Does entropic force always imply the Newtonian force law?

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Abstract

We study the entropic force by introducing a bound $S \leq A^{3/4}$ between entropy and area which was derived by imposing the non-gravitational collapse condition. In this case, applying a modified entropic force to this system does not lead to the Newtonian force law.

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1 Introduction

Recently, Verlinde has proposed the Newtonian force law as an entropic force by using the equipartition rule and the holographic principle [1]. After his work, the dynamics of apparent horizon in the Friedmann-Robertson-Walker universe [2], the Friedmann equations [3, 4], the connection in the loop quantum gravity [5], the accelerating surfaces [6], holographic actions for black hole entropy [7], and application to holographic dark energy [8] were considered from the entropic force. Furthermore, cosmological implications were reported in [9], the extension to Coulomb force [10], and the symmetry aspect of entropy force [11] were investigated. The entropic force was also discussed in the black hole spacetimes [12, 13]. Its connection to the uncertainty principle was considered in [14].

We briefly review what was going on the entropic force. Explicitly, when a test particle with mass m is close to a surface (holographic screen) with distance Δx (compared to the Compton wave length $\lambda_m = \frac{\hbar}{mc}$), the change of entropy on the holographic screen takes the form

$$\Delta S = 2\pi k_B \frac{\Delta x}{\lambda_m} \to 2\pi m \Delta x \tag{1}$$

in the natural units of $\hbar = c = k_B = 1$ and $G = l_{pl}^2$. Considering that the entropy of a system depends on the distance Δx, an entropic force F could be arisen from the thermodynamical conjugate of the distance

$$F\Delta x = T\Delta S \tag{2}$$

which is considered as an indication that the first law of thermodynamics may be realized on the holographic screen. Plugging (1) into (2) leads to a connection between entropic force and temperature

$$F = 2\pi m T \tag{3}$$

which implies that if one knows the temperature T, the entropic force is determined by (3). In order to define the temperature T on the screen, we assume that the energy E is distributed on a spherical shape of holographic screen with radius R and the mass M is located at the origin of coordinate as the source. Then, we introduce the holographic principle, the equality of energy and mass, and the equipartition rule [15, 16], respectively, as

$$N = \frac{A}{G} \tag{4}$$

$$E = M, \tag{5}$$

$$E = \frac{1}{2}NT \tag{6}$$

with the area of the holographic screen $A = 4\pi R^2$. These are combined to determine the temperature on the screen

$$T = \frac{GM}{2\pi R^2}. \tag{7}$$

Substituting (7) into (3), the entropic force is realized as the Newtonian force law

$$F = \frac{GmM}{R^2}. \tag{8}$$

In this work, we use a "modified entropic force" to derive the Newtonian force law by considering two entropy bounds of $S \leq A$ and $S \leq A^{3/4}$ instead of the "entropic force". The former bound leads to the Newtonian force law, while the latter does not provide the Newtonian force law. The latter accounts for the ordinary matter which is determined by the non-gravitational collapse condition.

2 Two issues on defining the temperature

It is well known that two unusual assumptions to derive the temperature were the holographic principle (4) and the equipartition rule (6). Concerning the former, an urgent issue is how one can construct a spherically holographic screen of radius R which encloses a source mass M located at the origin using the holographic principle. This is an important issue [13] because the holographic screen (an exotic description of spacetime) originates from relativistic approaches to black hole [22, 23] and cosmology [24, 25]. Verlinde has introduced this screen by analogy with an absorbing process of a particle around the event horizon of black hole. Considering a smaller test mass m located at Δx away from the screen and getting the change of entropy on the screen, its behavior should resemble that of a particle approaching a stretched horizon of a black hole, as was described by Bekenstein [17]. It is clear that Verlinde has introduced the holographic screen as a basic input to derive the entropic force.

The other issue is on the latter: why the equipartition rule could be applied to this non-relativistic surface to define the temperature without any justifications. For black holes, the equipartition rule becomes the Smarr formula of $E = NT/2 = 2ST$ when using the relation of $N = \frac{A}{G} = 4S$. Also, it can be derived from the first law of thermodynamics $dE = TdS$ for the Schwarzschild black hole where the Komar charge is just the ADM mass M. Even though the equipartition rule may be available for the classical (thermodynamic) system, the holographic principle of $N = A/G$ is not guaranteed to apply to any non-relativistic situations. In this sense, this issue is closely related to the first issue.

If the above two questions are answered properly, one would make a further step to understand the origin of Newtonian force via the entropic force. However, there remains a gap between non-relativistic approach (absence of horizons) and relativistic approach (presence of horizons).

3 Modified entropic force

In this section, we wish to develop another issue of modified entropic force. In deriving the non-relativistic Newtonian force law (8), we assume that the surface is between the test mass m and the source mass M, but the test mass is very close to the surface as compared to its Compton wavelength λ_m. According to Bekenstein's argument in deriving the area quantum of the Schwarzschild black hole [17], the test particle is indistinguishable from horizon itself if the test particle is on the order of Compton wavelength away from the event horizon. That is, a relativistic quantum particle cannot be localized to better than its Compton wavelength, yielding a lower bound on the increment of the black hole horizon area

$$\Delta A \geq (\Delta A)_{\min} = 8\pi l_p^2 \tag{9}$$

due to the assimilation of a neutral test particle. This implies, in turn, the increment of black hole entropy

$$\Delta S \approx \Delta A. \tag{10}$$

In other words, considering the event horizon as the holographic screen where relativistic effects dominate and the Newtonian approach is no longer a good scheme, the distance Δx of Compton wavelength is not a relevant requirement on increasing the area of event horizon [18].

At this stage, we remind the reader that the entropy increase (1) was derived from a simple analogy with entropic explanation of thermodynamically emergent forces on polymers immersed in a heat bath [1]. Therefore, it is not easy to see how the relation (1) works on the gravity side.

Hence, we have to develop another logic to explain a difference between (1) and (10). To this end, Smolin [5] has proposed that the information change in entropy carries the flux of a bit or byte across the surface which is necessarily discretized. He has proposed such a process that a small excitation initially in the interior region moves out to the exterior, where it may be interpreted as a massive particle. Jacobson's idea [19] has shown that any translation of an excitation across the boundary surface involves a change both of energy U and entropy S, where the latter implies a change of the area of the boundary. Further, Verlinde's idea [1] has implied that there must be a temperature T associated with this process since any change ΔU in energy is accompanied by ΔS. Smolin has proposed that the change ΔU in energy corresponds to this translational motion over a distance Δx. This means that there exists a force $F = \frac{\Delta U}{\Delta x}$ acting on the excitation. According to the first law of thermodynamics, this force takes the form

$$F\Delta x = \Delta U = T\Delta S \tag{11}$$

which implies *a modified entropic force*

$$F = T\frac{\Delta S}{\Delta x}. \tag{12}$$

Here, we mention that this modified equation differs from (2), even though their forms are the same [5, 20]. Importantly, if one gives up the linear relation (1) between ΔS and Δx, and then, it may be replaced by a relationship between entropy S and area A as

$$S = S(A). \tag{13}$$

4 $S \sim A$ versus $S \sim A^{3/4}$

From now on, we use the modified relation (12) to study the entropic force. It is well known that the entropy of Schwarzschild black hole is given by the Bekenstein-Hawking entropy [21, 17]

$$S_{BH} = \frac{A}{4l_p^2}. \tag{14}$$

However, the nature of this entropy is one of the greatest mysteries of modern physics because it scales as the area of black hole rather than its volume. This peculiar property has led to the holographic principle [22, 23, 24], stating that the number of degrees of freedom in any system including gravity effects grows only as the area of its boundary. If the entropy of surface is taken to be (14), one finds

$$\Delta S = \frac{\Delta A}{4l_p^2}. \tag{15}$$

It is noted that in (12), ΔS is one fundamental unit of entropy when $\Delta x \simeq \lambda_m = 1/m$ (if one considers (9) further, $\Delta S = 2\pi$) and the entropy gradient points radially from outside of the boundary surface to inside. We can easily check that Eq.(12) together with (15) leads to the Newtonian force law as in (8) when using the temperature (7).

On the other hand, it is known that gravitational collapse limits the entropy of a physical system. Information (entropy) requires the energy, while formation of a horizon by gravitational collapse restricts the amount of energy allowed in a finite region. Explicitly, 't Hooft has shown that if one excludes configuration whose energies are so large that they inevitably undergo gravitational collapse, one finds the *non-covariant entropy bound* [22]

$$S \leq \Big(\frac{A}{l_p^2}\Big)^{3/4} \tag{16}$$

which is clearly different from the *covariant entropy bound*

$$S \leq S_{BH} \sim \frac{A}{l_p^2}. \tag{17}$$

As a concrete example, we consider a thermal system of radius R and temperature T which implies that its entropy (energy) are given by $S \sim T^3R^3(E \sim T^4R^3)$. Requiring the non-gravitational collapse condition

$$E < \frac{R}{l_p^2}, \tag{18}$$

one obtains the temperature bound

$$T < \frac{1}{\sqrt{l_pR}}. \tag{19}$$

Then, the entropy bound appears as in (16)

$$S < \Big(\frac{R}{l_p}\Big)^{3/2} = \Big(\frac{A}{l_p^2}\Big)^{3/4}. \tag{20}$$

There are several arguments which support that the non-covariant bound has more application than the covariant entropy bound to a system of the ordinary matter. The authors [26] have proposed that the entropy bound (20) could be derived from the energy bound of $E \leq E_{BH}$, when getting rid of many states where the Schwarzschild radius is much larger than the system size. Hence the non-covariant entropy bound (20) is more restrictive than the covariant entropy bound (17) [27]. Buniy and Hsu [28] have shown that the entanglement entropy has the same bound as in (20) when imposing the non-gravitational collapse condition (18). Also, Chen and Xiao [29] have proved that under the condition of (18), the entropy bound for the local quantum field theory is $A^{3/4}$ but not A for considering either bosonic fields or fermionic fields in the system of size l. The authors [30] have confirmed that under the condition of (18), the entropy bound of the local quantum field theory has taken to be (20) when using the generalized uncertainty principle. Additionally, considering the spacetime foam uncertainty of $\delta l \geq l_{\rm p}^{\alpha} l^{\alpha-1}$ [31, 32], it was shown that the case of $\alpha = 2/3$ could explain the holographic model with infinite statistics whose entropy scales as $S \sim A$, while the case of $\alpha = 1/2$ could describe the ordinary matter with Bose-Fermi statistics whose entropy scales as $S \sim A^{3/4}$. Recently, the reasonable arguments has been given that the correct bound should be $S < A/4$, and not $S < A^{3/4}$ for the cosmological matter distributions [33]. However, that work conjectured that the stronger bound of $S < A^{3/4}$ does hold for static, weakly gravitating systems. Here, we consider the static, weakly gravitating systems because we are working with the non-gravitational collapse condition (18).

Finally, we would like to mention the possibility that the covariant entropy bound (17) works when it applies to general relativity. In a modified gravity of $f(R)$ theory [34], instead, the entropy takes the form [35, 36]

$$S_f = \frac{f'(R)A}{4l_p^2}. \tag{21}$$

Hence, the non-covariant entropy bound of (16) may be constructed from $f(R)$ gravities.

5 Entropic force on the ordinary matter

At this stage, we could not confirm that the non-covariant bound (16) takes into account the entropy of an ordinary matter including weakly gravitating effects exactly, instead of the covariant entropy bound (17) for the strongly gravitating systems of black hole, de Sitter cosmological horizon and apparent horizon in cosmology. However, there is no reason to prefer the maximum entropy of $S_{\rm max} \sim A$ to find the entropic force between ordinary matters (source particle and test particle). As was previously mentioned, there are several theoretical evidences which support that the bound (16) is better appropriate for describing the ordinary matter than the bound (17).

Hence, in this work, we use the maximum entropy of $S_{\rm max} \sim A^{3/4}$ in order to derive the entropy force for the system because the source mass M is not a black hole. Considering $S = \alpha(A/l_p^2)^{3/4}$, its variation is given by

$$\Delta S = \frac{\partial S}{\partial A}\Delta A = \frac{3\alpha}{4l_p^{3/2}}\frac{\Delta A}{A^{1/4}}. \tag{22}$$

In this case, the entropic force takes the form

$$F = T\frac{\Delta S}{\Delta x} = \frac{3\alpha T}{4l_p^{3/2}A^{1/4}}\frac{\Delta A}{\Delta x} \tag{23}$$

which leads to

$$F = G^{1/4}\frac{GmM}{R^{5/2}} \tag{24}$$

when choosing the temperature T in (8) and a coefficient α as

$$\alpha = \frac{(4\pi)^{1/4}}{3}. \tag{25}$$

It is evident that Eq.(24) does not represent the Newtonian force law. In order to derive the Newtonian force law, one has to explain why the entropy of the surface (holographic screen) should be given by the area-law. We note that (23) was used to make a correction to the Newtonian force law by considering the entropy corrections [20].

In addition to two issues of holographic principle and equipartition theorem on determining the temperature on the screen, there exists the third issue on a form of entropy of the screen when using the modified entropic force. It is clarified that if S does not satisfy an area-law (for example, $S \sim A^{3/4}$), one could not obtain the Newtonian force law from the modified entropic force (12).

6 Discussions

It is fare to say that the origin of the gravity is not yet fully understood. If the gravity is not a fundamental force, it may be emergent from the other approach to gravity. Newtonian force law may be emergent from the equipartition rule and the holographic principle [1].

As was mentioned previously, it may be not proper to use the linear relation (1) between ΔS and Δx on the gravity side. According to Bekenstein [17], a classical point particle could not increase the area of black hole horizon. On the other hand, a relativistic quantum particle cannot be localized to better than its Compton wavelength λ_m [18]. This yields a lower bound on the increment of the black hole area, due to the assimilation of a test particle. This is regarded as the origin of the entropy increase in the black hole. In order to explain this gab, Smolin has modified the linear relation (1) into a relation (15) between ΔS and ΔA [5].

Furthermore, we wish to point out that the source mass M behind the screen is not a black hole. Also the surface is not the event horizon of a black hole. Hence, the area-law entropy of $S \sim A$ is not justified to represent the entropy of ordinary matter. For this purpose, we introduced the non-covariant entropy of $S \sim A^{3/4}$ obtained when imposing the non-gravitational collapse condition (18) on the ordinary matter. Here, we found the non-appearance of Newtonian force law. In this sense, the non-appearance of Newtonian force may be related to the fact that entropy bound requests further modification.

Consequently, the appearance of Newtonian force law from the entropic force (2) seems not to be robust. The Newtonian force law was obtained when employing holographic principle and equipartition theorem to derive the temperature and using the modified entropic force (12) together with the area-law entropy. If one uses another entropy bound of $S \sim A^{3/4}$ together with (12), however, one fails to derive the Newtonian force law.

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