagrams of the illustrating model

Thus, various phenomena could happen depending on values of mass parameters, m, m_M, $m_{\bar{M}}$ and mB, that is, the unbounded-from-below direction, the symmetry breaking without the unbounded-from-below direction or no symmetry breaking. Indeed, this situation is quite similar to what happens in the Higgs scalar potential of the minimal supersymmetric standard model (MSSM).

4.2 Illustrating model

Here we give a simple example of theories, whose field contents are similar to the MSSM or its extensions and where symmetry breaking would happen.

We consider the gauge group $U(3) \times USp(6)_L \times USp(6)_R \times U(1)$ and three families of bifundamental fields, $\tilde{Q}_L : (3, 6, 1, 0)$, $\tilde{Q}_R : (\bar{3}, 1, 6, 0)$, $\tilde{L}_L : (1, 6, 1, -1)$ and $\tilde{L}_R : (1, 1, 6, 1)$ and the superpotential

$$W = h\tilde{Q}_L\tilde{Q}_R\tilde{L}_L\tilde{L}_R. \tag{46}$$

We expect that first the gauge couplings of $USp(6)_L \times USp(6)_R$ would approach to their non-trivial fixed point. Then, the $USp(6)_L \times USp(6)_R$

sector is dualized, that is, the gauge group is $U(3) \times USp(2)_L \times USp(2)_R \times U(1)$ as shown in Fig. 5. Note that $USp(2) \simeq SU(2)$. In addition we would have matter fields, $\hat{Q}_L : (\bar{3}, 2, 1, 0)$, $\hat{Q}_R : (3, 1, 2, 0)$, $L_L : (1, 2, 1, 1)$ and $L_R : (1, 1, 2, -1)$. Also, we would have several "meson fields" $M : (3, 1, 1, -1)$ and $\bar{M} : (\bar{3}, 1, 1, 1)$, which have mass terms $mM\bar{M}$ and Yukawa couplings with $\hat{Q}_L, \hat{Q}_R, L_L$ and L_R, but they can be integrated out because of heavy mass terms $mM\bar{M}$. Then, we obtain the superpotential

$$W = \hat{h}\hat{Q}_L\hat{Q}_R L_L L_R. \tag{47}$$

Next, we expect that the gauge coupling of $SU(3)$ approaches to the conformal fixed point. Then, the $U(3)$ sector is dualized. The gauge group is $U(3) \times USp(2)_L \times USp(2)_R \times U(1)$ and we would have matter fields, $Q_L : (3, 2, 1, 0)$, $Q_R : (\bar{3}, 1, 2, 0)$, $L_L : (1, 2, 1, 1)$ and $L_R : (1, 1, 2, -1)$ as well as several "Higgs fields" $H : (1, 2, 2, 0)$. The superpotential is obtained as

$$W = y_Q Q_L Q_R H + y_L L_L L_R H + mHH. \tag{48}$$

Note that the operator (47) corresponds to $y_L L_L L_R H$. However, the gauge symmetry $U(3) \times USp(2)_L \times USp(2)_R \times U(1)$ allows the mass terms mHH. Thus, we assume that such terms would be induced and we have added such terms. Then, if SUSY breaking terms induce a tachyonic mode of H, the symmetry $USp(2)_L \times USp(2)_R$ would be broken.

In this model, $USp(2)_L$ and $USp(2)_R$ symmetry breaking would happen at the same time. Although the left-right asymmetry is required for a realistic model, it would be difficult to generate such left-right asymmetry in this model. Some modification is necessary for a realistic model. At any rate, this model is an illustrating model for symmetry breaking. Such symmetry breaking by SUSY breaking terms in the duality cascade may be important, e.g. to realize the standard model at the bottom of the cascade. We would study model building towards more realistic models elsewhere.

5 Conclusion

We have studied the RG flow of softly broken supersymmetric theories showing the duality cascade. Gaugino masses and scalar masses are suppressed in most regime of the RG flow although they increase in a certain perturbative regime. After exponential damping, the gaugino mass $M_{1/2}^{(k)}$ corresponding to

the strongly coupled sector converges to a certain value, which is determined by the gauge coupling α_{k-1} and the gaugino mass $M_{1/2}^{(k-1)}$ in the weakly coupled sector. The scalar mass would also converge to the same order value. At the next stage of the cascade, the strongly and weakly coupled sectors are interchanged with each other and the gaugino mass $M_{1/2}^{(k-1)}$ would be suppressed. Thus, through the sequential cascade, the magnitude of gaugino masses and scalar masses would be suppressed.

The B-term may be important. In a certain parameter region, the B-term would induce tachyonic modes of M_{rs} and symmetry breaking would happen. Such an aspect would be important to realistic model building.

The RG flow of SUSY breaking terms in the cascading theory is quite non-trivial as the RG flow of gauge couplings. The gravity dual of the cascade rigid supersymmetric theory has been studied extensively. However, our analysis implies that the dilaton is also running as $e^{-\phi} \sim \alpha_k^{-1} + \alpha_{k-1}^{-1}$ [3], but the supergravity solution of the D3-brane does not admit this running behavior and most of the supergravity dual theories concentrate on the constant dilaton backgrounds. In this sense, the suppression of the gaugino masses would be a quite different mechanism from the suppression due to the warp factor as already pointed out in [32, 33, 34, 35]. The region of RG flow in our study might be outside of the supergravity approximation, but it would be quite interesting to study the gravity dual side corresponding to the RG flow of SUSY breaking terms including the dilaton running.

We have considered the scenario that supersymmetry is broken at high energy and investigated the RG flow of SUSY breaking terms. Alternatively, we could consider another scenario that supersymmetry would be broken at some stage of the cascade. For example, supersymmetry is broken dynamically through the cascade and such breaking is mediated to the visible sector. Such a study would also be important. We would study such a scenario elsewhere.

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