

Improved cosmological constraints on the curvature and equation of state of dark energy

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Abstract. We apply the Constitution compilation of 397 supernova Ia, the baryon acoustic oscillation measurements including the A parameter, the distance ratio and the radial data, the five-year Wilkinson microwave anisotropy probe and the Hubble parameter data to study the geometry of the universe and the property of dark energy by using the popular Chevallier-Polarski-Linder and Jassal-Bagla-Padmanabhan parameterizations. We compare the simple χ^2 method of joined contour estimation and the Monte Carlo Markov chain method, and find that it is necessary to make the marginalized analysis on the error estimation. The probabilities of Ω_k and w_a in the Chevallier-Polarski-Linder model are skew distributions, and the marginalized 1σ errors are $\Omega_m = 0.279_{-0.008}^{+0.015}$, $\Omega_k = 0.005_{-0.011}^{+0.006}$, $w_0 = -1.05_{-0.06}^{+0.23}$, and $w_a = 0.5_{-1.5}^{+0.3}$. For the Jassal-Bagla-Padmanabhan model, the marginalized 1σ errors are $\Omega_m = 0.281_{-0.01}^{+0.015}$, $\Omega_k = 0.000_{-0.006}^{+0.007}$, $w_0 = -0.96_{-0.18}^{+0.25}$, and $w_a = -0.6_{-1.6}^{+1.9}$. The equation of state parameter $w(z)$ of dark energy is negative in the redshift range $0 \leq z \leq 2$ at more than 3σ level. The flat Λ CDM model is consistent with the current observational data at the 1σ level.

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1. Introduction

The accelerating expansion of the universe was first discovered by the type Ia supernova (SN Ia) observations [1, 2]. The phenomena of acceleration could be explained straightforwardly by introducing an exotic source of matter with negative pressure, the so-called dark energy, which dominates the total matter content of the universe at the present epoch and causes the expansion to accelerate. During the past decade, in addition to the simple cosmological constant model, a lot of dynamical dark energy models, such as the quintessence [3], phantom [4], k-essence [5], tachyon [6], quintom [7], h-essence [8], Chaplygin gas [9], holographic dark energy [10], $f(R)$ [11], Dvali-Gabadadze-Porrati [12] models, etc, have been proposed. Although a lot of efforts have been made to understand the driving force of the accelerating expansion and the property of dark energy, whether dark energy is dynamical or not is still an open question. Therefore, it is necessary to study the nature of dark energy such as the evolutions of its energy density and equation of state.

Apart from phenomenological models, another effective approach to study dark energy is through the observational data. Recently, based on the popular Chevallier-Polarski-Linder (CPL) parametrization of dark energy [13], it was found that the flat Λ CDM model is inconsistent with the current data at more than 1σ level [14, 15]. In [14], it was suggested that the cosmic acceleration is slowing down from $z \sim 0.3$. In [15], it was claimed that dark energy suddenly emerged at redshift $z \sim 0.3$. Furthermore, possible oscillating behavior of dark energy was found in [16]. However, no evidence for dark energy dynamics was found in [17, 18, 19]. It was argued that the systematics in different data sets heavily affected the fitting results from observational data [18, 19]. To further study the dynamics of dark energy, it is necessary to apply more complimentary observational data. In this paper, we combine the Constitution sample of 397 SN Ia data [20], the model independent A parameter from the baryon acoustic oscillation (BAO) measurements [21], the two BAO distance ratios at $z = 0.2$ and $z = 0.35$ [22], the radial BAO measurements at $z = 0.24$ and $z = 0.43$ [23], the five-year Wilkinson microwave anisotropy probe data (WMAP5) [24], and the Hubble parameter $H(z)$ data [25, 26] to probe the geometry of the universe and the nature of dark energy by using the CPL and Jassal-Bagla-Padmanabhan (JBP) [27] parameterizations. We first use the simple χ^2 method of joined contour estimation to obtain the constraints on the model parameters. However, the simple χ^2 method by fixing other parameters at their best fit values has some drawbacks because we neglect the correlation effects between the parameters and the degeneracy between parameters was not considered. When the parameters are strongly correlated, the errors of some parameters will be under-estimated if we fix the other parameters at their best fit values. So we also apply the Monte Carlo Markov chain (MCMC) method to obtain the marginalized errors of the model parameters. The advantage of the MCMC method is that it considers the correlations between the model parameters and the result is more reliable.

The paper is organized as follows. In section 2, we present the SN Ia data [20], the

BAO data [21, 22, 23], the WMAP5 data [24] and the $H(z)$ data, and all the formulas related with these data. In section 3, We use the Λ CDM model as an example to show how to apply the data to constrain cosmological models. In section 4, we use the CPL model to study the geometry of the universe and the property of dark energy. The JBP model is used to probe the geometry of the universe and the evolution of dark energy in section 5. We conclude the paper in section 6.

2. Fitting procedure

To use the Constitution compilation of 397 SN Ia data [20], we minimize

$$\chi^2 = \sum_{i=1}^{397} \frac{[\mu(z_i) - \mu_{obs}(z_i)]^2}{\sigma_i^2}, \quad (1)$$

where the extinction-corrected distance modulus $\mu(z) = 5 \log_{10}[d_L(z)/\text{Mpc}] + 25$, σ_i is the total uncertainty in the SN Ia observation, the luminosity distance $d_L(z)$ is

$$d_L(z) = \frac{1+z}{H_0 \sqrt{|\Omega_k|}} \text{sinn} \left[\sqrt{|\Omega_k|} \int_0^z \frac{dz}{E(z)} \right], \quad (2)$$

the dimensionless Hubble parameter $E(z) = H(z)/H_0$, and

$$\frac{\text{sinn}(\sqrt{|\Omega_k|}x)}{\sqrt{|\Omega_k|}} = \begin{cases} \sin(\sqrt{|\Omega_k|}x)/\sqrt{|\Omega_k|}, & \text{if } \Omega_k < 0, \\ x, & \text{if } \Omega_k = 0, \\ \sinh(\sqrt{|\Omega_k|}x)/\sqrt{|\Omega_k|}, & \text{if } \Omega_k > 0. \end{cases} \quad (3)$$

Due to the arbitrary normalization of the luminosity distance, the nuisance parameter h in the SN Ia data is not the observed Hubble constant. So we marginalize the nuisance parameter h with a flat prior, after the marginalization, we get [28],

$$\chi_{sn}^2(\mathbf{p}) = \sum_{i=1} \frac{\alpha_i^2}{\sigma_i^2} - \frac{(\sum_i \alpha_i / \sigma_i^2 - \ln 10/5)^2}{\sum_i 1/\sigma_i^2} - 2 \ln \left(\frac{\ln 10}{5} \sqrt{\frac{2\pi}{\sum_i 1/\sigma_i^2}} \right), \quad (4)$$

where $\alpha_i = \mu_{obs}(z_i) - 25 - 5 \log_{10}[H_0 d_L(z_i)]$, and \mathbf{p} denotes the fitting parameters in the model. When using the SN Ia data, the radiation term can be neglected because its contribution is negligible.

In addition to the Constitution SN Ia data, we use the BAO distance measurements from the oscillations in the distribution of galaxies. From the BAO observation of the galaxy power spectra, Percival *et al* measured the distance ratio

$$d_z = \frac{r_s(z_d)}{D_V(z)} \quad (5)$$

at two redshifts $z = 0.2$ and $z = 0.35$ to be $d_{0.2}^{obs} = 0.1905 \pm 0.0061$, and $d_{0.35}^{obs} = 0.1097 \pm 0.0036$, respectively [22]. Here the effective distance is

$$D_V(z) = \left[\frac{d_L^2(z)}{(1+z)^2} \frac{z}{H(z)} \right]^{1/3}, \quad (6)$$

the drag redshift z_d is fitted as [29]

$$z_d = \frac{1291(\Omega_m h^2)^{0.251}}{1 + 0.659(\Omega_m h^2)^{0.828}} [1 + b_1(\Omega_b h^2)^{b_2}], \quad (7)$$

$$b_1 = 0.313(\Omega_m h^2)^{-0.419} [1 + 0.607(\Omega_m h^2)^{0.674}], \quad b_2 = 0.238(\Omega_m h^2)^{0.223}, \quad (8)$$

the comoving sound horizon is

$$r_s(z) = \int_z^\infty \frac{c_s(z) dz}{E(z)}, \quad (9)$$

the sound speed $c_s(z) = 1/\sqrt{3[1 + \bar{R}_b/(1+z)]}$, and $\bar{R}_b = 3\Omega_b h^2/(4 \times 2.469 \times 10^{-5})$. To use these BAO data, we calculate

$$\chi_{BAO2}^2(\mathbf{p}, \Omega_b h^2, h) = \Delta x_i \text{Cov}_1^{-1}(x_i, x_j) \Delta x_j, \quad (10)$$

where $x_i = (d_{z=0.2}, d_{z=0.35})$, $\Delta x_i = x_i - x_i^{obs}$ and $\text{Cov}_1(x_i, x_j)$ is the covariance matrix for the two parameters $d_{0.2}$ and $d_{0.35}$ [22]. Besides the model parameters \mathbf{p} , we need to add two more parameters $\Omega_b h^2$ and $\Omega_m h^2$ when we use the BAO data. In [22], they used the priors of $\Omega_b h^2 = 0.02273 \pm 0.00061$ and $\Omega_c h^2 = 0.1099 \pm 0.0063$.

From the measurement of the radial (line-of-sight) BAO scale in the galaxy power spectra, the cosmological parameters were determined from the measured values of

$$\Delta z_{BAO}(z) = \frac{H(z)r_s(z_d)}{c} \quad (11)$$

at two redshifts $z = 0.24$ and $z = 0.43$, which are $\Delta z_{BAO}(z = 0.24) = 0.0407 \pm 0.0011$ and $\Delta z_{BAO}(z = 0.43) = 0.0442 \pm 0.0015$, respectively [23]. Therefore, we add χ^2 with

$$\chi_{BAOz}^2(\mathbf{p}, \Omega_b h^2, h) = \left(\frac{\Delta z_{BAO}(0.24) - 0.0407}{0.0011} \right)^2 + \left(\frac{\Delta z_{BAO}(0.43) - 0.0442}{0.0015} \right)^2. \quad (12)$$

When we add these BAO data to the fitting, we also need to use the parameters $\Omega_b h^2$ and $\Omega_m h^2$. The values $\Omega_b h^2 = 0.02273 \pm 0.0066$ and $\Omega_m h^2 = 0.1329 \pm 0.0064$ were used in [23].

In addition to the above two BAO data sets, the BAO A parameter [30] is usually used. The BAO A parameter is defined as

$$A = \sqrt{\Omega_m} \frac{H_0 D_V(z = 0.35)}{z = 0.35} = \frac{\sqrt{\Omega_m}}{0.35} \left[\frac{0.35}{E(0.35)} \frac{1}{|\Omega_k|} \text{sinn}^2 \left(\sqrt{|\Omega_k|} \int_0^{0.35} \frac{dz}{E(z)} \right) \right]^{1/3}, \quad (13)$$

and it was measured to be $A = 0.493 \pm 0.017$ [21], so we add χ^2 with

$$\chi_{BAOa}^2(\mathbf{p}) = \left(\frac{A - 0.439}{0.017} \right)^2. \quad (14)$$

Note that the BAO A parameter depends on the model parameters \mathbf{p} only; it does not depend on the baryon density $\Omega_b h^2$ and the Hubble constant h . Although the radiation density depends on h , the contribution to the Hubble parameter $E(z)$ is negligible at the redshift $z = 0.35$, so we can neglect the radiation component when we use the BAO A data.

Both the SN Ia and the BAO data measure the distance up to redshift $z < 2$; we need to consider the distance at high redshift in order to determine the property of dark energy. Therefore, we implement the WMAP5 data. To use the full WMAP5 data, we need to add some more parameters which depend on inflationary models, and this will limit our ability to constrain dark energy models. So we only use the WMAP5 measurements of the derived quantities, such as the shift parameter $R(z^*)$, the acoustic scale $l_A(z^*)$ and the decoupling redshift z^* , to obtain

$$\chi_{CMB}^2 = \Delta x_i \text{Cov}_2^{-1}(x_i, x_j) \Delta x_j, \quad (15)$$

where the three parameters $x_i = (R(z^*), l_A(z^*), z^*)$, $\Delta x_i = x_i - x_i^{obs}$ and $\text{Cov}_2(x_i, x_j)$ is the covariance matrix for the three parameters [24]. The shift parameter R is expressed as

$$R(z^*) = \frac{\sqrt{\Omega_m}}{\sqrt{|\Omega_k|}} \text{sinn} \left(\sqrt{|\Omega_k|} \int_0^{z^*} \frac{dz}{E(z)} \right) = 1.710 \pm 0.019. \quad (16)$$

The acoustic scale l_A is

$$l_A(z^*) = \frac{\pi d_L(z^*)}{(1+z^*)r_s(z^*)} = 302.1 \pm 0.86, \quad (17)$$

and the decoupling redshift z^* is fitted by [31]

$$z^* = 1048[1 + 0.00124(\Omega_b h^2)^{-0.738}][1 + g_1(\Omega_m h^2)^{g_2}] = 1090.04 \pm 0.93, \quad (18)$$

$$g_1 = \frac{0.0783(\Omega_b h^2)^{-0.238}}{1 + 39.5(\Omega_b h^2)^{0.763}}, \quad g_2 = \frac{0.560}{1 + 21.1(\Omega_b h^2)^{1.81}}. \quad (19)$$

In [24], it was found that $\Omega_b h^2 = 0.02273 \pm 0.00062$ and $h = 0.719_{-0.027}^{+0.026}$.

The SN Ia data, the BAO data and the WMAP5 data use the distance measurement to determine the cosmological parameters. To get the distance scale, we need to integrate the equation of state parameter $w(z)$ twice, so the process of double integration smoothes out the variation of equation of state parameter $w(z)$ of dark energy. To alleviate the problem, we add the Hubble parameter $H(z)$ data. The Hubble parameter $H(z)$ at nine different redshifts was obtained from the differential ages of passively evolving galaxies in [25], and three more Hubble parameter data $H(z = 0.24) = 76.69 \pm 2.32$, $H(z = 0.34) = 83.8 \pm 2.96$ and $H(z = 0.43) = 86.45 \pm 3.27$ were determined recently in [26]. Therefore, we add these $H(z)$ data to χ^2 :

$$\chi_H^2(\mathbf{p}, h) = \sum_{i=1}^{12} \frac{[H(z_i) - H_{obs}(z_i)]^2}{\sigma_{hi}^2}, \quad (20)$$

where σ_{hi} is the 1σ uncertainty in the $H(z)$ data. The model parameters \mathbf{p} are determined by applying the maximum likelihood method of χ^2 fit. We use the publicly available MINUIT code for minimization and contour calculation [32]. Basically, The model parameters are determined by minimizing

$$\chi^2 = \chi_{sn}^2 + \chi_{Baoa}^2 + \chi_{Bao2}^2 + \chi_{Baoz}^2 + \chi_{CMB}^2 + \chi_H^2. \quad (21)$$

For the convenience of numerical fitting, we take $\Omega_b h^2 = 0.02273$ determined from the WMAP5 data [24]. For the Hubble constant h , two different values were observed.

The Hubble key project found that $h = 0.72 \pm 0.08$, and recently Riess *et al* obtained $h = 0.742 \pm 0.036$ by using a differential distance ladder method [34]. To account for the uncertainty of the Hubble constant, we treat it as a free parameter and then fix it at its best fit value.

3. Λ CDM model with curvature

For the cosmological constant, the equation of state parameter $w = p/\rho = -1$, and the energy density ρ_Λ is a constant. In a curved Λ CDM model, the curvature term $k \neq 0$, ordinary pressureless dust matter, radiation and the cosmological constant contribute to the total energy. The Friedmann equation is

$$E(z) = \frac{H(z)}{H_0} = [\Omega_k(1+z)^2 + \Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_\Lambda]^{1/2}, \quad (22)$$

where the Hubble constant $H_0 = 100 h \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_m = (8\pi G \rho_m)/(3H_0^2)$ is the current matter component, the current radiation component $\Omega_r = (8\pi G \rho_r)/(3H_0^2) = 4.1736 \times 10^{-5} h^{-2}$ [24], the current curvature component $\Omega_k = -k/(a_0^2 H_0^2)$ and $\Omega_\Lambda = 1 - \Omega_m - \Omega_k - \Omega_r$. In this model, we have two parameters $\mathbf{p} = (\Omega_m, \Omega_k)$ and one nuisance parameter h . For the fitting to the SN Ia data, the contribution to the Hubble expansion from the radiation is negligible and we usually neglect the radiation term.

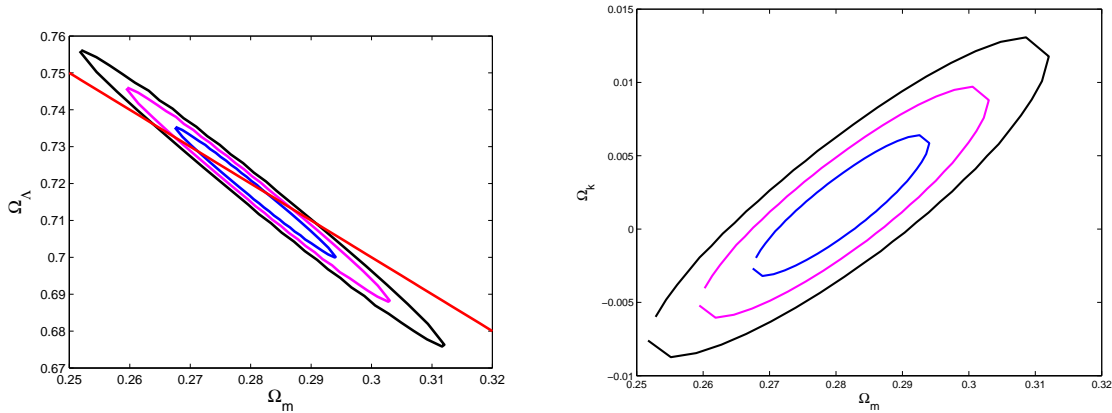


Figure 1. The 1σ , 2σ and 3σ joint contour plots of Ω_m and Ω_Λ (Ω_k) for the curved Λ CDM model. The straight line in the left panel denotes the flat Λ CDM model.

By fitting the Λ CDM model to the above observational data, we get $\chi^2 = 482.13$, $\Omega_m = 0.280_{-0.013}^{+0.014}$ and $\Omega_k = 0.001 \pm 0.005$. By fixing the nuisance parameter h at its best fit value $h = 0.702$, we obtain the contours of Ω_m and Ω_k . The joint contour plots of Ω_m and Ω_k or Ω_Λ are shown in figure 1. Compared with WMAP5 fitting results [24], we find that the current data make a little improvement on the constraints of Ω_m and Ω_k . The improvement is due to more SN Ia and BAO data in addition to the $H(z)$ data. The result tells us that the flat Λ CDM model is consistent with current observational data at the 1σ level.

Table 1. The marginalized estimates of the model parameters in CPL and JBP models.

Model	χ^2	Ω_m	Ω_k	w_0	w_a
CPL	481.27	$0.279^{+0.015}_{-0.008}$	$0.005^{+0.006}_{-0.011}$	$-1.05^{+0.23}_{-0.06}$	$0.47^{+0.28}_{-1.45}$
JBP	481.46	$0.281^{+0.015}_{-0.01}$	$0.000^{+0.007}_{-0.006}$	$-0.96^{+0.25}_{-0.18}$	$-0.6^{+1.9}_{-1.6}$

4. CPL parametrization with curvature

In order to investigate the equation of state of dark energy for a curved universe by observational data, in this section we study the popular CPL parametrization [13]

$$w(z) = w_0 + \frac{w_a z}{1+z}. \quad (23)$$

The dimensionless Hubble parameter including the contributions from dark energy, ordinary pressureless dust matter and radiation is

$$E(z) = \frac{H(z)}{H_0} = (\Omega_k(1+z)^2 + \Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_{DE})^{1/2}, \quad (24)$$

where the dimensionless dark energy density is

$$\Omega_{DE}(z) = (1 - \Omega_m - \Omega_k - \Omega_r) \times (1+z)^{3(1+w_0+w_a)} \exp[-3w_a z/(1+z)] \quad (25)$$

In this model, we have four model parameters $\mathbf{p} = (\Omega_m, \Omega_k, w_0, w_a)$. By applying the observational data discussed in the previous section to the CPL model, we are able to get the observational constraint on the model parameters $\mathbf{p} = (\Omega_m, \Omega_k, w_0, w_a)$. The best fit is $\chi^2 = 481.64$, $\Omega_m = 0.278$, $\Omega_k = 0.006$, $w_0 = -1.04$, $w_a = 0.42$ and $h = 0.70$. By fixing the parameters Ω_m , Ω_k and h at their best fit values, we obtain the contours of w_0 and w_a and they are shown in figure 2(a). From figure 2(a), we see that the Λ CDM model is excluded by the observational data at more than 3σ level. As we discussed in the previous section, we see that the Λ CDM model is consistent with the observational data. The totally different conclusions suggest that the simple χ^2 error estimation by fixing other parameters at their best fit values has some drawbacks because we neglect the correlation effects of the other parameters. The degeneracy between parameters was not considered in the above method. When the parameters are strongly correlated, the error of some parameters will be under-estimated if we fix the other parameters at their best fit values. To verify this point, we apply the MCMC method to constrain the parameter space \mathbf{p} and the nuisance parameters h and $\Omega_b h^2$. Our MCMC code [28] is based on the publicly available package COSMOMC [35]. By using the MCMC method, we get $\chi^2 = 481.27$; the marginalized 1σ errors are $\Omega_m = 0.279^{+0.015}_{-0.008}$, $\Omega_k = 0.005^{+0.006}_{-0.011}$, $w_0 = -1.05^{+0.23}_{-0.06}$ and $w_a = 0.5^{+0.3}_{-1.5}$. These results are summarized in table 1. The marginalized 1σ , 2σ and 3σ contour plots of w_0 and w_a are shown in figure 2(b). From figure 2(a) and 2(b), we find that the marginalized 1σ contour obtained by using the MCMC method includes the 3σ contour in figure 2(a). From figure 2(b), we see that the Λ CDM model is consistent with the CPL model at 1σ level.

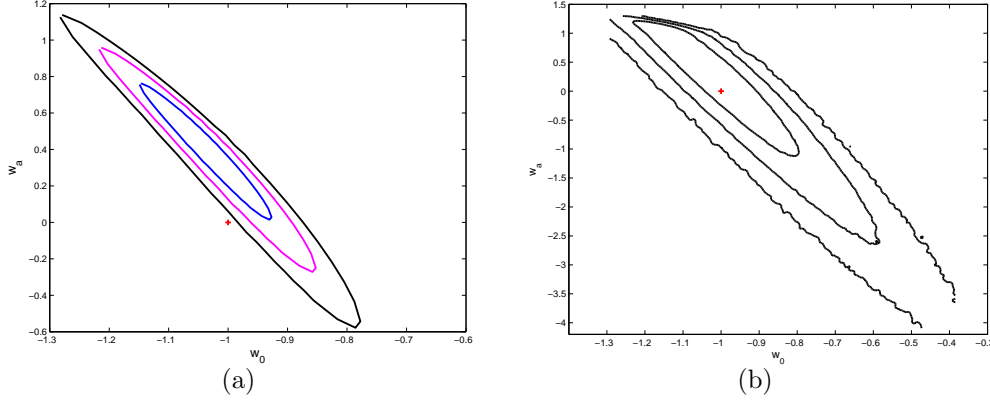


Figure 2. The 1σ , 2σ and 3σ contour plots of w_0 and w_a for the curved CPL parametrization. '+' denotes the point corresponding to the Λ CDM model. (a) Joint contours of w_0 and w_a by fixing the other parameters at their best fit values. (b) Marginalized contours of w_0 and w_a obtained from the MCMC method.

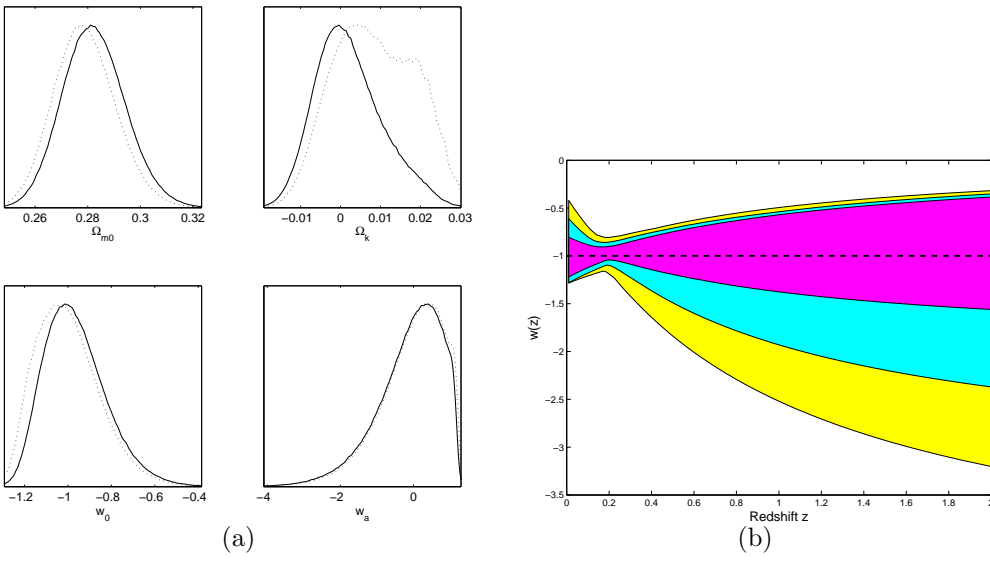


Figure 3. The marginalized distributions of the CPL model parameters \mathbf{p} are shown in (a). The solid lines are marginalized probabilities and the dotted lines are mean likelihoods. (b) Reconstructed evolution of $w(z)$ and the shaded areas are 1σ , 2σ and 3σ errors.

The marginalized distributions of the model parameters are shown in figure 3(a). The solid lines are marginalized probabilities and the dotted lines represent mean likelihoods. From figure 3(a), we find that the likelihood of Ω_k has a local maximum around $\Omega_k \sim 0.02$. Even we take $\Omega_k = 0.02$, the value of χ^2 is not far from the minimum value of χ^2 . Due to the degeneracy between model parameters, if we fix the other model parameters at their best fit values, then the joint contours of w_0 and w_a are underestimated, and the conclusion drawn from the under-estimated contours is not reliable. The results in figure 2(b) and 3(a) verify this point. By using the marginalized contours of w_0 and w_a , we reconstruct the evolution of $w(z)$ in figure 3(b). From figure 3(b), we

find that $w(z) < 0$ at more than 3σ confidence level up to redshift $z = 2$, and the Λ CDM model is consistent with the CPL model at the 1σ level. To account for the correlations between model parameters, we need to use the marginalized probability. To see whether this happens only for the CPL model, we analyze the JBP model in the next section.

5. JBP parametrization with curvature

In this section, we consider the JBP parametrization [27] for dark energy with the equation of state in the form below

$$w(z) = w_0 + \frac{w_a z}{(1+z)^2} . \quad (26)$$

The corresponding dimensionless dark energy density is then

$$\Omega_{DE}(z) = (1 - \Omega_m - \Omega_k - \Omega_r) \times (1+z)^{3(1+w_0)} \exp\left[3w_a z^2/2(1+z)^2\right] \quad (27)$$

In this model, we also have four parameters $\mathbf{p} = (\Omega_m, \Omega_k, w_0, w_a)$. We first use the simple χ^2 method to fit the model. The best fit is $\chi^2 = 481.84$, $\Omega_m = 0.281$, $\Omega_k = 0.0015$, $w_0 = -0.97$, $w_a = -0.03$ and $h = 0.70$. By fixing the parameters Ω_m , Ω_k and h at their best fit values, we obtain the contours of w_0 and w_a and they are shown by the solid lines in figure 4(a). Unlike the CPL model, the Λ CDM model is consistent with the JBP model at the 1σ level. To verify this conclusion, we also apply the MCMC method to the JBP model. By using the MCMC method, we get $\chi^2 = 481.46$; the marginalized 1σ errors are $\Omega_m = 0.281_{-0.01}^{+0.015}$, $\Omega_k = 0.000_{-0.006}^{+0.007}$, $w_0 = -0.96_{-0.18}^{+0.25}$ and $w_a = -0.6_{-1.6}^{+1.9}$. These results are summarized in table 1. The marginalized 1σ , 2σ and 3σ contour plots of w_0 and w_a are shown in figure 4(b). From figure 4(a) and 4(b), we see that the contours of w_0 and w_a are consistent although the constraints from the MCMC method are a little larger because we consider the correlations among all the parameters in the MCMC method. The Λ CDM model is also consistent with the JBP model at the 1σ level.

The marginalized distributions of the model parameters \mathbf{p} are shown in figure 5(a). The solid lines are marginalized probabilities and the dotted lines represent mean likelihoods. From figure 5(a), we find that the probability distributions of the parameters are more or less Gaussian. By using the marginalized contours of w_0 and w_a , we reconstruct the evolution of $w(z)$ in figure 5(b). From figure 5(b), we find that $w(z) < -0.2$ at more than 3σ confidence level up to redshift $z = 2$, and the Λ CDM model is consistent with the JBP model at the 1σ level.

6. Conclusions

Applying the simple χ^2 method, we fitted the CPL and JBP models to the combined SN Ia, BAO, WMAP5 and $H(z)$ data, and obtained the constraint on the property of dark energy. In both CPL and JBP models, there are four parameters $\mathbf{p} = (\Omega_m, \Omega_k, w_0, w_a)$. When we apply the BAO and WMAP5 data, we need to add two more parameters $\Omega_b h^2$ and h . If we make the joint error analysis, we have six parameters and it will

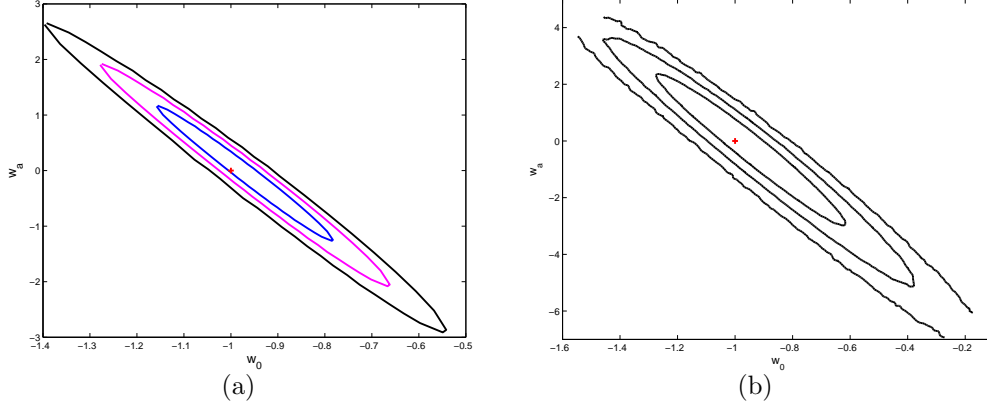


Figure 4. The 1σ , 2σ and 3σ contour plots of w_0 and w_a for the curved JBP parametrization. '+' denotes the point corresponding to the Λ CDM model. (a) Joint contours of w_0 and w_a by fixing the other parameters at their best fit values. (b) Marginalized contours of w_0 and w_a obtained from the MCMC method.

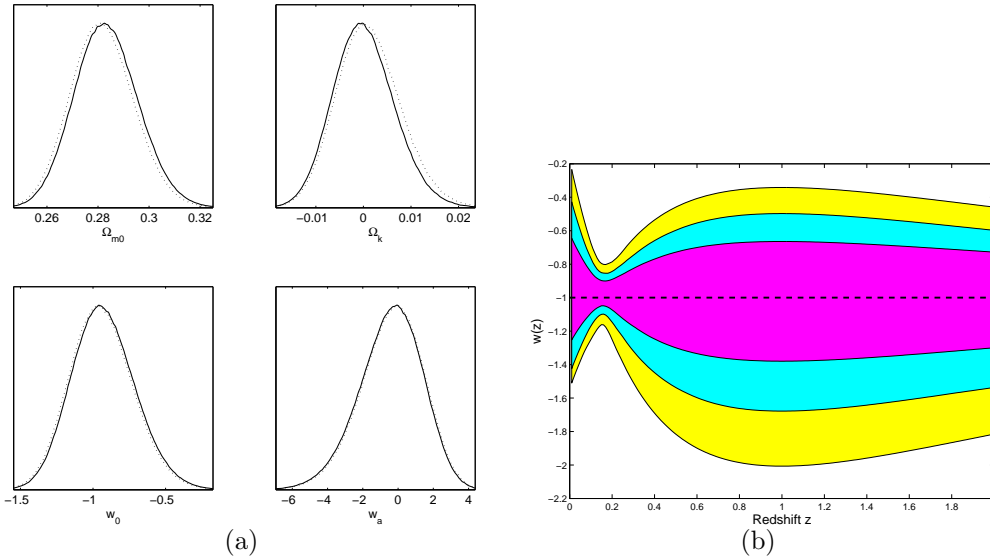


Figure 5. The marginalized distributions of the JBP model parameters \mathbf{p} are shown in (a). The solid lines are marginalized probabilities and the dotted lines are mean likelihoods. (b) Reconstructed evolution of $w(z)$ and the shaded areas are 1σ , 2σ and 3σ errors.

be hard to get a good joint constraint on all these parameters. Therefore, we take $\Omega_b h^2 = 0.02273$, and find out the best fit values of the parameters \mathbf{p} and h which minimize χ^2 ; then we fix the parameters Ω_m , Ω_k and h at their best fit values to obtain the joint constraints on w_0 and w_a . For the CPL model, the contours of w_0 and w_a (see figure 2(a)) show that the Λ CDM model is excluded at more than 3σ level. The JBP model is consistent with the Λ CDM model at the 1σ level. Since we get the contours of w_0 and w_a by fixing the other parameters at their best fit values, we neglect the correlation effects of the parameters and the conclusion based on this method may not

be reliable. To confirm this, we use the MCMC method to analyze the CPL and JBP models and obtain the marginalized probabilities of the parameters. For the CPL model, the probability distributions of Ω_k and w_a are skew distributions, and the marginalized 1σ errors are $\Omega_m = 0.279_{-0.008}^{+0.015}$, $\Omega_k = 0.005_{-0.011}^{+0.006}$, $w_0 = -1.05_{-0.06}^{+0.23}$ and $w_a = 0.5_{-1.5}^{+0.3}$. In the CPL model, the probability distributions of Ω_k has a local maximum in addition to a global maximum. The uncertainties in Ω_k and the degeneracies between Ω_k , w_0 and w_a lead to under-estimation of the error contours of w_0 and w_a if we fix Ω_k at its global best fit value, and the wrong conclusion that the Λ CDM model is excluded at more than 3σ level. However, this does not happen for the JBP model. For the JBP model, the parameters have Gaussian distributions, and the marginalized 1σ errors are $\Omega_m = 0.281_{-0.01}^{+0.015}$, $\Omega_k = 0.000_{-0.006}^{+0.007}$, $w_0 = -0.96_{-0.18}^{+0.25}$ and $w_a = -0.6_{-1.6}^{+1.9}$.

In summary, in addition to use the usual SN Ia, BAO A or BAO distance ratio, and WMAP data, we also use the radial BAO measurements and the $H(z)$ data to fit the CPL and JBP models. We find that the equation of state parameter of dark energy $w(z) < 0$ at more than 3σ level in the redshift range $0 \leq z \leq 2$, and the flat Λ CDM model is consistent with the current observational data at the 1σ level. Furthermore, we find that we need to do the marginalized analysis to estimate the errors of the model parameters.

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